This homework is due Nov 8, 2016, at 1PM.

1. **Homework process and study group**

   Who else did you work with on this homework? List names and student ID's. (In case of hw party, you can also just describe the group.) How did you work on this homework?
   Working in groups of 3-5 will earn credit for your participation grade.

   Solution: I worked on this homework with...

   I first worked by myself for 2 hours, but got stuck on Problem 5 so I went to office hours on...

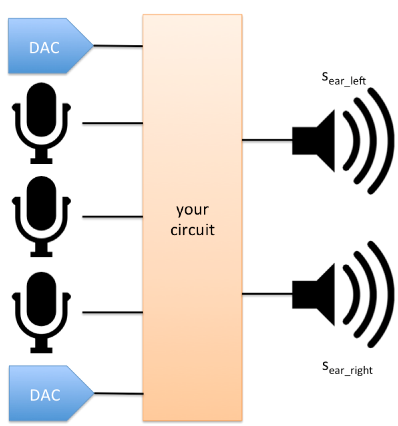
   Then I went to homework party for a few hours, where I finished the homework.

2. **Noice Cancelling Headphones**

   Implement the full noise-cancelling headphone amplifier seen in discussion. Recall that the stereo output is calculated using the following matrix equation:

$$\begin{bmatrix} s_{ear\_left} \\ s_{ear\_right} \end{bmatrix} = \begin{bmatrix} a_{1left} & a_{2left} & a_{3left} \\ a_{1right} & a_{2right} & a_{3right} \end{bmatrix} \cdot \begin{bmatrix} s_{mic1} \\ s_{mic2} \\ s_{mic3} \end{bmatrix} + \begin{bmatrix} s_{left} \\ s_{right} \end{bmatrix}$$

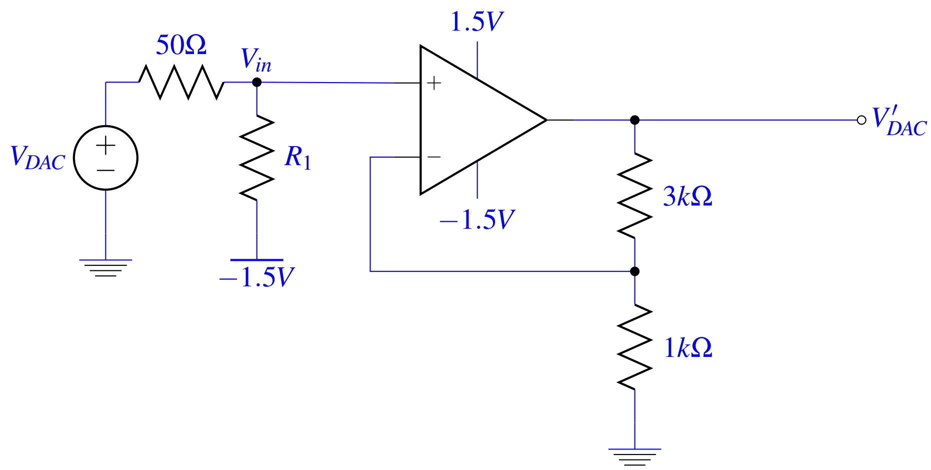
   A block diagram of the circuit is given below:



   Remember the outputs of the DAC range from $0-1V$ and outputs of the microphones range from $-1.5V$ to $1.5V$. The output should range from $-1.5V$ to $1.5V$. Refer back to discussion 9B and 10A for help.

   Solution: We already have a circuit that does subtraction from part (b) and a circuit that computes the noise cancelling signal in part (e). We just have to combine the two circuits such that it implements the matrix B in part (d).

   Recall our circuit from last discussion for voltage shifting and changing the range of a DAC:

