emcee: An Affine-Invariant Sampler

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Symmetries Graduate School 2023-01-30

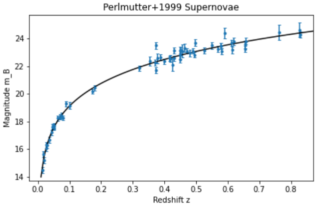
Borrowing heavily from Dan Foreman-Mackey's slides
https://speakerdeck.com/dfm/data-analysis-with-mcmc1
These slides are available at

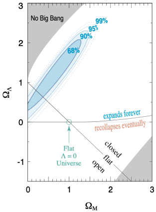
https://github.com/dstndstn/MCMC-talk/emcee-slides

Recap from last week's lecture (1)

- Markov Chain Monte Carlo (MCMC) *draws samples from a probability distribution* when you can *numerically evaluate* the probability function (up to a constant)
- Used extensively in data analysis: *inferring* parameters of models, given observed data
- *Usually* in a Bayesian context; the probability function we run MCMC on is the *posterior* probability:

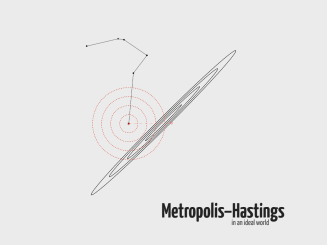
$$\text{posterior}(\text{params}|\text{data}) \propto$$
$$\quad \text{prior}(\text{params}) \times \text{likelihood}(\text{data}|\text{params})$$



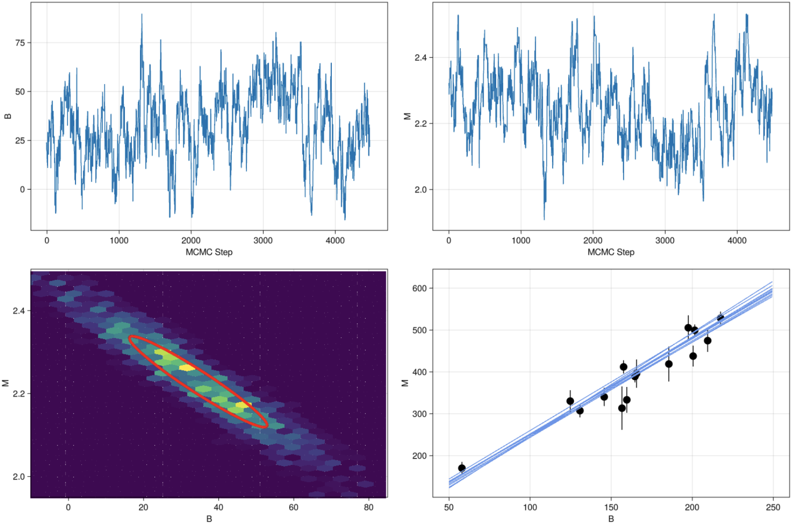


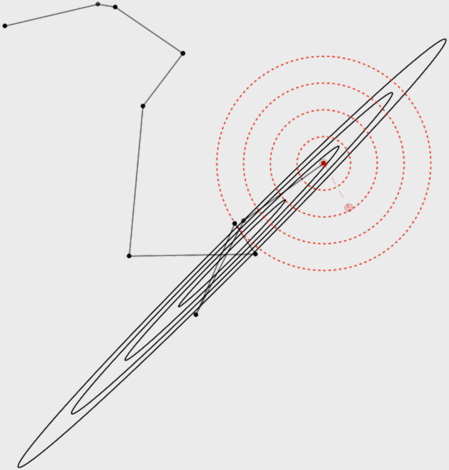
Recap from last week's lecture (2)

- The "classic" Markov Chain Monte Carlo algorithm is *Metropolis–Hastings*, which moves a *walker* or *particle* around the *state space* (*model parameter space*)
- A randomly-drawn *proposed* jump gets *evaluated* (by calling the probability function), and then *accepted*, or not
- A big difficulty is to *customize* the *proposal distribution* to get the algorithm to work efficiently

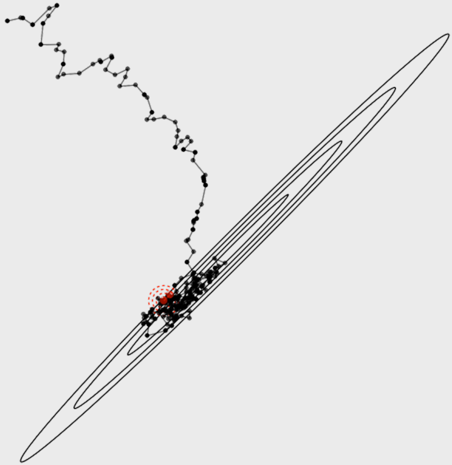


MCMC for model parameter inference





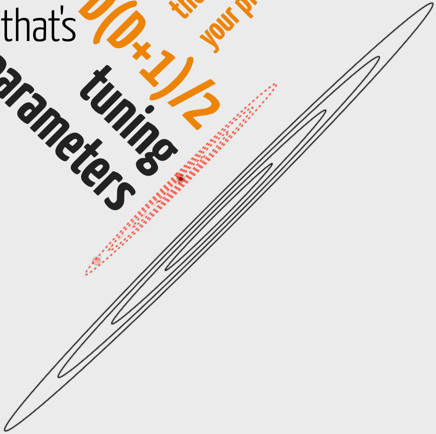
Metropolis–Hastings
in the real world



Metropolis–Hastings
in the real world

that's D(D+1)/2 tuning parameters

the dimension of your problem



Metropolis–Hastings

in the real world

that's

the dimension of
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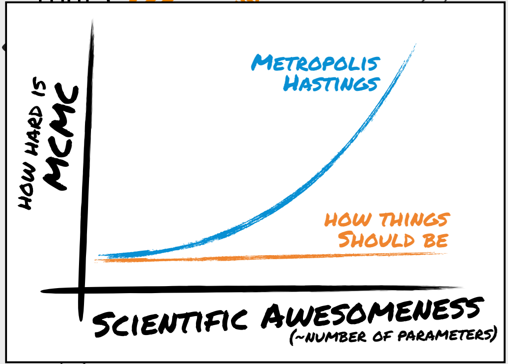


