Factor Analysis

- good for density estimation (when $n \gg m$) and dim. reduction

dims. ; # data points

Model

that is a MVN

posit that
- data lies along a lower dimensional (in dims) affine subspace w/ Gaussian, axis-aligned, noise. To generate a point in the lower dim. space w/ a MVN dist.

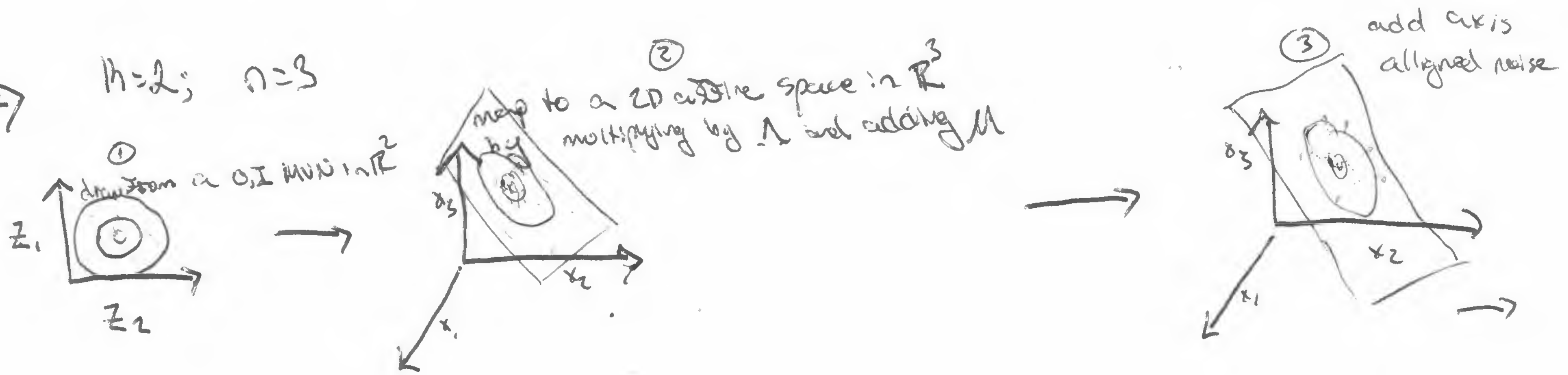
$$z^{(i)} \sim \mathcal{N}(0, I_k) \qquad \text{w/ } z^{(i)} \in \mathbb{R}^k;\ \mu, \varepsilon^{(i)} \in \mathbb{R}^n;\ \Lambda \in \mathbb{R}^{n \times k},\ \Psi \in \mathbb{R}^{n \times n}$$

(diagonal)

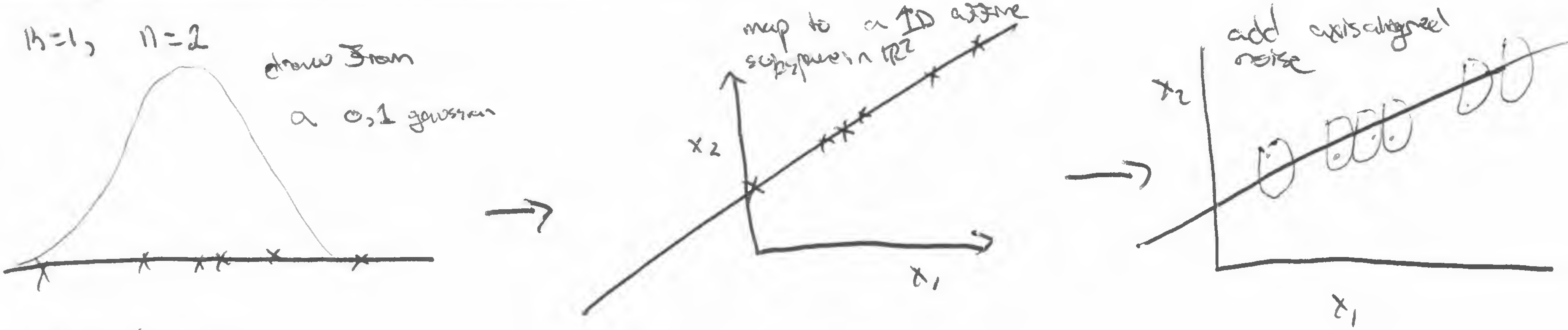
$$\varepsilon^{(i)} \sim \mathcal{N}(0, \Psi)$$

$$x^{(i)} = \mu + \Lambda z^{(i)} + \varepsilon^{(i)}$$

$\Rightarrow$ $k=2;\ n=3$



$k=1$, $n=2$



- k is chosen

- Λ, Ψ, & μ found via maximum likelihood

- get posteriors for free:

  $P(z^{(i)}|x^{(i)};\Theta)$ is a MVN, using the MAP estimate of $z^{(i)}$; we have that

  $x^{(i)}_{FA} = \mu_{z^{(i)}|x^{(i)}}$ = some combination of the MLE of Λ, Ψ, μ; where we use the EM algo. to find these estimates.