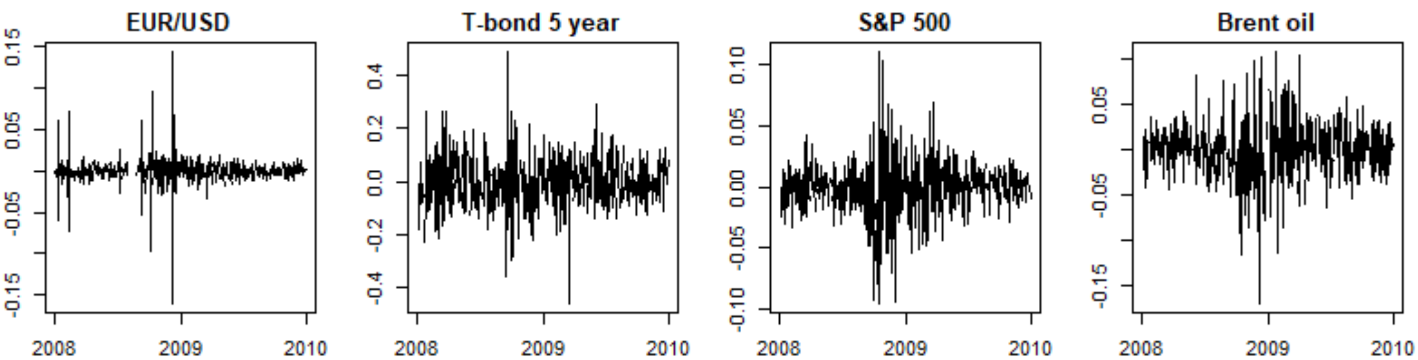
1. Volatility changes over time

What is financial risk?

Financial risk has many faces, and we measure it in many ways, but for now, let's agree that it is a measure of the possible loss on an investment. In financial markets, where we measure prices frequently, volatility (which is analogous to *standard deviation*) is an obvious choice to measure risk. But in real markets, volatility changes with the market itself.



In the picture above, we see the returns of four very different assets. All of them exhibit alternating regimes of low and high volatilities. The highest volatility is observed around the end of 2008 - the most severe period of the recent financial crisis.

In this notebook, we will build a model to study the nature of volatility in the case of US government bond yields.

In [1]:
```
# Load the packages
library(xts)
library(readr)

# Load the data
yc_raw <- read_csv("datasets/FED-SVENY.csv")

# Convert the data into xts format
yc_all <- as.xts(x = yc_raw[, -1], order.by = yc_raw$Date)

# Show only the 1st, 5th, 10th, 20th and 30th columns
yc_all_tail <- tail(yc_all[, c(1, 5, 10, 20, 30)])
yc_all_tail
```

```
Loading required package: zoo

Attaching package: 'zoo'

The following objects are masked from 'package:base':

    as.Date, as.Date.numeric

Parsed with column specification:
cols(
  .default = col_double(),
  Date = col_date(format = "")
)
See spec(...) for full column specifications.
```

	SVENY01	SVENY05	SVENY10	SVENY20	SVENY30
2019-03-22	2.4222	2.2613	2.4553	2.7661	3.0178
2019-03-25	2.3901	2.2281	2.4449	2.7610	3.0216
2019-03-26	2.3811	2.2016	2.4249	2.7508	3.0138
2019-03-27	2.3560	2.1931	2.4020	2.7092	2.9785
2019-03-28	2.3601	2.2137	2.4058	2.6907	2.9605
2019-03-29	2.3719	2.2398	2.4143	2.6939	2.9538

In [0]:
```
# These packages need to be loaded in the first @tests cell
library(testthat)
library(IRkernel.testthat)

soln_yc_raw <- read_csv("datasets/FED-SVENY.csv")
soln_yc_all <- as.xts(x = soln_yc_raw[, -1], order.by = soln_yc_raw$Da
te)

soln_yc_all_tail <- tail(soln_yc_all[, c(1, 5, 10, 20, 30)])

run_tests({
    # Packages loaded
    test_that("the correct package is loaded", {
        expect_true("xts" %in% .packages(),
                    info = "Did you load the xts package?")
        expect_true("readr" %in% .packages(),
                    info = "Did you load the readr package?")
    })

    # Date set loaded
    test_that("the dataset is loaded correctly", {
        expect_is(yc_raw, "tbl_df", info = "Did you read in the data w
ith read_csv() (not read.csv() )?")
        expect_equal(yc_raw, soln_yc_raw,
                     info = "yc_raw contains the wrong values. Did you
import the CSV file correctly?")
    })

    test_that("the xts object is created correctly", {
        expect_equal(yc_all, soln_yc_all,
                     info = "yc_all contains the wrong values. At the
conversion, did you filter out the first column? Did you apply the dat
e column as index?")
    })

    # Output shown
    test_that("output shown", {
            expect_equal(yc_all_tail, soln_yc_all_tail,
                    info = "The output shown is not correct. Did you c
hoose the correct columns? Did you use tail()?")
    })
})
```

