2017 Department of Statistics Knowledge Assessment
University of California, Irvine

September 13th, 2017

The following quiz will assess basic knowledge of statistical concepts. All of the questions that follow are multiple choice questions and are designed to assess knowledge of basic ideas learned prior to graduate school. For each question choose the answer which you believe to be correct. If you genuinely do not know the answer, or how to answer the question, choose **"I don't know"**.

Student ID Number: ______________________________

Student Name: ______________________________

Background Information

1. Which of the following best describes your undergraduate major?
   - A. Statistics
   - B. Mathematics
   - C. Engineering
   - D. Economics
   - E. Computer Science
   - F. Biological Sciences
   - G. Other ______________________________
2. Have you ever taken a basic statistics course (Non-calculus based)?
   - A. Yes
   - B. No
3. Have you ever taken any upper-division statistics theory courses (Calculus based)?
   - A. Yes
   - B. No
4. Have you ever taken any upper-division statistics methods courses (i.e. linear regression, ...)?
   - A. Yes
   - B. No
5. Approximately how much exposure to linear algebra have you had?
   - A. 0 to 0.5 years
   - B. 0.5 to 1 years
   - C. 1.0 to 1.5 years
   - D. 1.5 or more years
6. Have you ever taken a course on real-analysis?
   - A. Yes
   - B. No
7. How much exposure to programming courses have you had?
   - A. 0 to 0.5 years
   - B. 0.5 to 1 years
   - C. 1.0 to 1.5 years
   - D. 1.5 or more years
8. Are your familiar with the `R` programming language?
   - A. Yes
   - B. No
9. Did you work for any extended period of time in industry between your previous degree and starting your graduate education here at UC Irvine?
   - A. Yes
   - B. No
10. If so, how many years did you work? ______________________________
11. Also if so, What industry did you work in? ______________________________

Mathematics Questions

1. What is the solution to the following definite integral

$$\int_1^3 x^2 dx$$

   A. $\int_1^3 x^2 dx = 26$
   B. $\int_1^3 x^2 dx = x^3 + C$
   C. $\int_1^3 x^2 dx = \frac{26}{3}$
   D. $\int_1^3 x^2 dx = 8$
   E. I don't know

2. Consider the following function for scalar x,

$$f(n) = \left(1 - \frac{x}{n}\right)^n$$

   What is the limit of $f(n)$ as n tends to infinity?

   A. $\lim_{n\to\infty} f(n) = \log(x)$
   B. $\lim_{n\to\infty} f(n) = e^{-x}$
   C. $\lim_{n\to\infty} f(n) = \sin(x)$
   D. $\lim_{n\to\infty} f(n) = \arctan(x)$
   E. I don't know

3. Consider the following function

$$f(x) = \frac{x}{log(x)}$$

   Which of the following is the derivative of this function

   A. $f'(x) = \frac{1}{log(x)} - \frac{1}{log^2(x)}$
   B. $f'(x) = \frac{1}{1/x}$
   C. $f'(x) = \frac{x}{(log(x))^2}$
   D. $f'(x) = \frac{log(x)-1}{log(x)}$
   E. I don't know

4. Consider the following function

$$g(x) = xe^x$$

Which of the following is the derivative of $g(x)$

A. $g'(x) = \log(x)$

B. $g'(x) = xe^x + e^x$

C. $g'(x) = e^x$

D. $g'(x) = xe^x$

5. Consider the following function

$$y = -x^2 - x$$

for $x \in [-1, 2]$. At which points x are there extreme values of the function y?

A. $x_1^* = -1, x_2^* = 2$

B. $x_1^* = 0.5, x_2^* = 2$

C. $x_1^* = -1, x_2^* = 1$

D. $x_1^* = -1, x_2^* = 0.5$

E. $x_1^* = -0.5, x_2^* = 2$

6. Assume $t > 0$. Evaluate the following

$$\sum_{x=1}^{\infty} e^{-tx} \left(\frac{1}{2}\right)^x$$

A. ∞

B. 1

C. $\frac{1}{1-\frac{1}{2}e^{-t}}$

D. $\frac{1}{2e^t-1}$

7. Evaluate

$$\sum_{k=1}^{\infty} \frac{e^{-\beta}\beta^k}{k!}$$

A. ∞

B. 1

C. $1 - e^{-\beta}$

D. $e^{-\beta}$

E. $\frac{1}{1-e^{\beta}}$

F. I don't know

8. Let f be a real-valued function on $\mathcal{R}$, and suppose $x_0 \in \mathcal{R}$ is such that $f'(x_0) = 0$ and $f''(x_0) > 0$. Circle the statement below which is true.

