ONE-WAY ANOVA

ANOVA stands for "Analysis of Variance" and is an omnibus test, meaning it tests for a difference overall between all groups. The one-way ANOVA, also referred to as one factor ANOVA, is a parametric test used to test for a statistically significant difference of an outcome between 3 or more groups. Since it is an omnibus test, it tests for a difference overall, i.e. at least one of the groups is statistically significantly different than the others. However, if the ANOVA is significant one cannot tell which group is different. In order to tell which group is different, one has to conduct planned or post-hoc comparisons. As with all parametric tests, there are certain conditions that need to be met in order for the test results to be considered reliable.

The reason why it's called an one-way or one factor ANOVA even though there are 3 or more groups being tested is because those groups are under one categorical variable, such as race or education level, and the name is referring to the number of variables in the analysis and not the number of groups. If there are two variables being compared it would technically be called a two-way, or two factor, ANOVA if both variables are categorical, or it could be called an ANCOVA if the 2nd variable is continuous. The "C" doesn't stand for continuous, it stands for covariate.

When working from the ANOVA framework, independent variables are sometimes referred to as *factors* and the number of groups within each variable are called *levels*, i.e. one variable with 3 categories could be referred to as a factor with 3 levels.

Parametric test assumptions

- Population distributions are normal
- Samples have equal variances
- Independence

Hypothesis

$$H_0 : \bar{x}_1 = \bar{x}_2 = \bar{x}_3 = \ldots = \bar{x}_k$$
$$H_A : \text{At least one of the groups means differ}$$

The test statistic is the F-statistic and compares the mean square between samples (MS_B) to the mean square within sample (MS_W). This *F-statistic* can be calculated using the following formula:

$$F = \frac{MS_B}{MS_W}$$

Where,

$$MS_B = \frac{\text{Sum of square between sample } (SS_B)}{(k-1)}$$

$$MS_W = \frac{\text{Sum of square within sample } (SS_W)}{(n_T-k)}$$

k is the number of groups
n_T is the total number of observations

and where,

Sum of square between sample (SS_B) = $\sum_k n_k(\bar{x}_k - \bar{x})^2$
Sum of square within sample (SS_W) = $\sum_{i,k}(x_{i,k} - \bar{x}_k)^2$ or can be calculated as $\sum_k (n_k - 1)s_k^2$

One rejects the the null hypothesis, H_0, if the computed F-static is greater than the critical F-statistic. The critical F-statistic is determined by the degrees of freedom and alpha, α, value.

Reject H_0 if $\text{calucated F-statistic} > \text{critical F-statistic}$

Before the decision is made to accept or reject the null hypothesis the assumptions need to be checked. See this page on how to check the parametric assumptions in detail - how to check the assumptions for this example will be demonstrated near the end.

Let's make sense of all these mathmatical terms. In order to do that, let's start with a generic ANOVA table filled in with symbols and the data set used in this example for now.

ANOVA Table

Source	Sum of Squares	Degrees of Freedom	Mean Square	F-statistic
Between samples	SS_B	$k-1$	$MS_B = \frac{SS_B}{(k-1)}$	$\frac{MS_B}{MS_W}$
Within samples	SS_W	$n_T - k$	$MS_W = \frac{SS_W}{(n_T-k)}$	
Total	$TSS = SS_B + SS_W$	$n_T - 1$		
Note: TSS means total sum of squares				

Data Table

Drug Dose	Libido					Sample Size	Sample Means	Sample Variance
Placebo (k_1)	3	2	1	1	4	5 (n_1)	2.2 ($\bar{x}_1$)	1.7 (s_1^2)
Low (k_2)	5	2	4	2	3	5 (n_2)	3.2 ($\bar{x}_2$)	1.7 (s_2^2)
High (k_3)	7	4	5	3	6	5 (n_3)	5.0 ($\bar{x}_3$)	2.5 (s_3^2)
Total ($k = 3$)						15 (n_T)	3.5 ($\bar{x}$)	3.1 (s^2)

Now using the formulas from above, the ANOVA table can be filled in.

Between samples row

$$SS_B = \sum_k n_k(\bar{x}_k - \bar{x})^2 = 5(2.2 - 3.5)^2 + 5(3.2 - 3.5)^2$$

$$\text{Degrees of Freedom} = k - 1 = 3 - 1 = 2$$

$$\text{Mean square } (MS_B) = \frac{SS_B}{k-1} = \frac{20.15}{2} = 10.07$$

Within samples row

$$SS_W = \sum_{i,k}(x_{i,k} - \bar{x}_i)^2 \text{ or } \sum_k (n_k - 1)s_k^2 = (5-1)1.7 +$$

$$\text{Degrees of Freedom} = n_T - k = 15 - 3 = 12$$

$$\text{Mean square } (MS_W) = \frac{SS_W}{n_T - k} = \frac{23.6}{12} = 1.97$$

Total row

$$TSS = SS_B + SS_W = 20.15 + 23.6 = 43.75$$

$$\text{Degrees of Freedom} = n_T - 1 = 15 - 1 = 14$$

F-statistic

$$\text{F-statistic} = \frac{MS_B}{MS_W} = \frac{10.07}{1.97} = 5.11$$

ANOVA Table

Source	Sum of Squares	Degrees of Freedom	Mean Square	F-statistic
Between samples	20.15	2	10.07	5.11
Within samples	23.6	12	1.97	
Total	43.75	14		

In order to tell if the calcualted F-statistic is statistically significant, one would look up the F-statistic based on the degress of freedom and alpha level - using statistical software this doesn't need to be done since it'll be provided.

Fear not if math is not your strong suit. All this is being calucated when using the methods of a statistical software or programming language. It's good to know what is going on behind the scences. References for this section are provided at the end of the page.