2. Plotting the evolution of bond yields

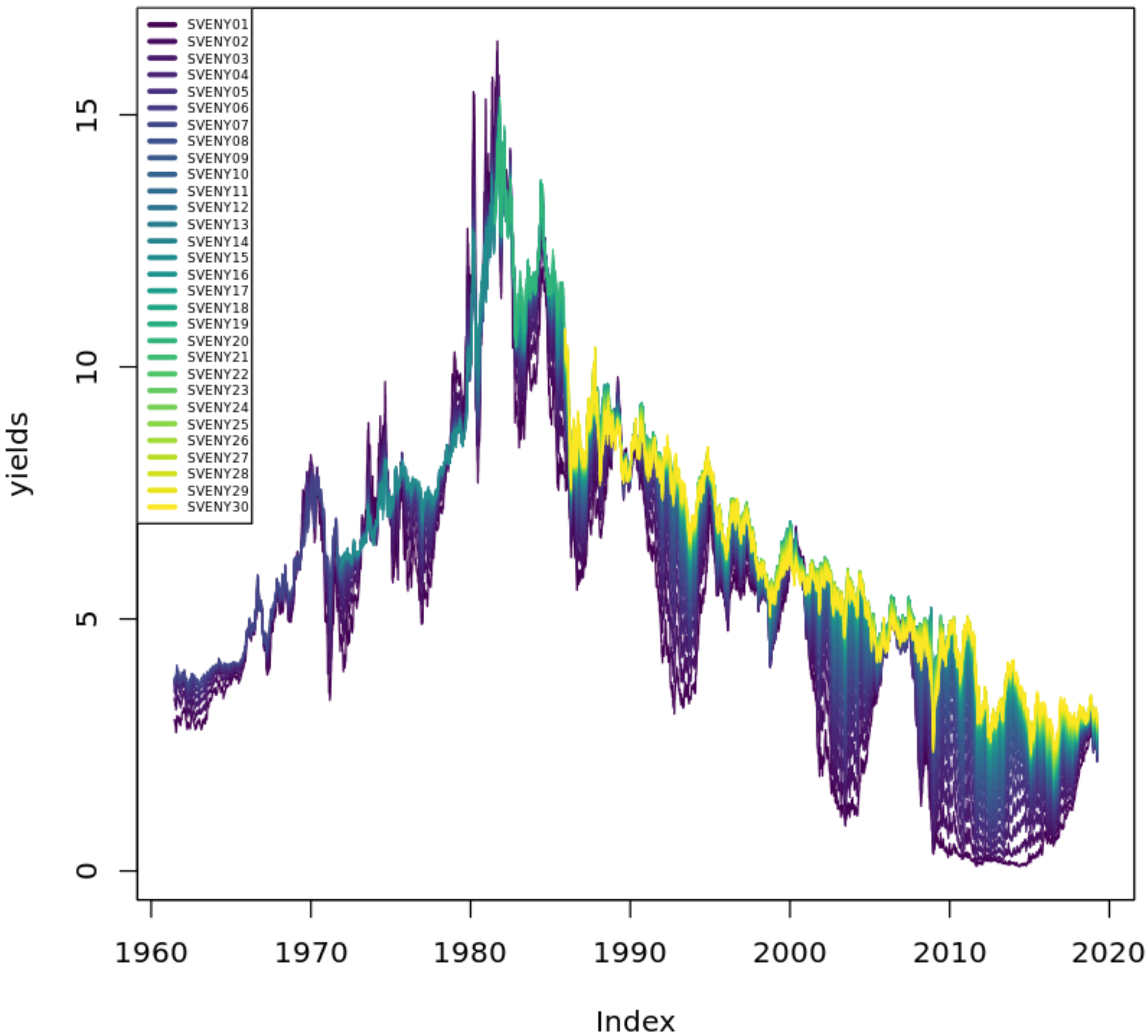
In the output table of the previous task, we see the yields for some maturities.

These data include the whole yield curve. The yield of a bond is the price of the money lent. The higher the yield, the more money you receive on your investment. The yield curve has many maturities; in this case, it ranges from 1 year to 30 years. Different maturities have different yields, but yields of neighboring maturities are relatively close to each other and also move together.

Let's visualize the yields over time. We will see that the long yields (e.g. SVENY30) tend to be more stable in the long term, while the short yields (e.g. SVENY01) vary a lot. These movements are related to the monetary policy of the FED and economic cycles.

In [2]:
```
library(viridis)

# Define plot arguments
yields  <- yc_all
plot.type  <- "single"
plot.palette <- viridis(30)
asset.names <- colnames(yc_all)

# Plot the time series
plot.zoo(x = yields, plot.type = plot.type, col = plot.palette)

# Add the legend
legend(x = "topleft", legend = asset.names,
       col = plot.palette, cex = 0.45, lwd = 3)
```

```
Loading required package: viridisLite
```


SVENY01
SVENY02
SVENY03
SVENY04
SVENY05
SVENY06
SVENY07
SVENY08
SVENY09
SVENY10
SVENY11
SVENY12
SVENY13
SVENY14
SVENY15
SVENY16
SVENY17
SVENY18
SVENY19
SVENY20
SVENY21
SVENY22
SVENY23
SVENY24
SVENY25
SVENY26
SVENY27
SVENY28
SVENY29
SVENY30
yields
0
5
10
15
1960
1970
1980
1990
2000
2010
2020
Index

In [0]:
```
soln_yields  <- soln_yc_all
soln_plot.type  <- "single"
soln_plot.palette <- viridis(30)
soln_asset.names <- colnames(soln_yc_all)

run_tests({
    # Plot parameters
    test_that("plot parameters are correct", {
        expect_equal(soln_yields, yields,
                     info = "The data are not correct. Check that you
used yc_all.")
    })
    test_that("plot parameters are correct", {
        expect_equal(soln_plot.type, plot.type,
                    info = "Did you set the plot.type correctly? It ca
n be either 'single' or 'mulitple'.")
    })
    test_that("plot parameters are correct", {
        expect_equal(soln_plot.palette, plot.palette,
                    info = "The color palette is not correct. Did you
use 30 colors from viridis?")
    })
    test_that("the legend was correctly defined", {
        expect_equal(soln_asset.names, asset.names,
                    info = "The the legend was not correctly defined.
It should be the vector of column names of yc_all.")
        })
    })
```

3. Make the difference

In the output of the previous task, we see the level of bond yields for some maturities, but to understand how volatility evolves we have to examine the changes in the time series. Currently, we have yield levels; we need to calculate the changes in the yield levels. This is called "differentiation" in time series analysis. Differentiation has the added benefit of making a time series independent of time.

In [3]:
```
# Differentiate the time series
ycc_all <- diff.xts(yc_all)

# Show the tail of the 1st, 5th, 10th, 20th and 30th columns
ycc_all_tail <- tail(ycc_all[, c(1, 5, 10, 20, 30)])
ycc_all_tail
```

```
           SVENY01 SVENY05 SVENY10 SVENY20 SVENY30
2019-03-22 -0.0412 -0.1039 -0.0878 -0.0924 -0.0864
2019-03-25 -0.0321 -0.0332 -0.0104 -0.0051  0.0038
2019-03-26 -0.0090 -0.0265 -0.0200 -0.0102 -0.0078
2019-03-27 -0.0251 -0.0085 -0.0229 -0.0416 -0.0353
2019-03-28  0.0041  0.0206  0.0038 -0.0185 -0.0180
2019-03-29  0.0118  0.0261  0.0085  0.0032 -0.0067
```

In [0]:
```
soln_ycc_all <- diff.xts(soln_yc_all)

# Show only the 1st, 5th, 10th, 20th and 30th columns
soln_ycc_all_tail <- tail(soln_ycc_all[, c(1, 5, 10, 20, 30)])


run_tests({
    # Differentiation is correct
    test_that("ycc_all is correct", {
        expect_equal(ycc_all, soln_ycc_all,
                     info = "You did not differentiate the time series
correctly. Did you use the diff.xts() function on yc_all?"
                     )
    })

     # Output shown
    test_that("output shown", {
                expect_equal(ycc_all_tail, soln_ycc_all_tail,
                     info = "The output shown is not correct. Did you c
hoose the correct columns? Did you use tail()?" )
    })
})
```

