EE 16A: Homework 8

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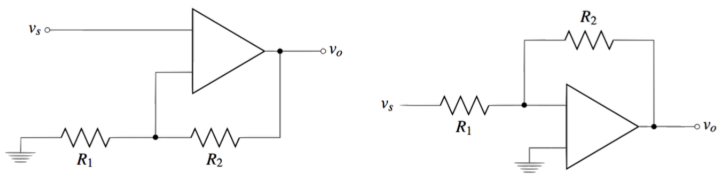
October 25, 2016

1. **Worked With...**

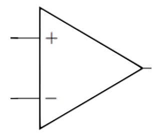
   Ilya (3031806896), James Zhu (3031793129)
   I worked alone on Friday morning, then met up with Ilya and James to discuss on Saturday afternoon.

2. **Basic Amplifier Building Blocks**



(a) (b)

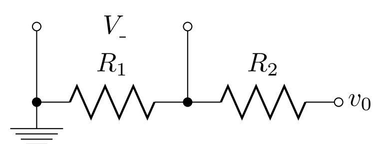
(a) *Label the input terminals of the Op-amp so it is in negative feedback. Then, derive the voltage gain of the non-inverting amplifier using the Golden Rules. Explain the origin of the name of the amplifier.*



From the Golden Rules, we know that

$$I_+ = I_- = 0$$
$$V_+ = V_-$$
$$V_+ = v_s$$

Thus we can reduce the bottom half circuit to a simple voltage divider to get



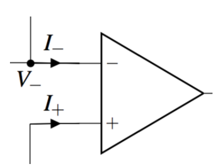
$$V_- = v_s = \frac{R_1}{R_1 + R_2} v_o$$

which gives the voltage gain of

$$\boxed{\frac{v_o}{v_s} = 1 + \frac{R_2}{R_1}}$$

Name makes sense from the fact that the gain is non-negative and >1.

(b) *Label the input terminals of the Op-amp so it is negative feedback. Then, derive the voltage gain of the inverting amplifier using the Golden Rules. Explain the origin of the name of the amplifier.*



First, we use the Golden Rules, and the fact that the opamp is connected in negative feedback, to get

$$V_- = V_+ = 0$$

Also, since we know that the current through each terminal of the opamp will be 0 (fromt the Golden Rules), we can write the following nodal analysis equation at the - terminal of the opamp:

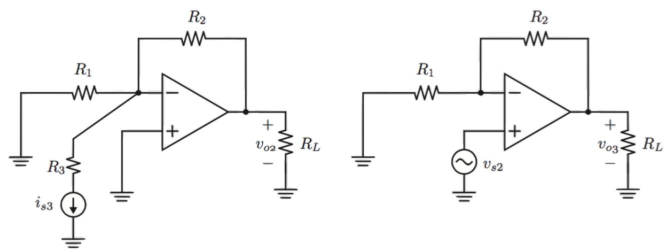
$$\frac{v_s - V_{-}}{R_1} = \frac{V_{-} - v_o}{R_2}$$

which gives the voltage gain of

$$\boxed{\frac{v_o}{v_s} = -\frac{R_2}{R_1}}$$

Name makes sense from the fact that the gain is negative.

3. **Amplifier with Multiple Inputs**



(a) *Use the Golden Rules to find v_{o2} for the first circuit.*

$$V_- = V_+ = 0$$

Conduct nodal analysis at the - terminal of the opamp:

$$\frac{0-0}{R_1} + \frac{0-v_{o2}}{R_2} + i_{s3} = 0$$
$$-\frac{v_{o2}}{R_2} + is3 = 0$$
$$\boxed{v_{o2} = R_2 i_{s3}}$$