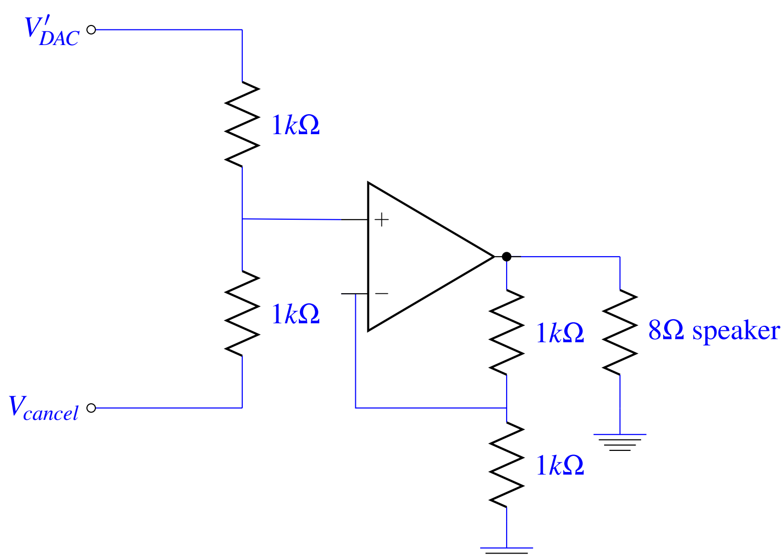


Now we want

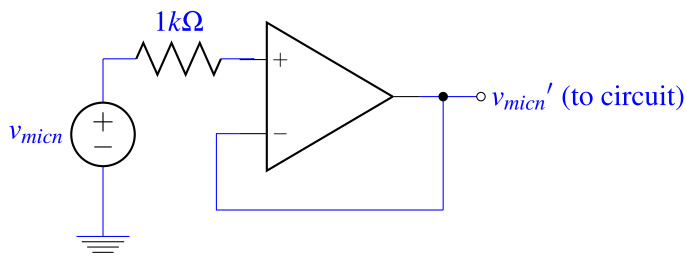
$$v_{left} = V'_{DAC,left} + v_{cancel\_left}$$

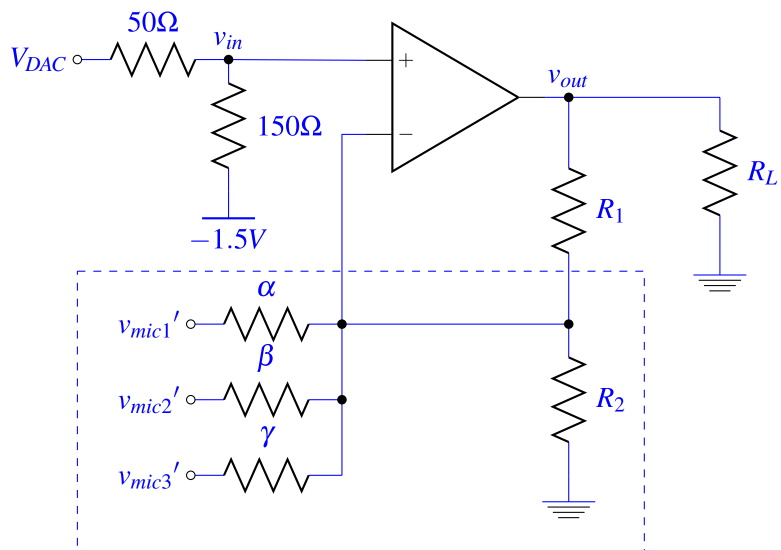
$$v_{right} = V'_{DAC,right} + v_{cancel\_right}$$

Note that v_{cancel} already inverted the microphone signal so we are adding it to V_{DAC}. To sum, we can use a noninverting summer. Again, we consider the channels independently.



Below is alternative circuit that does the same with one op-amp. The approach here is to start with by attenuating the mic voltages and summing them with a voltage divider and solving for the correct resistances. Then we feed the the microphone voltage into the negative terminal and the DAC signal into the positive terminal of an op amp to subtract the signals.





Recall that the v_{in} range is $-0.375V$ to $0.375V$, and it has to be amplified 4 times. We can write the KCL equation in the inverting input of the op-amp.

$$\frac{v_{mic1}-v_{in}}{\alpha}+\frac{v_{mic2}-v_{in}}{\beta}+\frac{v_{mic3}-v_{in}}{\gamma}+\frac{v_{out}-v_{in}}{R_1}+\frac{0-v_{in}}{R_2}=0$$

$$\frac{v_{out}}{R_1}=v_{in}\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{R_1}+\frac{1}{R_2}\right)-\frac{v_{mic1}}{\alpha}-\frac{v_{mic2}}{\beta}-\frac{v_{mic3}}{\gamma}$$

$$v_{out}=v_{in}\left(\frac{R_1}{\alpha}+\frac{R_1}{\beta}+\frac{R_1}{\gamma}+1+\frac{R_1}{R_2}\right)-\frac{R_1}{\alpha}v_{mic1}-\frac{R_1}{\beta}v_{mic2}-\frac{R_1}{\gamma}v_{mic3}$$

Just as before, we can compare this formula to the output we want. In this case, we want $v_{out}=4v_{in}-a_1\,v_{mic1}-a_2\,v_{mic2}-a_3\,v_{mic3}$. Thus,

$$\frac{R_1}{\alpha}+\frac{R_1}{\beta}+\frac{R_1}{\gamma}+1+\frac{R_1}{R_2}=4 \qquad \frac{R_1}{\alpha}=a_1 \qquad \frac{R_1}{\beta}=a_2 \qquad \frac{R_1}{\gamma}=a_3$$