4. The US yields are no exceptions, but maturity matters

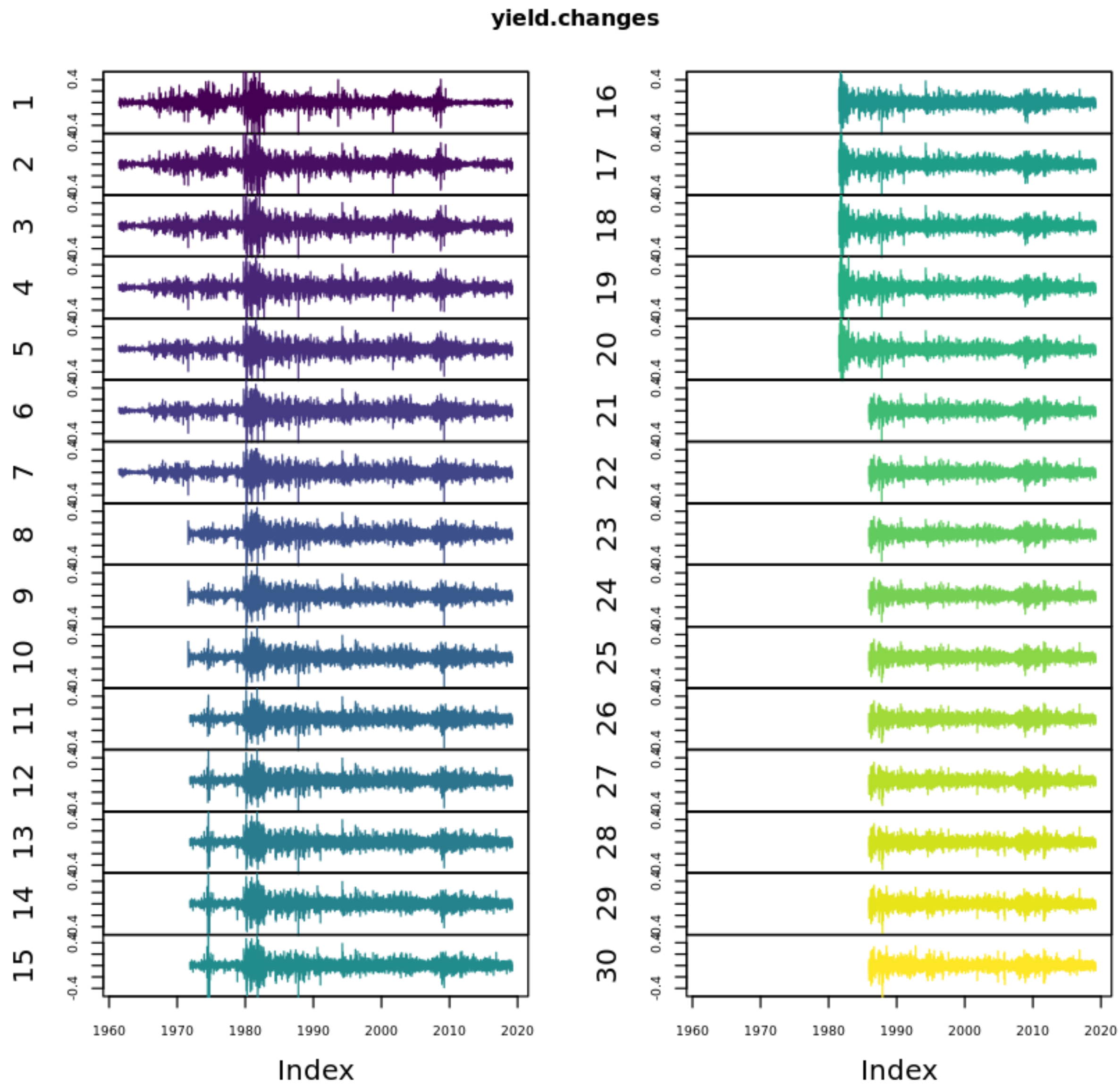
Now that we have a time series of the changes in US government yields let's examine it visually.

By taking a look at the time series from the previous plots, we see hints that the returns following each other have some unique properties:

- The direction (positive or negative) of a return is mostly independent of the previous day's return. In other words, you don't know if the next day's return will be positive or negative just by looking at the time series.
- The magnitude of the return is similar to the previous day's return. That means, if markets are calm today, we expect the same tomorrow. However, in a volatile market (crisis), you should expect a similarly turbulent tomorrow.

In [4]:

```
# Define the plot parameters
yield.changes <- ycc_all
plot.type <- "multiple"


# Plot the differtianted time series
plot.zoo(x = yield.changes, plot.type = plot.type,
     ylim = c(-0.5, 0.5), cex.axis = 0.7,
     ylab = 1:30, col = plot.palette)
```


yield.changes
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
1960 1970 1980 1990 2000 2010 2020
Index
Index

In [0]:
```
# One or more tests of the student's code
# The @solution should pass the tests
# The purpose of the tests is to try to catch common errors and
# to give the student a hint on how to resolve these errors

soln_yield.changes <- soln_ycc_all
soln_plot.type <- "multiple"

run_tests({
    # Plot parameters
    test_that("the plotted data are correct", {
        expect_equal(soln_yield.changes, yield.changes,
                     info = "Did you set the parameter x correctly? It should be the differentiated data.")
    })

    test_that("plot.type set correctly", {
        expect_equal(plot.type, soln_plot.type,
                     info = "Did you set the plot.type correctly? It can be either 'single' or 'mulitple'.")
    })
})
```

