EE 16A: Homework 10

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1. **Worked With...**

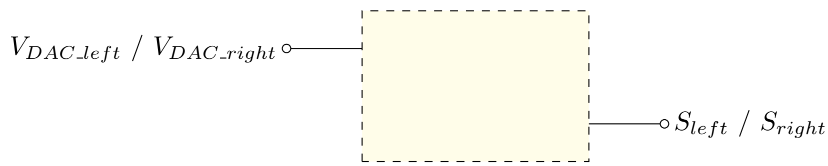
   Ilya (3031806896), James Zhu (3031793129)
   I worked alone on Friday morning, then met up with Ilya and James to discuss on Saturday afternoon.

2. **Noice Cancelling Headphones**
So we have

$$\begin{bmatrix} S_{ear\_left} \\ S_{ear\_right} \end{bmatrix} = A \begin{bmatrix} S_{mic1} \\ S_{mic2} \\ S_{mic3} \end{bmatrix} + B \begin{bmatrix} S_{mic1} \\ S_{mic2} \\ S_{mic3} \end{bmatrix} + \begin{bmatrix} S_{left} \\ S_{right} \end{bmatrix},$$

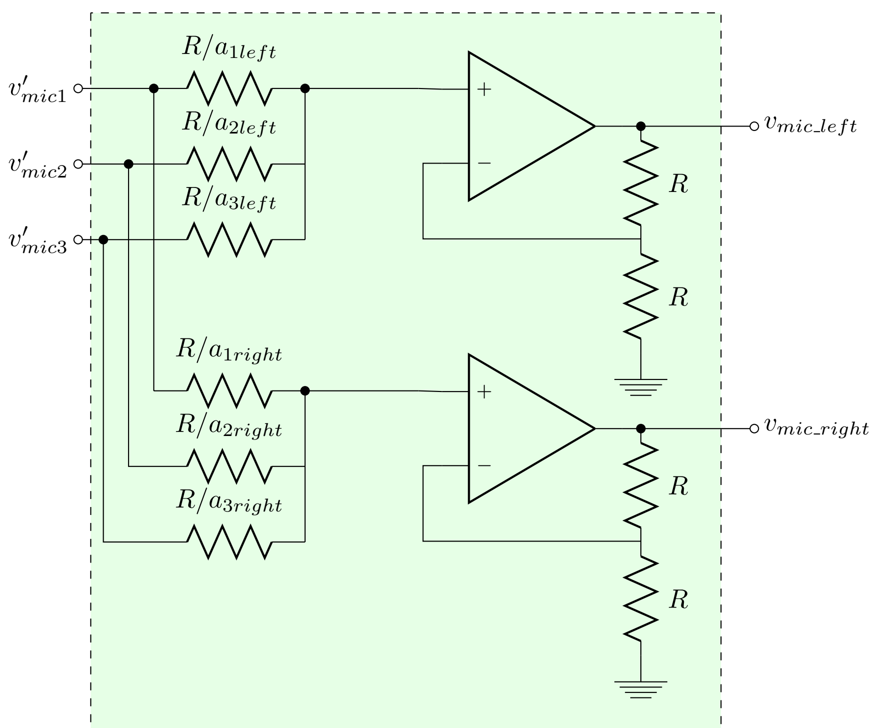
where we define B as the matrix operation implemented by the active noise cancellation circuitry:

Part 1 Build a circuit to drive DAC outputs to the specified $-1.5V$ - $1.5V$ range.
To achieve the goal, we need 3 things:
1. Shift the signal$(0V - 1V)$ to center at $0V$.
2. Provide gain to the signal to go from a $1V$ range to a $3V$ range.
But we already did this in discussion 9B so just use the circuitry from there for both DACs.
We will represent that circuit as like a black box below for later use:



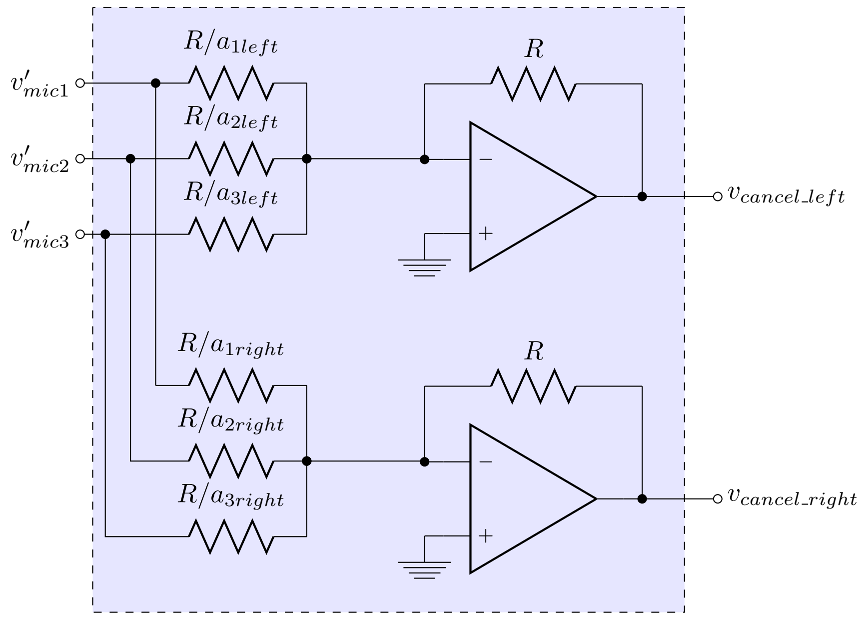
(1)

Part 3 Build the mic voltage separation / summer circuitry to implement the matrix operation A:



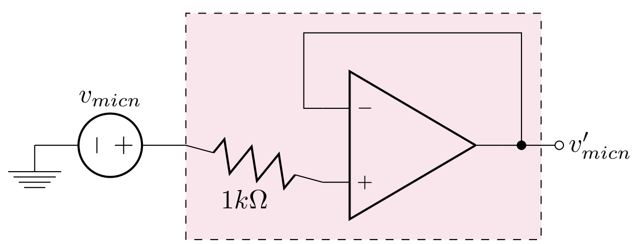
(2)

Part 2 Build the external noise canceling circuitry:
In order to ensure that the user doesnt hear any of the sounds picked up by the microphones, we want $B = -A$.
And so we get a circuit that implements B:



(3)

for any R, and where each v'_{micn} is connected to the output of each microphone buffer as below:



(4)

because we have to take into account the fact that our microphone voltages have source resistances of $1k\Omega$.

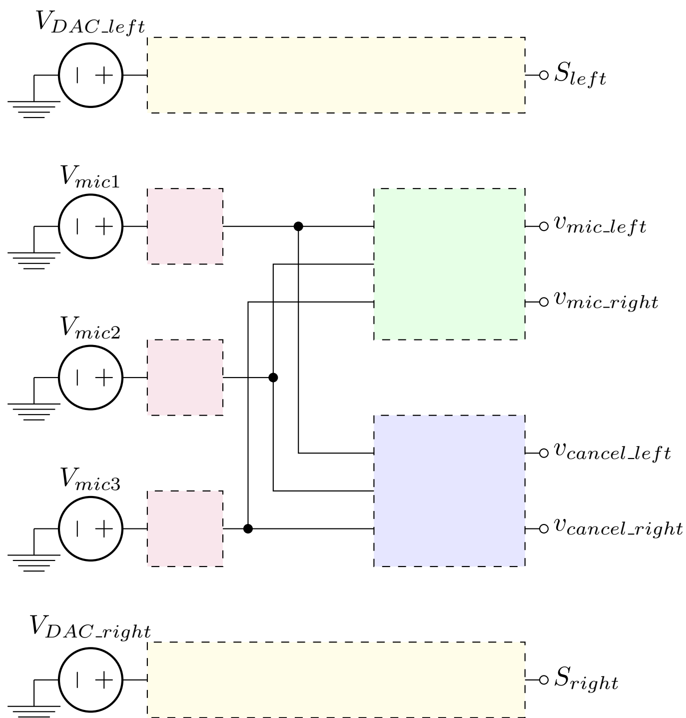
Part 4
Now just add a summer for adding in the audio signal/output of all the above circuits for each side to get S_{ear_left} and S_{ear_right}!
The circuit from figure (1) provides