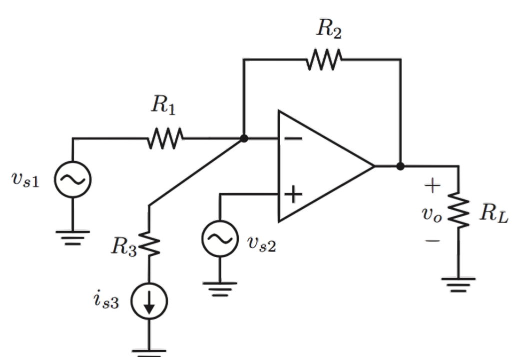
(b) *Use the Golden Rules to find v_{o3} for the second circuit.*

$$V_- = V_+ = v_{s2}$$

Nodal analysis at the - terminal of the opamp:

$$\frac{v_{s2}-0}{R_1} + \frac{v_{s2}-v_{o3}}{R_2} = 0$$
$$-\frac{R_2}{R_1} = \frac{-v_{s2}+v_{o3}}{v_{s2}}$$
$$\boxed{v_{o3} = v_{s2}(1+\frac{R_2}{R_1})}$$

(c) *Use the Golden Rules to find the output voltage v_o for the circuit.*



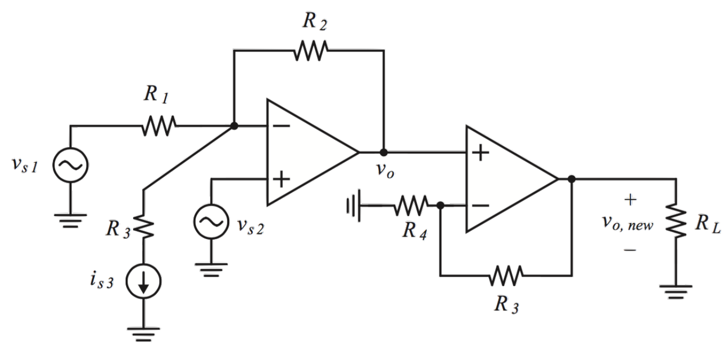
$$V_- = V_+ = v_{s2}$$

Nodal analysis at the - terminal of the opamp:

$$\frac{v_{s2}-v_{s1}}{R_1}+\frac{v_{s2}-v_o}{R_2}+i_{s3}=0$$
$$i_{s3}+\frac{v_{s2}-v_{s1}}{R_1}=\frac{v_o-v_{s2}}{R_2}$$

$$\boxed{v_o=R_2 i_{s3}+\frac{R_2}{R_1}(v_{s2}-v_{s1})+v_{s2}} \tag{1}$$

(d) *Now add a second stage. What is $v_{o,new}$? Does v_o change between the last part and this part? Does the voltage $v_{o,new}$ depend on R_L?*



$$V_- = V_+ = v_o$$

Nodal analysis at the - terminal of the opamp:

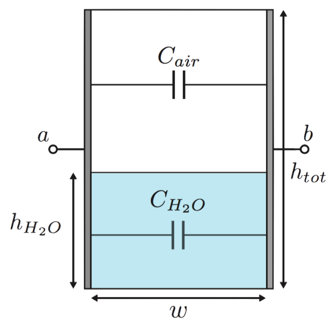
$$\frac{v_o-0}{R_4}+\frac{v_o-v_{o,new}}{R_3}=0$$
$$v_{o,new}=v_o\left(1+\frac{R_3}{R_4}\right)$$

where v_o stays the same $(R_2 i_{s3}+\frac{R_2}{R_1}(v_{s2}-v_{s1})+v_{s2})$ from equation (1), and thus

$$\boxed{v_{o,new}=\left(1+\frac{R_3}{R_4}\right)\left(R_2 i_{s3}+\frac{R_2}{R_1}(v_{s2}-v_{s1})+v_{s2}\right)}$$

We can also see that $v_{o,new}$ does not depend on R_L.

4. **It's finally raining!** *A lettuce farmer in the Salinas valley has grown tired of weather.coms imprecise rain measurements. So, she decided to take matters into her own hands by building a rain sensor. She placed a rectangular tank outside and attached two metal plates to two opposite sides in an effort to make a capacitor whose capacitance varies with the amount of water inside.*
The width and length of the tank are both w (i.e. the base is square) and the height of the tank is h_{tot}.



