## 2019 Statistics Graduate Bootcamp University of California, Irvine

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Updated - September 9, 2019

## 1 Day 1 - Basic Probability Theory

## 1.1 Overview

- Goal is to serve as a reminder of basic probability theory
- Reintroduce probability and its axioms
- Reintroduce random variables
- Very fundamental material for first year classes.

## 1.2 Introduction to Probability

- 1. Consider an experiment where a 6-sided die is rolled ten times, and let the random variable X equal the number of sixes.
  - (a) What is the sample space of this experiment?
  - (b) Calculate P(X=0)
  - (c) P(X = 1)
  - (d) P(X = 4)
  - (e) Show that  $\sum_{x \in Supp\{X\}} P(X = x) = 1$ .
- 2. Consider an experiment where a 6-sided die is rolled until the first six occurs, and let the random variable X equal the number of rolls required.
  - (a) What is the sample space of this experiment?
  - (b) Calculate P(X=0)
  - (c) P(X = 1)
  - (d) P(X = 4)
  - (e) Show that  $\sum_{x \in Supp\{X\}} P(X = x) = 1$ .

- 3. Four red, 8 yellow, and 5 green balls are randomly arranged in a line.
  - (a) What is the probability the first 5 balls are yellow?
  - (b) What is the probability the first two balls are red or the last two balls are green?
  - (c) What is the order of the sample space?
- 4. A 52-card deck contains 4 suits (hearts, diamonds, clubs, spades) of 13 cards each. If we draw 5 cards randomly without replacement, what is the probability that all cards drawn are hearts?
- 5. Prove using the axioms of probability that  $P(A^C) = 1 P(A)$ .
- 6. Prove  $P(A \cup B) = P(A) + P(B) P(A \cap B)$ .
- 7. Prove Boole's inequality: For events  $A_i$ ,  $i = 1, \dots, \infty$ ,

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i).$$

- 8. Consider a group of N people. Let B be the event that two individuals in the group share a birthday. Assuming that birthdays are uniformly distributed over the year and ignoring leap years, derive N such that P(B) > 0.5.
- 9. The Smiths have two children. At least one of them is a boy. What is the probability that both children are boys?
- 10. Two litters of anteaters have been born at the zoo. Litter 1 has two brown-haired and one gray-haired, Litter 2 has three brown-haired and two-gray haired. We select a litter at random and then select an anteater pup at random from the selected litter.
  - a) What is the probability the animal chosen is brown-haired?
  - b) Given that a brown-haired offspring was selected, what is the probability that the sampling was from Litter 1?
- 11. Show that if  $P(\cdot)$  is a legitimate probability function and B is a set with P(B) > 0, then  $P(\cdot|B)$  satisfies the axioms of probability. Explain why we need to assume P(B) > 0.