$$\begin{bmatrix} S_{left} \\ S_{right} \end{bmatrix},$$

and the circuits from figures (2) to (4) combined give us

$$A \begin{bmatrix} S_{mic1} \\ S_{mic2} \\ S_{mic3} \end{bmatrix} + B \begin{bmatrix} S_{mic1} \\ S_{mic2} \\ S_{mic3} \end{bmatrix},$$

which would visually look like

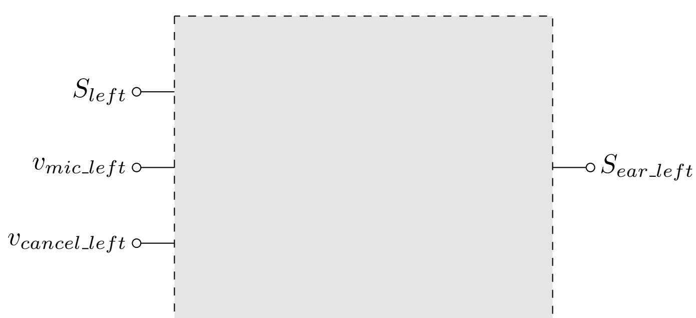


(5)

By adding two traditional summers (similar in concept to the green boxes) to the above diagram, we can get

$$S_{ear\_left} = v_{mic\_left} + v_{cancel\_left} + S_{left}$$
$$S_{ear\_right} = v_{mic\_right} + v_{cancel\_right} + S_{right}$$

For example, to get S_{ear_left},

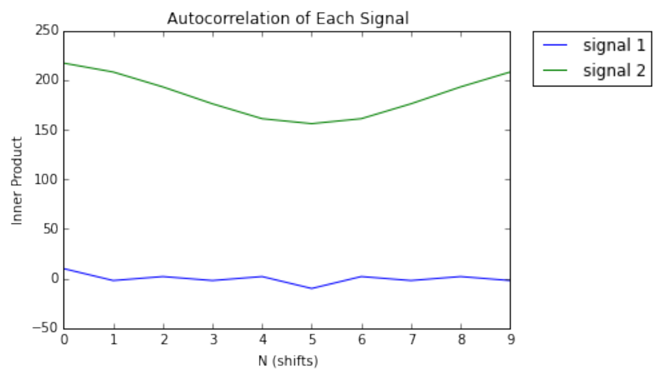


where the black box represents the traditional summer.

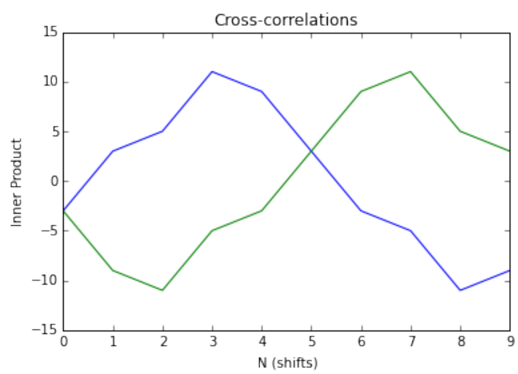
3. **Mechanical: Correlation** *see attached iPython notebook*

$$\vec{s_1} = (1, -1, 1, -1, -1, -1, 1, -1, 1, 1)$$
$$\vec{s_2} = (1, 2, 3, 4, 5, 6, 7, 6, 5, 4)$$

(a) *autocorrelation*



(b) *cross-correlation*



4. **Inner products** *Use the Cauchy-Schwarz inequality to verify (i.e. prove or derive) the triangle inequality:*

$$\|\vec{x}+\vec{y}\| \leq \|\vec{x}\| + \|\vec{y}\|.$$

Proof. Observe the following manipulation

$$\begin{aligned}\|x+y\|^2 &= \langle x+y, x+y\rangle \\ &= \langle x,x\rangle + \langle x,y\rangle + \langle y,x\rangle + \langle y,y\rangle \\ &= \|x\|^2 + 2\langle x,y\rangle + \|y\|^2.\end{aligned}$$

