Helios distribution fitting

David Stansby

August 22, 2017

Abstract

A technical summary of the fitting method applied to Helios distribution functions.

1 1D distributions

The 1D distributions are reduced 3D distributions, which were integrated over all solid angles on board the spacecraft before being transmitted. They are given in the raw files as distribution function[1] as a function of speed. The particles were measured in energy per charge (E/q) bins, and the speeds calculated assuming all particles were protons; for bin with a retarding potential V:

$$Vq_p = \frac{1}{2}m_p v^2 \tag{1}$$

$$v = \sqrt{\frac{2Vq_p}{m_p}} \tag{2}$$

Particles with a differing m/q ratios will therefore appear at a velocity that is $\sqrt{m/q}$ higher than their true velocity.

1.1 I1a

Figure 1 shows an example of a 1D I1a distribution function.

1.2 I1b

Figure 1 shows an example of a 1D I1b distribution function. Because I1b measured current and not particle counts, a particle with a charge q will contribute a factor q more to I1b compared to I1a. When the proton core of each distribution is normalised to one, a second population at higher velocities lies a factor of 2 higher in I1b, consistent with the population being alpha particles.

1.3 1D Fits

- If there are no points in (I1a/I3) **or** I1b no fitting is done (error codes 7 or 8).

Points in (I1a/I3) that are greater than the lowest count rate measured are removed. Points in I1b that are greater than 2 times the lowest count rate measured are removed. This removes points that are at the noise floor.

[1] We are unsure of the units, but know by comparision to the 3D distribution functions that they are dimensions T/L^4, as a standard 1D reduced distribution function is expected to have.

- If there are now no points in (I1a/I3) **or** I1b no fitting is done (error codes 7 or 8).
- If there are now less than 6 points in (I1a/I3) **or** I1b no fitting is done (error code 5).

In order to filter out corrupted distribution functions the following criteria are applied:

- I1a
  - If the maximum I1b distribution function value is more than 10 times the maximum I1a distribution function value no fitting is done (error code 9)
- I3
  - If the maximum I3 distribution function value is more than 5 times the maximum I1b distribution function value no fitting is done (error code 9)

Initial guesses for the proton core are taken as

- A: The maximum of (I1a/I3)
- v: Velocity at maximum value of (I1a/I3)
- v_{th}: 40 km/s

The I1b distribution is then fitted to one Maxwellian:

$$f\left(v; n, v_0, v_{th}\right) = n \cdot 4\pi v^2 \cdot \left(\frac{1}{\pi v_{th}^2}\right)^{3/2} e^{-\frac{v-v_0}{v_{th}}} \tag{3}$$

The fitting is done using least squares minimisation[2] of the residuals calculated from $(f_{data} - f_{fit})$ to get best estimates of the 3 parameters. Possible error codes are given in table 1. The output variables of each individual fit is given in table 2. If the fitting fails at any stage, all the parameters are set to *nan* and still saved.

The only fit parameter used for the subsequent 3D fits is the thermal speed.

2 3D distributions

The 3D distributions given as distribution function values as a function of v_x, v_y, v_z.

- If the 1D fit indicates a corruped I1a distribution (error code 9, table 1), no 3D fitting is done.

Initially the following points are removed from all distribution functions:

1. Points with a single count
2. Points with counts ≥ 32768
3. Energy bins 1, 2 and 3 (assumed to consist of only noise)
4. Points with $|\mathbf{v}|$ greater than 1,000 times the greatest velocity bin in the 1D I1a distribution

- If the distribution now has ≤ 6 points, no 3D fitting is done (error code 5)

[2] `https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.least_squares.html`

- If the distribution now has less than 3 bins resolution in the ϕ direction, no 3D fitting is done (error code 12)

- If the distribution now has less than 3 bins resolution in the θ direction, no 3D fitting is done (error code 12)

- If the minimum $|\mathbf{v}|$ is greater than the velocity at which the I1a 1D distribution is a maximum, no 3D fitting is done (error code 10)

A check is then done for magnetic field data availability. If 4Hz data is available when the distribution was measured, that is used. Otherwise if 6s data is available that is used. An average ($\mathbf{B}_0$)[3] and standard deviation of each component taken. The velocities are then rotated into the frame aligned with $\mathbf{B}_0$. If no magnetic field data is present, fitting is done in the original spacecraft frame and only the bulk velocity parameters saved.