Metropolis–Hastings
in the real world

Jonathan Goodman

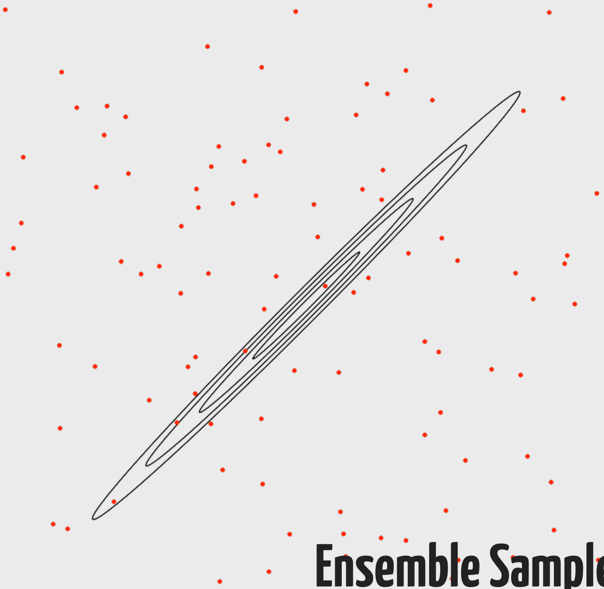
Jonathan Weare

"Ensemble samplers with affine invariance"

(dfm.io/mcmc-gw10)

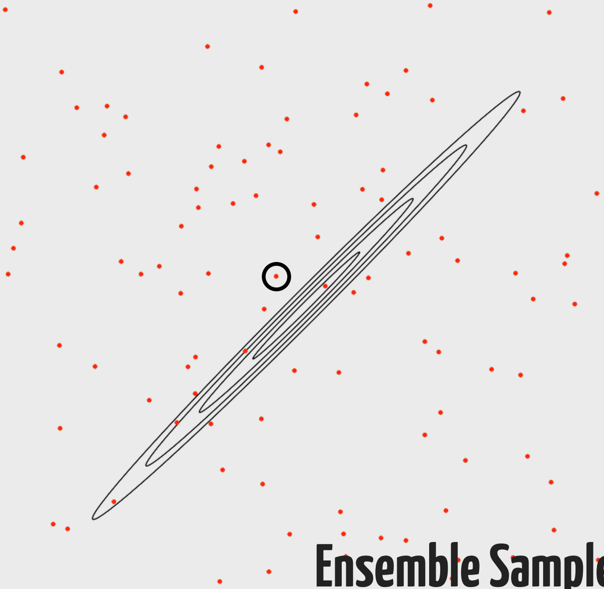
introducing **emcee** the MCMC Hammer

arxiv.org/abs/1202.3665
dan.iel.fm/emcee



Ensemble Samplers

in the **real** world



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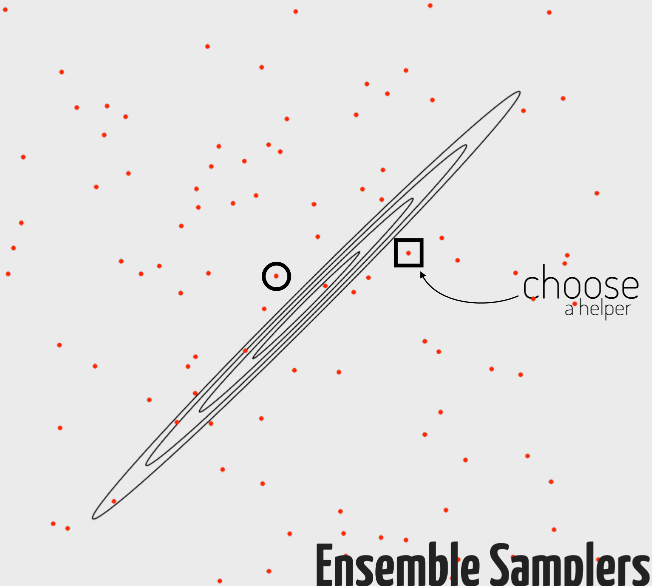

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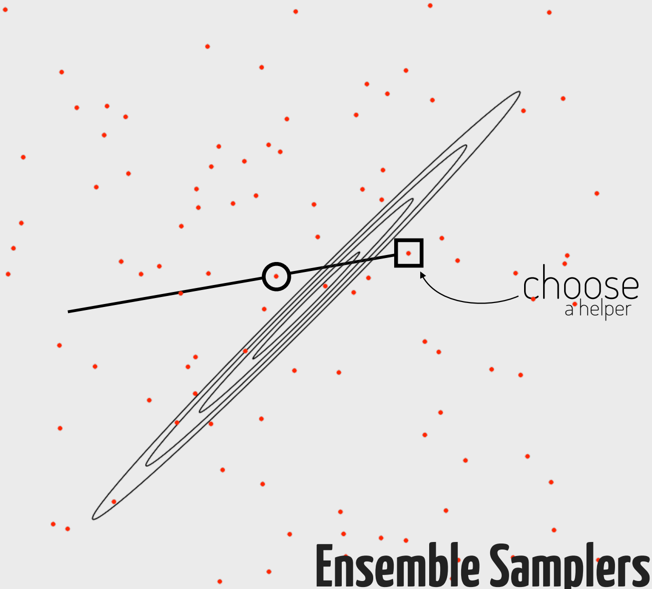