(a) *What is the capacitance between terminals a and b when the tank is full? What about when it is empty?*

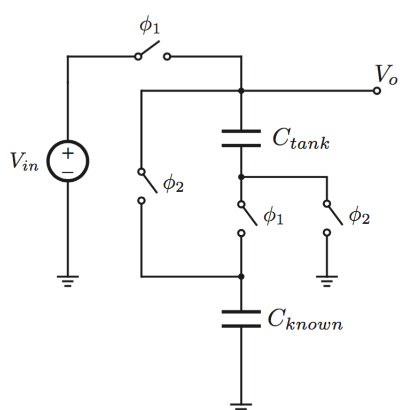
Full: $81\epsilon \cdot \frac{w \cdot h_{tot}}{w} = \boxed{81\epsilon h_{tot}}$

Empty: $\boxed{\epsilon h_{tot}}$

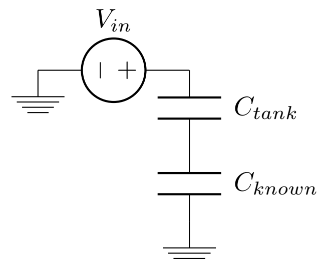
(b) *Suppose the heightofthewaterinthetankis h_{H_2O}. Modeling the tankasapairofcapacitors in-parallel, find the total capacitance between the two plates, C_{tank}.*

$$\begin{aligned} C_{air} &= \epsilon \frac{w \cdot (h_{tot} - h_{H_2O})}{w} \\ &= \epsilon(h_{tot} - h_{H_2O}) \\ C_{H_2O} &= 81\epsilon \frac{w \cdot h_{H_2O}}{w} \\ &= 81\epsilon h_{H_2O} \\ C_{tank} = C_{air} + C_{H_2O} &= \boxed{\epsilon(80h_{H_2O} + h_{tot})} \end{aligned}$$

(c) *Find the voltage V_o in phase Φ_2 as a function of the height of the water.*



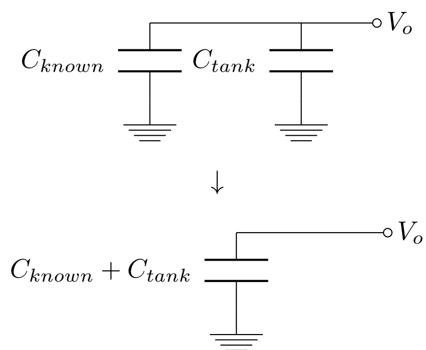
Phase 1:



where the charge on each place is

$$Q = (C_{tank} || C_{known}) \cdot V_{in} \tag{2}$$

Phase 2:



where the charge on the effective plate $C_{known} + C_{tank}$ is $2Q$, thus

$$V_o = \frac{2Q}{C_{known} + C_{tank}}$$

and plugging in equation (2) from above, we get

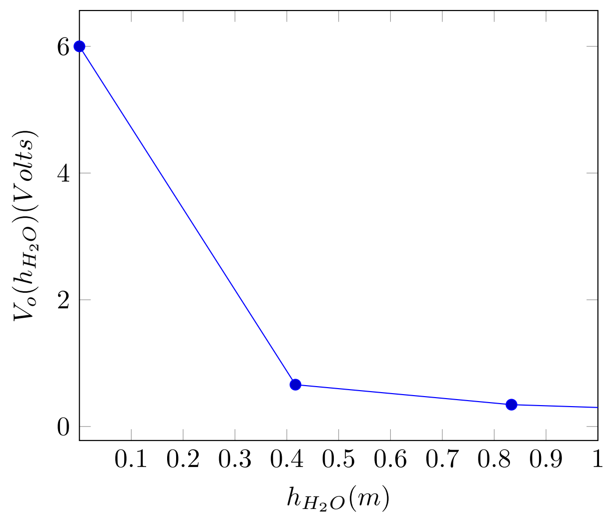
$$= \frac{2V_{in}(C_{known} || C_{tank})}{C_{known} + C_{tank}}$$

$$\boxed{V_o(h_{H_2O}) = 2V_{in}\frac{C_{known}C_{tank}}{(C_{known} + C_{tank})^2}}$$

where $C_{tank} = \epsilon(80h_{H_2O} + h_{tot})$

(d) *Plot this voltage V_o as a function of the height of the water. Vary the tank from empty to full. Use values of $V_{in} = 12V, w = 0.5m, h_{tot} = 1m$, and $\epsilon = 8.8541012F/m$. For C_{known} use a similar tank that is known to always be empty.*

