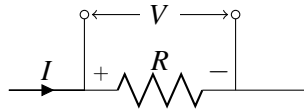


Power

Power P is the time rate of change of energy. Let ΔE be the change in energy. Recall that voltage V is the amount of energy needed to move a unit charge between two points, i.e., $V = \frac{\Delta E}{\Delta q}$. Hence, we have

$$P = \frac{\Delta E}{\Delta t} = \frac{V \Delta q}{\Delta t} = V \frac{\Delta q}{\Delta t} = V \times I. \quad (1)$$

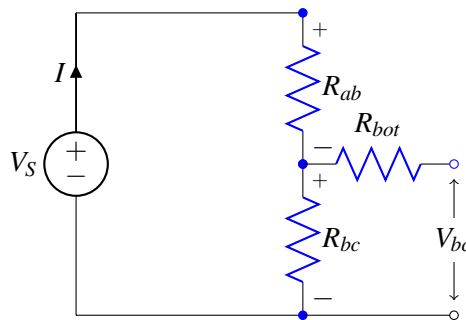
The power dissipated(positive) or supplied(negative) by a component with voltage V and current I is thus equal to their product $V \times I$. Note that according to passive sign convention, positive current goes into the positive terminal of the component.



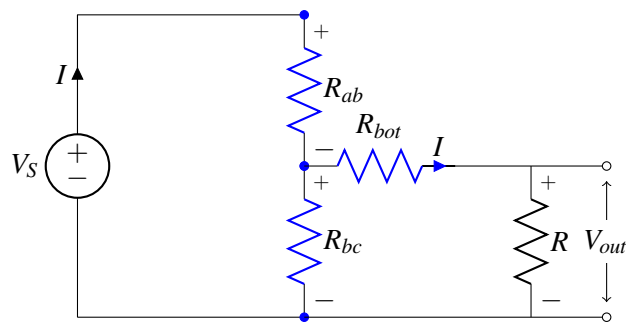
When $P = IV$ is positive, power is being dissipated; when $P = IV$ is negative, power is being generated.

Equivalence

In the previous note we introduced how a resistive touchscreen works. Essentially, we can model the touchscreen as a voltage divider in each direction when your finger touches the screen. To figure out the vertical position, recall that we have the following voltage divider.

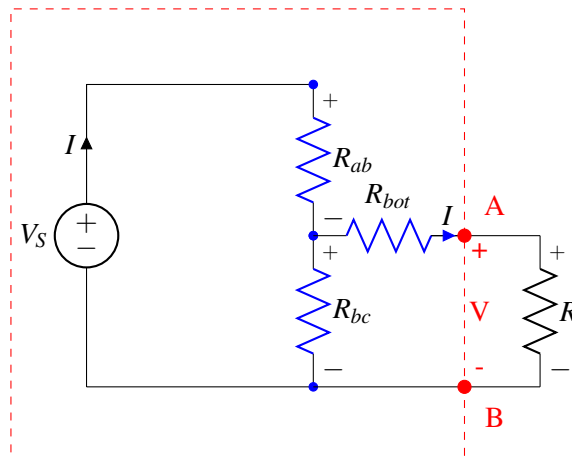


Note that R_{bot} is the resistor in the bottom plate. We can measure the voltage V_{bc} to figure out the position at which you touched. Now what if the device you're using to measure the voltage has a resistance R associated with it? Then the circuit becomes

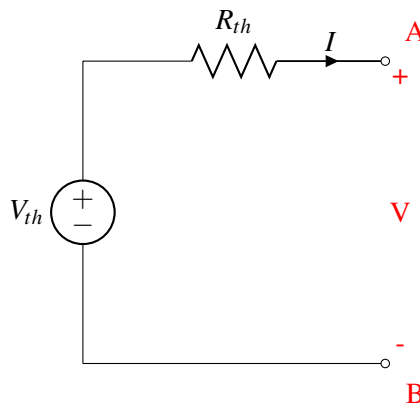


First observe that the output voltage V_{out} is not equal to V_{bc} because there will be a current I flowing through resistors R and R_{out} . Suppose we want to find out what the output voltage V_{out} would be in this case. We could figure it out by writing down a system of linear equations using KCL, KVL, and Ohm's law. However, what if the measurement device changed and its internal resistance is now equal to R' . Do we have to redo everything again? It turns out there's a faster way to do that by using the concept of an equivalent circuit.

