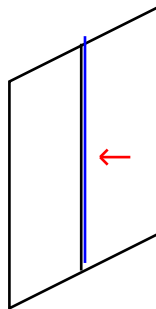


Design Process

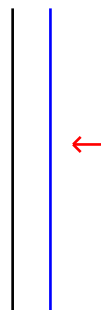
When we build a system or device, we often want to systematically figure out how to achieve our goal. At a high level, the process involves the following steps:

- Step 1: Concretely state your goal for the system
- Step 2: Describe a strategy for achieving your goal
- Step 3: Implement the components within your strategy
- Step 4: Verify your design meets the original requirements
- Iterate through these more than once

We will apply the design method above to the development of a touchscreen. Now for simplicity, let's only consider the one-dimensional case. Suppose we have a touchscreen as follows



The touchscreen has two layers, the vertical blue wire and the black plane. The red arrow indicates the user and the user is only allowed to touch somewhere on the vertical blue wire. Looking closer, the portion of the touch screen we are concerned with is



When we press the screen, the above becomes



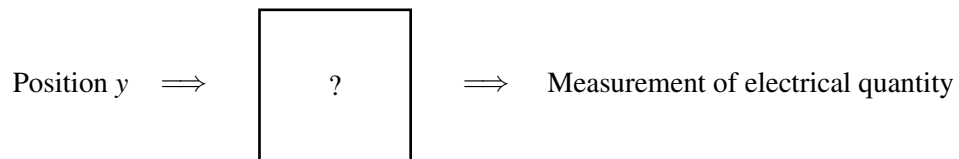
Let's apply the design method to this problem.

- **Step 1: Concretely state your goal for the system**

We would like to find out the vertical position at which the finger touches the screen.

- **Step 2: Describe a strategy for achieving your goal**

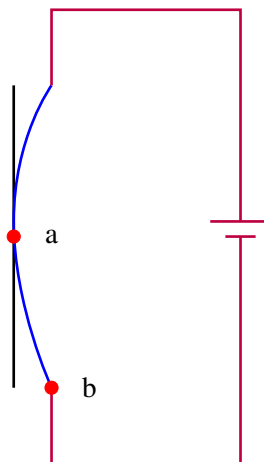
We can't directly measure the state of the finger. However, we will see that the finger touch can affect certain electrical quantity associated with the touchscreen. By measuring that electrical quantity and figuring out how it maps to what we originally want to detect, we can figure out what position on the screen the finger is touching.



As depicted above, we need to figure out a way to relate what we're looking for position y with the measurement of the electrical quantity.

- **Step 3: Implement the components within your strategy**

It turns out what we can do is to connect the touch screen with a battery as follows



It turns out we can figure out the state y if we measure the voltage V_{ab} across point a and b ! However, before we can understand why, we need some basic knowledge of circuits. We will return to this at the end of the lecture.

Basic Circuits

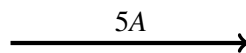
The first thing we need to know in designing the touch screen is that applying **voltage** across a metal (or any conductive material) causes **currents** to flow through the material. Currents are **charges** moving in a particular direction. We can think of voltage as providing the energy to force the charges to move in a particular direction, just like we need to exert force on a box to push it. Let's formally explain these basic circuit components below:

Charge: There are two types of charges, positive charge (+) and negative charge (−). We measure charges with the unit **coulomb (C)**.

Current: Current is the net amount of charge crossing a surface in a unit time. We usually use the symbol **I** to denote current. The unit for currents is **ampere (A)**, which is equivalent to 1 coulomb per second. If ΔQ is the net charge crossing surface in Δt , then the current is

$$I = \frac{\Delta Q}{\Delta t}.$$

We typically specify the direction of the net positive charge flow using an arrow. For example, the following arrow



represents that net +5 coulombs of charges are crossing the surface per second to the right. We could also say that net −5 coulombs of charges are crossing the surface per second to the left.

Voltage: Voltage is the amount of energy needed to move a unit charge between two points. We usually denote voltage with symbol **V**. It has unit **volt(V)**, which is voltage difference between two points that will require 1 joule of energy to move 1 coulomb unit of charge between the points, i.e. $1V = 1J/C$. Voltage is a relative quantity defined between two points. An absolute voltage is meaningless and usually is implicitly referenced to a known point in the circuit (ground) or in some cases a point at infinity.