$V_{in} = 12V$
$w = 0.5m$
$h_{tot} = 1m$
$\epsilon = 8.8541012F/m$
$C_{known} = \epsilon h_{tot} = \epsilon$



(e) *What does V_o represent? Its something we can measure! Our original goal was to determine what the height of the water in the tank without having to look inside it. Rewrite the last part to solve for h_{water}.*
V_o represents the output voltage that is measured in a voltmeter or some other device of the lettuce farmer's liking. With that measurement and with some simple rearrangement, the farmer can know what the precise rain measurements are.

$$\begin{aligned} V_o &= 2V_{in}\frac{C_{known}C_{tank}}{(C_{known}+C_{tank})^2} \\ \frac{V_o}{2V_{in}} &= \frac{C_{known}C_{tank}}{(C_{known}+C_{tank})^2} \\ \frac{V_o}{2V_{in}}\cdot(C_{known}+C_{tank})^2 &= C_{known}C_{tank} \end{aligned}$$

which becomes a messy quadratic in the form ax^2+bx+c

$$\frac{V_o}{2V_{in}}C_{tank}^2+(\frac{V_o}{V_{in}}-1)C_{known}C_{tank}+\frac{V_o}{2V_{in}}C_{known}^2=0$$

Solving for C_{tank}, we get

$$\begin{aligned} C_{tank} &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} \\ &= \frac{-(\frac{V_o}{V_{in}}-1)C_{known}+\sqrt{((\frac{V_o}{V_{in}}-1)C_{known})^2-(\frac{V_o}{V_{in}}C_{known})^2}}{\frac{V_o}{V_{in}}} \\ &= \frac{V_{in}}{V_o}C_{known}-C_{known}+C_{known}\sqrt{(\frac{V_{in}}{V_o})^2-2\frac{V_{in}}{V_o}} \\ &= C_{known}(\frac{V_{in}}{V_o}-1+\sqrt{(\frac{V_{in}}{V_o})^2-2\frac{V_{in}}{V_o}}) \end{aligned}$$

And knowing that $C_{tank}=\epsilon(80h_{H_2O}+h_{tot})$, we solve for h_{H_2O} to get

$$\boxed{h_{H_2O}=\frac{C_{known}(\frac{V_{in}}{V_o}-1+\sqrt{(\frac{V_{in}}{V_o})^2-2\frac{V_{in}}{V_o}})-\epsilon h_{tot}}{80\epsilon}}$$

(f) *What are the units of your result for V_o and for h_{water}?*

For V_o:

$$V_o(h_{H_2O}) = 2V_{in}\frac{C_{known}C_{tank}}{(C_{known}+C_{tank})^2}$$

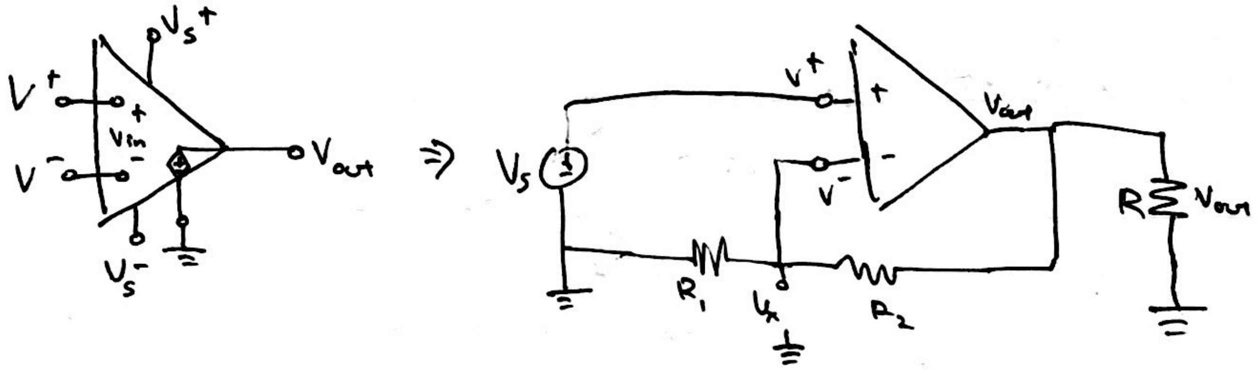
Capacitances cancel out to give us a unit of $\boxed{Volts}$.

