0. Identifying Information

a. **Name:** David Lee

b. **SID:** 3031796951

c. **Discussion GSI(s) (optional):**

d. **Lab GSI (optional):**

1. Study Group

Solution
William Song - SID: 3031799759
Matthew Soh - SID: 3032109159

2. Stojanovic's Optimal Smoothies

a. What were Professor Ayazifar's ratings for each fruit? **Work this problem out by hand.**

Solution (see scanned attachment for work)

Strawberries: 6

Bananas: 5

Mangos: 8

Blueberries: 8

Initial Set-up: $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{2}{5} & \frac{3}{5} & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 6\frac{1}{3} \\ 6\frac{1}{3} \\ 6\frac{4}{5} \\ 5\frac{2}{3} \end{bmatrix}$,

where a, b, c, d are Strawberries, Bananas, Mangos, and Blueberries respectively.

Answer: $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 8 \\ 8 \end{bmatrix}$

b. What mystery fruit combination should Professor Stojanovic put in Professor Ayazifar's personalized smoothie? What score would Professor Ayazifar give for this smoothie? (There may be more than one correct answer)

Solution For one possible answer, Professor Stojanovic should combine $\frac{1}{2}$ mango and $\frac{1}{2}$ blueberries in Professor Ayazifar's personalized smoothie for the maximum score.

Professor Ayazifar would give a score of 8 for that smoothie.

3. Finding charges from voltage measurements

Write the system of linear equations relating the voltages to charges, and solve the system to find the charges Q_1, Q_2, Q_3. You may use your IPython notebook to solve the system.

Solution Set-up: $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{5}} & \frac{1}{2} \\ 1 & \frac{1}{\sqrt{2}} & 1 \\ \frac{1}{2} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} \frac{4+3\sqrt{5}+\sqrt{10}}{2\sqrt{5}} \\ \frac{2+4\sqrt{2}}{\sqrt{2}} \\ \frac{4+\sqrt{5}+3\sqrt{10}}{2\sqrt{5}} \end{bmatrix}$,

where Q_1, Q_2, Q_3 are the three point charges whose positions are known.

Answer: $Q_1 = 1.0 Q_2 = 2.0000000000000013 Q_3 = 2.9999999999999999$

or $\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(see attached ipynb pdf for code)

4. The Framingham Risk Score

a. The intern dug up some of the records for patients in the study group who fit the criteria of the formula in question. The records are summarized in the table. Use these records to devise a system of linear equations where a, b, c and d are the unknowns.

Solution Set-Up: $\begin{bmatrix} 4.18965474 & 5.28826703 & 4.00733319 & 4.88280192 \\ 4.11087386 & 5.19295685 & 3.8501476 & 4.82028157 \\ 4.09434456 & 5.19295685 & 3.91202301 & 4.78749174 \\ 3.13549422 & 4.88280192 & 3.80666249 & 4.88280192 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 26.84889282 \\ 26.48830875 \\ 26.31468674 \\ 24.07908834 \end{bmatrix}$,

where a, b, c, d are constant coefficients for the equation for R.

(see attached ipynb pdf for code)

b. Solve the system of linear equations that you devised in question (a) of this problem. For this question, you can use IPython.

Solution [2.3098569091997123 1.1695549079666905 -0.6945169529064997 2.8200267513575112] or

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2.3098569091997123 \\ 1.1695549079666905 \\ -0.6945169529064997 \\ 2.8200267513575112 \end{bmatrix}$$

(see attached ipynb pdf for code)

5. Filtering out the Troll

a. Using the notation above, let the important speaker be speaker A (with signal $\vec{a}$) and let the person trolling be "speaker" B (with signal $\vec{b}$). Express the recordings of the two microphones $\vec{m_1}$ and $\vec{m_2}$ (i.e. the signals recorded by the first and the second microphones, respectively) as a linear combination of $\vec{a}$ and $\vec{b}$.

Solution

$$\vec{m_1} = \frac{\sqrt{2}}{2}\vec{a} + \frac{\sqrt{3}}{2}\vec{b}$$

$$\vec{m_2} = \frac{\sqrt{2}}{2}\vec{a} - \frac{1}{2}\vec{b}$$

(see scanned attachment for work)

b. Recover the important speech $\vec{a}$, as a weighted combination of $\vec{m_1}$ and $\vec{m_2}$. In other words, write $\vec{a} = u \cdot \vec{m_1} + v \cdot \vec{m_2}$ (where u and v are scalars). What are the values of u and v?

Solution

$$\vec{a} = \frac{2}{\sqrt{2}+\sqrt{6}}\vec{m_1} + \frac{2\sqrt{3}}{\sqrt{2}+\sqrt{6}}\vec{m_2}$$

$$u = \frac{2}{\sqrt{2}+\sqrt{6}}$$

$$v = \frac{2\sqrt{3}}{\sqrt{2}+\sqrt{6}}$$

(see scanned attachment for work)

c. Partial IPython code can be found in `prob1.ipynb`. Complete the code to get a clean signal of the important speech. What does the speaker say? (Optional: Where is the speech taken from?)

