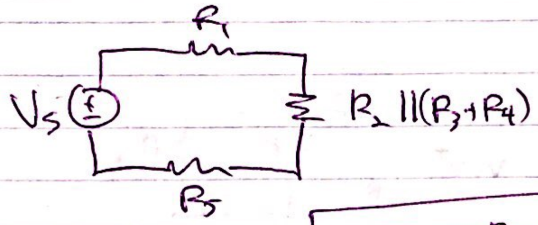
MT3) a) $X \xrightarrow{R_{eq}} B$ $\quad \boxed{R_{eq} = R_1 + (R_2 \| (R_3 + R_4))}$

As $R_4 \to \infty$, $\boxed{R_{eq} = R_1 + R_2}$

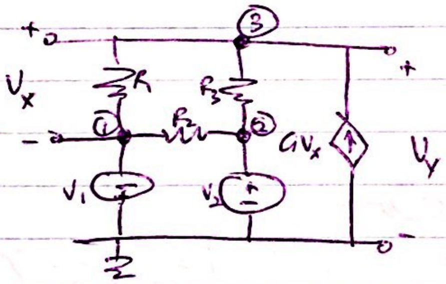
b)



$\therefore i_{total} = \dfrac{V_S}{R_1 + R_5 + R_2 \| (R_3 + R_4)}$

$$\therefore \boxed{i_2 = \frac{R_3 + R_4}{R_2 + R_3 + R_4} \left(\frac{V_S}{R_1 + R_5 + R_2 \| (R_3 + R_4)} \right)}$$

MT 4)



Label nodes ①, ②, ③.

$V_③ - V_① = V_x = \underbrace{V_y - (-V_1)}_{}$

$V_③ = V_y$

$V_① = -V_1$ $\qquad V_y = V_x + V_1$

$V_② = V_2$

Nodal Analysis @ ~~②~~③:

$$\cancel{\frac{V_② - V_①}{R_2} + \frac{V_② - V_③}{R_3} + V_②}$$

$$\frac{V_③ - V_①}{R_1} + \frac{V_③ - V_②}{R_3} + -GV_x = 0$$

$$\frac{V_x}{R_1} + \frac{V_y - V_2}{R_3} - GV_x = 0$$

$$\frac{V_x}{R_1} + \frac{V_x + \cancel{V_1 - V_2}^{\,0}}{R_3} - GV_x = 0$$

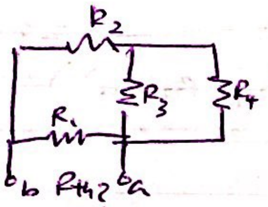
$$\frac{1}{10} V_x + \frac{1}{40} V_x - \frac{1}{4} V_x = 0$$

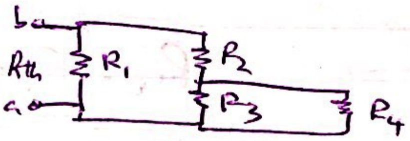
$$\frac{4}{40} V_x + \frac{1}{40} V_x - \frac{10}{40} V_x = 0$$

$$-\frac{1}{8} V_x = 0$$

$$\boxed{V_x = -8\text{ V}} \longrightarrow \boxed{V_y = -3\text{V}}$$

HW 5) a)

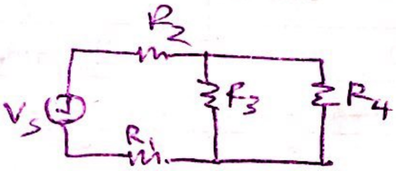




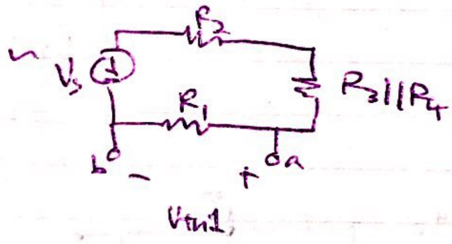
$$\therefore \boxed{R_{th} = R_1 \,\|\, (R_2 + (R_3 \| R_4))}$$

b) $V_{th} = ?$

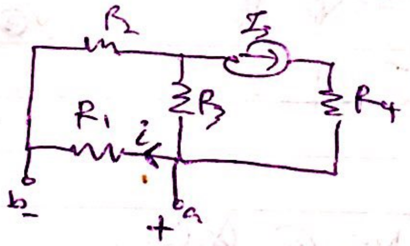
First, null I_s:



$$\therefore V_{th1} = \frac{R_1}{R_1+R_2+(R_3\|R_4)} V_s$$



Next, null V_s:

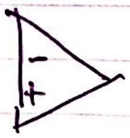


$$i = \frac{R_3}{R_1+R_2+R_3} I_s$$

$$V_{th_2} = i R_1 = \frac{R_1 R_3}{R_1+R_2+R_3} I_s$$

$$\therefore \boxed{V_{th} = V_{th1} + V_{th_2} = \frac{R_1}{R_1+R_2+(R_3\|R_4)} V_s + \frac{R_1 R_3}{R_1+R_2+R_3} I_s}$$

6) a)



b) From the golden rules, we get

$$V^+ = V^- = V_{in_2}$$

And using the knowledge that $I^- = I^+ = 0$, we can use KVL to perform nodal analysis @ the V^- node:

$$\frac{V_{in_2} - V_{in_1}}{2} + \frac{V_{in_2} - V_o}{6} = 0$$

$$\frac{V_o - V_{in_2}}{\cancel{3}} = 3(V_{in_2} - V_{in_1})$$

$$\therefore \boxed{V_o = 4V_{in_2} - 3V_{in_1}}$$

7) a) $F = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$

b) Put F into RREF: $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

We can easily see that there are 2 lin. independent cols,

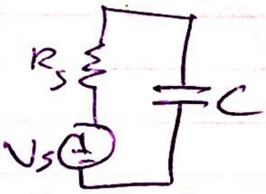
$$\therefore \boxed{\text{rank}(F) = 2}$$

To get null space, solve for $Fx = b$

$$1\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 1\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 1\begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = 0$$

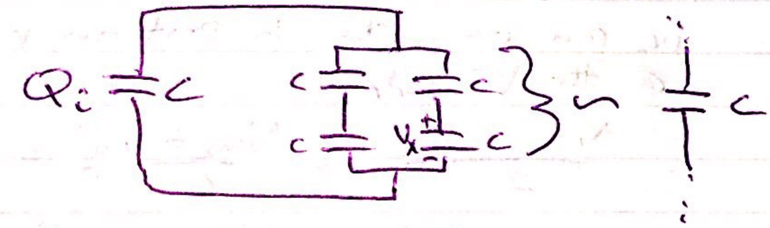
$$\therefore \boxed{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ is the basis of } N(F)}$$

8) When in phase 1:



At steady state, C is charged to V_s,
so $Q_i = CV_s$

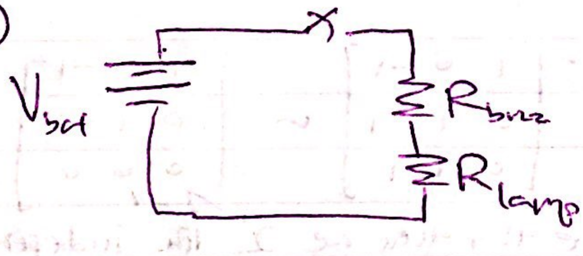
In phase 2:



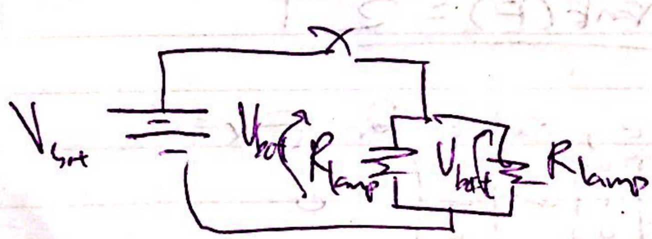
Due to charge sharing, and conservation of charge, on each side there is $Q_i/2$ charges.
That means that on each of the four capacitors on the right, there are $Q_i/4$ charges.

$\therefore V_x = \frac{Q}{C} = \boxed{\frac{Q_i}{4C}}$

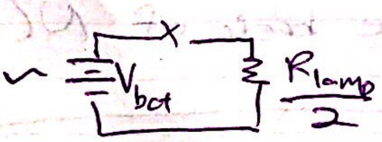
9) a)



b)



Between the choice of putting the lamps in series or parallel, I would put them in parallel to maximize power dissipated.



