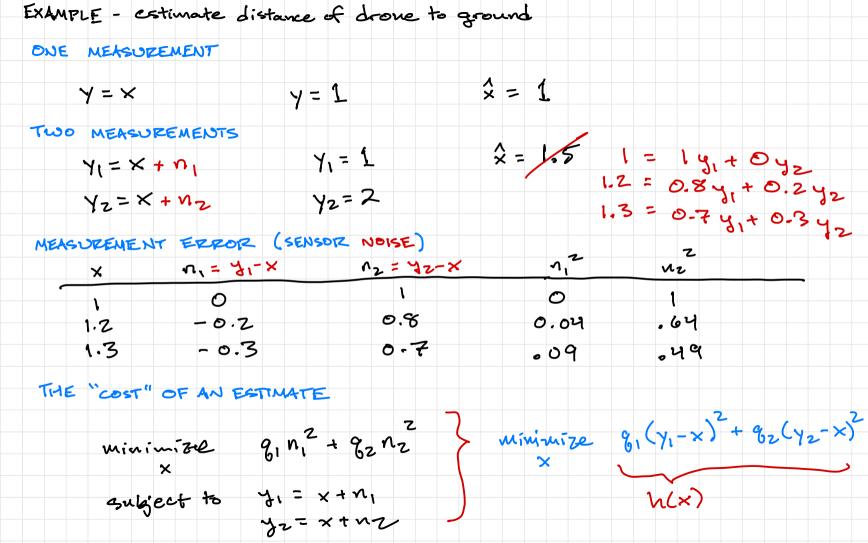
Optimal observers

check on Edri

AE353 Spring 2025 Bret1

< nx number of states x = Ax+Bu na number of inputs y = Cx My number of outputs OPTIMAL CONTROLVER where K=lgr (A, B, Qc, Rc) u=-Kx diagonal w/ positive { Qc is nx x nx & evror numbers { Rc is nu x nx & effort OPTIMAL OBSERVER 2= A2 + Bu-L(C2-y) where L= lg- (AT, CT, Ro, Qo) diagonal w/ positive { Rois nx x nx - Moise a disturbance



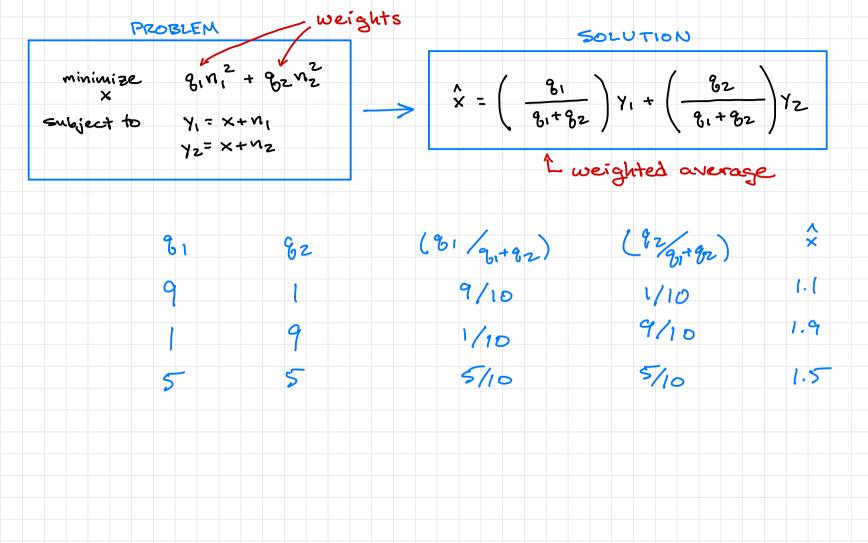
minimize
$$g_1 r_1^2 + g_2 r_2^2$$
 minimize $g_1 (y_1 - x)^2 + g_2 (y_2 - x)^2$
subject to $y_1 = x + r_1$
 $y_2 = x + r_2$

$$0 = \frac{\partial h}{\partial x} = -2g_1(y_1 - x) - 2g_2(y_2 - x)$$

$$= -2(g_1y_1 - g_1x + g_2y_2 - g_2x)$$

$$= -2((g_1y_1 + g_2y_2) - (g_1 + g_2)x)$$

$$x = \frac{g_1 y_1 + g_2 y_2}{g_1 + g_2} = \left(\frac{g_1}{g_1 + g_2}\right) y_1 + \left(\frac{g_2}{g_1 + g_2}\right) y_2$$



(x(t) TQ x(t) + u(t) TR u(t)) dt
effort minimi ze u=-Kx u[+0,∞) x(+) = Ax(+) + Bu(+) for +∈[+0, ∞) subject to OPTIMAL $x(10) = x_0$ CONTROLLER OPTIMAL OBSERVER (n(t) TQon(t) + d(t) TRod(t)) dt - so sensor noise process disturbance minimize x(+,) x(+) = Ax(+) + Bu(+)+d(+) } for + E(-0, +,]
y(+) = Cx(+) + n(+) subject to SUPPOSE: THEN: the solution to $x(t_1) = \alpha$ if $t_1 = t_{\alpha}$ $|\hat{x} = A\hat{x} + Bu - L(C\hat{x} - y)|, \hat{x}(t_{\alpha}) = \alpha$ $X(t_1) = \beta$ if $t_1 = t_{\beta}$ is x(tB) = B

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minimize
$$x(t_1) = \begin{cases} (n(t)^T Q_0 n(t) + d(t)^T R_0 d(t)) dt \\ x(t_1) = \\ x(t_1) \end{cases}$$
subject to
$$x(t) = A \times (t) + B u(t) + d(t)$$
for $t \in (-\infty, t_1]$

$$y(t) = C \times (t) + u(t)$$
for $t \in (-\infty, t_1]$

$$y(t) = C \times (t) + u(t)$$
for $t \in (-\infty, t_1]$

$$y(t) = C \times (t) + u(t)$$

$$y(t) = u(t) + d(t)$$

$$y(t) = v(t) + u(t)$$

How exactly should we choose Q, and Ro? Define: n = y - (Cx+Du) Q = diag(q1, q2, ...)

Suppose of is the standard deviation of Ni. Then a good choice is:

Why: If o: is small, then gi will be big and so you are saying you trust the ith sensor a lot - which is exactly what

you should do if your model of

this sensor has low noise.

LOR is a standard way to choose L minimize $\int_{-\infty}^{\tau_i} \left(n(t)^T Q_{i} u(t) + d(t)^T R_{i} d(t) \right) dt$ subject to x(+) = Ax(+) + Bu(+) + d(+) y(+) = Cx(+)+Du(+)+ n(+) The solution is: \$ = A\$ + Bu-L (C\$+Du-y)

L = lgr (AT, CT, Ro, Qo) 1 L Q big => n small => TRUST SENSORS Robig => d small => TRUST DYNAMIC MODEL 8; = (1/o;) = inverse of error variance Choose Ro in exactly the some way, by looking at the error x - (Ax+Bu)

this is just intuition - it is possible to prove

that this choice of Qis, in fact, optimal

in the linearized EOMs.