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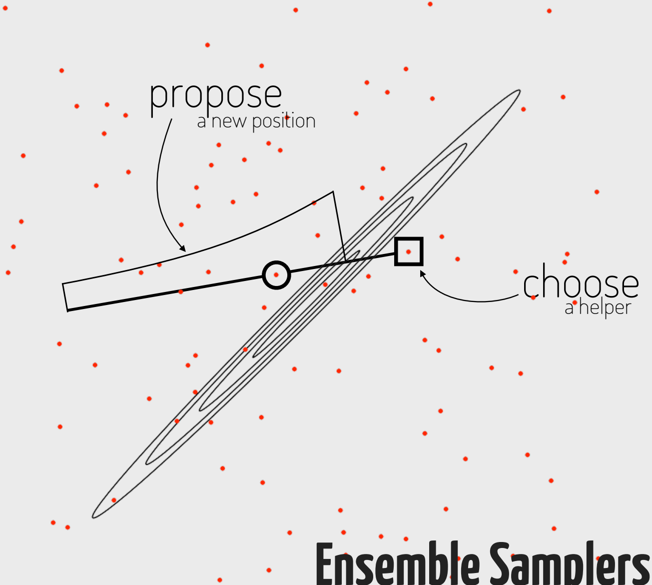
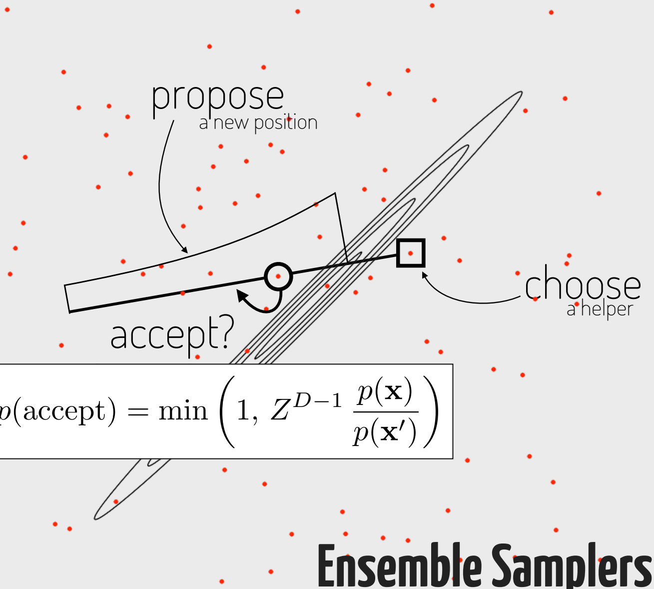
in the **real** world

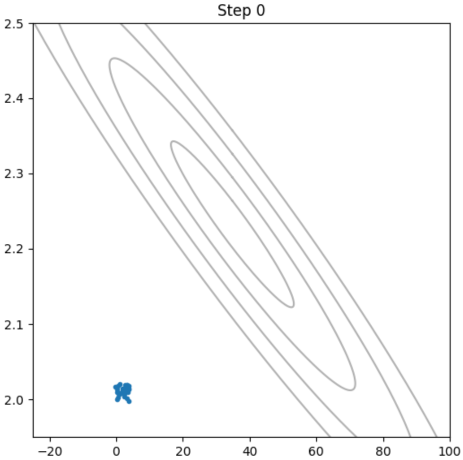


Ensemble Samplers
in the **real** world



$$p(\text{accept}) = \min\left(1,\ Z^{D-1}\,\frac{p(\mathbf{x})}{p(\mathbf{x}')}\right)$$

Emcee demo



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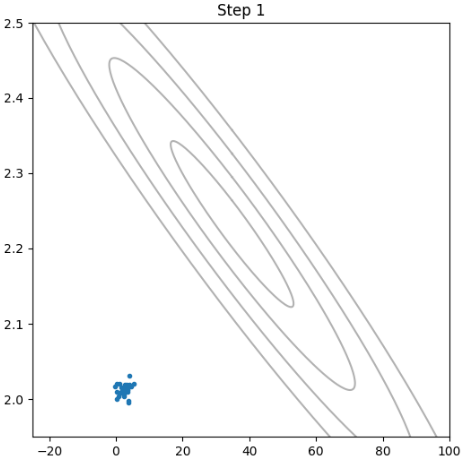

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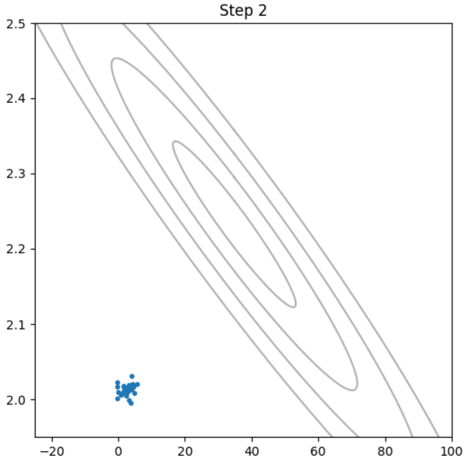