Then by the Cauchy-Schwartz inequality we have that

$$\|x+y\|^2 \leq \|x\|^2 + 2\|x\|\|y\| + \|y\|^2 = (\|x\| + \|y\|)^2$$

Thus we have

$$\|x+y\| \leq \|x\| + \|y\|.$$

□

5. **Midterm Review**
   See attached scans

6. **No more circuits :))!!!!** Replicate the projection equation from discussion in LaTeX without the ugly arrows on top like they do in textbooks, which would make Babak happy (not as easy as it sounds)

$$\begin{aligned} proj_{\mathbf{b}}\mathbf{a} &= \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{b}\|^2}\mathbf{b} \\ &= \frac{\langle \mathbf{a}, \mathbf{b} \rangle}{\langle \mathbf{b}, \mathbf{b} \rangle}\mathbf{b} \end{aligned}$$

hw10

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1 2. Mechanical: Correlation

```
In [42]: %pylab inline
         import numpy as np
         import matplotlib.pyplot as plt
         from scipy.linalg import circulant
```

```
Populating the interactive namespace from numpy and matplotlib
```

```
In [43]: s1 = np.array([1,-1,1,-1,-1,-1,1,-1,1,1])
         s2 = np.array([1,2,3,4,5,6,7,6,5,4])
         print("signal 1 = ", s1)
         print("signal 2 = ", s2)
```

```
signal 1 =  [ 1 -1  1 -1 -1 -1  1 -1  1  1]
signal 2 =  [1 2 3 4 5 6 7 6 5 4]
```

1.1 (a) autocorrelation

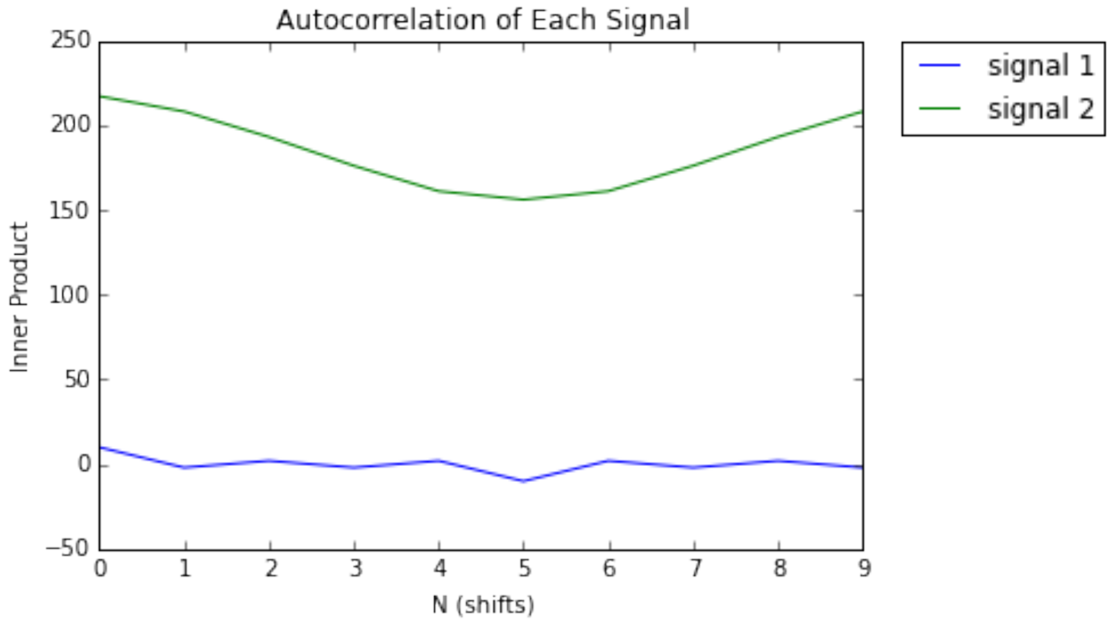
```
In [46]: c1 = circulant(s1).transpose()
         c2 = circulant(s2).transpose()
```

```
In [78]: auto1 = np.dot(c1,s1)
         print ("Autocorrelation of signal 1: ", auto1)
         auto2 = np.dot(c2,s2)
         print ("Autocorrelation of signal 2: ", auto2)
```

```
Autocorrelation of signal 1:  [ 10  -2   2  -2   2 -10   2  -2   2  -2]
Autocorrelation of signal 2:  [217 208 193 176 161 156 161 176 193 208]
```