A 3D bi-Maxwellian fit is done of the following form (fit parameters in bold):

$$f\left(v_{\|}, v_{\perp 1}, v_{\perp 2}\right) = \mathbf{A} \cdot \exp - \left\{ \left(\frac{v_{\|} - \mathbf{u}_{\|}}{\mathbf{w}_{\|}} \right)^2 + \left(\frac{v_{\perp 1} - \mathbf{u}_{\perp 1}}{\mathbf{w}_{\perp}} \right)^2 + \left(\frac{v_{\perp 2} - \mathbf{u}_{\perp 2}}{\mathbf{w}_{\perp}} \right)^2 \right\} \tag{4}$$

The 6 fit parameters are amplitude (A), 3 velocities ($u_{\|}$, $u_{\perp 1}$, $u_{\perp 2}$), and 2 thermal speeds ($w_{\perp}$, $w_{\|}$). Initial guesses for these parameters are given in table 3.

The fit is done using least squares minimisation[4] of the residuals calculated from ($f_{data} - f_{fit}$).

- If the least squares fitting function doesn't return a success status code, results are ignored (error code 6)

- If the amplitude returned is greater than 10 times the maximum value of the distribution function, it is physically unrealistic and results are ignored (error code 11)

- If the amplitude returned is less than 0.1 times the maximum value of the distribution function, it is physically unrealistic and results are ignored (error code 11)

- If the amplitude returned is negative, results are ignored (error code 11)

- If the bulk velocity is outside the 3D array of velocities where the distribution function was measured, results are ignored (error code 4)

The number density is calculated using

$$n = A \cdot \pi^{3/2} w_{\perp} w_{\perp} w_{\|} \tag{5}$$

and temperatures calculated using

$$T_{\perp/\|} = \frac{m_p w_{\perp/\|}^2}{2k_B} \tag{6}$$

Possible error codes are given in table 4.

The output dictionary of each individual fit is given in table 5. If the fitting fails at any stage, all the parameters are set to *nan* and still saved. In addition to the fitted parameters, some orbital data from the merged data files is added to the output.

Figure 2 shows an example of 3D distributions and fitting. The bottom panel illustrates that the fitting method is not sensitive to the beam population.

[3]For Helios 2 the B_y and B_z 4Hz components are flipped to agree with the 6s magnetic field data

[4]`https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.least_squares.html`

3 Caveats

A Tables and figures

Code	Explanation	% points
1	Fitting successful	86.1
5	Less than 6 points available for fitting	12.9
7	No non-noise I1a data available	0.19
8	No non-noise I1b data available	0.14
9	Distribution file is corrupted	0.18

Table 1: Possible error codes returned by 1D fitting process. Percentage points are for Helios 2 entire mission, with total number of distributions = 892,905.

Parameter	Parameter name	Units
Proton number density	n_p	cm^{-3}
Proton velocity	v_p	$km \cdot s^{-1}$
Proton thermal speed	vth_p	$km \cdot s^{-1}$
Proton temperature	T_p	Kelvin
Time	Time	
Fitting status	status	
Instrument	Instrument	

Table 2: Variables output by the 1D fitting process

Parameter	Initial guess 1	Initial guess 2
Amplitude	Maximum 3D distribution function value	
Bulk velocity	Numerical moment from whole 3D distribution function	
Thermal speeds	1D thermal speed fit value	40 km/s

Table 3: Initial guesses for the 3D fit parameters

Code	Explanation	% points
1	Fitting successful	65.6
2	No magnetic field data available	30.4
4	Fitted velocity outside measurement array	0.33
5	Less than 6 points available for fitting	0.40
9	Distribution file is corrupted	0.18
10	1D proton peak not present in 3D distribution	1.39
11	Number density physically unrealistic	0.88
12	Less than 3 angular bins present in either direction	0.69

Table 4: Possible error codes returned by 3D fitting process. Percentage points are for Helios 2 entire mission, with total number of distributions = 892,905.