For h_{water}:

$$h_{H_2O} = \frac{C_{known}(\frac{V_{in}}{V_o} - 1 + \sqrt{1 - 2\frac{V_o}{V_{in}}}) - \epsilon h_{tot}}{80\epsilon}$$

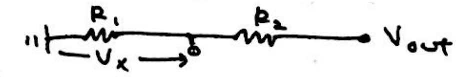
The numerator is in Farads, the denominator ϵ is in $\frac{F}{m}$, giving us a unit of $\boxed{meters}$.

I'M DONE W LATEX

5) 

a) $V_{out} = A(\underbrace{V^+}_{V^+ = V_s} - \underbrace{V^-}_{V^- = V_x})$

$I^+ = I^- = 0$

∴ 

$= A(V_s - V_x)$ $\qquad V_x = \frac{R_1}{R_1+R_2} V_{out}$

$= A\left(V_s - \frac{R_1}{R_1+R_2} V_{out}\right)$

$= A V_s - A\left(\frac{R_1}{R_1+R_2}\right) V_{out}$

Rearranging,

$V_{out}\left(1 + A\left(\frac{R_1}{R_1+R_2}\right)\right) = A V_s$

$$\therefore \boxed{V_{out} = \frac{A}{1 + A\left(\frac{R_1}{R_1+R_2}\right)} V_s} \rightarrow \boxed{V_x = \underbrace{\frac{A\left(\frac{R_1}{R_1+R_2}\right)}{1 + A\left(\frac{R_1}{R_1+R_2}\right)}} V_s}$$

★ None of these values depend on R.

★ This implies that the magnitude of V_x is smaller than that of V_s.

b) As $A \to \infty$, the ratios $\to 1$, so

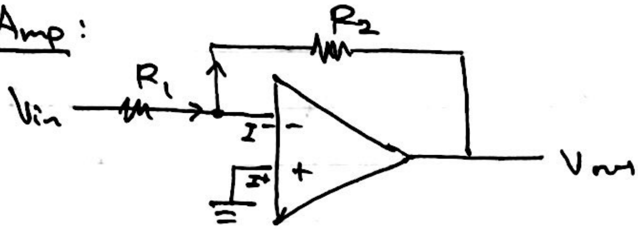
~~both V_{out} and~~ $\boxed{V_x = V_s}$

Yes, we get the same answers $V_+ = V_- = V_s$ when NFB.

$V_{out} = \left(\frac{R_1+R_2}{R_1}\right) V_x$

$$\boxed{V_{out} = \left(\frac{R_1+R_2}{R_1}\right) V_s}$$

6) a) Inverting Amp:

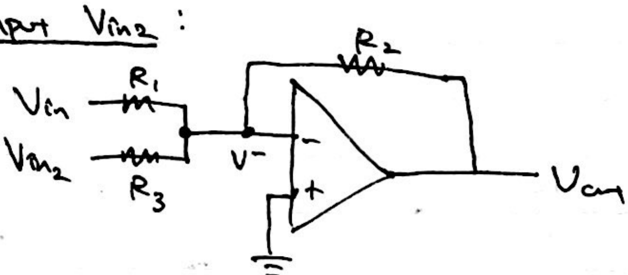


$I^- = I^+ = 0$, $V^+ = V^- = 0$ }, so nodal analysis @ V^- node gives

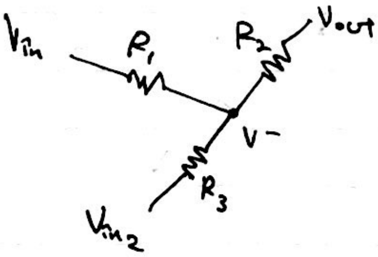
$$\frac{V_{in} - 0}{R_1} = \frac{0 - V_{out}}{R_2}$$