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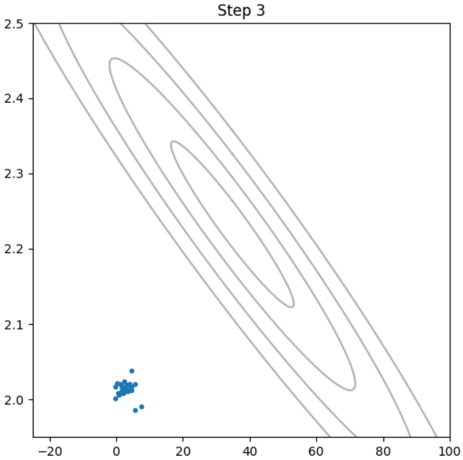

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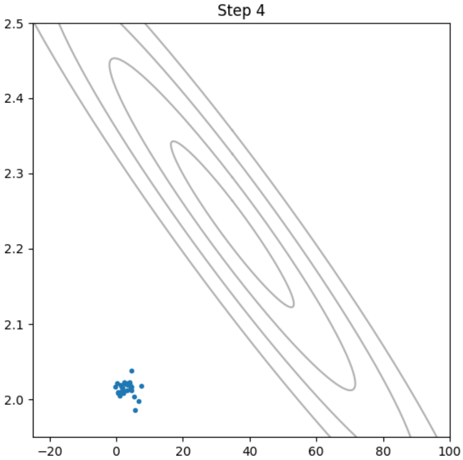

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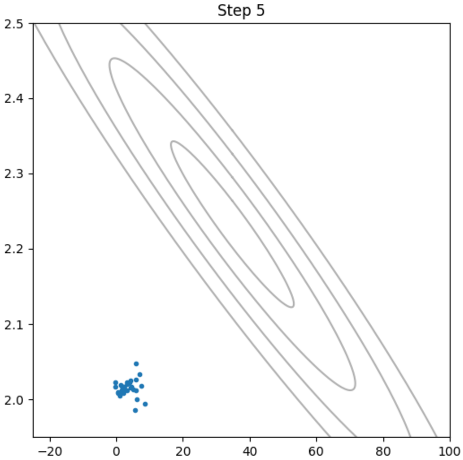

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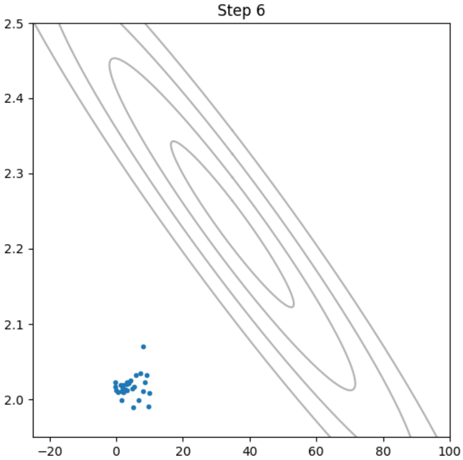

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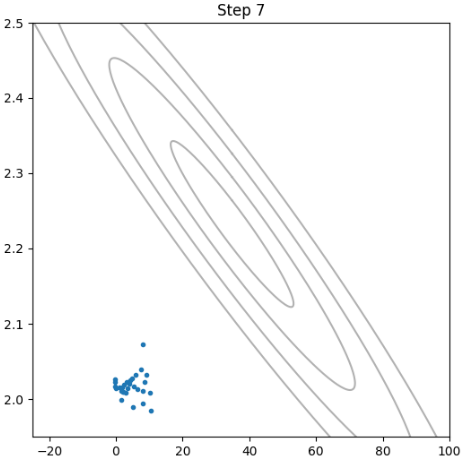

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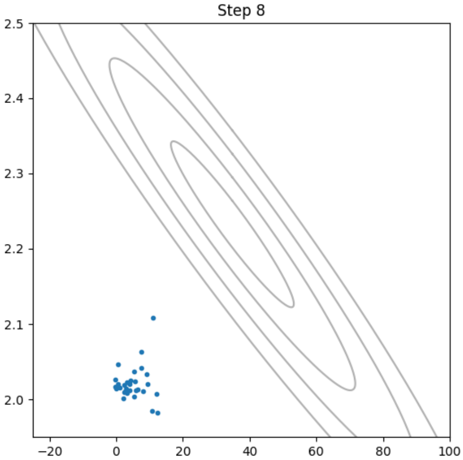

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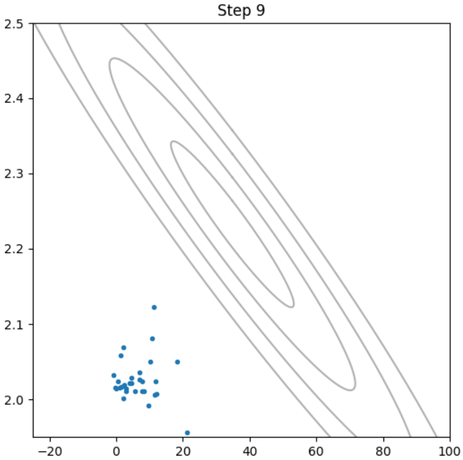

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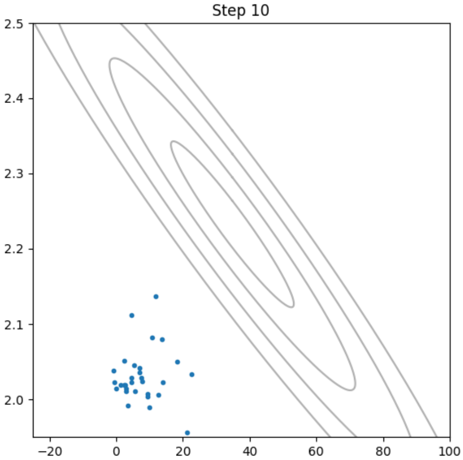

