Notes on Pearl 1995: Causal Diagrams for Empirical Research

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Discussion Questions

- Pearl states that whether the "three rules" of the calculus of interventions are sufficient to derive all identifiable causal effects remains an open question. Is that still true? Do the given inference rules align with our intuition about causality?
- Pearl suggests that "standard probability theory" is too weak to "describe the precise experimental conditions that prevail in a given study". Do we agree with this statement? How does this align with Rubin's potential outcome notation?
- The title of the paper explicitly applies causal diagrams to empirical research. Is the relatively abstract "calculus of intervention" put to good use by researchers in the field? If so, where? If not, what obstacles to adoption are there?

1. Introduction

- Classic Cochran example of eelworm control via fumigants can be expressed/analyzed via DAGs
- Causal diagrams help to: 1. Explicitly encode causal assumptions underlying model. 2. Decide if assumptions are sufficient to consistently estimate target causal effect. 3. If yes, gives closed form expression for estimator, If no suggests what observations are necessary.
- Hollow circle indicates unobserved quantity, full circle indicates observed.
- Dashed arrows connect nodes where at least one quantity is not observed, full arrows if both observed.
- Causal assumptions about how one quantity affects another are encoded by present arrows + direction. Missing arrow indicates that one quantity cannot directly affect another.

- Check notation: $\check{x}$ signifies that random variable X is set to fixed value x by external intervention.

2. Graphical Models and the Manipulative Account of Causation

2.1 Graphs and conditional independence.

- DAGs useful for representing conditional independence assumptions, as can identify conditional independence restrictions that are implicit in factorization of joint probability:

$$P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i | pa_i)$$

  where pa_i is some subset of (X_1, X_{i-1}). If construct DAG where variables are in pa_i are represented by the parents of node X_i, then independences implicit in the above factorization can be recovered by d-separation test.

- D-separation: X, Y, Z are three disjoint node sets in a DAG, and p is an (undirected) path between a node in X and on in Y. Z is said to block p if either some node w on p has converging arrows along p and neither w or its descendent are in Z, or there is some node w on p without converging arrows along p and $w \in Z$.

- If for every p between X and Y, we have that Z blocks p, then Z d-separates X, Y: $(X \perp Y | Z)_G$.

- There is 1-1 correspondence between conditional independence assumptions, and triples of nodes that are d-separated.

- Alternative test for d-separation: Delete all nodes from G except those in X, Y, Z and their ancestors. If any two nodes share a common ancestor, connect with undirected arrow. Remove all direction from arrows. If Z is a cut-set of the graph which separates X, Y, then Z d-separates X, Y.

2.2 Graphs as Models of Intervations

- Non parametric structural equations: for each RV, have $X_i = f_i(pa_i, \epsilon_i)$ where f_i is some fixed function and all the ϵ_i are assumed mutually independent disturbances (exogenous factors). If any ϵ_i could plausibly affect multiple X_i, then it should be included in model as unobserved variable with those X_i as children.

- Correspondence between causal diagrams and potential outcomes: Read equality as "is determined by".

- Because each ϵ_i is independent of non-descendants of X_i, child-parent characterization as determinstic function instead of conditional probability leads to same independence constraints. But also provides language of how to specify resulting distribution changes with respect to external interventions.
- Atomic intervention $\text{Set}(X_i = x_i)$ changes functional mechanism determining X_i from $X_i = f_i(pa_i, \epsilon_i)$ to $X_i = x_i$ while keeping all other mechanisms the same.
- Causal effect: for disjoint sets of variables, X, Y, causal effect of X on Y is function from $\mathcal{X}$ to space of distributions on $\mathcal{Y}$. For each $x \in \mathcal{X}$, $P(y|\check{x}) = P(Y = y)$ under new collection of mechanisms which arises from deleting all equations for X_i in X and substituting values x_i into all other mechanisms.
- Graphically, $\text{Set}(X_i = x_i)$ amounts to removing lines from pa_i to X_i and keeping rest of network the same.

3. Controlling Confounding Bias

- Setup is: DAG G, observed variables V_0 from non-experimental context, want to estimate impact of $\text{Set}(X_i = x_i)$ on X_j: $P(x_j|\check{x}_i)$.
- Set of variables Z satisfies back-door criterion relative to X_i and X_j if: No node in Z is a descendent of X_i, and Z blocks every path between X_i and X_j which contains an arrow into X_i (i.e. the back door)
- Point of back-door is not every path between X_i, and X_j need be blocked. Only those which enter from the back-door.
- If Z satisfies back-door criterion for X, Y, then causal effect of X on Y is identifiable, and given by

$$P(y|\check{x}) = \sum_z P(y|x, z)p(z).$$

- Front door criterion: If Z intercepts all directed paths between X and X and: No back door paths between X and Z, every back door path between Z, X is blocked by X. Then the causal effect between X and X is identifiable and given by

$$P(y|\check{x}) = \sum_x P(z|x) \sum_x^{'} P(y|x', z)P(x')$$

- Figure 3 looks nearly identical to the diagram of confounding we teach in intro stats courses. I (Austin) am having a tough time buying that front door criterion holds. If the unobserved affects both X and Y, how can we possibly derive a form for the causal effect from X to Y? I am missing something from this portion.

4. A Calculus of Intervention

- Use diagrams to manipulate causal effects $P(y|\check{x})$
- If can reduce $P(y|\check{x})$ into an expression involving standard (unchecked) probabilities of observed quantities, then causal effects of X on Y is identifiable.
- Three rules governing: insertion/deletion of observables, action/observation exchange, insertion/deletion of actions.
- Not clear if those three rules are sufficient to derive all identifiable causal effects
- Example of using those three rules to derive the front-door expression.
- If we want to estimate $P(y|\check{x})$, but not identifiable. If we can't run experiment on X directly, can we identify another set of variable Z which can be controlled?
- If we can transform $P(y|\check{x})$ into expression with only z checked, then can recover causal effect of X on Y via experiment using Z.

5. Graphical Tests of Identifiability

- Bow pattern is and equation $Y = f_Y(X, U, \epsilon_Y)$ where U is unobserved. It does not permit causal inference.
- Presence of a variable $Z \to X$. Where Z is connected to X, but not to U. This will facilitate the relationship.
- Confounding arc: If there is a back-door path which contains only unobserved variables and no converging arrows along the path, can replace whole path with confounding arc.
- If confounding arc present between X, Y, then $P(y|\check{x})$ cannot be identified, except when linearity assumptions are made (Instrumental Variables)
- For non-parametric models, addition of arcs can impede but not help identifiability, bc reduces possible d-separation.
- Examples + analysis of simple graphs where $P(y|\check{x})$ is identifiable.
- Example similar to Rubin, where identifiability of $P(y|\check{x})$ requires adjusting for concomitant RV, but adjustment has unusual form.
- Local identifiability not necessary for global identifiability.
- Also examples of non-identifiable DAGs.