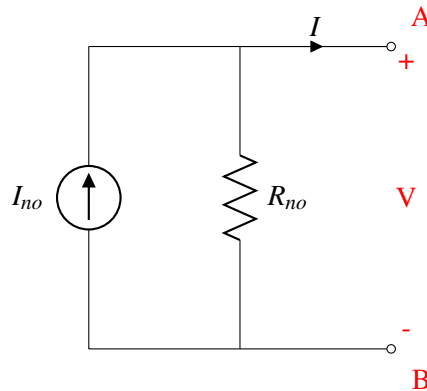
Let's focus our attention on the original circuit we had, which is illustrated as follows with the red dotted lines.



It turns out we can replace everything in the red box with either



or

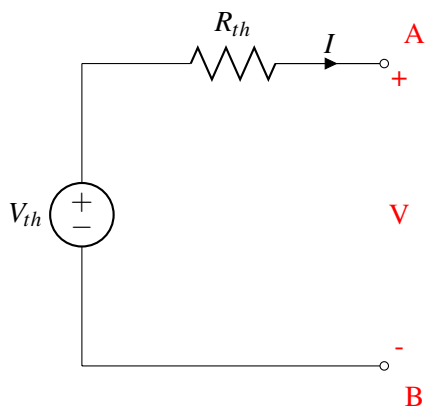


We call the first circuit containing a voltage source and a resistor the **Thevenin equivalent circuit**; we call the second circuit containing a current source and a resistor the **Norton equivalent circuit**. Once we simplify the original circuit to one of the above, we can easily figure out V_{out} no matter what resistor it is connected to on the right. In fact, we can convert **any** circuit into any one of the equivalent forms above.

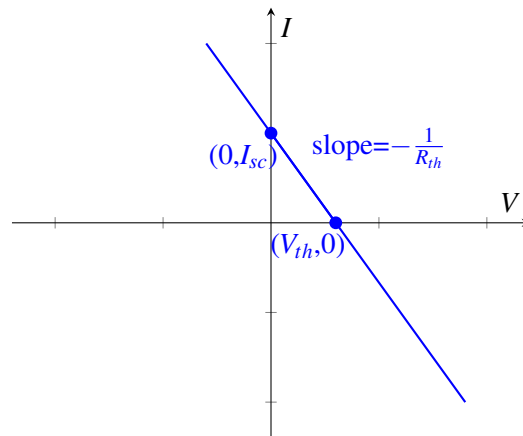
Before we move on, let's clarify what we mean by "equivalent"? Two circuits are equivalent if the $I - V$ relationship of one is exactly identical to that of the other circuit. (As a reminder, the simplest I-V relationship we've seen so far is for a resistor, i.e., $V = IR$ or $I = \frac{V}{R}$).

Now why is this possible intuitively? Since voltage and current are governed by linear relationship for all of the circuit elements we've learned about and a line can be uniquely determined by exactly two points, we can capture the original circuit with a simplified circuit with exactly two components: a voltage (current) source and a resistor.

Thevenin Equivalent Circuit

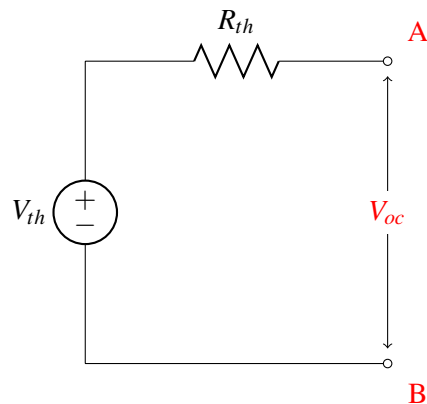


Now how would you figure out V_{th} and R_{th} for the Thevenin equivalent circuit? If we plot the I v.s. V relationship where V is the voltage drop from A to B and I is the current flowing into A (as shown in the figure), we would get a graph that looks like the following.