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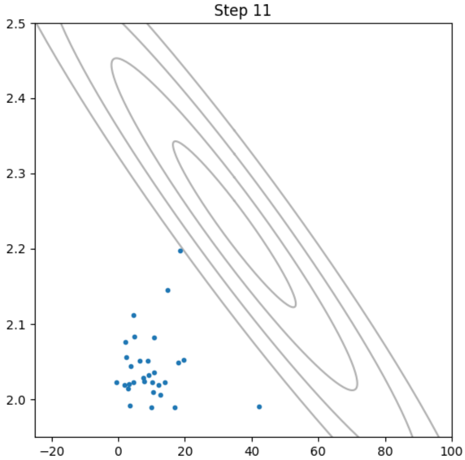

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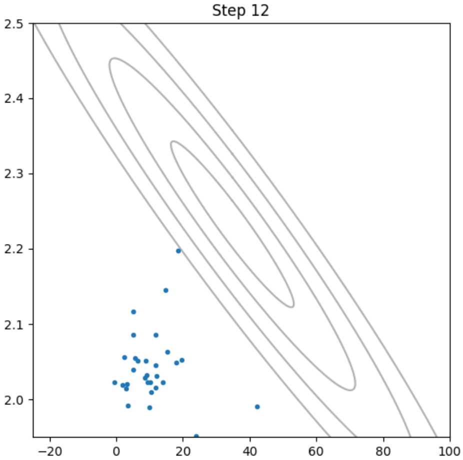

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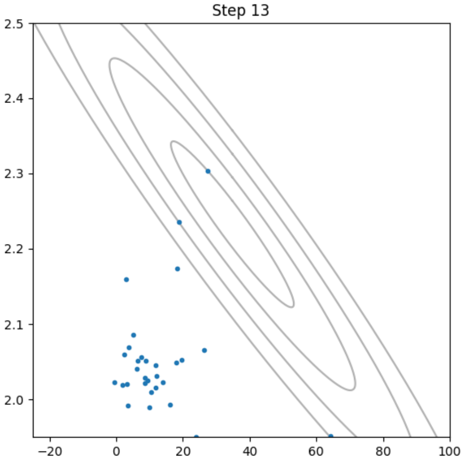

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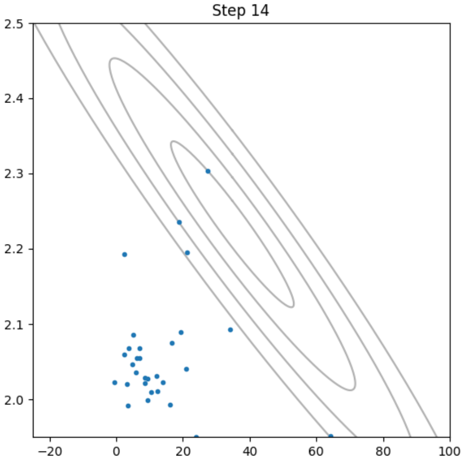

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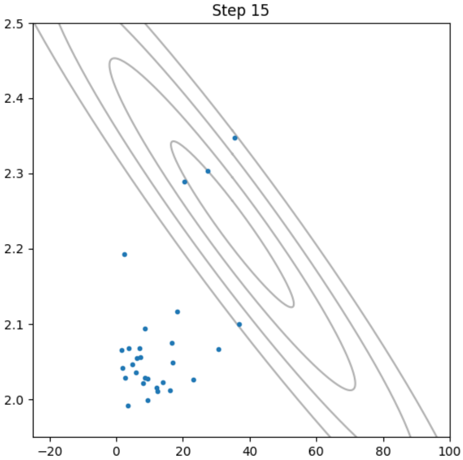

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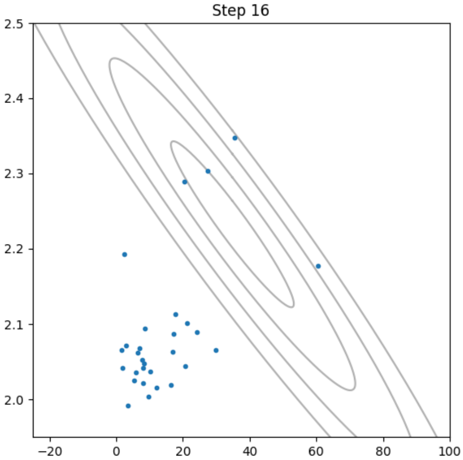

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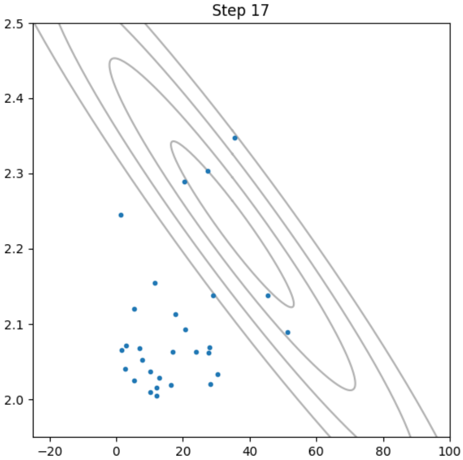