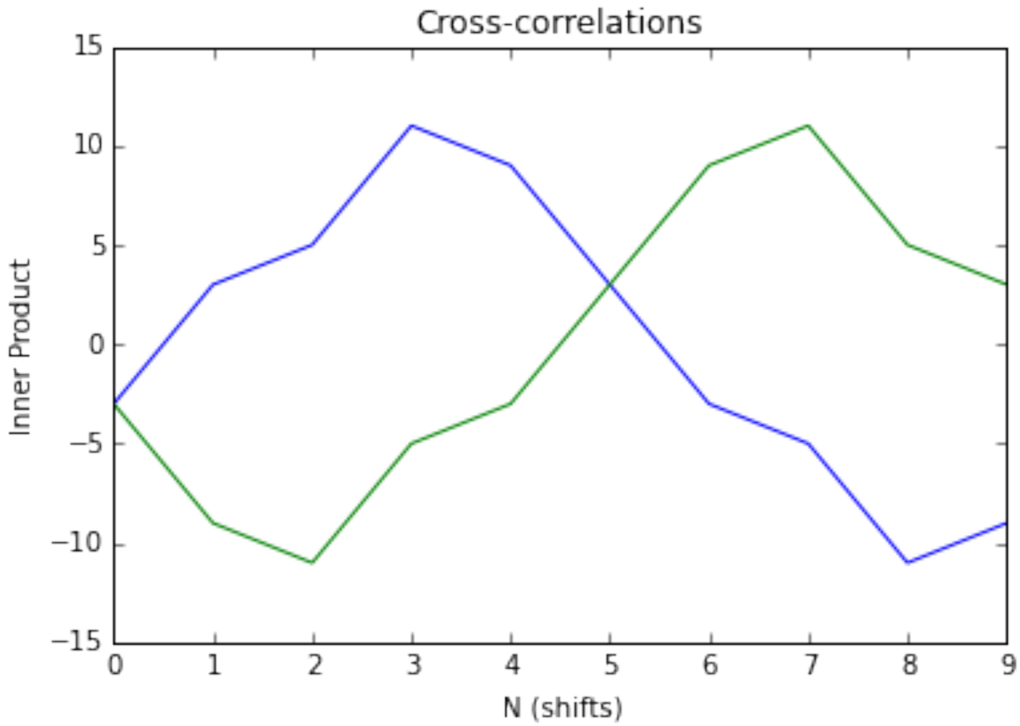
```
In [79]: plt.title('Autocorrelation of Each Signal')
         plt.plot(auto1, label='signal 1')
         plt.plot(auto2, label='signal 2')
         plt.xlabel('N (shifts)')
         plt.ylabel('Inner Product')

         plt.legend(bbox_to_anchor=(1.05, 1), loc=0, borderaxespad=0.)
```

```
Out[79]: <matplotlib.legend.Legend at 0x10c450fd0>
```



1.2 (b) cross-correlation

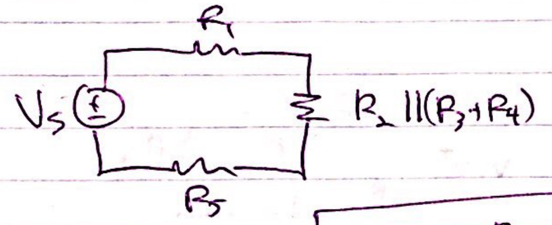
```
In [77]: cross1 = np.dot(c2,s1)
         print ("Cross-correlation 1: ", cross1)
         cross2 = np.dot(c1,s2)
         print ("Cross-correlation 2: ", cross2)

Cross-correlation 1:  [ -3   3   5  11   9   3  -3  -5 -11  -9]
Cross-correlation 2:  [ -3  -9 -11  -5  -3   3   9  11   5   3]

In [80]: plt.title('Cross-correlations')
         plt.plot(cross1)
         plt.plot(cross2)
         plt.xlabel('N (shifts)')
         plt.ylabel('Inner Product')

Out[80]: <matplotlib.text.Text at 0x10c461a20>
```