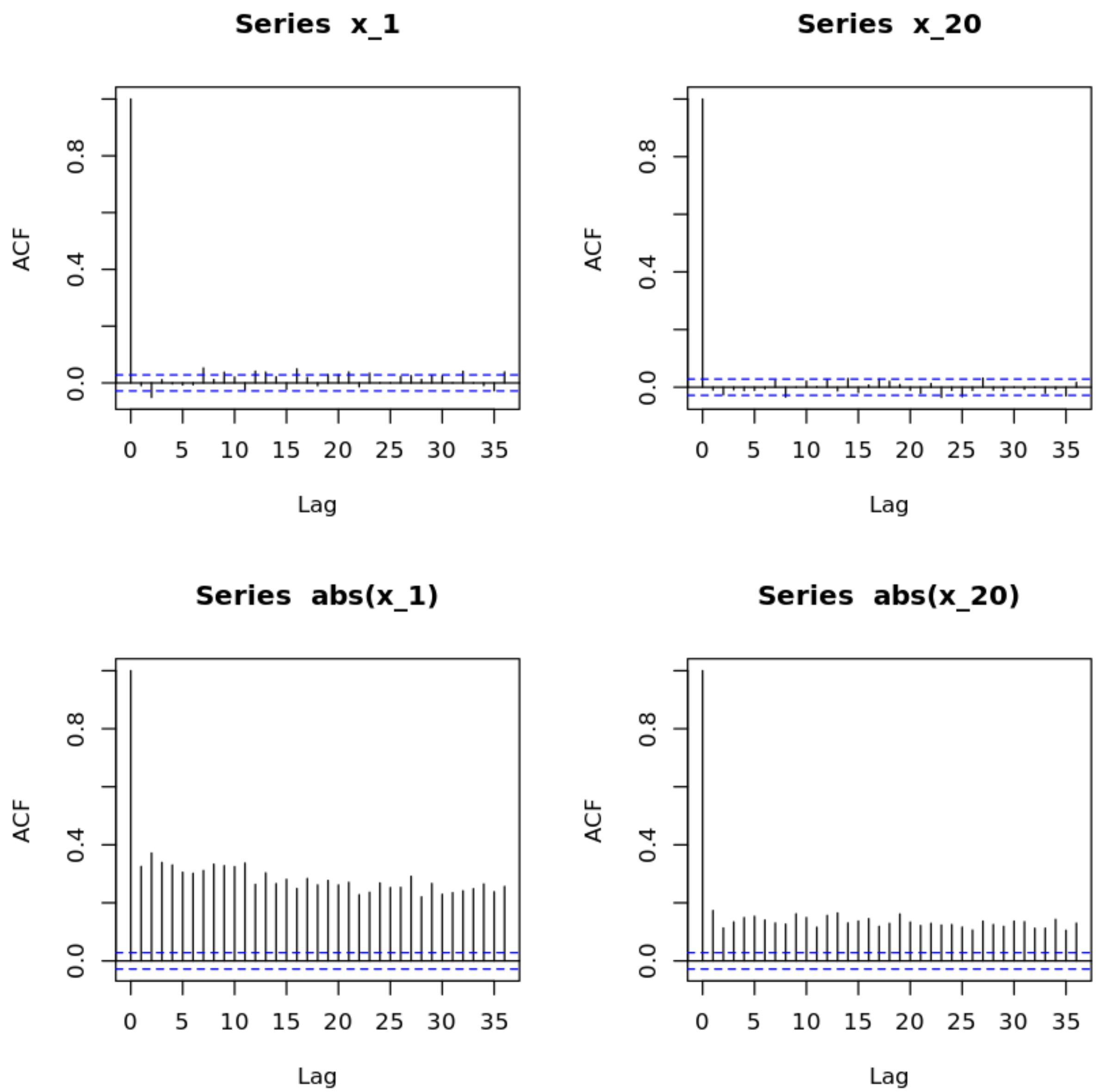
5. Let's dive into some statistics

The statistical properties visualized earlier can be measured by analytical tools. The simplest method is to test for autocorrelation. Autocorrelation measures how a datapoint's past determines the future of a time series.

- If the autocorrelation is close to 1, the next day's value will be very close to today's value.
- If the autocorrelation is close to 0, the next day's value will be unaffected by today's value.

Because we are interested in the recent evolution of bond yields, we will filter the time series for data from 2000 onward.

In [5]:
```
# Filter for changes in and after 2000
ycc <- ycc_all["2000/", ]

# Save the 1-year and 20-year maturity yield changes into separate var
iables
x_1 <- ycc[, "SVENY01"]
x_20 <- ycc[, "SVENY20"]

# Plot the autocorrelations of the yield changes
par(mfrow=c(2,2))
acf_1 <- acf(x_1)
acf_20 <- acf(x_20)

# Plot the autocorrelations of the absolute changes of yields
acf_abs_1 <- acf(abs(x_1))
acf_abs_20 <- acf(abs(x_20))
```



In [0]:
```
# Filter for changes in and after 2000
soln_ycc <- soln_ycc_all["2000/", ]

# Save the 1-year and 20-year maturity yield changes into separate var
iables
soln_x_1 <- soln_ycc[, "SVENY01"]
soln_x_20 <- soln_ycc[, "SVENY20"]

# Plot the autocorrelations of the changes of yields
par(mfrow=c(2,2))
soln_acf_1 <- acf(soln_x_1, plot = FALSE)
soln_acf_20 <- acf(soln_x_20, plot = FALSE)
# Plot the autocorrelations of the absolute changes of yields, too
soln_acf_abs_1 <- acf(abs(soln_x_1), plot = FALSE)
```

```
soln_acf_abs_20 <- acf(abs(soln_x_20), plot = FALSE)



run_tests({
    test_that("ycc is correct", {
        expect_equal(ycc, soln_ycc,
                     info = "ycc is not correct. Did you filter for th
e time period in and after 2000 correctly?"
                    )
    })
    test_that("x_1 is correct", {
        expect_equal(x_1, soln_x_1,
                     info = "x_1 is not correct. Did you select the 1s
t column?"
                    )
    })
    test_that("x_20 is correct", {
        expect_equal(x_20, soln_x_20,
                     info = "x_20 is correct. Did you select the 20th
column?"
                    )
    })
    test_that("acf_1 is correct", {
        expect_equal(acf_1$acf, soln_acf_1$acf,
                     info = "Autocorrelation of x_1 is not correct. Di
d you use the acf() function on x_1?"
                    )
    })

    test_that("acf_20` is correct", {
        expect_equal(acf_20$acf, soln_acf_20$acf,
                     info = "Autocorrelation of x_20 is not correct. D
id you use the acf() function on x_20?"
                    )
    })
    test_that("acf_abs_1 is correct", {
        expect_equal(acf_abs_1$acf, soln_acf_abs_1$acf, label = "acf_a
bs_1", expected.label = "acf(abs(x_1))",
                     info = "Autocorrelation of absolute values of x_1
is not correct. Did you use the acf() and abs() functions on x_1?"
                    )
    })
    test_that("acf_abs_20 is correct", {
        expect_equal(acf_abs_20$acf, soln_acf_abs_20$acf, label = "acf
_abs_20", expected.label = "acf(abs(x_20))",
                     info = "Autocorrelation of absolute values of x_2
0 is not correct. Did you use the acf() and abs() functions on x_20?"
                    )
    })
```

```
})
```

6. GARCH in action

A Generalized AutoRegressive Conditional Heteroskedasticity (GARCH (https://en.wikipedia.org/wiki/Autoregressive_conditional_heteroskedasticity)) model is the most well known econometric tool to handle changing volatility in financial time series data. It assumes a hidden volatility variable that has a long-run average it tries to return to while the short-run behavior is affected by the past returns.