Parameter	Parameter label	Units
Proton number density	n_p	cm^{-3}
Proton x velocity	vp_x	$km \cdot s^{-1}$
Proton y velocity	vp_y	$km \cdot s^{-1}$
Proton z velocity	vp_z	$km \cdot s^{-1}$
Proton perpendicular thermal speed	vth_p_perp	$km \cdot s^{-1}$
Proton parallel thermal speed	vth_p_par	$km \cdot s^{-1}$
Proton perpendicular temperature	Tp_perp	Kelvin
Proton parallel temperature	Tp_par	Kelvin
B_x	Bx	nT
B_y	By	nT
B_z	Bz	nT
Magnetic field standard deviation	sigma B	nT
Time	Time	
Fitting status	status	
Instrument	Instrument	
Magnetic field instrument	B instrument	
Sun-spacecraft distance	r_sun	AU
Carrington longitude	clong	Degrees
Carrington lattitude	clat	Degrees
Carrinton rotation number	carrot	
Spacecraft-Earth angle	earth_he_angle	Degrees

Table 5: Final data output. The top set of values are the fitted parameters, the middle set give information on the fit, and the bottom set are Helios orbital parameters.

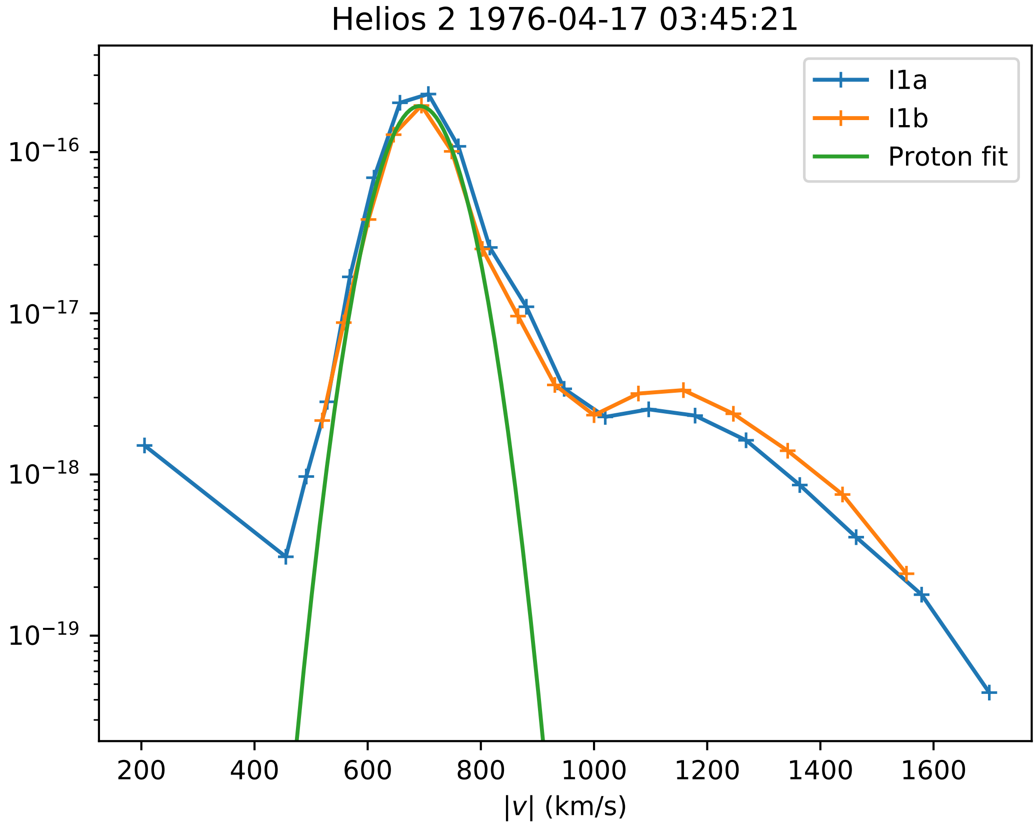

Figure 1: 1D distribution function examples. Blue line shows I1a distribution function and orange line shows I1b distribution for the same interval. Green line shows the result of 1D Maxwellian fitting.