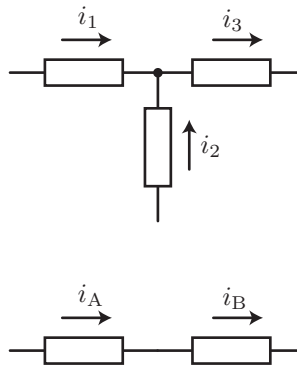
Kirchoff's Current Law (KCL)

KCL states that the net charge flowing into any node of a circuit is identically zero. This follows from the conservation of charge – charge cannot vanish or appear at any node. For example, in the following circuit, the currents must satisfy

$$i_1 + i_2 - i_3 = 0. \tag{1}$$

In addition, when current is travelling through elements in series, the currents must be equal.

$$i_A = i_B. \tag{2}$$



Kirchoff's Voltage Law (KVL)

Kirchoff's Voltage Law states that the net potential around any loop in a circuit is zero.

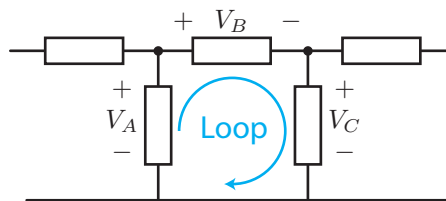


Figure 1: KVL illustration

$$\sum_{\text{Loop}} V_k = 0. \quad (3)$$

This follows from the conservation of energy – the net energy in going around a loop is zero. This is just like what you may have seen in physics – if you're climbing a hill, the net potential energy you gain climbing up the hill and going back down the hill is zero because the amount of potential energy gained climbing up the hill is equal to the amount of energy lost when you go down the hill.

Let's look at the example in figure ???. Adding the voltage "drops" around the loop, we have

$$-V_A + V_B + V_C = 0. \quad (4)$$

Note that if we had defined the loop in the opposite direction,

$$V_A - V_C - V_B = 0. \quad (5)$$

Resistors

Real conductors have "resistance", which impedes the flow of current by producing a voltage drop across the path of the current. The higher the current, the higher the voltage drop. Remarkably, the voltage drop is exactly proportional to the current. This is a statement of **Ohm's Law**:

$$V = IR. \quad (6)$$

The proportionality constant is known as the resistance of the conductor. The unit of R is Voltage/Ampere, or more commonly "Ohms" (Ω). Such a conductor is known as a resistor. The value of resistance can be altered by using different materials in the construction of the component, or by modifying the physical dimensions of the conductor.



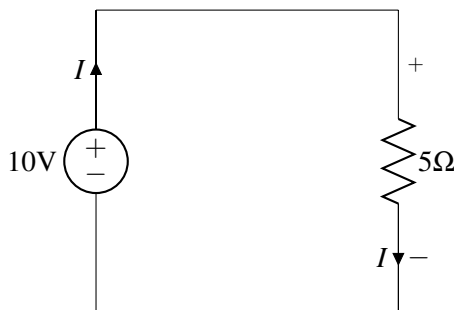
Figure 2: Current flow into a resistor results in a voltage drop proportional to the current.

For a wire of uniform cross area, the resistance is calculated as follows

$$R = \frac{\rho l}{A}.$$

It's proportional to the length l of the wire, inversely proportional to the cross sectional area A , and proportional to the material resistivity ρ . Intuitively, this makes sense because the longer the wire, the more impedance the current face flowing through, and the larger the cross sectional area, the easier it is for current to flow through.

Let's look at the following example involving a voltage source and a resistor:



Using Ohm's law, we know that the voltage drop across the resistor on the right is equal to $\Delta V = 5I$ and the voltage gain when going through the voltage source on the left is 10V. Using KVL, we have

$$10 - 5I = 0. \tag{7}$$

Hence, we have that the current going through the loop (in the specified direction) is $I = 2A$ and the voltage drop across the resistor is $5\Omega \times 2A = 10V$.

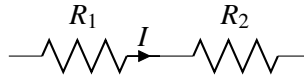
Passive sign convention

Notice the $+$ and $-$ signs we draw across resistors. In electrical engineering, we often adopt the passive sign convention – positive current should go into the positive terminal of

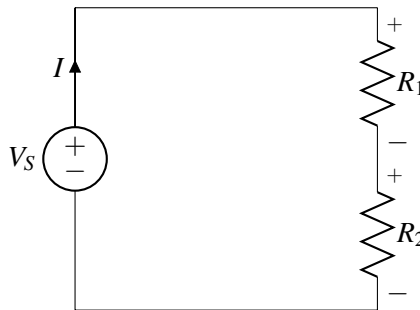
A circuit often consists of many resistors. The way the resistors are connected can often be categorized into the two types: (1)series, (2)parallel. Let's get some intuition and perform analysis on each type below:

Resistors in Series

Now, let's see what happens if we connect resistors in series.