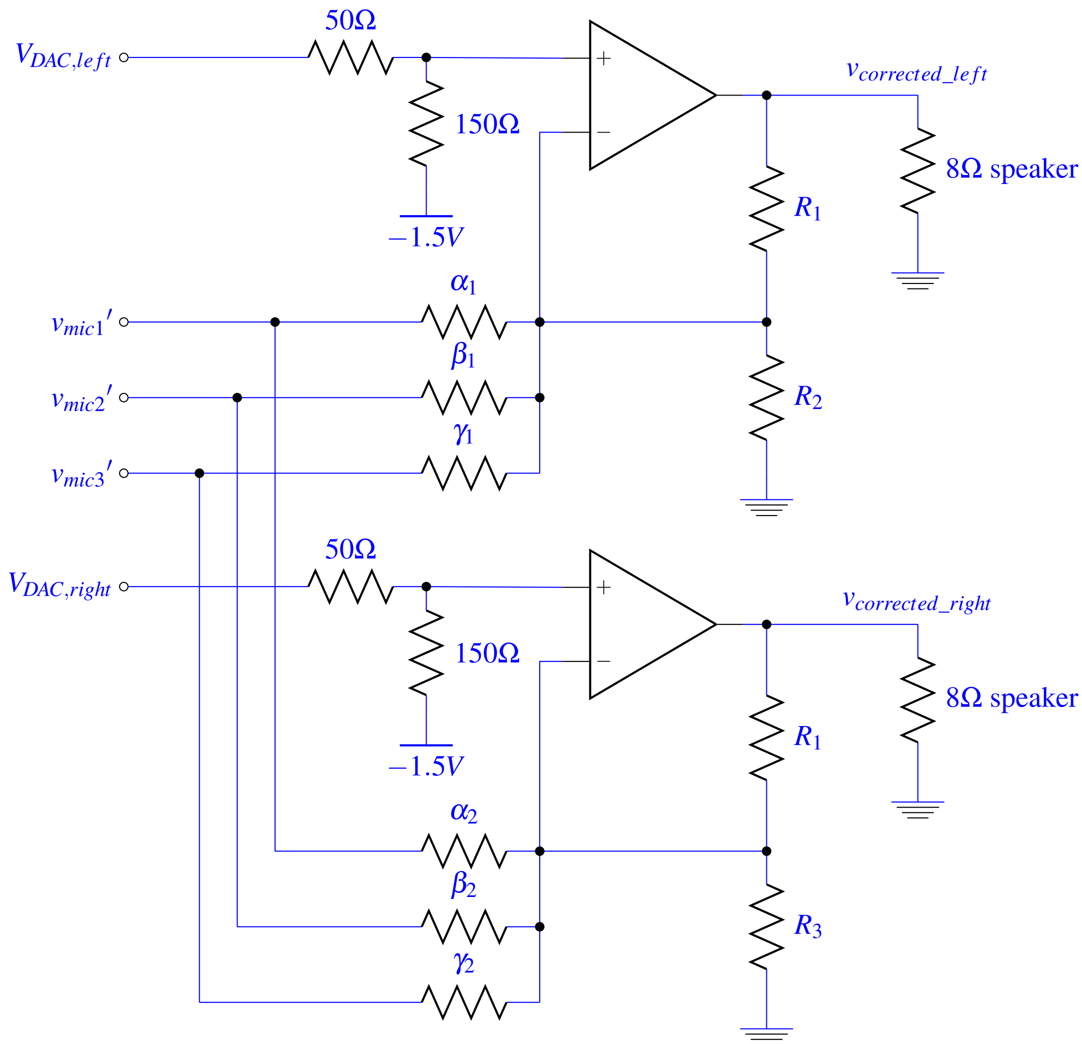
$$\alpha=\frac{R_1}{a_1} \qquad \beta=\frac{R_1}{a_2} \qquad \gamma=\frac{R_1}{a_3}$$

From the first equation,

$$a_1+a_2+a_3+1+\frac{R_1}{R_2}=4$$

$$R_2=\frac{R_1}{3-a_1-a_2-a_3}$$

Thus, if we pick a value for R_1, we can use the formulas above to calculate α, β, γ and R_2.
Now that we have a working circuit for one speaker, we can duplicate this circuit to have two speakers. Notice that in the circuit below we can use the same value for R_1 in the two channels, but we have to keep R_2 as a variable (hence it is replaced with R_3 in the right channel). This is because R_1 is a free variable. If we choose a value for R_1 arbitrarity, we can calculate what the other resistor values have to be with the equations we have derived.

