2019 Statistics Graduate Bootcamp
University of California, Irvine

TA: Derenik Haghverdian
Department of Statistics

Updated - September 16, 2019

Consider a random sample $x_1, x_2, \cdots, x_n \overset{iid}{\sim} Unif(0, \theta)$.

1. Find the method of moments estimator of θ, call it $\hat{\theta}_{MoM}$.
2. Show that $\hat{\theta}_{MoM}$ is unbiased.
3. What is the asymptotic distribution of the $\hat{\theta}_{MoM}$?
4. Find the likelihood function $\mathcal{L}(\theta|\mathbf{x})$.
5. Find the maximum likelihood estimator of θ, call it $\hat{\theta}_{MLE}$.
6. Use `R` to generate a random sample $x_1, x_2, \cdots, x_{20}$ from $Unif(0, 10)$. Use `set.seed(1234)` before you generate your random sample. Plot the likelihood function. Calculate the $\hat{\theta}_{MLE}$ for this sample. Indicate this value on the plot along with the true value $\theta = 10$.
7. Generate $B = 1000$ samples each of size $n = 20$ from a $Unif(0, 1)$ distribution. Plot the empirical sampling distribution of $\hat{\theta}_{MoM}$.
8. The MLE $\hat{\theta}_{MLE}$ is biased. Write an `R` simulation to approximate the bias of $\hat{\theta}_{MLE}$ for a sample of size $n = 20$. Use $B = 1000$ samples.
9. Use the simulation to estimate the variance of $\hat{\theta}_{MoM}$ and the variance of $\hat{\theta}_{MLE}$.
10. The *Mean-squared Error* (MSE) of an estimator $\tilde{\theta}$ of parameter θ is defined as

$$MSE_\theta(\tilde{\theta}) = Bias^2_\theta(\tilde{\theta}) + Var_\theta(\tilde{\theta}).$$

Approximate $MSE_{\theta=1}(\hat{\theta}_{MoM})$ and $MSE_{\theta=1}(\hat{\theta}_{MLE})$ using your `R` simulation.