Solution "All human beings are born free and equal in dignity and rights"

6. Your Own Problem

Solution David, Damian, and Dirk went to the Asian Ghetto to buy some food. They were advised not to order anything but the Godfather from Gypsies, the Eggplant(Item #5 on the menu) from Thai Basil, or the Kimchi Pork Rice (Item #40 on the menu) from Bears Ramen. Hungry as they were, David ordered two Godfathers and one Rice, Damian ordered one of each (for a total of 3 dishes), and Dirk ordered three Eggplants and two Rice. David, Damian, and Dirk each paid \$35, \$32, and \$49 respectively. Unfortunately, they all forgot the price of each dish when they finished eating. Can you solve for the price of each dish with the given information?

Solve by setting up a system of equations with the given information.

Answer: Godfather = \$12, Eggplant = \$9, Rice = \$11

Reminder: *Make sure to attach a pdf version of your iPython code below!*

$\frac{1}{3}a + \frac{1}{3}b + 0c + \frac{1}{3}d = 6\frac{1}{3}$

$\frac{1}{3}a + \frac{1}{3}b + \frac{1}{3}c + 0d = 6\frac{1}{3}$

2)
$$\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{2}{5} & \frac{3}{5} & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 6\frac{1}{3} \\ 6\frac{1}{3} \\ 6\frac{4}{5} \\ 5\frac{2}{3} \end{bmatrix}$$

a = strawberries
b = bananas
c = mangos
d = blueberries

$$\cancel{\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 19 \\ 0 & 0 & \frac{1}{3} & -\frac{1}{3} & 0 \\ 0 & 2 & 3 & & \\ & & & & \end{array}\right]}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 19 \\ 1 & 1 & 1 & 0 & 19 \\ 0 & 2 & 3 & 0 & 34 \\ 2 & 1 & 0 & 0 & 17 \end{array}\right] \begin{array}{c} E_1 \\ E_2 \\ E_3 \\ E_4 \end{array}$$

$$\sim E_2 - \tfrac{1}{2}E_4 \quad \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 19 \\ 0 & \frac{1}{2} & 1 & 0 & \frac{21}{2} \\ 0 & 2 & 3 & 0 & 34 \\ 2 & 1 & 0 & 0 & 17 \end{array}\right]$$

$$(E_3 - 4E_2) \sim \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 19 \\ 0 & \frac{1}{2} & 1 & 0 & \frac{21}{2} \\ 0 & 0 & +1 & 0 & +8 \\ 2 & 1 & 0 & 0 & 17 \end{array}\right]$$

$$\sim (E_4 - 2E_1) \quad \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 19 \\ 0 & \frac{1}{2} & 1 & 0 & \frac{21}{2} \\ 0 & 0 & 1 & 0 & 8 \\ 0 & -1 & 0 & -2 & -21 \end{array}\right]$$

$$E_4 + 2E_2 \sim \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 19 \\ 0 & \frac{1}{2} & 1 & 0 & \frac{21}{2} \\ 0 & 0 & 1 & 0 & 8 \\ 0 & 0 & 2 & -2 & 0 \end{array}\right]$$

$$\sim 2(E_2), \ \tfrac{1}{2}(E_4) \quad \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 19 \\ 0 & 1 & 2 & 0 & 21 \\ 0 & 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

$$E_4 - E_3 \sim \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 19 \\ 0 & 1 & 2 & 0 & 21 \\ 0 & 0 & 1 & 0 & 8 \\ 0 & 0 & 0 & +1 & 8 \end{array}\right]$$

$c = d = 8$

$a + b + d = 19$, $\quad b + 2c = 21$

$a + 5 + 8 = 19$, $\quad b = 21 - 2(8)$

$a = 6$, $\quad b = 5$

a) $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 8 \\ 8 \end{bmatrix}$

b) $\frac{1}{2}$ mango and $\frac{1}{2}$ Blueberries, Score = 8

5) a) $\vec{m_1} = \cos(45°)\vec{a} + \cos(-30°)\vec{b}$

$\vec{m_2} = \sin(45°)\vec{a} + \sin(-30°)\vec{b}$

$\therefore \boxed{\begin{aligned} \vec{m_1} &= \frac{\sqrt{2}}{2}\vec{a} + \frac{\sqrt{3}}{2}\vec{b} \\ \vec{m_2} &= \frac{\sqrt{2}}{2}\vec{a} - \frac{1}{2}\vec{b} \end{aligned}}$ $\begin{matrix}(E_1) \\ (E_2)\end{matrix}$

b) $\frac{1}{2}\vec{b} = -\vec{m_2} + \frac{\sqrt{2}}{2}\vec{a}$

$\vec{b} = \sqrt{2}\vec{a} - 2\vec{m_2}$

plug into E_1

$\vec{m_1} = \frac{\sqrt{2}}{2}\vec{a} + \frac{\sqrt{6}}{2}\vec{a} - \sqrt{3}\vec{m_2}$

$\frac{\sqrt{2}+\sqrt{6}}{2}\vec{a} = \vec{m_1} + \sqrt{3}\vec{m_2}$

$\vec{a} = \frac{2}{\sqrt{2}+\sqrt{6}}\vec{m_1} + \frac{2\sqrt{3}}{\sqrt{2}+\sqrt{6}}\vec{m_2}$

