

2016 Statistics Graduate Bootcamp

University of California, Irvine

TA: Dustin Pluta
Department of Statistics

Updated - September 2, 2016

1 Day 1 - Basic Probability Theory

1.1 Overview

- Goal is to serve as a reminder of basic probability theory
- Reintroduce probability and its axioms
- Reintroduce random variables
- Very fundamental material for first year classes.

1.2 Introduction to Probability

1. Consider an experiment where a 6-sided die is rolled ten times, and let the random variable X equal the number of sixes.
 - (a) What is the sample space of this experiment?
 - (b) Calculate $P(X = 0)$
 - (c) $P(X = 1)$
 - (d) $P(X = 4)$
 - (e) Show that $\sum_{x \in \text{Supp}\{X\}} P(X = x) = 1$.
2. Consider an experiment where a 6-sided die is rolled until the first six occurs, and let the random variable X equal the number of rolls required.
 - (a) What is the sample space of this experiment?
 - (b) Calculate $P(X = 0)$
 - (c) $P(X = 1)$
 - (d) $P(X = 4)$
 - (e) Show that $\sum_{x \in \text{Supp}\{X\}} P(X = x) = 1$.
3. Four red, 8 yellow, and 5 green balls are randomly arranged in a line.
 - (a) What is the probability the first 5 balls are yellow?
 - (b) What is the probability the first two balls are red or the last two balls are green?
 - (c) What is the order of the sample space?

4. A 52-card deck contains 4 suits (hearts, diamonds, clubs, spades) of 13 cards each. If we draw 5 cards randomly without replacement, what is the probability that all cards drawn are hearts?
5. Prove using the axioms of probability that $P(A^C) = 1 - P(A)$.
6. Prove $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
7. Prove Boole's inequality: For events $A_i, i = 1, \dots, \infty$,

$$P(\cup_{i=1}^{\infty} A_i) \leq \sum_{i=1}^{\infty} P(A_i).$$

8. Consider a group of N people. Let B be the event that two individuals in the group share a birthday. Assuming that birthdays are uniformly distributed over the year and ignoring leap years, derive N such that $P(B) > 0.5$.
9. The Smiths have two children. At least one of them is a boy. What is the probability that both children are boys?
10. Two litters of anteaters have been born at the zoo. Litter 1 has two brown-haired and one gray-haired, Litter 2 has three brown-haired and two-gray haired. We select a litter at random and then select an anteater pup at random from the selected litter.
 - a) What is the probability the animal chosen is brown-haired?
 - b) Given that a brown-haired offspring was selected, what is the probability that the sampling was from Litter 1?
11. Show that if $P(\cdot)$ is a legitimate probability function and B is a set with $P(B) > 0$, then $P(\cdot|B)$ satisfies the axioms of probability. Explain why we need to assume $P(B) > 0$.