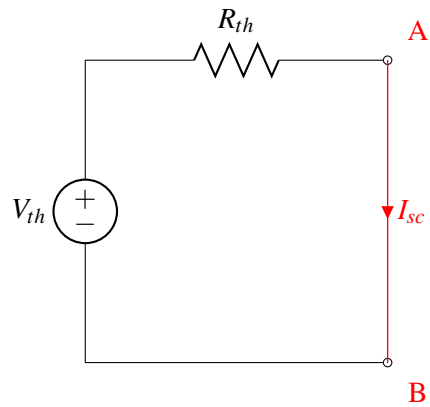


We know once we pin down the values of two points on a line, we can uniquely determine the line. In particular, we look at the following two points

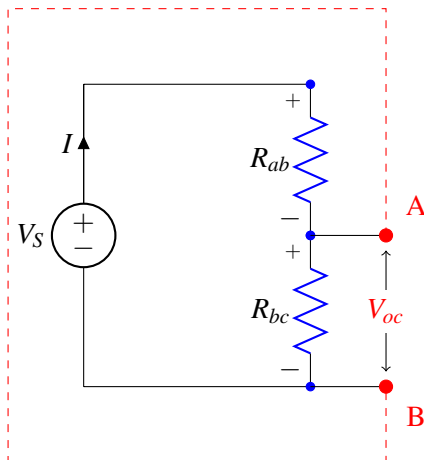
- Intersection with the V-axis: At this point, the current flowing into A is equal to zero, which is equivalent to leaving the circuit open between A and B . So to find V_{th} , we leave the circuit open between A and B as follows. Let the voltage drop from point A to B be V_{oc} in this case. Since there is no current flowing through the circuit, we know that $V_{th} = V_{oc}$.



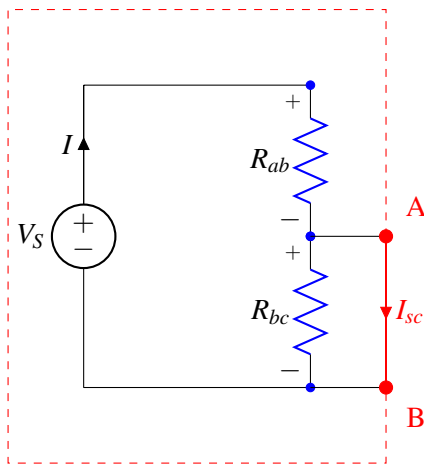
- Intersection with the I-axis: At this point, the voltage across A and B is equal to zero, which is equivalent to shorting A and B . We denote the current at this point be I_{sc} . Then we know that $I_{sc} = \frac{V_{th}}{R_{th}}$ by Ohm's law. Hence, we can find R_{th} by computing $R_{th} = \frac{V_{th}}{I_{sc}}$.



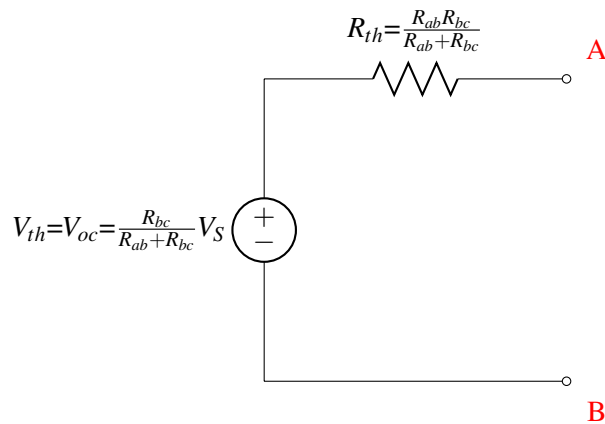
Now let's apply our analysis above to a voltage divider circuit shown below (which is very similar to the touchscreen). To figure out V_{th} , we solve for V_{oc} in the following circuit