The most popular form of the GARCH model assumes that the volatility follows this process:

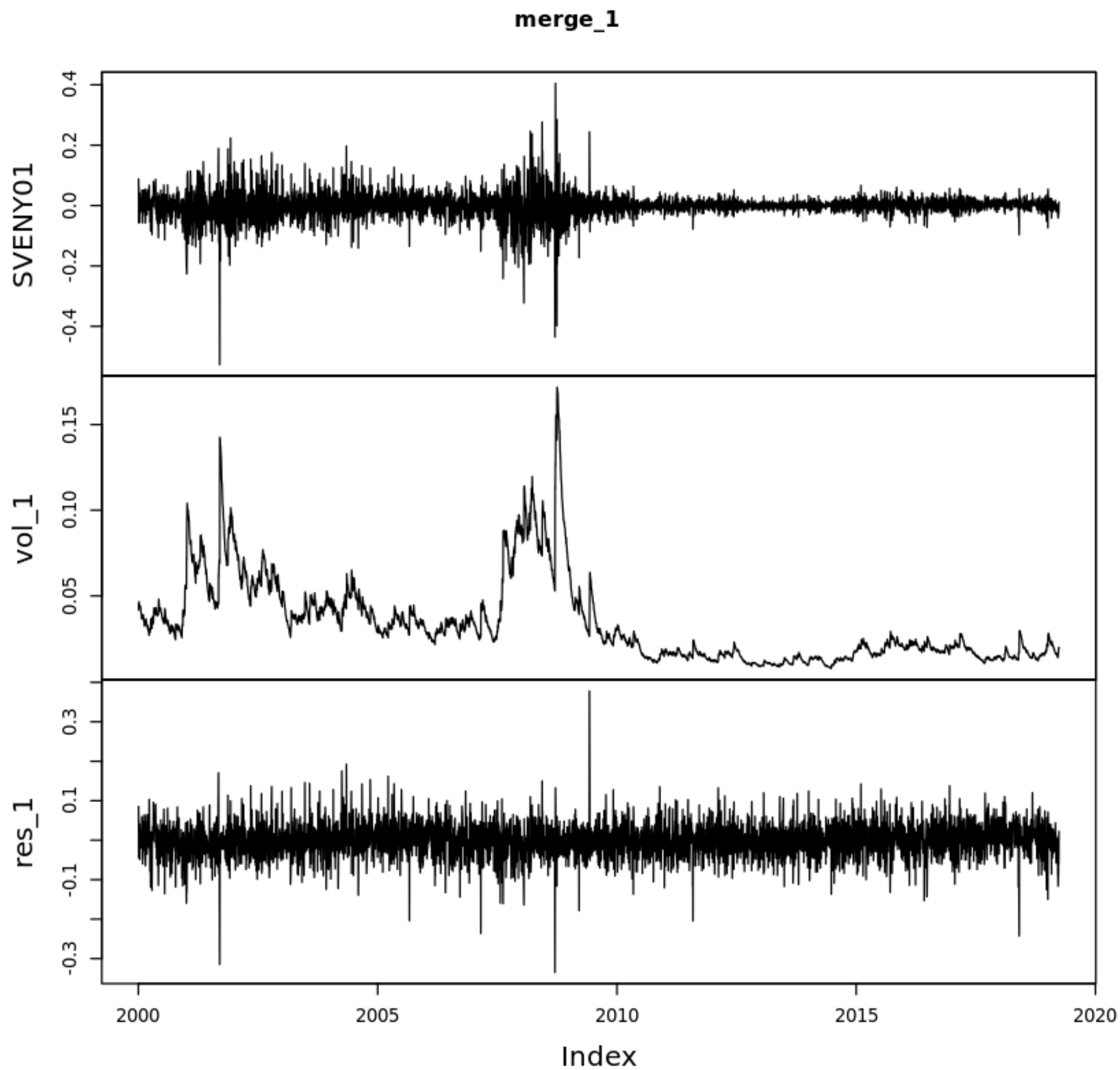
$^2{}_t = \omega + \alpha \cdot \varepsilon^2{}_{t-1} + \beta \cdot \sigma^2{}_{t-1}$

$_{t-1}$ the last day's volatility and ε_{t-1} is the last day's return. The estimated parameters are ω, α, and β.

For GARCH modeling we will use `rugarch` (https://cran.r-project.org/web/packages/rugarch/index.html) package developed by Alexios Ghalanos.

In [6]:
```
library(rugarch)
# Specify the GARCH model with the skewed t-distribution
spec <- ugarchspec(distribution.model = "sstd")

# Fit the model
fit_1 <- ugarchfit(x_1, spec = spec)

# Save the volatilities and the rescaled residuals
vol_1 <- sigma(fit_1)
res_1 <- scale(residuals(fit_1, standardize = TRUE)) * sd(x_1) + mean(
x_1)

# Plot the yield changes with the estimated volatilities and residuals
merge_1 <- merge.xts(x_1, vol_1, res_1)
plot.zoo(merge_1)
```

```
Loading required package: parallel

Attaching package: 'rugarch'

The following object is masked from 'package:stats':

    sigma
```



In [0]:
```
# Specify the GARCH model with the skewed t-distribution
soln_spec <- ugarchspec(distribution.model = "sstd")

# Fit the model
soln_fit_1 <- ugarchfit(soln_x_1, spec = soln_spec)
```