A. x_0 is a local maximum of f

B. x_0 is a local minimum of f

C. x_0 is a root of f

D. x_0 is a saddle point of f

E. x_0 is a global maximum of f

F. I don't know

9. Use the fact that

$$\int_0^{\infty} x^{\alpha-1} e^{-\beta x}\, dx = k$$

for some constant k, to evaluate

$$\int_0^{\infty} x^{\alpha} e^{-\beta x}\, dx.$$

A. ∞

B. k

C. $\frac{-1}{\beta} + k\alpha\beta$

D. $\frac{k\alpha}{\beta}$

E. $\frac{k\alpha}{\beta^2}$

F. I don't know

10. Assume $A \in \mathcal{R}^{n\times n}$ is a nonzero positive semidefinite matrix. Circle the correct statement.

    A. $AA^{-1} = I$

    B. $\det(A) > 0$

    C. A is invertible

    D. For $x \in \mathcal{R}^n$, $x^T Ax \geq 0$

    E. I don't know

11. A matrix A is a projection matrix if it is symmetric positive semidefinite and $A^2 = A$. Assume A is $n \times m$ and $\text{rank}(A) = m$. Select the projection matrices.

    A. AA^T

    B. $A(A^T A)^{-1}A^T$

    C. $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

    D. $\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

    E. I don't know

Probability Questions

12. If $P(A \cap B^c) = \frac{1}{3}$, what is the range of possible values for $P(B)$?

    A. $[0, 1]$

    B. $(\frac{1}{3}, 1]$

    C. $[0, \frac{1}{3}]$

    D. $[\frac{2}{3}, 1]$

    E. $\left[0, \frac{2}{3}\right]$

    F. I don't know

13. Let events A and B be mutually exclusive. Which of the following is definitely true? ($\cap \equiv$ intersection, $\cup \equiv$ union)

    A. $P(A \cup B) = 0$

    B. $P(A \cap B) = 0$

    C. $P(A \cap B) = P(A)P(B)$

    D. $P(A \cup B) = P(A)P(B)$

    E. I don't know

14. Select the scenario for which a Poisson distribution would be appropriate to model the random variable X.

    A. Flip a coin until three heads occur, let X be the total number of flips.

    B. X is the length of time until a continuously used hard drive fails.

    C. X is the number of calls received at a company help desk in an hour period.

    D. X is the number of failed attempts before finding the correct password for a computer account using random guesses.

    E. I don't know

For the next few questions (15-18), consider the following probability mass function and corresponding calculations:

$X = x$	0	1	2	3	4
$p(X = x)$	0.1	0.2	0.3	0.3	0.1
$x \times p(x)$	0	0.2	0.6	0.9	0.4
$x^2 \times p(x)$	0	0.2	1.2	2.7	1.6

15. What is the probability that X is less than or equal to 2.

    A. $p(X \leq 2) = 0.3$

    B. $p(X \leq 2) = 0.7$

    C. $p(X \leq 2) = 0.6$

    D. $p(X \leq 2) = 0.7$

    E. I don't know

16. What is the expected value of X, i.e. $E(X)$, of this probability mass function?

    A. $E(X) = 2$

    B. $E(X) = 2.1$

    C. $E(X) = 2.5$

    D. $E(X) = 3$

    E. I don't know

17. What is the expected value of X^2, i.e. $E(X^2)$, of this probability mass function?

a) $E(X^2) = 5.7$
b) $E(X^2) = 2.1$
c) $E(X^2) = 4.5$
d) $E(X^2) = 1.0$
e) I don't know

