EE16A MIDTERM 2 STUDY GUIDE

WK4 - GRAPHS, CIRCUITS, AND KIRCHHOFF'S LAW

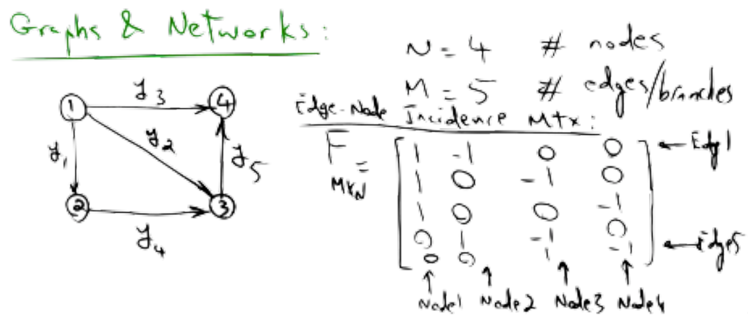
Fund Thm of Linear Algebra (Important Review)

I. Deals with the dimensions of the four fundamental subspaces
- *dim C*(A) + *dim N*(A^T) = M
- *dim C*(A^T) + *dim N*(A) = N
- Basically, rank = dim row(# of lin indp rows) = dim col(# of lin indp cols)

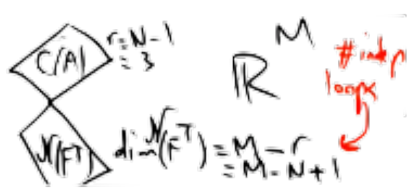
II. Deals with orthogonalities of subspaces
- $C(A)$ orthogonal to $N(A^T)$ (stronger statement: $C(A)$ is the orthogonal complement of the Left Null Space, meaning, if you put these two subspaces together, they fill up M)
- $C(A^T)$ orthogonal to $N(A)$

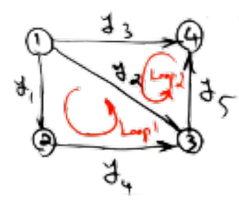
EE Conventions on IncidenceMtx and Connecting to 4 Subspaces

Incidence Matrix (*B*) will be in the form of ***Edge-Node Incidence Matrix***, where nodes are represented in the columns and edges as rows



Left Nullspace tells us the # of independent loops in a circuit/network/etc:





Basic Circuits

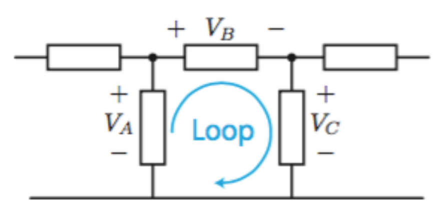
Charge (+/-): measure with the unit **coulomb (C)**

Current: net amount of charge crossing a surface in a unit time; unit = **ampere (A)** = 1C/s

Voltage: amt of energy needed to move a unit charge btwn two points; **volt (V)** = 1J/C
Voltage is a relative quantity defined between two points. An absolute voltage is meaningless and usually is implicitly referenced to a known point in the circuit (ground) or in some cases a point at infinity.

Kirchhoff's CL and VL

KCL states the net charge flowing into any node of a circuit is 0.
In Lin Alg terms: $y^TF = 0^T \longrightarrow F^Ty = 0$ (Left nullspace)
KVL states the net potential around any loop in a circuit is 0.



$-V_A + V_B + V_C = 0.$

Resistors

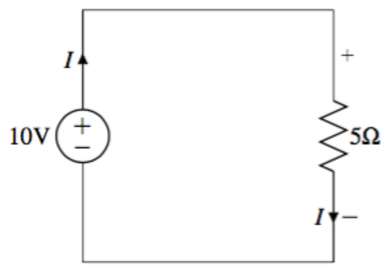
Ohm's Law: $V = IR$

Resistance: unit = V/A = Ω "**Ohms**"

For a wire of uniform cross area, the resistance is calculated as follows

$$R = \frac{\rho l}{A}.$$

It's proportional to the length *l* of the wire, inversely proportional to the cross sectional area *A*, and pro- portional to the material resistivity *ρ*.



