In this project, you will work with a small group of peers to select an engineering-relevant initial boundary value problem (IBVP), decide on an appropriate numerical method to solve that problem, and perform systematic studies that let you explore something that excites you about the IBVP that you've chosen.

Your project grade will be based on an evaluation of a technical report that you submit. Your report should be typeset in Latex, and your submission should be the .pdf output of that document. The grade will be based on the following components. You have freedom in what report structure you choose, but may wish to adopt a report structure that mirrors the evaluation criteria below:

1. (22 points) Present an IBVP that is relevant to engineering. (i) Describe why that problem is interesting/important to understand, (ii) synthesize a question or set of questions that you want to ask of that IBVP, and (iii) convey your IBVP mathematically. Be sure that you specify any parameters that the IBVP depends on, and which parameter value ranges are relevant to the specific questions you want to explore. **Since you already covered the heat equation in your individual project, do not choose this problem unless you pursue significant extensions to what a typical individual project output would have looked like**. Similarly, be thoughtful about your choice of initial boundary value problem. The Euler Bernoulli beam equation could be interesting but has more spatial derivatives than you are used to dealing with. A nonlinear IBVP could introduce complexities from the purely linear cases we've considered. Extensions of IBVPs to higher spatial dimensions could be interesting, but that added spatial dimension adds significant additional complexity. A key part of the process to this first part is an iterative process where you give your group structured space to explore problems that excite you, find intractable challenges, scope down, change which IBVP you are considering, and so on. Finding an ambitious but doable problem should take time and struggle. Embrace that!

2. (22 points) Present a numerical method that is appropriate to study the questions that you've asked of the IBVP. Your presentation will be evaluated based on (i) your justification for why the method is appropriate to address the questions you want to probe, using accuracy, stability, and cost considerations; (ii) your mathematical derivation of the method; (iii) an algorithmic summary of how the method computes the numerical solution as a function of space and time. Be sure to leverage course concepts in presenting and deriving the numerical method. **Methods derived using ChatGPT, that let ChatGPT use a derivation completely different from the approach used in class will be penalized**. By all means, use ChatGPT to explore and present new methods, but be sure to iterate with chatGPT to be sure the methods you use are cast within the framework and language we've built in the class over an entire semester.

3. (22 points) Demonstrate correct implementation of the method. This demonstration should include (i) appropriate error convergence studies that shows your method scales at the expected convergence rate; (ii) an evaluation of which simulation parameters (e.g., time step size) your method needs to use so that you can accurately study the problem of interest.

4. (22 points) Present results that meaningfully address the question you ask. The results should utilize professional, concise text along with clear companion figures that get at the heart of the questions you pose in part 1. Be thoughtful about these results! Raw results that plot your solution at various time instances are rarely the most direct way of addressing the specific questions you posed.

5. (6 points) Provide a summary description of everyone's roles in the project. These summary descriptions need not provide a rough estimate of the percentage of the project that each group member contributed. But a clear, synthesized summary of the tasks performed by each team member should be provided. There is flexibility in how the group structure is set up. Everyone can work equally on every part of the project, or you can provide team leads for things like defining the problem statement, picking and justifying the method, coding the method, etc. But each student must meaningfully contribute to portions of the project that pertain to the technical content of the course: numerical methods. Insufficient contribution from a team member could lead to point deductions from the nominal group grade.

6. (6 points) Reproducibility. You should provide a link to your code that contains the full version of the code, including whatever is needed to run the convergence tests and generate any results

that you use in your report. For full credit, the code must be well organized, ideally distinguishing between the code that implements the numerical method (perhaps as a function that time marches the solution), and the other codes that perform convergence tests and generate results. **Code that is hosted as a well-constructed repository on Github can be awarded up to 3 bonus points on the group project grade.**

Remember that the group portion of project 2 is worth 17.5% of your semester grade. Beyond that: if your group project 2 grade is higher than your project 1 grade, then I will redistribute the weight of the projects to place more emphasis on project 2 as follows:

Grading scheme from Canvas: Group project 1 = 15%, group project 2 = 17.5%.

If your group project 2 grade is higher: Group project 1 = 7.5%, group project 2 = 25%.

Between the significant weight of the project, the ability to leverage multiple team mates in creating your report, and the added time you will have to complete it, I am looking for you to go well beyond what is expected from a homework submission. Going "beyond what is expected" will mean different things to different people. I want to build in flexibility for that "above and beyond" component to be driven by something you're passionate about. Just be sure to work hard and have fun!

As always, chatGPT usage is encouraged. It may not be tractable to provide a single chat transcript or set of chat transcripts from your interactions with chatGPT. Instead, please create an Acknowledgments section where you make it clear the role that chatGPT had in your project. The role chatGPT plays should reflect that your group drove the project directions, pushing chatGPT to complete tasks and fact checking what it does!