Cross-correlations
Inner Product
N (shifts)
15
10
5
0
−5
−10
−15
0
1
2
3
4
5
6
7
8
9

MT3) a) $X \xrightarrow{R_{eq}} B$ $\boxed{R_{eq} = R_1 + (R_2 \| (R_3 + R_4))}$

As $R_4 \to \infty$, $\boxed{R_{eq} = R_1 + R_2}$

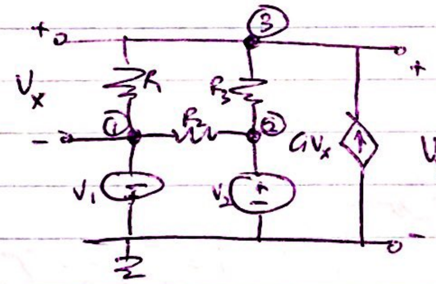
b)



$\therefore i_{total} = \dfrac{V_s}{R_1 + R_5 + R_2 \| (R_3 + R_4)}$

$$\therefore \boxed{i_2 = \frac{R_3 + R_4}{R_2 + R_3 + R_4}\left(\frac{V_s}{R_1 + R_5 + R_2 \| (R_3 + R_4)}\right)}$$

MT 4)



Label nodes ①, ②, ③.

$V_③ - V_① = V_x = \underbrace{V_y - (-V_1)}_{}$

$V_③ = V_y$ $\quad V_y = V_x + V_1$

$V_① = -V_1$

$V_② = V_2$

Nodal Analysis @ ~~②~~ ③ :

~~$\dfrac{V_② - V_①}{R_2} + \dfrac{V_② - V_③}{R_3} + \dfrac{V_②}{}$~~

$$\frac{V_③ - V_①}{R_1} + \frac{V_③ - V_②}{R_3} + -GV_x = 0$$

$$\frac{V_x}{R_1} + \frac{V_y - V_2}{R_3} - GV_x = 0$$

$$\frac{V_x}{R_1} + \frac{V_x + \cancel{V_1 - V_2}^{\,0}}{R_3} - GV_x = 0$$

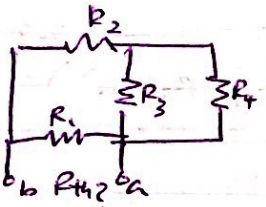
$$\frac{1}{10}V_x + \frac{1}{40}V_x - \frac{1}{4}V_x = 0$$

$$\frac{4}{40}V_x + \frac{1}{40}V_x - \frac{10}{40}V_x = 0$$

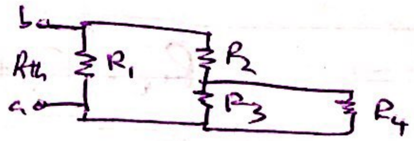
$$-\frac{1}{8}V_x = 0$$

$\boxed{V_x = -8\text{ V}} \longrightarrow \boxed{V_y = -3\text{V}}$

HW 5) a)



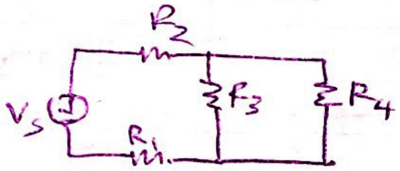
$\rightarrow$



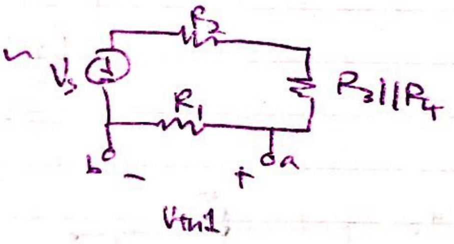
$$\therefore \boxed{R_{th} = R_1 \parallel (R_2 + (R_3 \parallel R_4))}$$

b) $V_{th} = ?$

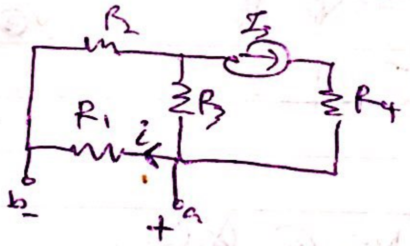
First, null I_s :



