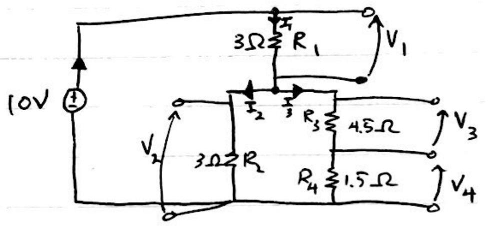
EE 16A HOMEWORK 5 DAVID J. LEE-3031796951

1) Matthew Soh - 3032109159
William Song - 3031799759
I worked alone for about 3 hours - less this time around b/c of time constraints - and discussed with group on Sunday.

2) a)



Ohm's Law:

$V_1 = 3I_1$

$V_2 = 3I_2$

$V_3 = 4.5I_3$

$V_4 = 1.5I_3$

applying KVL multiple times we get:

$$V_2 = V_3 + V_4$$
$$V_1 + V_2 = 10$$
$$V_1 + V_3 + V_4 = 10$$

applying KCL on the junction we get:

$$I_1 = I_2 + I_3$$

Combining the three sets of equations, we get:

$$\begin{aligned} 3I_2 - 4.5I_3 - 1.5I_3 &= 0 \\ 3I_1 + 3I_2 &= 10 \\ 3I_1 + 4.5I_3 + 1.5I_3 &= 10 \\ I_1 - I_2 - I_3 &= 0 \end{aligned}$$

Solving for the currents, we get:

$$\left[\begin{array}{ccc|c} 0 & 3 & -6 & 0 \\ 3 & 3 & 0 & 10 \\ 3 & 0 & 6 & 10 \\ 1 & -1 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & \frac{4}{3} \\ 0 & 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 & 0 \end{array}\right] \therefore \boxed{\begin{aligned} I_1 &= 2\text{ A} \\ I_2 &= \tfrac{4}{3}\text{ A} \\ I_3 &= \tfrac{2}{3}\text{ A} \end{aligned}}$$

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We can now solve for the voltages across the resistors by plugging in the I_1, I_2, I_3 into the Ohm's Law equations:

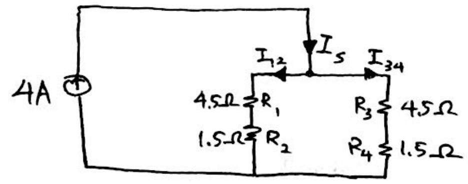
$$V_1 = 3I_1 = 3(2) = \boxed{6\ V}$$
$$V_2 = 3I_2 = 3(\tfrac{4}{3}) = \boxed{4V}$$
$$V_3 = 4.5I_3 = 4.5(\tfrac{2}{3}) = \boxed{3V}$$
$$V_4 = 1.5I_3 = 1.5(\tfrac{2}{3}) = \boxed{1\ V}$$

Thus, the voltages across and currents flowing through each resistor is given by:

R_1 : $6V$, $2\ A$	R_3 : $3V$, $\frac{2}{3}A$
R_2 : $4V$, $\frac{4}{3}\ A$	R_4 : $1V$, $\frac{2}{3}A$

b) Disregarding R_5, we can see that (R_1+R_2) parallels and is symmetrical to (R_3+R_4). Thus we can safely analyze the circuit as if R_5 wasn't there.

Also, the symmetry tells us that $I_S = 4A$ splits at the first junction evenly and $\boxed{I_{12} = I_{34} = 2A}$:



Voltages across each resistor now can be calculated easily applying Ohm's Law,

$$V_1 = I_{12}R_1 = 2(4.5) = \boxed{9\ V = V_1}$$
$$V_2 = I_{12}R_2 = 2(1.5) = \boxed{3\ V = V_2}$$
$$V_3 = I_{34}R_3 = 2(4.5) = \boxed{9V = V_3}$$
$$V_4 = I_{34}R_4 = 2(1.5) = \boxed{3V = V_4}$$