Intuitively, the amount of impedance the current faces by going through the two resistors in series would just be the sum of the impedance faced by the current going through each individual of the resistor. Let's analyze this rigorously.



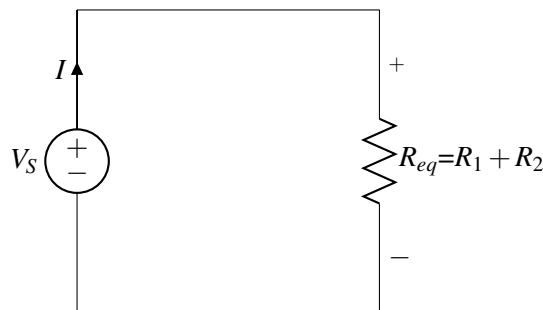
We know by Ohm's law that the voltage drop across R_1 is IR_1 and the voltage drop across R_2 is IR_2 . Similarly, applying KVL on the loop, we have

$$V_S - IR_1 - IR_2 = 0 \quad (8)$$

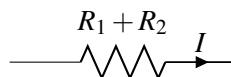
which leads us to

$$V_S - I(R_1 + R_2) = V_S - IR_{eq} = 0 \quad (9)$$

where $R_{eq} = R_1 + R_2$. So essentially, we can think of the circuit as

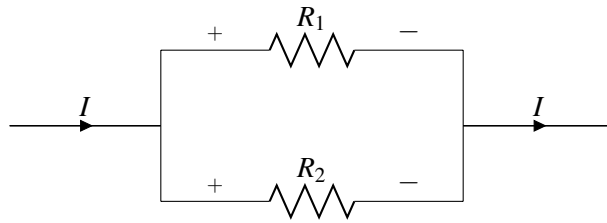


where the two resistors in series are replaced by a resistor with resistance $R_{eq} = R_1 + R_2$. We say that the **equivalent resistance** for resistors in series is the sum of their resistance.

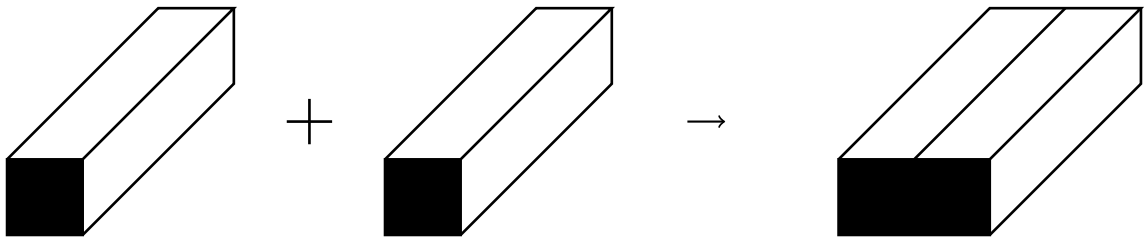


Resistors in Parallel

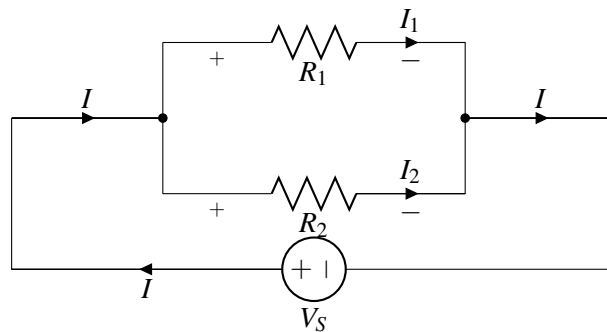
How about resistors in parallel?



Let's think about this intuitively. Having resistors in parallel is like taking two boxes and combining them via their sides. This increases the cross-sectional area in the front, as illustrated below. So intuitively, the equivalent resistance of resistors in parallel would be smaller than the resistance of the original resistors. In addition, intuitively, more current would choose to go to the side with less resistance. We can show this rigorously.