$$\therefore V_{th1} = \frac{R_1}{R_1 + R_2 + (R_3 \parallel R_4)} V_s$$



Next, null V_s :



$$i = \frac{R_3}{R_1 + R_2 + R_3} I_s$$

$$V_{th_2} = i R_1 = \frac{R_1 R_3}{R_1 + R_2 + R_3} I_s$$

$$\therefore \boxed{V_{th} = V_{th1} + V_{th_2} = \frac{R_1}{R_1 + R_2 + (R_3 \parallel R_4)} V_s + \frac{R_1 R_3}{R_1 + R_2 + R_3} I_s}$$

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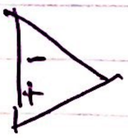
6) a)



b) From the golden rules, we get

$$V^+ = V^- = V_{in_2}$$

And using the knowledge that $I^- = I^+ = 0$, we can use KVL to perform nodal analysis @ the V^- node:

$$\frac{V_{in_2} - V_{in_1}}{2} + \frac{V_{in_2} - V_o}{6} = 0$$

$$\frac{V_o - V_{in_2}}{\cancel{6}} = 3(V_{in_2} - V_{in_1})$$

$$\therefore \boxed{V_o = 4V_{in_2} - 3V_{in_1}}$$

7) a)

$$F = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

b) Put F into RREF: $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

We can easily see that there are 2 lin. independent cols,

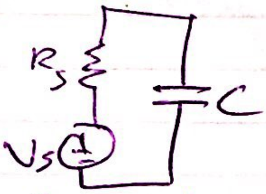
$$\therefore \boxed{\text{rank}(F) = 2}$$

To get null space, solve for $Fx = b$

$$1\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 1\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 1\begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = 0$$

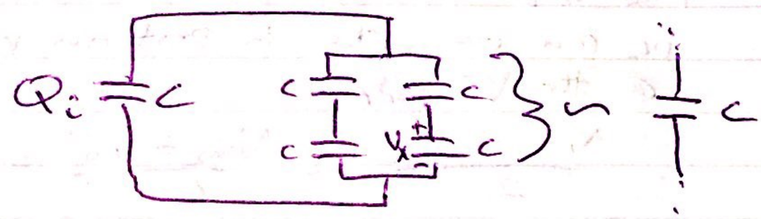
$\therefore$ $\boxed{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ is the basis of } N(F)}$

8) When in phase 1:



At steady state, C is charged to V_s,
so $Q_i = CV_s$

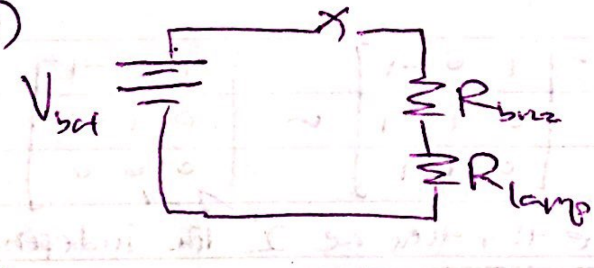
In phase 2:



Due to charge sharing, and conservation of charge, on each side there is $Q_i/2$ charges.
That means that on each of the four capacitors on the right, there are $Q_i/4$ charges.

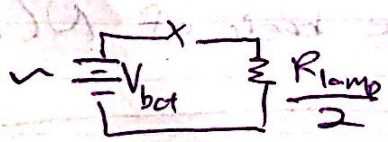
$\therefore V_x = \frac{Q}{c} = \boxed{\frac{Q_i}{4C}}$

9) a)



b)

Between the choice of putting the lamps in series or parallel, I would put them in parallel to maximize power dissipated.