And simply, $\boxed{V_5 = 0}$

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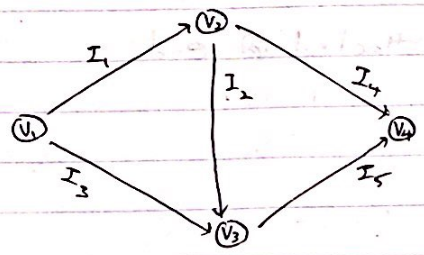
3) a)



$$F = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

b) let $\vec{v}$ represent the voltage nodes.

$$F\vec{v} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} V_1 - V_2 \\ V_2 - V_3 \\ V_1 - V_3 \\ V_2 - V_4 \\ V_3 - V_4 \end{bmatrix}$$ is a vector of ΔV, the voltage differences.

since $R = \begin{bmatrix} R_1 & & & & \\ & R_2 & & & \\ & & R_3 & & \\ & & & R_4 & \\ & & & & R_5 \end{bmatrix}$ is the diagonal matrix of branch matrices and $\vec{i} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix}$ the current vector,

We can rewrite the Ohm's Law ($V = IR$) as:

$$\boxed{F\vec{v} = R\vec{i}}$$

c) First, taking $F\vec{v} = R\vec{i}$ and solving for $\vec{i}$, $\vec{i} = R^{-1}F\vec{v}$.

also, $f = \begin{bmatrix} -I \\ 0 \\ 0 \\ I \end{bmatrix}$

We know that R, the diagonal matrix, is invertible, and $R^{-1} = G$ where G is the conductance matrix. So we can write $\vec{i} = GF\vec{v}$.

Now, using $F^T\vec{i} + f = 0$, we can write $\vec{v}$ in terms of known quantities by plugging in $\vec{i}$:

$$\boxed{F^T G F \vec{v} = -f}$$

which is in the form $Ax = b$, so $\vec{v}$ can be solved.

d) Solving for $\vec{i}$ from the rewritten Ohm's Law, $\vec{i} = GF\vec{v}$. Then using the above $\vec{v}$,

$$\vec{i} = -GF(F^TGF)^{-1}f = -GFF^{-1}G^{-1}F^{T^{-1}}f$$

$$\boxed{\vec{i} = -F^{T^{-1}}f}$$

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e) See IPython Notebook attached @ end.

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4) a) $P = IV \rightarrow I = \frac{P}{V} = \frac{0.4\,W}{3.8\,V} \approx 0.1053$ Amps

Assume the Galaxy S3's average power draw rate is 0.1053 Amps. Then a full charge of 2200 mAh would last:

$$\frac{2200\,mAh}{105.2632\,mA} \approx \boxed{20.9 \text{ hours}}$$

b) Energy for full charge:

$$0.4\,\frac{J}{s} \times 20.9\,hr \times \frac{3600\,s}{1\,hr} = \boxed{30{,}096\,J},$$

which is $\frac{30096\,J}{3.8\,V} = \boxed{7{,}920\ C}$ Char

c) For a month of 31 days:

$$\$0.16/kWh \times \frac{1\,kW}{10^3\,W} \times 3.8\,V \times 2.2\,Ah \times 31 = \boxed{\$0.040128}$$

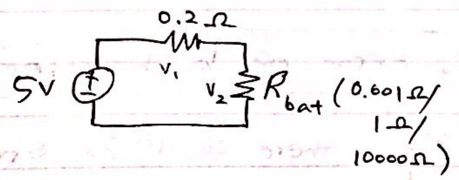
d) In 16 hours, you get $16\,h \times 40\,mA = 640\,mAh$, which is not enough.