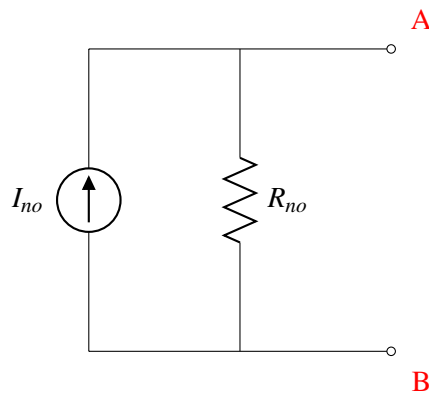
To figure out R_{th} , we short A and B in the original circuit, find the current I_{sc} going from A to B, and compute $R_{th} = \frac{V_{th}}{I_{sc}}$.



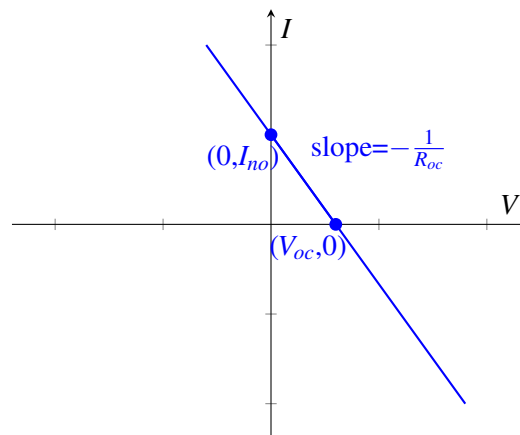
In the specific example we worked with, we can see that $V_{th} = V_{oc} = \frac{R_{bc}}{R_{ab} + R_{bc}} V_S$ and $R_{th} = \frac{V_{th}}{I_{sc}} = \frac{\frac{R_{bc}}{R_{ab} + R_{bc}} V_S}{\frac{V_S}{R_{ab}}} = \frac{R_{ab} R_{bc}}{R_{ab} + R_{bc}}$. So the Thevenin equivalent circuit is



Norton Equivalent Circuit



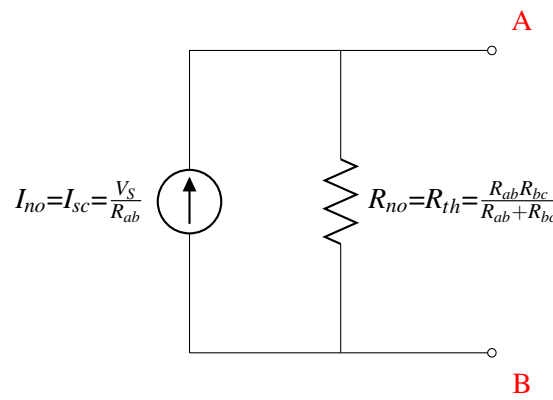
In the Norton equivalent circuit, instead of a voltage source, we have a current source instead. However, the analysis follows very similarly to what we did for Thevenin circuit.



Again, we are specifically interested in the following two points on the line:

- Intersection with the x-axis: There is no current flowing through A and B , which corresponds to the case where the circuit between A and B is open. We let V_{oc} represents the voltage across A and B in this scenario. We know by Ohm's law, $V_{oc} = I_{no}R_{no}$.
- Intersection with the y-axis: The current flowing from A to B is equal to I_{no} , which corresponds to the case where A and B are shorted.