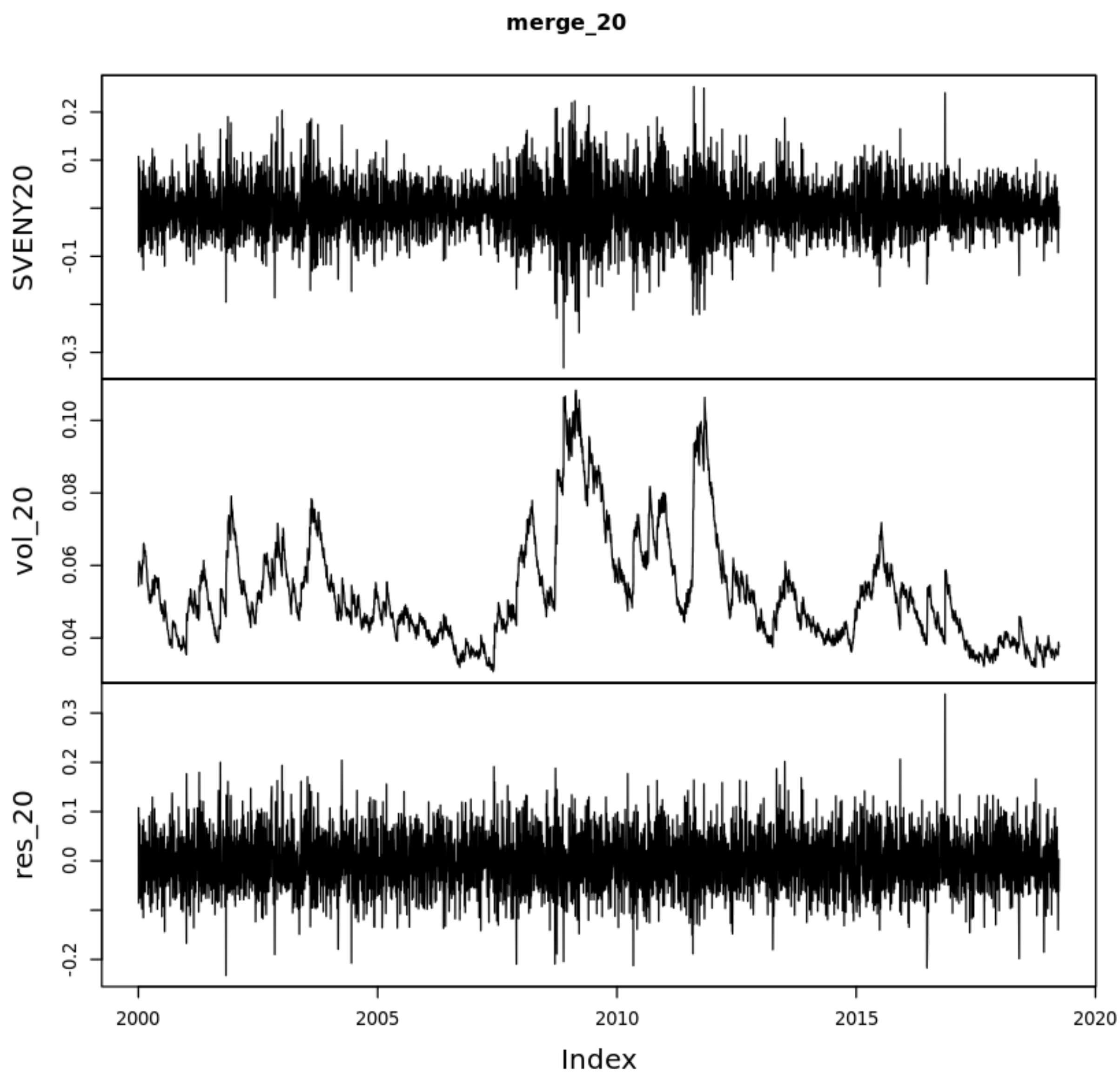
```
# Save the volatilities and the rescaled residuals
soln_vol_1 <- sigma(soln_fit_1)
soln_res_1 <- scale(residuals(soln_fit_1, standardize = TRUE)) * sd(so
ln_x_1) + mean(soln_x_1)

# Plot the yield changes with the estimated volatilities and residuals
.
soln_merge_1 <- merge.xts(soln_x_1, vol_1 = soln_vol_1, res_1 = soln_r
es_1)

run_tests({
    test_that("spec is correct", {
        expect_equal(spec, soln_spec,
            info = "The ugarch specification is not correct. Did you u
se the ugarchspec() function and the 'sstd' parameter?"
        )
    })

    test_that("fit_1 is correct", {
        expect_equal(fit_1@fit$residuals, soln_fit_1@fit$residuals,
            info = "The fitted model is not correct. Did you apply the
ugarchfit() function on x_1 with specification defined earlier?"
        )
    })

    test_that("vol_1 is correct", {
        expect_equal(vol_1, soln_vol_1,
            info = "The volatility is not correct. Did you use the sig
ma() function on the fitted model (fit_1?"
        )
    })

    test_that("res_1 is correct", {
        expect_equal(res_1, soln_res_1,
            info = "The residuals are not correct. Did you use the res
iduals() function on the fitted model (fit_1? The standardize paramete
r can be either TRUE or FALSE."
        )
    })

    test_that("merge_1 is correct", {
        expect_equal(merge_1, soln_merge_1,
            info = "The merged object is not correct. Did you applied
the merge.xts() function on the original series (x_1), the volatilitie
s (vol_1) and the residuals (res_1)?"
        )
    })
})
```

7. Fitting the 20-year maturity

Let's do the same for the 20-year maturity. As we can see in the plot from Task 6, the bond yields of various maturities show similar but slightly different characteristics. These different characteristics can be the result of multiple factors such as the monetary policy of the FED or the fact that the investors might be different.

Are there differences between the 1-year maturity and 20-year maturity plots?

In [7]:
```
# Fit the model
fit_20 <- ugarchfit(x_20, spec = spec)

# Save the volatilities
vol_20 <- sigma(fit_20)
res_20 <- scale(residuals(fit_20, standardize = TRUE)) * sd(x_20) + me
an(x_20)

# Plot the yield changes with the estimated volatilities and residuals
merge_20 <- merge.xts(x_20, vol_20, res_20)
plot.zoo(merge_20)
```



In [0]:
```
# Specify the GARCH model with the skewed t-distribution
soln_spec <- ugarchspec(distribution.model = "sstd")

# Fit the model
soln_fit_20 <- ugarchfit(soln_x_20, spec = soln_spec)

# Save the volatilities and the rescaled residuals
soln_vol_20 <- sigma(soln_fit_20)
soln_res_20 <- scale(residuals(soln_fit_20, standardize = TRUE)) * sd(
soln_x_20) + mean(soln_x_20)

# Plot the yield changes with the estimated volatilities and residuals
.
soln_merge_20 <- merge.xts(soln_x_20, vol_20 = soln_vol_20, res_20 = s
```

```
oln_res_20)


run_tests({
    test_that("spec is correct", {
        expect_equal(spec, soln_spec,
            info = "The ugarch specification is not correct. Did you u
se the ugarchspec() function and the 'sstd' parameter?"
        )
    })

    test_that("fit_20 is correct", {
        expect_equal(fit_20@fit$residuals, soln_fit_20@fit$residuals,
            info = "The fitted model is not correct. Did you apply the
ugarchfit() function on x_20 with specification defined earlier?"
        )
    })

    test_that("vol_20 is correct", {
        expect_equal(vol_20, soln_vol_20,
            info = "The volatility is not correct. Did you use the sig
ma() function on the fitted model (fit_20)?"
        )
    })

    test_that("res_20 is correct", {
        expect_equal(res_20, soln_res_20,
            info = "The residuals are not correct. Did you use the res
iduals() function on the fitted model (fit_20? The standardize paramet
er can be either TRUE or FALSE."
        )
    })

    test_that("merge_20 is correct", {
        expect_equal(merge_20, soln_merge_20,
            info = "The merged object is not correct. Did you applied
the merge.xts() function on the original series (x_20), the volatiliti
es (vol_20) and the residuals (res_20)?"
        )
    })
})
```