18. What is the variance, i.e. $Var(X)$, of this probability mass function?

a) $Var(X) = E(X^2) + (E(X))^2 = 10.11$
b) $Var(X) = (E(X^2) - E(X))^2 = 12.96$
c) $Var(X) = E(X^2) - E(X) = 3.6$
d) $Var(X) = E(X^2) - (E(X))^2 = 1.29$
e) I don't know

Consider the following joint probability mass function for the random variables X and Y

$$P(Y=y, X=x) = \begin{array}{c|cc} & X=0 & X=1 \\ \hline Y=0 & 0.1 & 0.3 \\ Y=1 & 0.1 & 0.5 \end{array}$$

19. What is the marginal distribution of X?
    - A. $P(X=0) = 0.4$ & $P(X=0) = 0.6$
    - B. $P(X=0) = 0.2$ & $P(X=0) = 0.8$
    - C. $P(X=0) = 0.1$ & $P(X=0) = 0.3$
    - D. $P(X=0) = 0.1$ & $P(X=0) = 0.5$
    - E. I don't know

20. Evaluate the following: X and Y are independent.
    - A. TRUE
    - B. FALSE
    - C. I don't know

21. Which of the following is the Bernoulli Distribution

    A. $f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$
    B. $f(x;p) = p^x(1-p)^{1-x}$
    C. $f(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$
    D. $f(x;p,n) = \binom{n}{x} p^x(1-p)^{n-x}$
    E. $f(x;\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1}\exp(-\beta x)$
    F. $f(x;\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$
    G. Not shown above
    H. I don't know

22. Which of the following is the Normal Distribution

    A. $f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$
    B. $f(x;p) = p^x(1-p)^{1-x}$
    C. $f(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$
    D. $f(x;p,n) = \binom{n}{x} p^x(1-p)^{n-x}$
    E. $f(x;\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1}\exp(-\beta x)$
    F. $f(x;\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$
    G. Not shown above
    H. I don't know

23. Which of the following is the Poisson Distribution

    A. $f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$
    B. $f(x;p) = p^x(1-p)^{1-x}$
    C. $f(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$
    D. $f(x;p,n) = \binom{n}{x} p^x(1-p)^{n-x}$
    E. $f(x;\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1}\exp(-\beta x)$
    F. $f(x;\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$
    G. Not shown above
    H. I don't know

24. Let a random variable Y be distributed as a standard normal, i.e. $Y \sim N(0,1)$, and let X be distributed as a Chi-Squared random variable with d degrees of freedom, i.e.$X \sim \chi^2_d$, and finally let Y and X be independent. Which is following is the correct definition of a random variable T so that it follows a t-distribution,

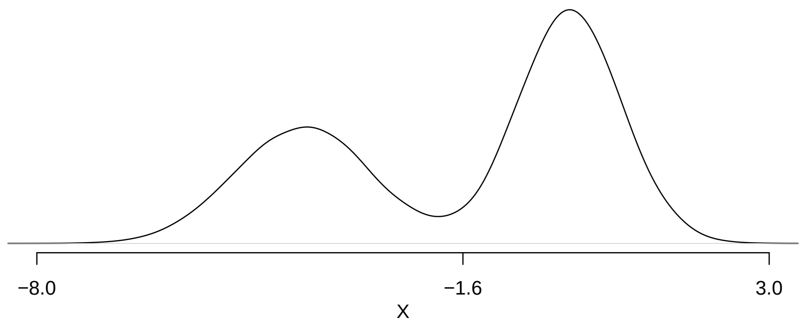
    A. $T = \frac{Y}{X}$
    B. $T = \frac{Y}{\sqrt{X/d}}$
    C. $T = \frac{Y}{\sqrt{X}}$
    D. $T = \frac{Y}{\sqrt{dX}}$
    E. Not shown above
    F. I don't know

25. Let X be a normal random variable, i.e. $X \sim N(\mu, \sigma^2)$ and let $Y = aX + b$, where a and b are scalar values. What is the distribution of Y?