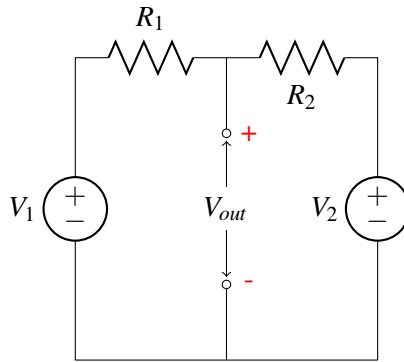
Notice the procedure for finding I_{no} and R_{no} is exactly the same as that of finding V_{th} and R_{th} in Thevenin equivalent circuit. So I_{no} is just equal to I_{sc} and V_{oc} is just equal to V_{th} . Hence, $R_{no} = \frac{V_{oc}}{I_{no}} = \frac{V_{th}}{I_{sc}} = R_{th}$. The Norton equivalent circuit for our example is



Superposition

We've started to build a library of things we can use in the future. If we want to take a voltage source V and output a voltage within that range (αV where $0 \leq \alpha \leq 1$), what can we do? We can use a voltage divider! Now what if we want to take two voltage sources and add them together? In this section, we are going to look at circuits with multiple voltage or current sources. In particular, we would like to introduce a very useful idea in working circuits of this type – superposition.

We know that everything we've learned so far in circuit is linear. Because of the linearity property of circuits consisting of only resistors and voltage(current) sources, we can add the response of each source to get the actual response. The idea is we can compute the response of each source one at a time, ignoring the response of the rest of the sources. Then we add all of them together to get the final result.



The procedure to do this (which is known as superposition) is as follows:

For each source k (either voltage source or current source)

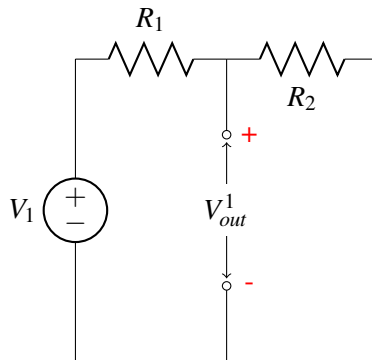
Set all other sources to 0

- **Voltage source: replace with a short circuit**
- **Current source: replace with an open circuit**

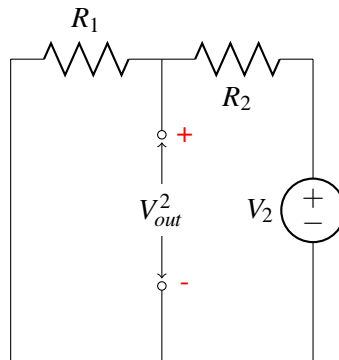
Compute $V_{out,k}$ due to this source k

Compute V_{out} by summing the $V_{out,k}$ s for all k .

We will illustrate this idea on the circuit above: We would like to figure out V_{out} . We first compute the output voltage due to V_1 and hence source V_2 will be replaced with a short circuit:

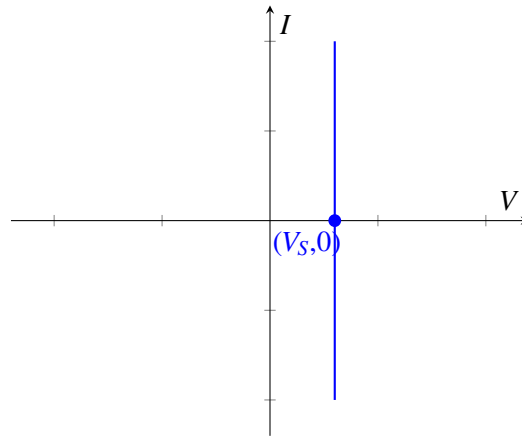


We know that $V_{out}^1 = \frac{R_2}{R_1 + R_2} V_1$. Next we compute the output voltage due to and hence source V_1 will be replaced with a short circuit.

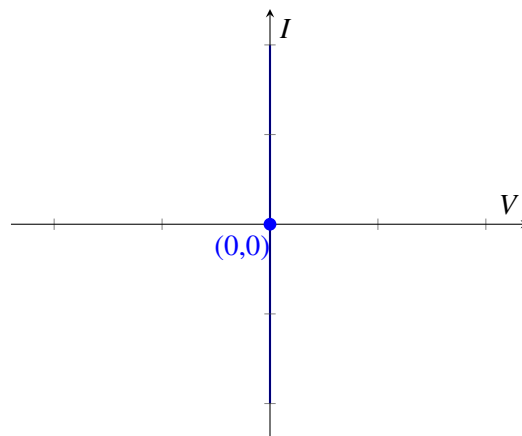


We can see that $V_{out}^2 = \frac{R_1}{R_1+R_2}V_2$. Finally, to get the output voltage V_{out} of the original circuit, we add the two output voltages based on the response of each voltage source together $V_{out} = V_{out}^1 + V_{out}^2 = \frac{R_2}{R_1+R_2}V_1 + \frac{R_1}{R_1+R_2}V_2$.