Ex) Using Olm's law, we know that the voltage drop across the resistor is ΔV = 5I and the voltage gain when going through the voltage source on the left is 10V.

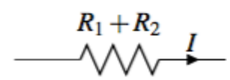
Using KVL, we have 10 – 5I = 0.

Hence, the current going through the loop is I = 2A and the voltage drop across the resistor is 5Ω × 2A = 10V.

Resistors in Series

The **equivalent resistance(R_{eq})** for resistors in series is the SUM of their resistance.

$$R_{eq} = R_1 + R_2$$



Resistors in Parallel

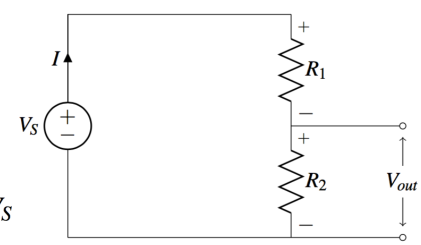
$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

Voltage Divider

Use if we want a different potential difference across a device (not the voltage source).

Notice that we can create any output voltage of $V_{out} = \alpha V_S$ for any $\alpha \in [0,1]$ by varying the resistance R_1 and R_2.

$$V_{out} = IR_2 = \frac{R_2}{R_1 + R_2} V_S$$



Nodal Analysis

Use KCL/KVL to perform nodal analysis on each node to solve for node voltages.

Should get n equations for n nodes to be able to solve for each node.

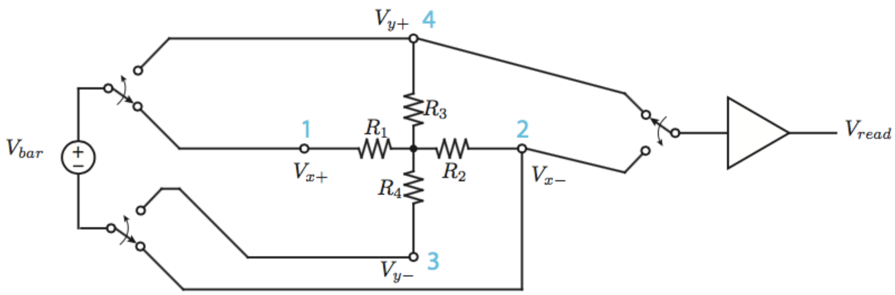
If not enough equations, ground a node.

Circuit Analysis

In order to analyze circuits with resistors and capacitors after a very long time, just treat all capacitors as open circuits, bc after a very long time, a capacitor charges up, and acts like an open circuit.

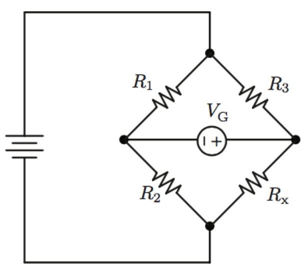
WK5 - DESIGN AND TOUCHSCREEN

Resistive Touchscreen



Schematic of a 4-wire resistive touchscreen. Two sets of switches are used to alternatively connect a voltage source across one set of Rs while the other set is used to read voltage.

Wheatstone Bridge - to measure unknown R



R1, R3 = known.

R2 = Adjustable

Rx =   ?

The value of R2 is varied until the circuit is balanced, when *I* through galvanometer V_G is 0.

R_x = (R3/R1) * R2

WK6 - EQUIVALENCE, SUPERPOSITION, POWER

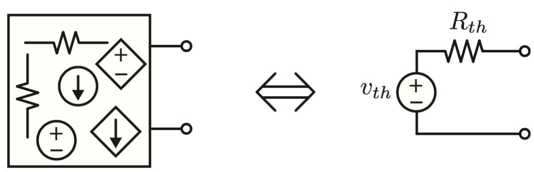
Power

P is the time rate of change of energy.

$$P = \frac{\Delta E}{\Delta t} = \frac{V\Delta q}{\Delta t} = V\frac{\Delta q}{\Delta t} = V \times I.$$

When *P* = *IV* is (+), power's being dissipated; when (-), power is being generated.