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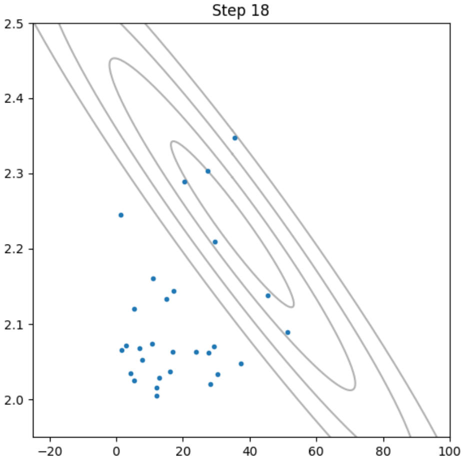

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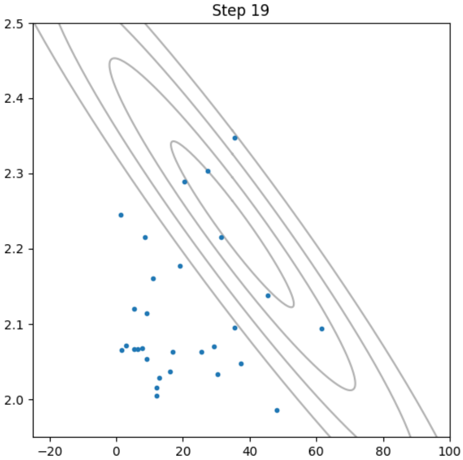

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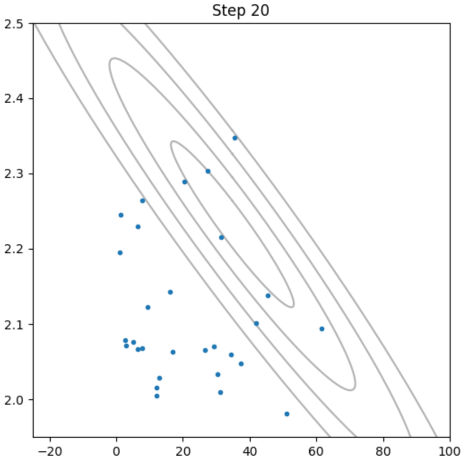

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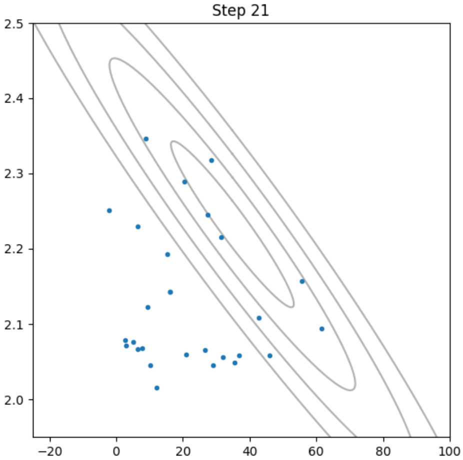

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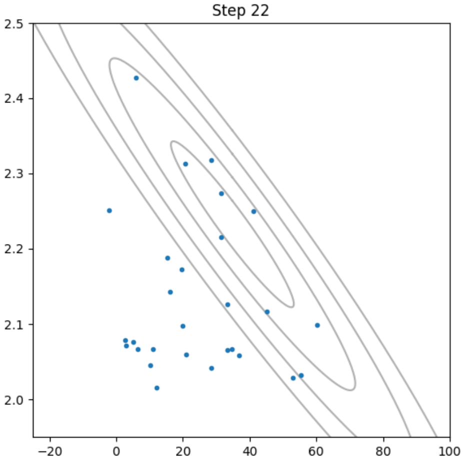

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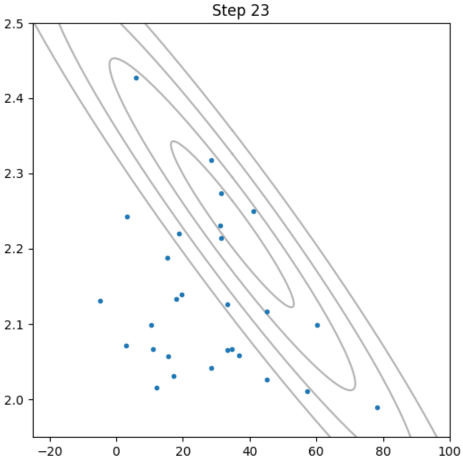

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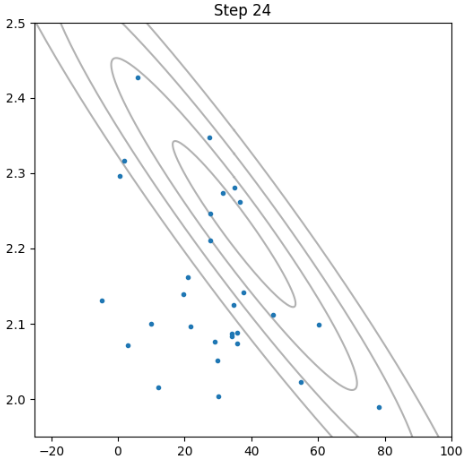

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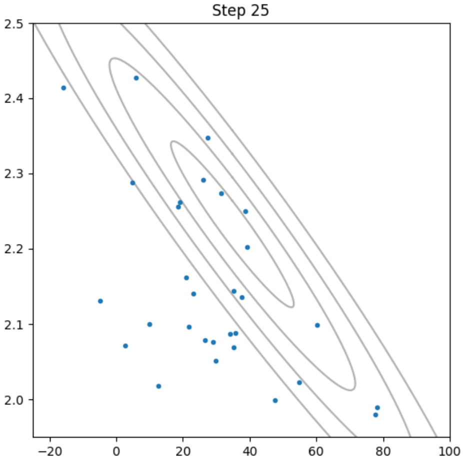