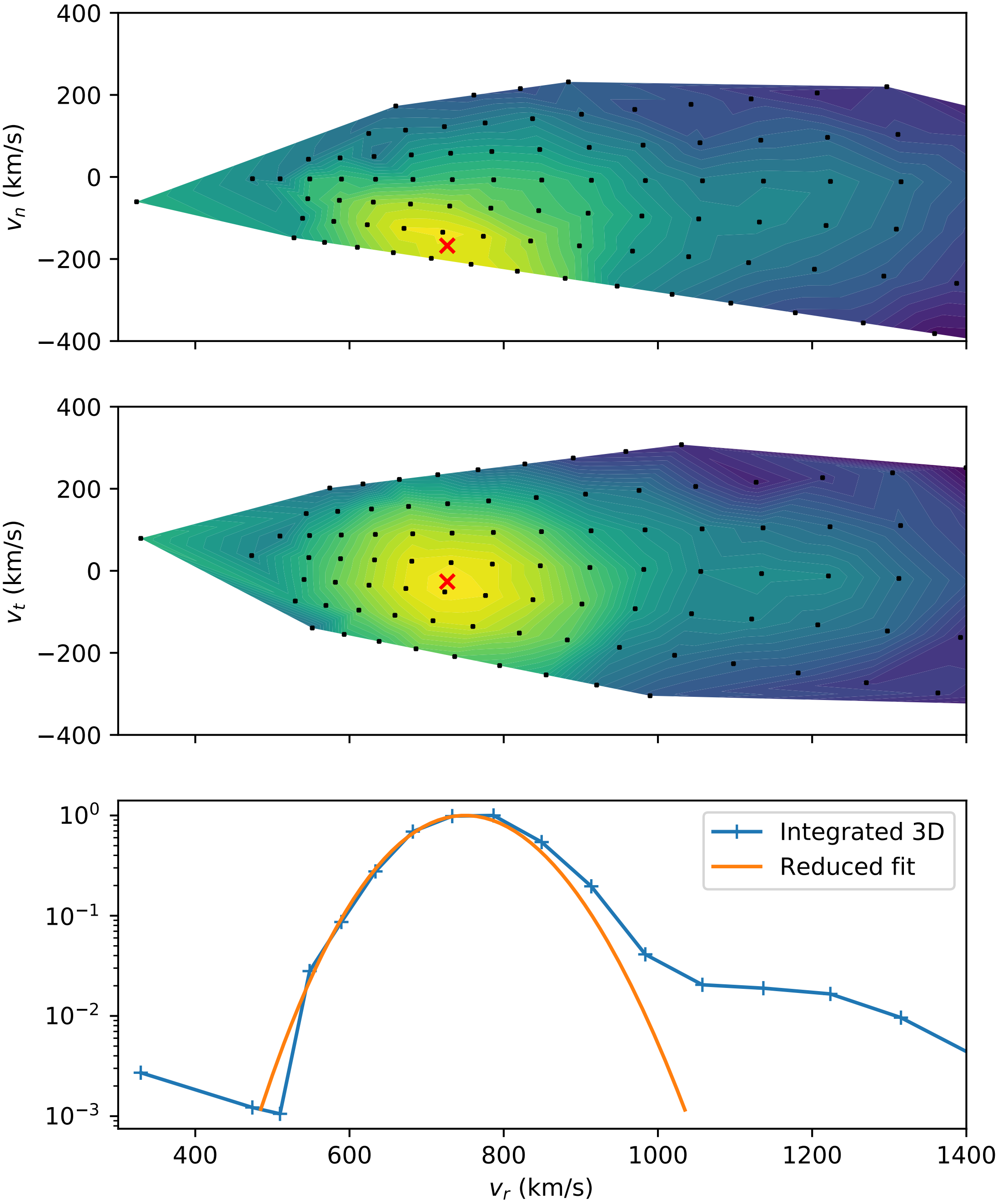

Figure 2: Top panel shows reduced 2D distribution function in the R-N plane in a RTN coordinate system. Middle panel shows reduced 2D distribution function in the R-T plane. Red crosses denote the bulk velocity of the fitted bi-Maxwellian. Bottom panel shows reduced 1D distribution function in blue, and the fitted bi-Maxwellian numerically reduced to 1D in orange.