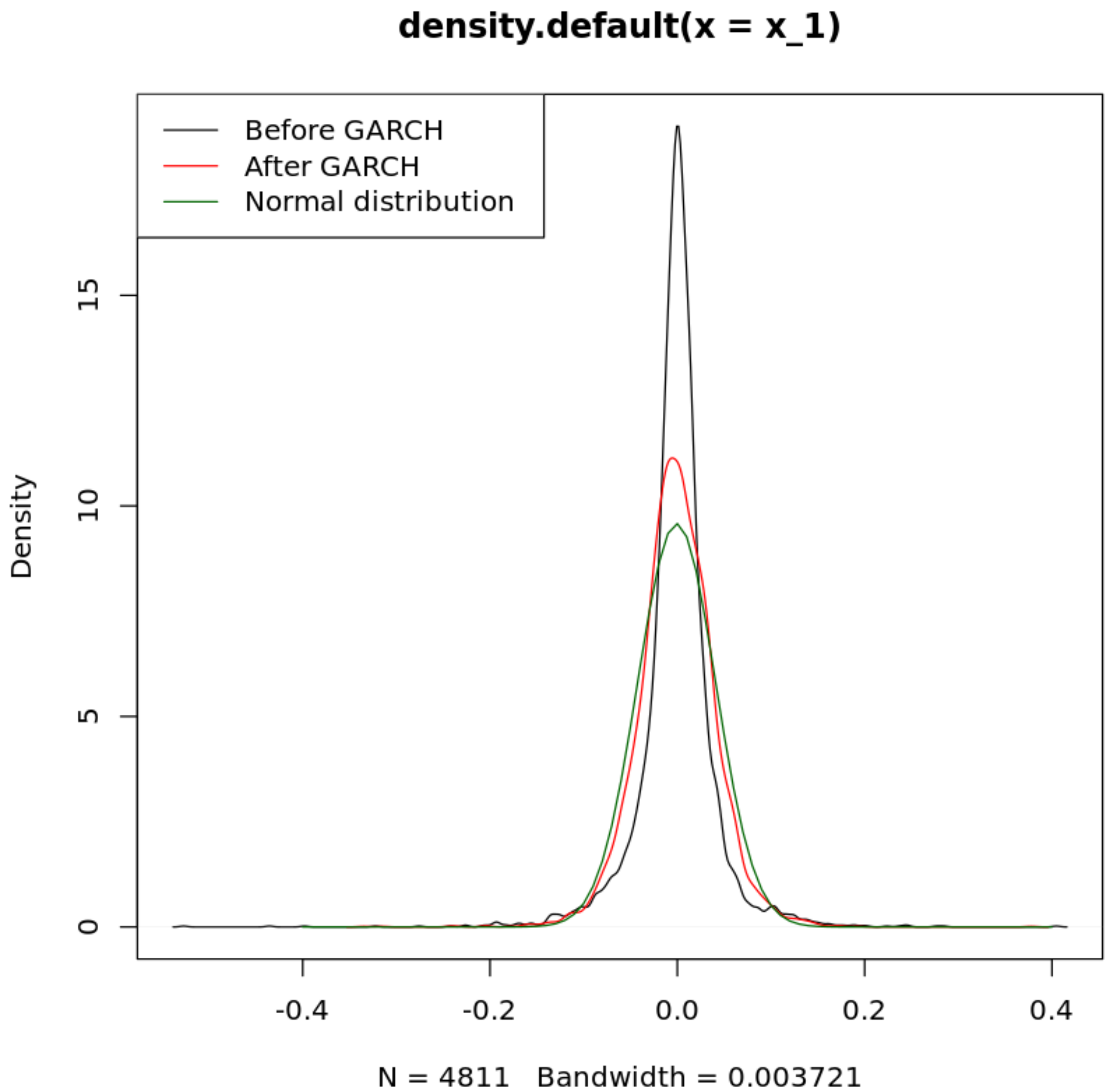
8. What about the distributions? (Part 1)

From the plots in Task 6 and Task 7, we can see that the 1-year GARCH model shows a similar but more erratic behavior compared to the 20-year GARCH model. Not only does the 1-year model have greater volatility, but the volatility of its volatility is larger than the 20-year model. That brings us to two statistical facts of financial markets not mentioned yet.

- The unconditional (before GARCH) distribution of the yield differences has heavier tails than the normal distribution.
- The distribution of the yield differences adjusted by the GARCH model has lighter tails than the unconditional distribution, but they are still heavier than the normal distribution.

Let's find out what the fitted GARCH model did with the distribution we examined.

In [8]:
```
# Calculate the kernel density for the 1-year maturity and residuals
density_x_1 <- density(x_1)
density_res_1 <- density(res_1)

# Plot the density digaram for the 1-year maturity and residuals
plot(density_x_1)
lines(density_res_1, col = "red")

# Add the normal distribution to the plot
norm_dist <- dnorm(seq(-0.4, 0.4, by = .01), mean = mean(x_1), sd = sd
(x_1))
lines(seq(-0.4, 0.4, by = .01),
      norm_dist,
      col = "darkgreen"
     )

# Add legend
legend <- c("Before GARCH", "After GARCH", "Normal distribution")
legend("topleft", legend = legend,
       col = c("black", "red", "darkgreen"), lty=c(1,1))
```



In [0]:

```
# One or more tests of the student's code
# The @solution should pass the tests
# The purpose of the tests is to try to catch common errors and
# to give the student a hint on how to resolve these errors


# Plot the density diagram for 1-year maturity
soln_density_x_1 <- density(soln_x_1)

# Add the density of the residuals
soln_density_res_1 <- density(soln_res_1)


# And finally add the normal distribution to the plot
```