How many cells? $2200\,mA \times \frac{1}{40\,mA} \times 3.8\,V \times \frac{1}{0.5\,V} = \boxed{418 \text{ cells}}$

~~e)~~ $\$1.50 \times \frac{1}{0.02\,W} \times \frac{1}{16\,hr \times 365 \times 60 \times 60 \times 10} = \boxed{\$0.00000570776\,/J}$

This is a very silly idea. Who wants the hassle?

e)



$5 = V_1 + V_2$; $V_1 = 0.2I$, $V_2 = R_{bat} I$

$5 = 0.2I + R_{bat} I$

$$I = \frac{5}{0.2 + R_{bat}}$$

@ $R_{bat} = 0.001\Omega$: $I = 24.87562\,A$, $P = I^2R = 0.6188\,W$,

Time to Charge $= 8.6/0.62 \approx 13.5\,hr$

@ $R_{bat} = 1\Omega$: $I = 4.166\bar{6}\,A$, $P = I^2R = 17.36\bar{1}\,W$,

Time to Charge $= 0.48\,hr$

@ $R_{bat} = 10000\Omega$: $I = 0.00049990\,A$, $P = I^2R = 0.00249990\,W$,

Time to Charge ≈ 139 days

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5) a) Similar to question 2) b), we can see that if the current across the galvanometer is 0, the circuit must by symmetrical, and (R_1+R_2) be parallel to (R_3+R_x), and I splits evenly to each parallel branch.

The total resistance can be written as

$$R_{eq} = \frac{1}{\frac{1}{R_1+R_2} + \frac{1}{R_3+R_x}} = \frac{1}{\frac{R_1+R_2+R_3+R_x}{(R_1+R_2)(R_3+R_x)}}$$

$$R_{eq}(R_1+R_2+R_3+R_x) = R_1R_3 + R_1R_x + R_2R_3 + R_2R_x$$

or,

Since $V = IR_{eq} \Rightarrow R_{eq} = \frac{V}{I}$,

$$\boxed{\frac{I}{V} = \frac{1}{R_1+R_2} + \frac{1}{R_3+R_x}}$$

b) The maximum for R_x is $\boxed{250\Omega}$

c) Procedure: Subtract R_x by 100Ω, and multiply by the inverse of the temperature coefficient of PT100.:

$$\text{Temperature of } R_x\ (°C) = (R_x - 100) \times \frac{1\ °C}{0.366\ \Omega} = \frac{R_x - 100}{0.366}.$$

Maximum temp would be @ the max R_x, which is

$$\frac{250-100}{0.366} \approx \boxed{409.8361\ °C}$$

d) $\left(\frac{430.5 - 410}{410}\right) \times 100\% = \boxed{5\% \text{ error on both sides}}$

e) With the 5% max error in R_3, there is 10% error in R_1 $\left(\frac{5}{50} = 0.1\right)$. $\boxed{10\%}$

f) Not very accurate for sure!

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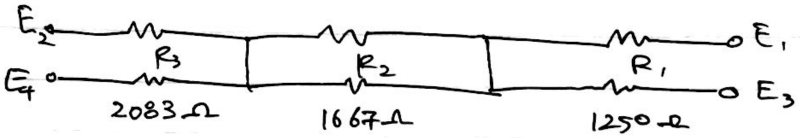
6) a) $R = 8000\,\Omega$

b)



c) Across $E_4 - E_3$:

$R_{eq} = R_1 + R_3 + \frac{R_2}{2}$

$= 4167\,\Omega$

$\therefore V_{E4-E3} = I R_{eq} = \boxed{4.167\,V}$

d) $R_{eq} = R_{plate}\left(\frac{Y_1}{H} + \frac{Y_2 - Y_1}{2H} + \frac{H - Y_2}{H}\right)$

$V = I R_{plate}\left(\frac{Y_1}{H} + \frac{Y_2 - Y_1}{2H} + \frac{H - Y_2}{H}\right)$

$\hookrightarrow Y_2 - Y_1 = 2H\left(1 - \frac{V}{I R_{plate}}\right)$

$\therefore$ It is easy to see that this gives us the information about the difference in height of the two touch positions.

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HW5 Q3

EECS 16A: Designing Information Devices and Systems I, Fall 2016

Circuit solver

In this question we will write a program that solves circuits methodically.

In [89]:
```
import numpy as np
from numpy import linalg
from __future__ import print_function
```

(i) Write the incidence matrix F for the graph.