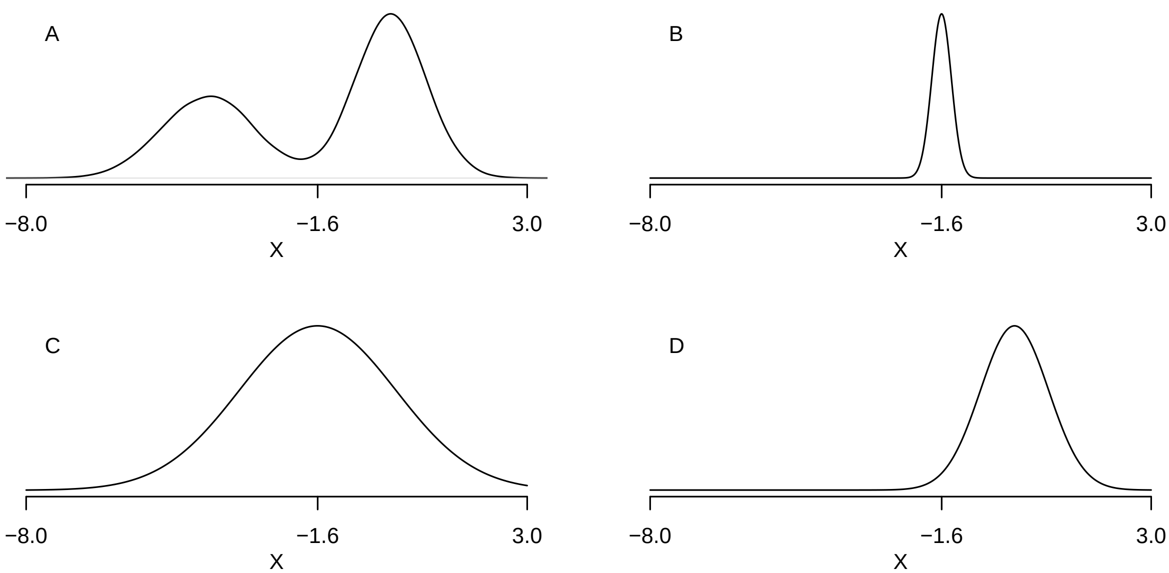
    A. $Y \sim N(a\mu + b, a\sigma^2)$
    B. $Y \sim N(a\mu, b + \sigma^2)$
    C. $Y \sim N(a\mu, b^2\sigma^2)$
    D. $Y \sim N(a\mu + b, a^2\sigma^2)$
    E. I don't know

Statistics Questions

26. Consider the following distribution,



The mean of this distribution is $\mu = -1.6$ and shown on the figure. Based upon the Central Limit Theorem, which of the following best represents the distribution of the sample mean from this distribution when $n = 100$?



A. Plot A
B. Plot B
C. Plot C
D. Plot D
E. I don't know

27. Let X_1, X_2, X_3 be a random sample from $N(\mu, 1)$. Select the estimator for μ which is unbiased and has the smallest variance.

    A. X_1

    B. $\frac{X_1+X_2}{2}$

    C. $\frac{2X_1+2X_2+X_3}{5}$

    D. $\frac{X_1+X_2+X_3}{3}$

    E. I don't know

28. Suppose you suspect a coin is weighted so that heads comes up with probability $p > 0.5$, but you do not know the exact value of p. To test the hypothesis $H_0 : p = 0.5$ against the alternative $H_1 : p > 0.5$, you flip the coin 100 times and record 65 heads. Based upon this data you obtain the following estimate for the proportion of heads and a test statistic. This is shown below,

$\hat{p}$	z-score
$\frac{65}{100} = 0.65$	$z = \frac{0.65-0.5}{\sqrt{\frac{0.5(1-0.5)}{100}}} = 3.0$

Additionally from a table you find that $z_{0.01} = -2.33$, select the conclusion you should make based upon testing the hypothesis at the $\alpha = 0.01$ significance level for this one sided test.

    A. Reject H_0, there is sufficient evidence to conclude that $p > 0.5$ at the 0.01 level of significance.

    B. Fail to reject H_0, there is insufficient evidence to conclude that $p > 0.5$ at the 0.01 level of significance.

    C. Reject H_1, there is sufficient evidence to conclude that $p = 0.5$ at the 0.01 level of significance.

    D. Not enough information given to perform the test.

    E. I don't know