Thevenin Equivalent Circuit



Step 1 - Finding *V_th*: Open circuit where load is, then find *V_oc* across it

Finding *R_th* (method 1 - finding current first)

Step 2: Short where load is, then find *I_sc* across it

Step 3: Ohm's law to solve = *V_th / I_sc*

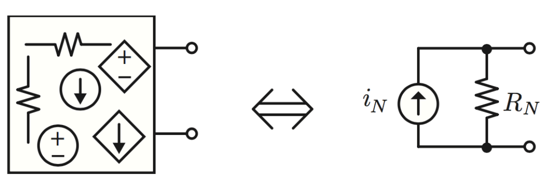
Finding *R_th* (direct method - If no dependent sources in the circuit under consideration)

Step 2: Zero out power sources (like Superposition; *Vs -> sc*, *Is -> oc*)

Step 3: Find *R_th* by observing the equivalent resistance

(TIP: do loop-tracing starting from smallest loop to analyze circuit)

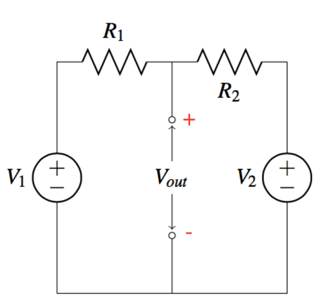
Norton Equivalent Circuit



Step 1 - Finding *i_N*: Short where load is, then find *I_sc* across it

Step 2 - Finding *R_N*: *R_n = R_thc*

Superposition



Finding Vout for circuits with multiple voltage or current:

The procedure to do this (which is known as superposition) is as follows:

For each source k (either voltage source or current source)

Set all other sources to 0

- **Voltage source: replace with a short circuit**
- **Current source: replace with an open circuit**

Compute $V_{out,k}$ due to this source k

Compute V_{out} by summing the $V_{out,k}s$ for all k.

WK7 - CAPACITORS

Q = CV

*C = ε A/d (unit = **Farad**)*

Permitivity of air: ε = 8.85×10−12F/m

E(of fully charged C) = 1/2 CV^2

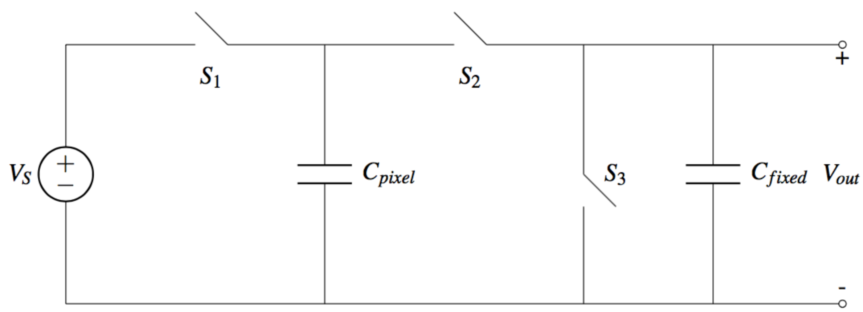
Capacitors in Parallel

Ceq = C1 +C2

Capacitors in Series

Ceq = C1C2 / C1 + C2

Charge Sharing



Phase 1:

We close switches S1 and S3, and leave switch S2 open, which charges Cpixel, discharges Cfixed

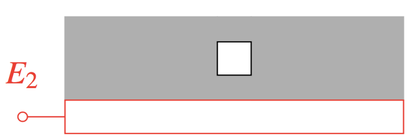
Phase 2 - Where Charge Sharing occurs:

We open switches S1 and S3 and close switch S2, charges will flow from Cpixel to Cfixed until the voltage across both capacitors is the same

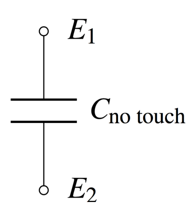
We can solve for Cpixel by conservation of charge:

$Q_{before} = Q_{pixel} + Q_{fixed} = Q_{pixel}' + Q_{fixed}' = Q_{after}$

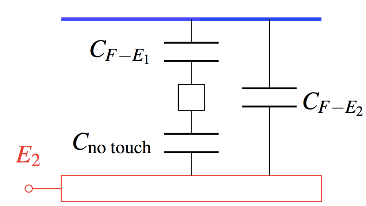
2D Capative Touchscreen



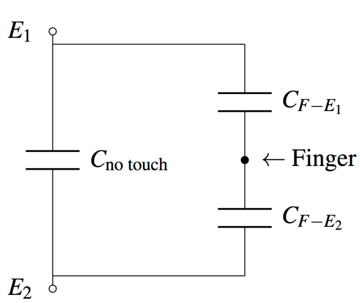
No Touch:



Touch:



With Finger Touch in circuit form:



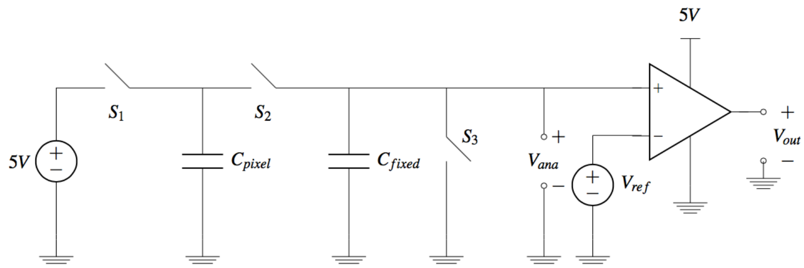
So when there is a touch, the capacitance is equal to:

$$C_{E_1-E_2} = C_{\text{no touch}} + C_{F-E_1} || C_{F-E_2}$$
$$= C_{\text{no touch}} + \frac{C_{F-E_1} C_{F-E_2}}{C_{F-E_1} + C_{F-E_2}}$$

Now, use capacitor measurement circuit with Op-Amp (in next section) to determine the present of touch.

Wk8/9 - Op-Amps

Capacitor Measurement Circuit - w Op-Amp as Comparator



Choose *C_fixed* and *V_ref* s.t. *V_ana* > *V_ref* when touch, and *V_ana* < *V_ref* when no touch.

Negative Feedback

When we want to get a system to have a desired output, negative feedback loops can help re-adjust to the value of the desired output when the output is too high or too low relative to the target value.

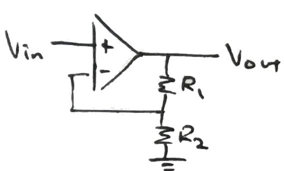
Golden Rules

1) $I_+ = I_- = 0$, holds regardless of NFB
2) $V_+ = V_-$, when NFB

Use these rules and KCL to solve many Op-Amp circuits

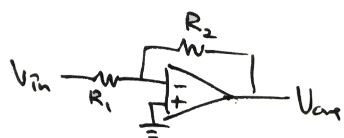
Types of Op-Amps

None-Inverting



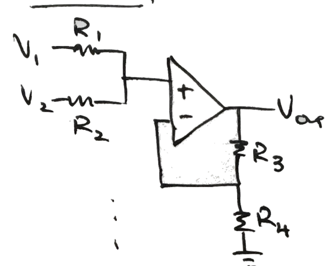
$$V_{out} = \left(1 + \frac{R_1}{R_2}\right) V_{in}$$

Inverting



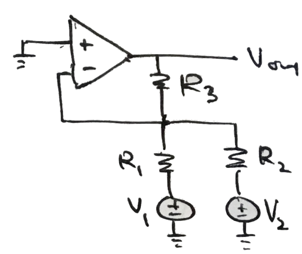
$$V_{out} = -\frac{R_2}{R_1} V_{in}$$

Summing



$$V_{out} = \underbrace{\frac{R_3+R_4}{R_4}}_{\text{then use } R_3 \text{ \& } R_4 \text{ to scale that } V^+}\underbrace{\left(\frac{R_2}{R_1+R_2} V_1 + \frac{R_1}{R_1+R_2} V_2\right)}_{\text{use } R_1 \text{ \& } R_2 \text{ to get wanted } V^+}$$

Invert-Summing



$$V_{out} = -\frac{R_3}{R_1} V_1 - \frac{R_3}{R_2} V_2$$

Wk9 - Circuit Design Examples

Voltage-Controlled Current Source