$\boxed{u = \frac{2}{\sqrt{2}+\sqrt{6}}, \quad v = \frac{2\sqrt{3}}{\sqrt{2}+\sqrt{6}}}$

c) "All are created equal"

EE16A: Homework 1

Problem 3: Finding Charges from Voltage Measurements

In [1]:

```python
import numpy as np
a = np.array([
        [1/np.sqrt(2), 1/np.sqrt(5), 1/2],
        [1, 1/np.sqrt(2), 1],
        [1/2, 1/np.sqrt(5), 1/np.sqrt(2)]
    ])
b = np.array([(4+3*(np.sqrt(5))+np.sqrt(10))/(2*(np.sqrt(5))),(2+4*(np.sqrt(2)))/(np.sqrt(2)),(4+n
p.sqrt(5)+3*(np.sqrt(10)))/(2*(np.sqrt(5)))])
x = np.linalg.solve(a,b)
var = [u'Q\u2081', u'Q\u2082', u'Q\u2083']

for i in range(0,3):
    print('{:.4f}{} + {:.4f}{} + {:.4f}{} = {:.4f}'.format(a[i][0], var[0], a[i][1], var[1], a[i][
2], var[2], b[i]))
print('')
for i in range(0,3):
    print('{} = {}'.format(var[i], x[i]))
```

```
0.7071Q₁ + 0.4472Q₂ + 0.5000Q₃ = 3.1015
1.0000Q₁ + 0.7071Q₂ + 1.0000Q₃ = 5.4142
0.5000Q₁ + 0.4472Q₂ + 0.7071Q₃ = 3.5157

Q₁ = 1.0
Q₂ = 2.0000000000000013
Q₃ = 2.999999999999999
```

Problem 4: The Framingham Risk Score ¶

In [2]:
```
# Tip: np.log works element-wise on an np.array

import numpy as np
a = np.array([
        np.log([66,198,55,132]),
        np.log([61,180,47,124]),
        np.log([60,180,50,120]),
        np.log([23,132,45,132])
    ])
def R(p):
    return np.log(np.log(1-p)/np.log(0.95)) + 25.66
b = np.array([R(0.1550),R(0.1108),R(0.0940),R(0.0105)])
x = np.linalg.solve(a,b)

print('Part a)')
for i in range(0,4):
    print ('{:.4f}a + {:.4f}b + {:.4f}c + {:.4f}d = {:.4f}'.format(a[i][0], a[i][1], a[i][2], a[i]
[3], b[i]))
print('\nPart b)')
for i in range(0,4):
    print('{} = {}'.format('abcd'[i], x[i]))
```

```
Part a)
4.1897a + 5.2883b + 4.0073c + 4.8828d = 26.8489
4.1109a + 5.1930b + 3.8501c + 4.8203d = 26.4883
4.0943a + 5.1930b + 3.9120c + 4.7875d = 26.3147
3.1355a + 4.8828b + 3.8067c + 4.8828d = 24.0791

Part b)
a = 2.3098569091997123
b = 1.1695549079666905
c = -0.6945169529064997
d = 2.8200267513575112
```

Problem 5: Filtering out the troll

In [3]:
```
import numpy as np
import matplotlib.pyplot as plt
import wave as wv
import scipy
from scipy import io
import scipy.io.wavfile
from scipy.io.wavfile import read
from IPython.display import Audio
import warnings
warnings.filterwarnings('ignore')
sound_file_1 = 'm1.wav'
sound_file_2 = 'm2.wav'
```

Let's listen to the recording by the first microphone (it can take some time to load the sound file).

In [22]:
```
Audio(url='m1.wav', autoplay=False)
```

Out[22]: Loading

And this is the recording by the second microphone (it can take some time to load the sound file).

In [23]:
```
Audio(url='m2.wav', autoplay=False)
```

Out[23]: Loading

We read the first recording to `corrupt1` and second recording to `corrupt2` variables.

In [24]:
```
rate1,corrupt1 = scipy.io.wavfile.read('m1.wav')
rate2,corrupt2 = scipy.io.wavfile.read('m2.wav')
```

Enter the gains to combine the two recordings to get the clean speech.

In [28]:
```
# enter the gains u to weight recording 1 and v to weight recording 2
u = 2/(np.sqrt(2)+np.sqrt(6))
v =2*np.sqrt(3)/(np.sqrt(2)+np.sqrt(6))
```

Weighted combination of the two recordings

In [29]:
```
s1 = u*corrupt1 + v*corrupt2
```

Let's listen to the resulting sound file (make sure your speaker's volume is not very high, the sound may be loud if things go wrong).

In [30]:
```
Audio(data=s1, rate=rate1)
```

Out[30]:

Problem 6: My Own Problem

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Unfortunately, they all forgot the price of each dish when they finished eating. Can you solve for the price of each dish with the given information?

Solve by setting up a system of equations with the given information.

Answer: Godfather = $12, Eggplant = $9, Rice = $11