In [90]:
```
F = np.matrix([[1, -1, 0, 0],
               [0, 1, -1, 0],
               [1, 0, -1, 0],
               [0, 1, 0, -1],
               [0, 0, 1, -1]])
print('\nF:\n',F)
```

```
F:
 [[ 1 -1  0  0]
 [ 0  1 -1  0]
 [ 1  0 -1  0]
 [ 0  1  0 -1]
 [ 0  0  1 -1]]
```

(ii) Specify the resistance matrix R.

In [91]:
```
R1, R2, R3, R4, R5 = 100000, 200, 100, 100000, 100
R = np.matrix([[R1,0,0,0,0],
               [0,R2,0,0,0],
               [0,0,R3,0,0],
               [0,0,0,R4,0],
               [0,0,0,0,R5]])

# For convenience, we will define the conductance matrix G as the inverse
G = np.linalg.inv(R)

print('\nR:\n',R)
print('\nG:\n',G)
```

```
R:
 [[100000      0      0      0      0]
 [     0    200      0      0      0]
 [     0      0    100      0      0]
 [     0      0      0 100000      0]
 [     0      0      0      0    100]]

G:
 [[  1.00000000e-05   0.00000000e+00   0.00000000e+00   0.00000000e+00
    0.00000000e+00]
 [  0.00000000e+00   5.00000000e-03   0.00000000e+00   0.00000000e+00
    0.00000000e+00]
 [  0.00000000e+00   0.00000000e+00   1.00000000e-02   0.00000000e+00
    0.00000000e+00]
 [  0.00000000e+00   0.00000000e+00   0.00000000e+00   1.00000000e-05
    0.00000000e+00]
 [  0.00000000e+00   0.00000000e+00   0.00000000e+00   0.00000000e+00
    1.00000000e-02]]
```

(iii) Write down the vector f so that KCL is satisfied as: $F^T i + f = 0$

In [92]:
```
I = 3
f = np.array([[-I],
              [0],
              [0],
              [I]])
print('\nf:\n', f)
```

```
f:
 [[-3]
 [ 0]
 [ 0]
 [ 3]]
```

(iv) Setting a potential in v to 0 corresponds to deleting a column of F. Write down our new "grounde

In [93]:
```
F_grounded = np.delete(F, np.s_[0:1] ,1)
f_1 = np.delete(f, 0, 0)
print('\nF_grounded:\n', F_grounded)
```

```
F_grounded:
 [[-1  0  0]
 [ 1 -1  0]
 [ 0 -1  0]
 [ 1  0 -1]
 [ 0  1 -1]]
```

(v) Implement your algebraic solution to compute v in terms of F_grounded, G, and f.

Hint: if you have an equation Av=b where A is a square matrix and b is a vector, use np.linalg.solve t

In [94]:
```
A = np.dot(F_grounded.T, np.dot(G,F_grounded))
v = np.linalg.solve(A,-f_1)
print('\nv:\n', v)
```

```
v:
 [[-299.7002997]
 [-299.7002997]
 [-599.4005994]]
```

(vi) Compute $\vec{i}$ with your solution of $\vec{v}$.

In [96]:
```
i = np.linalg.solve(F.T, -f)

print('\ni:\n', i)
```

```
---------------------------------------------------------------------------
LinAlgError                               Traceback (most recent call last
<ipython-input-96-8d19665f8ca0> in <module>()
----> 1 i = np.linalg.solve(F.T, -f)
      2 
      3 print('\ni:\n', i)

/Users/Dawvid/anaconda3/lib/python3.4/site-packages/numpy/linalg/linalg.py
    353     a, _ = _makearray(a)
    354     _assertRankAtLeast2(a)
--> 355     _assertNdSquareness(a)
    356     b, wrap = _makearray(b)
    357     t, result_t = _commonType(a, b)

/Users/Dawvid/anaconda3/lib/python3.4/site-packages/numpy/linalg/linalg.py
(*arrays)
    210     for a in arrays:
    211         if max(a.shape[-2:]) != min(a.shape[-2:]):
--> 212             raise LinAlgError('Last 2 dimensions of the array must
    213 
    214 def _assertFinite(*arrays):

LinAlgError: Last 2 dimensions of the array must be square
```

In [ ]:
```
```