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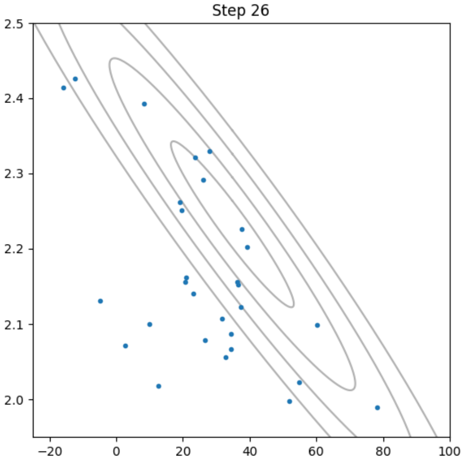

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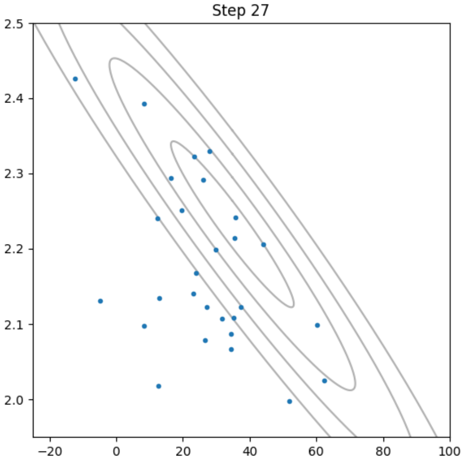

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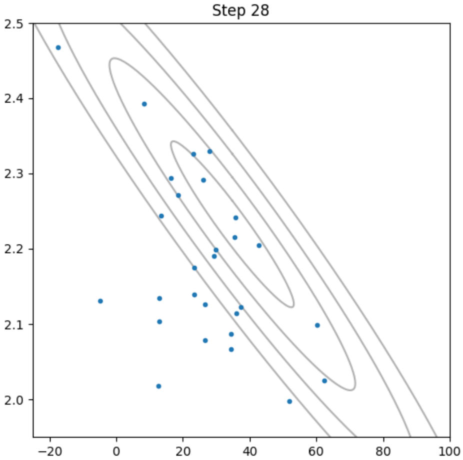

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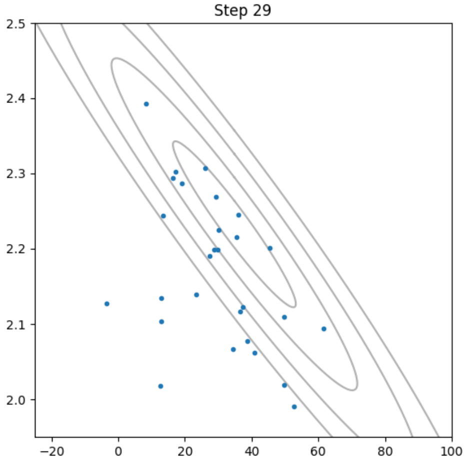

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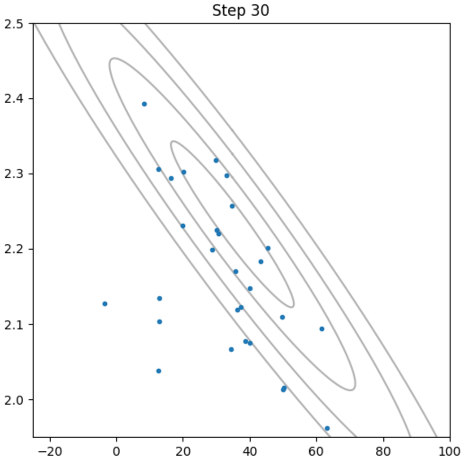

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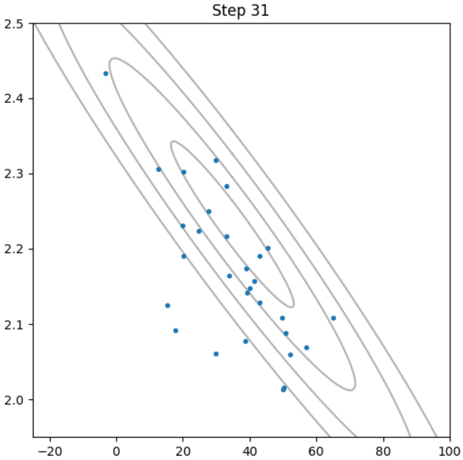

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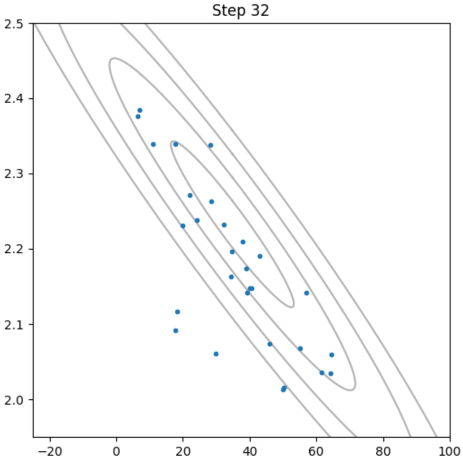