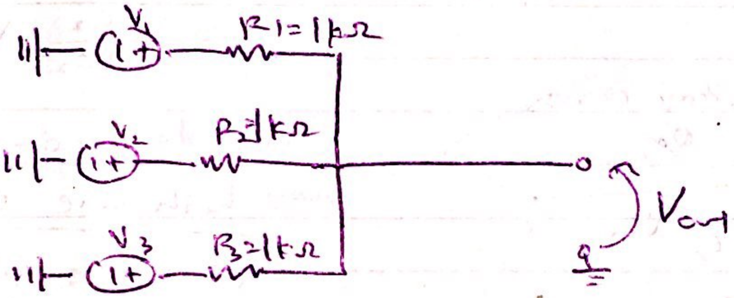
$\therefore I = \frac{2V_{bat}}{R_{lamp}}$ $\quad \therefore P = IV = \boxed{\frac{2(V_{bat})^2}{R_{lamp}}}$

10) a) $$\frac{V_{out}-V_1}{R_1} + \frac{V_{out}-V_2}{R_2} + \frac{V_{out}-V_3}{R_3} = 0$$

$$\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)V_{out} = \frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}$$

$$\boxed{V_{out} = (R_1 \| R_2 \| R_3)\left(\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}\right)}$$

b)



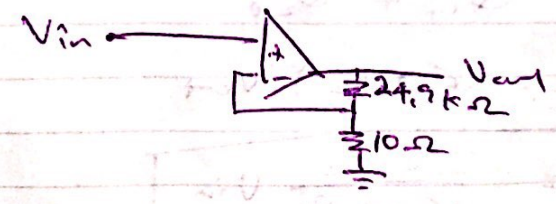
$$\therefore \boxed{V_{out} = \frac{1}{\cancel{R}}(V_1+V_2+\dots+V_n)}$$

which in this case is

$$\boxed{V_{out} = \frac{1}{3}(V_1+V_2+V_3)}$$

c) the gain is $\cancel{\frac{10\,mV}{2.5\,V}}$ $\frac{2.5\,V}{0.01\,V} = \times 250$

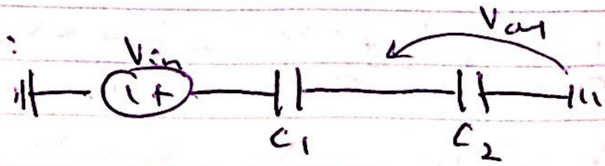
We can use:



$$\therefore V_{out} = \left(1+\frac{24.9k\Omega}{10\Omega}\right)V_{in}$$

$$\boxed{V_{out} = 250\,V_{in}}$$

11) a) in phase I :



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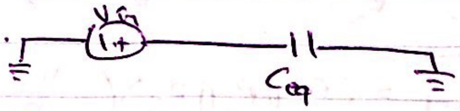


$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \qquad \therefore Q_i = C_{eq} V_{in}$$

$$= \frac{C_1 C_2}{C_1 + C_2} V_{in}$$

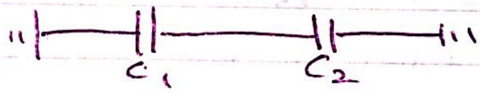
Since the capacitors are in series, they both have equal charge of

$$\boxed{Q_1 = Q_2 = \frac{C_1 C_2}{C_1 + C_2} V_{in}}$$

To find voltage across each, $V_n = Q/C_n$

$$\boxed{\begin{aligned} V_1 &= \frac{C_2}{C_1 + C_2} V_{in} \\ V_2 &= \frac{C_1}{C_1 + C_2} V_{in} \end{aligned}}$$

b) In phase 2 :



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As a function of V_{out}, C_1, & C_2,

$$\cancel{Q_1 = Q_2} = \boxed{Q_1 = C_1 V_{out} \quad , \quad Q_2 = C_2 V_{out}}$$

c) $Q_i = Q_f$

$$Q_{1i} + Q_{2i} = Q_{1f} + Q_{2f}$$

$$\frac{2 C_1 C_2}{C_1 + C_2} V_{in} = (C_1 + C_2) V_{out}$$

$$\therefore \boxed{V_{out} = \frac{2 C_1 C_2}{(C_1 + C_2)^2} V_{in}}$$

d) Efficiency $= E_2 / E_1 = \dfrac{\cancel{C\left(\frac{V_{in}}{2}\right)^2} \; C\left(\frac{V_{in}}{2}\right)^2}{C\left(\frac{V_{in}}{2}\right)^2}$

$$= \boxed{1}$$