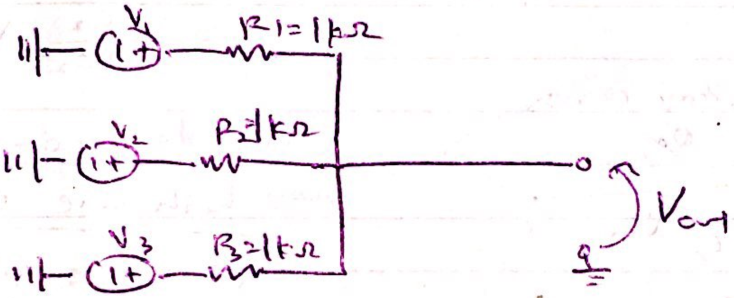
$\therefore I = \frac{2V_{bat}}{R_{lamp}}$ $\therefore P = IV = \boxed{\frac{2(V_{bat})^2}{R_{lamp}}}$

10) a) $$\frac{V_{out} - V_1}{R_1} + \frac{V_{out} - V_2}{R_2} + \frac{V_{out} - V_3}{R_3} = 0$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) V_{out} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

$$\boxed{V_{out} = (R_1 \| R_2 \| R_3)\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)}$$

b)



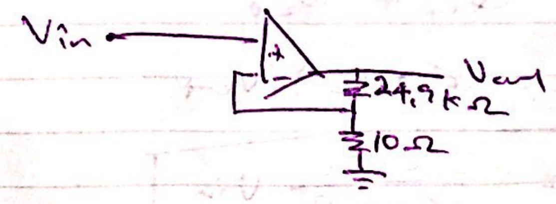
$$\therefore \boxed{V_{out} = \frac{1}{\cancel{R}}(V_1 + V_2 + \dots + V_n)}$$

which in this case is

$$\boxed{V_{out} = \frac{1}{3}(V_1 + V_2 + V_3)}$$

c) the gain is $\cancel{\frac{10\,mV}{2.5\,V}}$ & $\frac{2.5\,V}{0.01\,V} = \times 250$

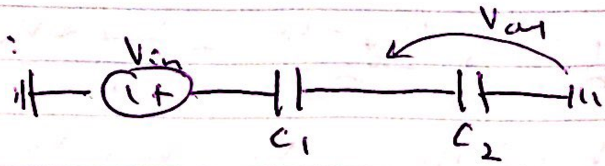
We can use :



$$\therefore V_{out} = \left(1 + \frac{24.9k\Omega}{10\Omega}\right) V_{in}$$

$$\boxed{V_{out} = 250\, V_{in}}$$

11) a) in phase I:



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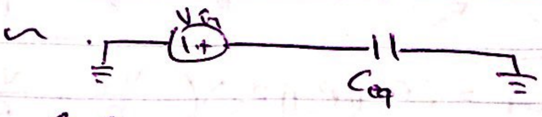


$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$ $\therefore Q_i = C_{eq} V_{in}$

$= \frac{C_1 C_2}{C_1 + C_2} V_{in}$

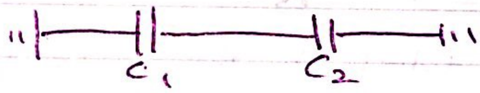
Since the capacitors are in series, they both have equal charge of

$$\boxed{Q_1 = Q_2 = \frac{C_1 C_2}{C_1 + C_2} V_{in}}$$

To find voltage across each, $V_n = Q/C_n$

$$\boxed{V_1 = \frac{C_2}{C_1 + C_2} V_{in} \qquad V_2 = \frac{C_1}{C_1 + C_2} V_{in}}$$

b) In phase 2:



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As a function of V_{out}, C_1, & C_2,

$$\cancel{Q_1 = Q_2} = \boxed{Q_1 = C_1 V_{out}, \quad Q_2 = C_2 V_{out}}$$

c) $Q_i = Q_f$

$$Q_{1i} + Q_{2i} = Q_{1f} + Q_{2f}$$

$$\frac{2 C_1 C_2}{C_1 + C_2} V_{in} = (C_1 + C_2) V_{out}$$

$$\therefore \boxed{V_{out} = \frac{2 C_1 C_2}{(C_1 + C_2)^2} V_{in}}$$

d) Efficiency $= E_2 / E_1$ $= \frac{\cancel{C\left(\frac{V_{in}}{2}\right)^2}\ C\left(\frac{V_{in}}{2}\right)^2}{C\left(\frac{V_{in}}{2}\right)^2}$

$= \boxed{1}$

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