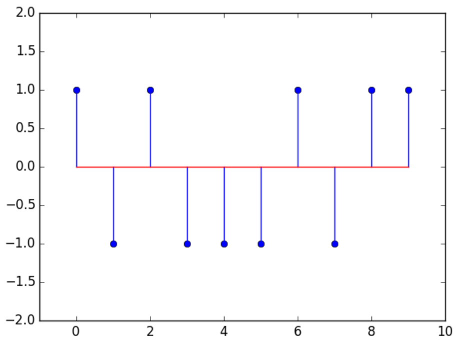


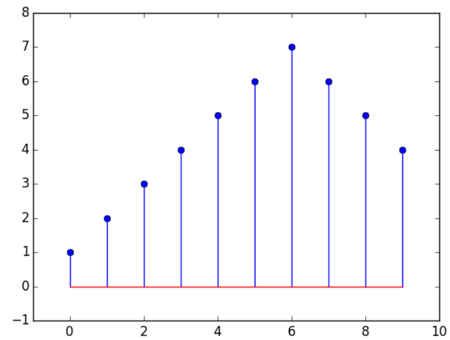
We have seen that if we choose values for R_1 and R_3 arbitrarily, we can find the other resistor values.

$$\alpha_1 = \frac{R_1}{a_{1left}} \qquad \beta_1 = \frac{R_1}{a_{2left}} \qquad \gamma_1 = \frac{R_1}{a_{3left}} \qquad R_2 = \frac{R_1}{3 - a_{1left} - a_{2left} - a_{3left}}$$

$$\alpha_2 = \frac{R_1}{a_{1right}} \qquad \beta_2 = \frac{R_1}{a_{2right}} \qquad \gamma_2 = \frac{R_1}{a_{3right}} \qquad R_3 = \frac{R_1}{3 - a_{1right} - a_{2right} - a_{3right}}$$

3. Mechanical: Correlation





(a) Calculate and plot the **autocorrelation** (the inner products of one period of the signal with all the possible shifts of one period of the same signal) of each of the above signals. Each signal is periodic with a period of 10 (one period is shown).

Solution: The solution is given in `sol9.ipynb`. (It is fine if you did the calculations and plotting by hand.)

(b) Calculate and plot the **cross-correlation** (the inner products of one period of the first signal with all possible shifts of one period of the second signal) of the two signals. Each signal is periodic with a period of 10 (one period is shown).

Solution: The solution is given in `sol9.ipynb`. (It is fine if you did the calculations and plotting by hand.)

4. Inner products

The Cauchy-Schwarz inequality states that for two vectors $\vec{x}, \vec{y} \in \mathbb{R}^n$:

$$|\langle \vec{x}, \vec{y} \rangle| = |\vec{x}^T \vec{y}^*| \leq ||\vec{x}|| \cdot ||\vec{y}||$$

Use the Cauchy-Schwarz inequality to verify (i.e. prove or derive) the triangle inequality:

$$||\vec{x} + \vec{y}|| \leq ||\vec{x}|| + ||\vec{y}||$$

(Hint: Start with $||\vec{x} + \vec{y}||^2$)

Solution: We consider the Euclidean 2-norm here. By Cauchy-Schwarz inequality we have

$$|\langle \vec{x}, \vec{y} \rangle| \leq \|\vec{x}\|_2 \cdot \|\vec{y}\|_2$$

$$|\langle \vec{y}, \vec{x} \rangle| \leq \|\vec{y}\|_2 \cdot \|\vec{x}\|_2$$

Therefore,

$$\begin{aligned}
\|\vec{x} + \vec{y}\|_2^2 &= (\vec{x} + \vec{y})^T (\vec{x} + \vec{y}) \\
&= \|\vec{x}\|_2^2 + \|\vec{y}\|_2^2 + \langle \vec{x}, \vec{y} \rangle + \langle \vec{y}, \vec{x} \rangle \\
&\leq \|\vec{x}\|_2^2 + \|\vec{y}\|_2^2 + 2\|\vec{y}\|_2 \cdot \|\vec{x}\|_2 \\
&= (\|\vec{x}\|_2 + \|\vec{y}\|_2)^2
\end{aligned}$$

Taking square root on both sides, we get

$$\|\vec{x}+\vec{y}\|_2 \leq \|\vec{x}\|_2 + \|\vec{y}\|_2$$

5. **Redo the midterm.**
Redo the midterm problems (released Thursday after the midterm). **Solution:** See midterm solutions.

6. **Your Own Problem** Write your own problem related to this week's material and solve it. You may still work in groups to brainstorm problems, but each student should submit a unique problem. What is the problem? How to formulate it? How to solve it? What is the solution?