2019 Statistics Graduate Bootcamp
University of California, Irvine

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1 Day 1 - Basic Probability Theory

1.1 Overview

- Goal is to serve as a reminder of basic probability theory
- Reintroduce probability and its axioms
- Reintroduce random variables
- Very fundamental material for first year classes.

1.2 Introduction to Probability

1. Consider an experiment where a 6-sided die is rolled ten times, and let the random variable X equal the number of sixes.
   (a) What is the sample space of this experiment?
   (b) Calculate $P(X = 0)$
   (c) $P(X = 1)$
   (d) $P(X = 4)$
   (e) Show that $\sum_{x \in Supp\{X\}} P(X = x) = 1$.
2. Consider an experiment where a 6-sided die is rolled until the first six occurs, and let the random variable X equal the number of rolls required.
   (a) What is the sample space of this experiment?
   (b) Calculate $P(X = 0)$
   (c) $P(X = 1)$
   (d) $P(X = 4)$
   (e) Show that $\sum_{x \in Supp\{X\}} P(X = x) = 1$.

3. Four red, 8 yellow, and 5 green balls are randomly arranged in a line.

   (a) What is the probability the first 5 balls are yellow?

   (b) What is the probability the first two balls are red or the last two balls are green?

   (c) What is the order of the sample space?

4. A 52-card deck contains 4 suits (hearts, diamonds, clubs, spades) of 13 cards each. If we draw 5 cards randomly without replacement, what is the probability that all cards drawn are hearts?

5. Prove using the axioms of probability that $P(A^C) = 1 - P(A)$.

6. Prove $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

7. Prove Boole's inequality: For events $A_i, i = 1, \cdots, \infty$,

$$P\left(\cup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i).$$

8. Consider a group of N people. Let B be the event that two individuals in the group share a birthday. Assuming that birthdays are uniformly distributed over the year and ignoring leap years, derive N such that $P(B) > 0.5$.

9. The Smiths have two children. At least one of them is a boy. What is the probability that both children are boys?

10. Two litters of anteaters have been born at the zoo. Litter 1 has two brown-haired and one gray-haired, Litter 2 has three brown-haired and two-gray haired. We select a litter at random and then select an anteater pup at random from the selected litter.
    a) What is the probability the animal chosen is brown-haired?
    b) Given that a brown-haired offspring was selected, what is the probability that the sampling was from Litter 1?

11. Show that if $P(\cdot)$ is a legitimate probability function and B is a set with $P(B) > 0$, then $P(\cdot|B)$ satisfies the axioms of probability. Explain why we need to assume $P(B) > 0$.