Constraining the Hemispherical Structure in the Hidden Layer At the Top of the Earth's Inner Core

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January 20, 2015

Abstract

Since its discovery in 1936, the Earth's inner core has been well documented by both body wave and normal mode studies. However, one area where properties are not yet well measured is the top of the inner core. The upper region of the inner core is of particular interest as it is thought that as the outer core freezes onto the inner core the variable environment at this boundary is encoded in the properties of the frozen material.

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1 Introduction

2 Theoretical Background

This project centres around using seismic body wave analysis in order to investigate the velocity structure of the upper inner core. These are elastic waves, caused by earthquakes, that travel through the interior of the Earth. Under the assumptions of a continuous, linearly elastic medium, infinitesimal strains and constant medium properties one can derive the elastic wave equation[1]

$$\rho \frac{\partial^2 \vec{u}}{\partial t^2} = (\lambda + \mu) \nabla (\nabla \cdot \vec{u}) + \mu \nabla^2 \vec{u} \tag{1}$$

where u is the displacement vector, ρ the density of the medium and λ and μ Lamé parameters of the medium. A general displacement vector can be decomposed into a irrotational scalar and solenoidal vector potentials such that

$$u = \nabla \phi + \nabla \times \vec{\psi} \tag{2}$$

Solving this equation can be achieved for high-frequency waves, or rays, yielding the Eikonal equation which describes how the rays propagate through a given velocity field. This high frequency ray approximation is valid for $\delta (\Delta c (\vec{x})) \ll c (\vec{x})$.

Substituting (2) in to (1) yields two independent wave equations, one for ϕ and one for $\vec{\psi}$, which describe P-waves and S-waves respectively. These equations have different but constant phase velocities. P-waves are compressional with motion occurring parallel to the wave vector, whereas S-waves are transverse with motion occurring perpendicular to the wave vector.

In general we cannot make the assumption that the medium has constant parameters; this certainly is not the case for the Earth! To take varying properties into account the constant phase velocity wave equation can be generalised to have a position dependant velocity

$$\frac{\partial^2 \phi}{\partial t^2} = c^2 (\vec{x}) \nabla^2 \phi \tag{3}$$

the condition for this approximation to be valid is

Because the outer core is liquid with $\mu \approx 0$ and thus does not transmit S-waves, it is P-waves that are used to sample the inner core.

3 Papers

- Nissen-Meyer et al. (2014) - Describes the AxiSEM waveform modelling software. Discusses the need for full waveform modelling, and the computational constraints that AxiSEM overcomes.
- Waszek & Deuss (2013) - Calculate attenuation properties, after taking into account velocity structure from Waszek & Deuss (2011).
  - Large enough earthquakes to provide visible signal.
  - Deep enough to prevent surface reflection interference.
  - Filter from 0.7 Hz to 2 Hz.

[1] This section is essentially summarised from Shearer (2009)

References

Nissen-Meyer, T., van Driel, M., Stähler, S. C., et al. 2014, Solid Earth, 5, 425

Shearer, P. M. 2009, Introduction to Seismology, 2nd edn. (Cambridge University Press)

Waszek, L., & Deuss, A. 2011, Journal of Geophysical Research, 116, B12313

—. 2013, Geophysical Journal International, 195, 2005