1. *Type I Error:*

$$\alpha := P(\text{Reject } H_0 | H_0 \text{ true})$$

Power:

$$1 - \beta := P(\text{Reject } H_0 | H_0 \text{ false})$$

   (a) Consider a one-sample z-test of the mean with a sample $X_i \overset{iid}{\sim} N(\mu, 1), i = 1, \cdots n$. Draw the reference distribution and shade the rejection region for testing the hypothesis $H_0 : \mu = 0$ against $H_1 : \mu > 0$ with significance level α.

   (b) Assuming $\mu = 0$, what is $P(\frac{\bar{X}}{1/\sqrt{n}} > z_{1-\alpha})$?

   (c) Assuming $\mu = 1$, what is $P(\frac{\bar{X}}{1/\sqrt{n}} > z_{1-\alpha})$? Draw the reference and true distributions and shade the calculated probability.

   (d) Write an `R` function to calculate the power for this test. The function should have arguments for the true value of μ, the sample size n, and the value for α. Set defaults $\mu = 0, \alpha = 0.05, n = 30$. Use the function to plot the power curve for $n = 30$.

2. (a) Load the `PlantGrowth` data set in `R` with the command

```
data("PlantGrowth")
```

   You can read about the data set with the command

```
?PlantGrowth
```

   (b) Find the mean of the three treatment groups using the `aggregate` function.

   (c) Similarly, find the sample standard deviation of the three treatment groups.

   (d) Write a function in `R` to perform a two-sample t-test to test whether the means from group 1 and group 2 are significantly different under the assumption of equal variance. Use the following function definition:

```
my.ttest <- function(grp1, grp2, alpha = 0.05) {
# Code to perform calculations goes here...
...
# Return these values
return(data.frame(t.diff, P, df)
}
```

   (e) Use your t-test function to test whether the plants in treatment group 1 had significantly different growth on average compared to the plants in the control group.

   (f) Compare the results from the previous part to the results given by the `t.test` function with the argument $var.equal = TRUE$.

   (g) Plot the reference distribution and indicate the value of the test statistic on the plot.

   (h) State the conclusion of the hypothesis test in context.

3. Consider a population distributed as $Pois(\lambda)$. We wish to test the hypothesis $H_0 : \lambda = 1$ against $H_0 : \lambda > 1$. To test this hypothesis, a sample $X_i \overset{iid}{\sim} Pois(\lambda), i = 1, \cdots, n = 10$ is collected, but we are only given the information $\sum X_i$.

   (a) What is the distribution of $\sum_{i=1}^{10} X_i$?

   (b) What is the Type I Error if we reject H_0 when $\sum X_i > 10$? Plot the reference distribution and shade the rejection region.

   (c) Assuming $\lambda = 1.5$, what is the power if we reject H_0 when $\sum X_i > 10$?

   (d) What is the most appropriate decision rule if we want a Type I Error of $\alpha = 0.01$?

   (e) For the decision rule in the previous part, find the power if $\lambda = 1.5$. Plot the reference and true distributions, shade the area under the true distribution over the rejection region.