We have the following circuits



First, let's apply KCL on the circuit. We see that I_1 and I_2 branch off from I and converge back together. We have

$$I = I_1 + I_2. \quad (10)$$

Now, applying KVL on the loop that contains R_1 and V_S , we have

$$V_S - I_1 R_1 = 0. \quad (11)$$

Applying KVL again on the loop that contains R_2 and V_S , we have

$$V_S - I_2 R_2 = 0. \quad (12)$$

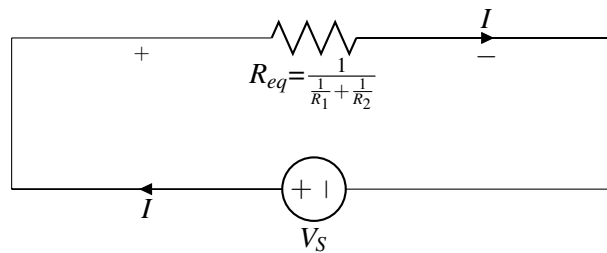
Now substituting $I_1 = \frac{V_S}{R_1}$ and $I_2 = \frac{V_S}{R_2}$ in the first equation, we have

$$I = I_1 + I_2 = \frac{V_S}{R_1} + \frac{V_S}{R_2} = V_S \left(\frac{1}{R_1} + \frac{1}{R_2} \right). \quad (13)$$

This is equivalent to

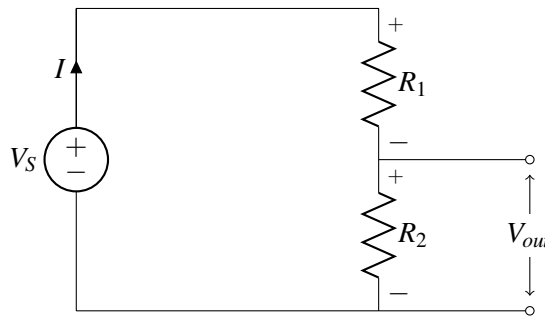
$$V_S = I \left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \right) = I R_{eq}. \quad (14)$$

where we set $R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$. Hence, the resistors in parallel circuit shown earlier is equivalent to



Voltage Divider

Notice that in the circuits we've seen so far, the voltage source always creates a set value of potential difference across itself. What if we need to a different potential difference across a device? We can achieve this using a voltage divider. Recall we have the resistors in series circuit



We figured out earlier that

$$V_S = I(R_1 + R_2). \quad (15)$$

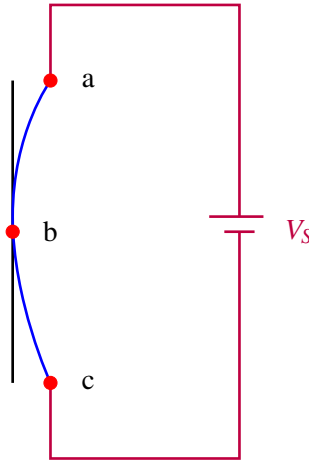
Hence, $I = \frac{V_S}{R_1 + R_2}$. So the voltage drop across R_2 is

$$V_{out} = I R_2 = \frac{R_2}{R_1 + R_2} V_S. \quad (16)$$

Notice that we can create any output voltage of $V_{out} = \alpha V_S$ for any $\alpha \in [0, 1]$ by varying the resistance R_1 and R_2 .

Resistive Touchscreen Revisited

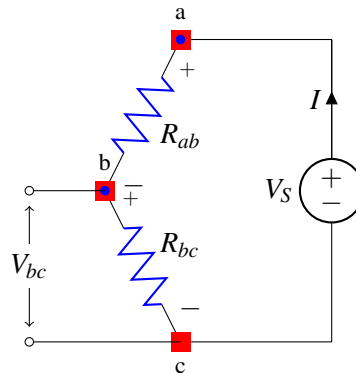
We now have the knowledge we need to design resistive touchscreen! Continuing with step (3) of our design process in the first section, we have the following design



Now that you've been introduced to some knowledge in circuits, what does the above look like to you? A voltage divider! Concretely, we can think of the thin piece of metal (blue line) as a resistor of resistance R and length l . We let l_{ab} denote the length of the metal between point (a) and (b), and l_{bc} denote the length of the metal between point (b) and (c). First, we know that

$$l_{ab} + l_{bc} = l. \quad (17)$$

We can essentially model the above touch screen with the following circuit



We know that resistance is proportional to its length. Hence, we have

$$\frac{R_{ab}}{R_{bc}} = \frac{l_{ab}}{l_{bc}}. \quad (18)$$

From the voltage divider in the last section, we know that

$$V_{bc} = \frac{R_{bc}}{R_{ab} + R_{bc}} V_S = \frac{l_{bc}}{l_{ab} + l_{bc}} V_S = \frac{l_{bc}}{l} V_S. \quad (19)$$

Reorganizing the terms, we have that

$$l_{bc} = \frac{V_{bc}}{V_S} l. \quad (20)$$

Since we know our input voltage V_S and the length of the entire metal l , we can completely figure out the state l_{bc} by measuring the output voltage v_{bc} ! l_{bc} would just be the position at which our finger touches the touchscreen, with the reference point being node c the bottom of the metal.