Now we ask the question: why does it make sense to replace voltage sources with short circuits? If we look at the I-V plot of a voltage source V_S , where I is the current going through the voltage source, then the plot would be a vertical line:

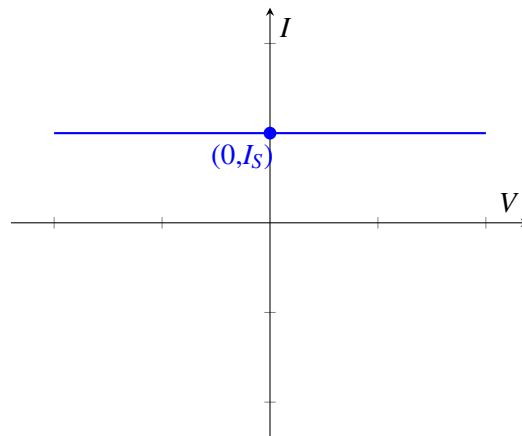


Now if we want to ignore the contribution of this voltage source, we are setting $V_S = 0$. Then the I-V plot is exactly the y-axis.

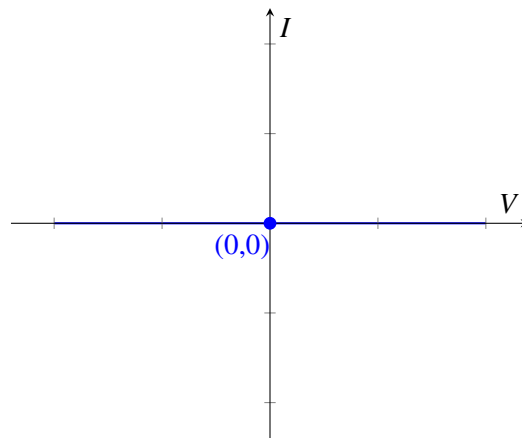


What does this mean? This means that it allows any current to go through, however the voltage drop always remains zero. This is exactly what a short circuit does.

Now let's look at why we replace current sources with open circuits. If we plot the I-V graph of a current source I_S , we get the following:

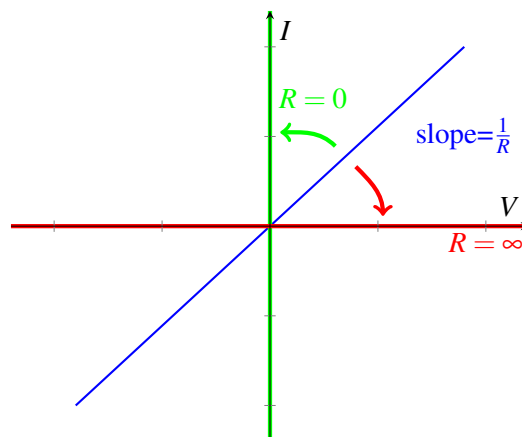


What if we turn off the current source? Then the I-V graph becomes the x-axis, i.e., the line $I = 0$.



What does this mean? This means no matter what voltage you apply, there will be no current. This is equivalent to an open circuit.

Now let's apply this graph interpretation to resistors as well! Recall that by Ohm's law, the I-V graph across a resistor looks like



We know that the slope of the line is equal to $\frac{1}{R}$. What happens when R becomes infinite? Then the line becomes the x-axis, which corresponds to an open circuit as we've seen earlier. Now what happens when R is zero? The line becomes the y-axis, which corresponds to a short circuit.

To summarize, **zero voltage source and zero resistance are equivalent to short circuits; zero current source and infinite resistance are equivalent to open circuits.**