Lagrangian Equations

Chapter 6

October 25, 2003

The Lagrangian is

$$L = \frac{m}{2} g_{ij} \frac{dx^i}{dt} \frac{dx^j}{dt} \tag{1}$$

and the Euler-Lagrange equations are

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}^i} - \frac{\partial L}{\partial x^i} = 0 \tag{2}$$

where $\dot{x}^i = dx^i/dt$

Now,

$$\frac{\partial L}{\partial \dot{x}^i} = m g_{ij} \dot{x}^j \tag{3}$$

Then we put this into the equation:

$$\frac{d}{dt}(m g_{ij} \dot{x}^j) - \frac{m}{2} \dot{x}^k \dot{x}^j g_{kj,i} = 0 \tag{4}$$

You should be able to show that this is equivalent to the metric tensor. But for your question, you should put in the expression for the metric tensor and the three components of this equation become the desired results.

OK?