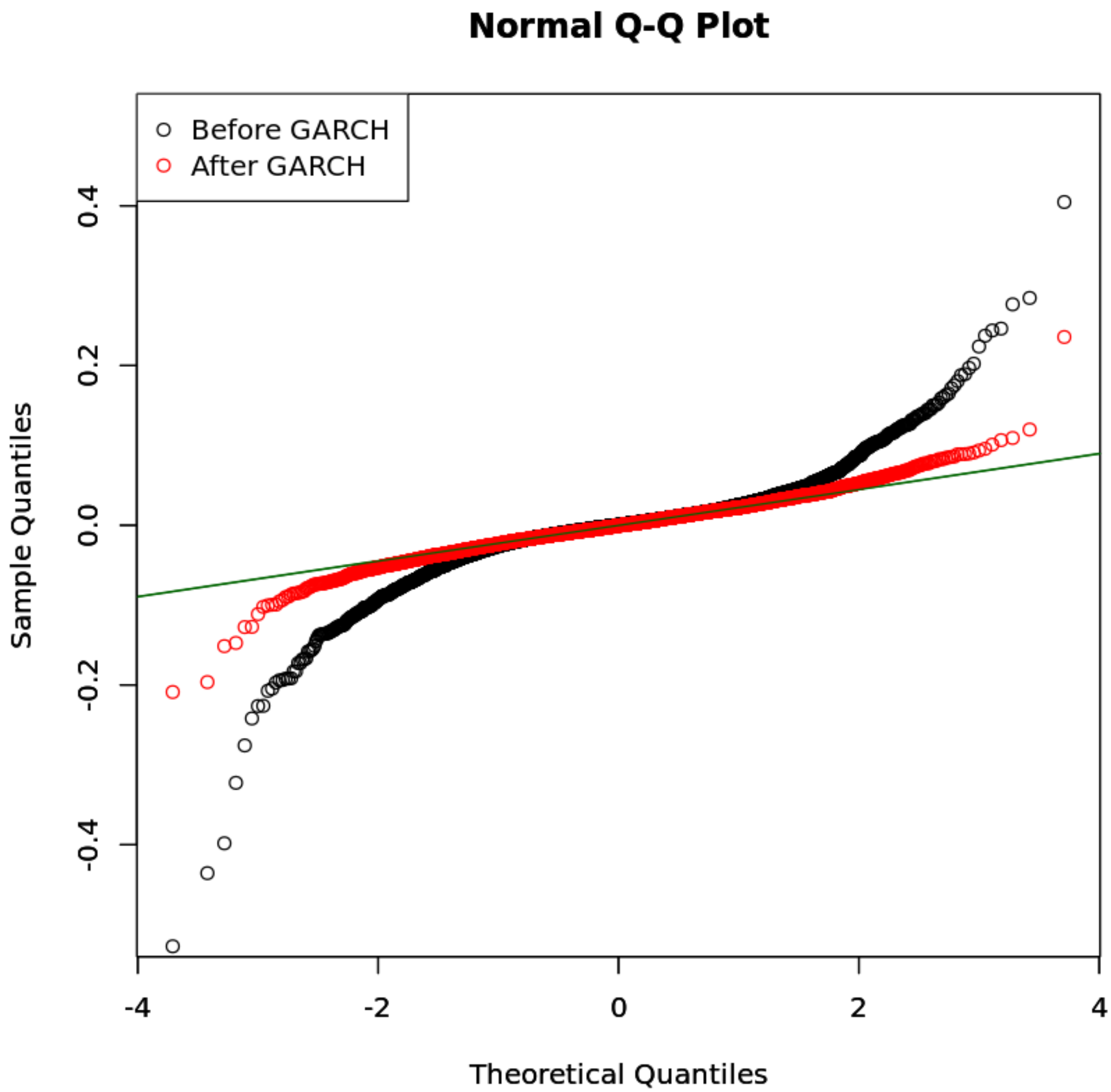
```
soln_norm_dist <- dnorm(seq(-0.4, 0.4, by = .01), mean = mean(soln_x_1
), sd = sd(soln_x_1))

# Add legend
soln_legend <- c("Before GARCH", "After GARCH", "Normal distribution")


run_tests({
    test_that("density_x_1 is correct", {
        expect_equal(density_x_1$x, soln_density_x_1$x,
            info = "Density of original time series is not correct. Di
d you apply the density() function on x_1?"
        )
        expect_equal(density_x_1$y, soln_density_x_1$y,
            info = "Density of original time series is not correct. Di
d you apply the density() function on x_1?"
        )
    })

    test_that("soln_res_1 is correct", {
        expect_equal(density_res_1$x, soln_density_res_1$x,
            info = "Density of residuals is not correct. Did you apply
the density() function on res_1?"
        )
        expect_equal(density_res_1$y, soln_density_res_1$y,
            info = "Density of residuals is not correct. Did you apply
the density() function on res_1?"
        )
    })

    test_that("normal distribution is correct", {
        expect_equal(norm_dist, soln_norm_dist,
            info = "The normal distrubution in the plot is not correct
. Did you define the mean and sd correctly?"
        )
    })

    test_that("legend is correct", {
        expect_equal(legend, soln_legend,
            info = "The legend is not correct. Did you listed the labe
ls in the instructions correctly?"
        )
    })

})
```

9. What about the distributions? (Part 2)

In the previous plot, we see that the two distributions from the GARCH models are different from the normal distribution of the data, but the tails, where the differences are the most profound, are hard to see. Using a Q-Q plot will help us focus in on the tails.

You can read an excellent summary of Q-Q plots [here (https://stats.stackexchange.com/questions/101274/how-to-interpret-a-qq-plot)](https://stats.stackexchange.com/questions/101274/how-to-interpret-a-qq-plot).

In [9]:
```
# Define plot data: the 1-year maturity yield changes and the residual
s
data_orig <- x_1
data_res <- res_1

# Define the benchmark distribution (qnorm)
distribution <- qnorm

# Make the Q-Q plot of original data with the line of normal distribut
ion
qqnorm(data_orig, ylim = c(-0.5, 0.5))
qqline(data_orig, distribution = distribution, col = "darkgreen")

# Make the Q-Q plot of GARCH residuals with the line of normal distrib
ution
par(new=TRUE)
qqnorm(data_res * 0.623695122815242, col = "red", ylim = c(-0.5, 0.5))
qqline(data_res * 0.623695122815242, distribution = distribution, col
= "darkgreen")
legend("topleft", c("Before GARCH", "After GARCH"), col = c("black", "
red"), pch=c(1,1))
```



Normal Q-Q Plot
Before GARCH
After GARCH
Sample Quantiles
Theoretical Quantiles
0.4
0.2
0.0
-0.2
-0.4
-4
-2
0
2
4

In [0]:
```
# Define plot data: the 1-year maturity yield changes and the residual
s
soln_data_orig <- x_1
soln_data_res <- res_1

# Define the benchmark distribution (qnorm)
soln_distribution <- qnorm


run_tests({
    test_that("the data_orig is correct", {
        expect_equal(data_orig, soln_data_orig,
            info = "The data_orig is not correct. Is it equal to the o
riginal series of 1-year yield changes?"
        )
    })

    test_that("the distribution is correct", {
        expect_equal(distribution, soln_distribution,
            info = "The distribution is not correct. Is it based on th
e normal distribution?"
        )
    })

    test_that("the data_res is correct", {
        expect_equal(data_res, soln_data_res,
            info = "The data_res is not correct. Is it equal to the re
siduals of the model fitted on 1-year yield changes?"
        )
    })
})
```

10. A final quiz

In this project, we fitted a GARCH model to develop a better understanding of how bond volatility evolves and how it affects the probability distribution. In the final task, we will evaluate our model. Did the model succeed, or did it fail?

In [10]:
```
# Q1: Did GARCH revealed how volatility changed over time? # Yes or N
o?
(Q1 <- "Yes")

# Q2: Did GARCH bring the residuals closer to normal distribution? Yes
or No?
(Q2 <- "Yes")

# Q3: Which time series shows the most erratic behaviour? Choose 1 or
20.
(Q3 <- 1)
```

'Yes'

'Yes'

1

In [0]:
```
run_tests({
    test_that("the Q1 is correct", {
        expect_equal(tolower(Q1), "yes",
            info = "The 1st answer is not correct. If GARCH could not
estimate the volatility over time, its plot would be a constant line."
        )
    })

    test_that("the Q2 is correct", {
        expect_equal(tolower(Q2), "yes",
            info = "The 2nd answer is not correct. The residuals from
the examples are still not normal, but they are less erratic than befo
re."
        )
    })

    test_that("the Q3 is correct", {
        expect_equal(Q3, 1,
            info = "The 3rd answer is not correct. Which time series s
howed rapid changes in behavior?"
        )
    })
})
```