$\therefore$ volt. gain $\boxed{\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}}$

b) Second Input V_{in2}:



Nodal analysis @ V^- node:



$$\frac{V^- - V_{in}}{R_1} + \frac{V^- - V_{in2}}{R_3} + \frac{V^- - V_{out}}{R_2} = 0$$

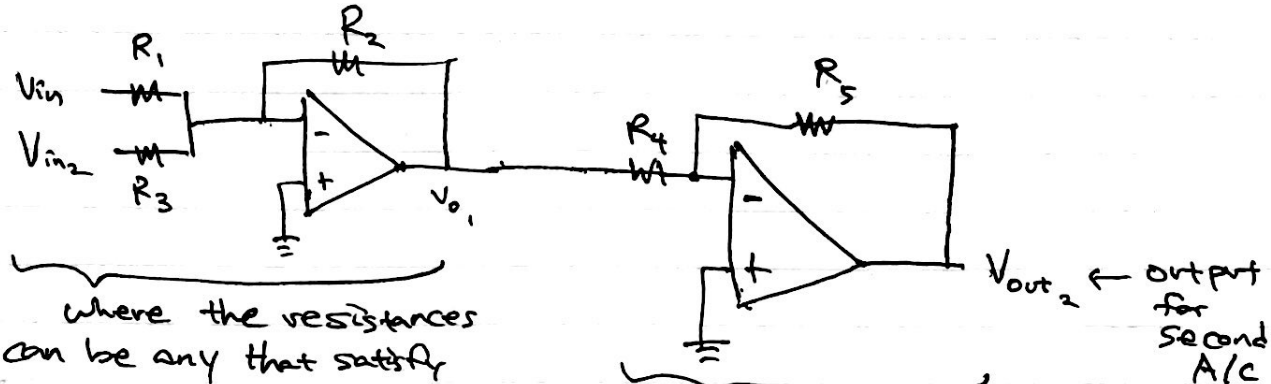
Since $V^- = V^+ = 0$,

$$\frac{V_{out}}{R_2} = -\frac{V_{in}}{R_1} - \frac{V_{in2}}{R_3}$$

$$\boxed{V_{out} = -\left(\frac{R_2}{R_1} V_{in} + \frac{R_2}{R_3} V_{in2}\right)}$$

c c) To get $V_{out} = -(\frac{1}{4} V_{s1} + 2 V_{s2})$, we set

$\boxed{R_1 = 8\,k\Omega, \quad R_2 = 2\,k\Omega, \quad R_3 = 1\,k\Omega}$ so that $\frac{R_2}{R_1} = \frac{8}{2} = \frac{1}{4}$, $\frac{R_2}{R_3} = \frac{2}{1} = 2$

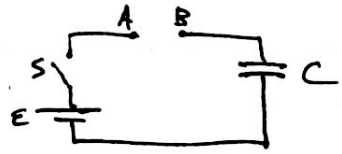
d) 

where the resistances can be any that satisfy the ratio $R_1 : R_2 : R_3$
$8 : 2 : 1$

set $R_4 = R_5$ to invert V_{o_1} so that $V_{out_2} = -V_{o_1}$

7) THINK ABOUT IT!

Question: Consider the following circuit qualitatively:



How will you charge the capacitor?

Answer: Case 1

Attach a wire btwn A & B, then close S. If we assume resistancelessness of the wires, charge flows into the plates super quick that the ~~diff~~ ΔV_c across the capacitor "instantly" becomes equal to ε across the battery.

Thus $Q = C\Delta V_c = \boxed{C\varepsilon}$

Case 2

Insert a resistor btwn A & B, then close S.
We know that at any time, $\varepsilon = \Delta V_R + \Delta V_c$.
@ $t = 0$, $\Delta V_c = 0$, so the current, initially, can just be described as $I_i = \Delta V_R / R = \varepsilon / R$.
As time goes on, ΔV_c increases while ΔV_R decreases, meaning I decreases, and as $I \to 0$, $\Delta V_R \to 0$, $\Delta V_c \to \varepsilon$.
Circuit is now static.