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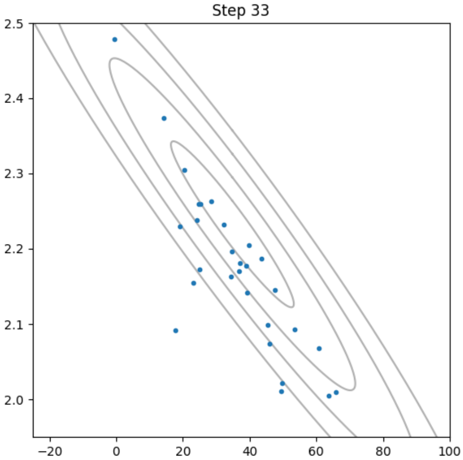

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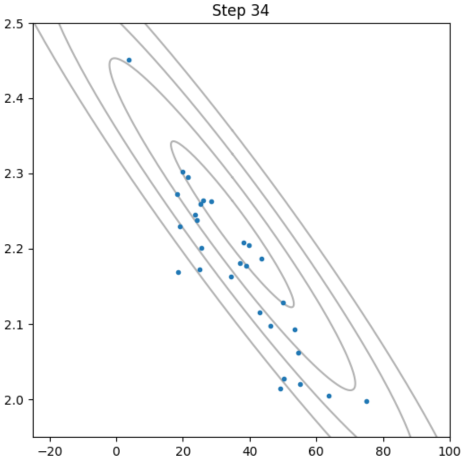

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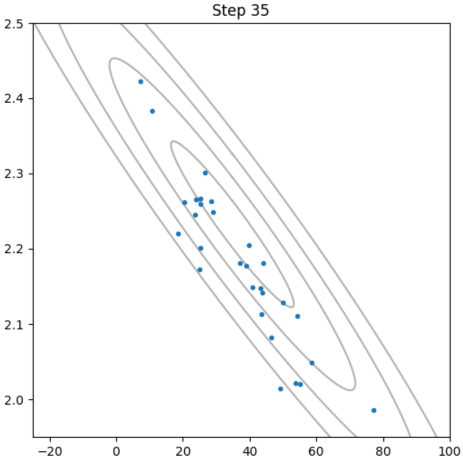

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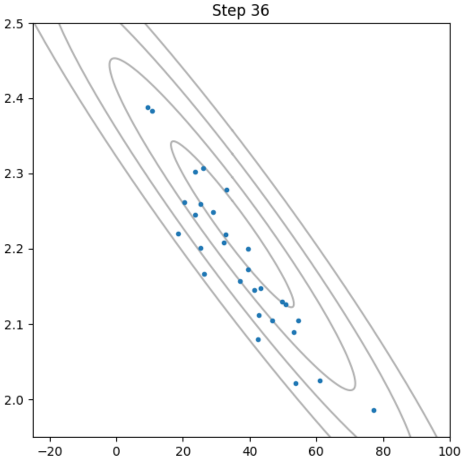

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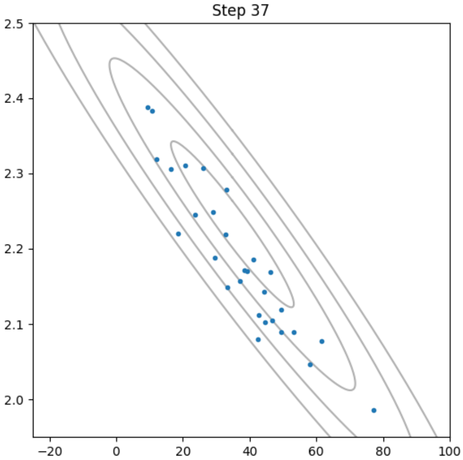

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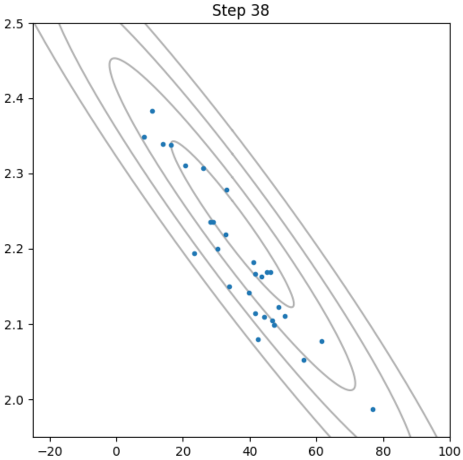

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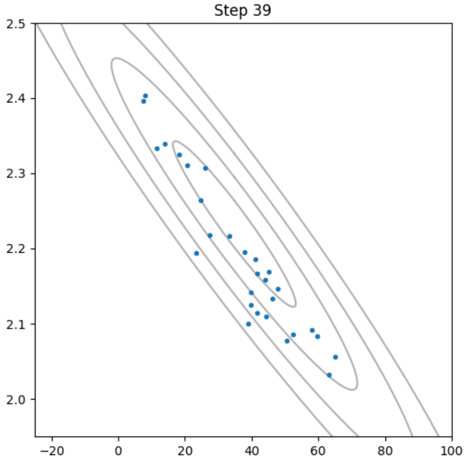

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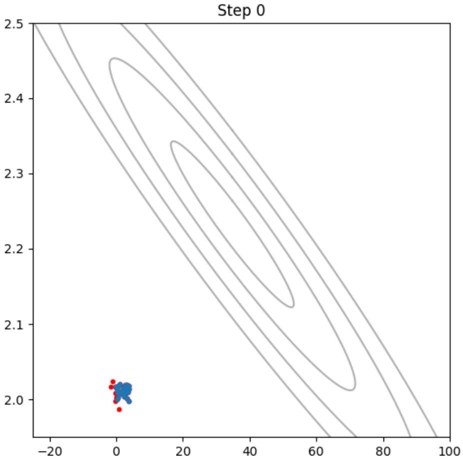

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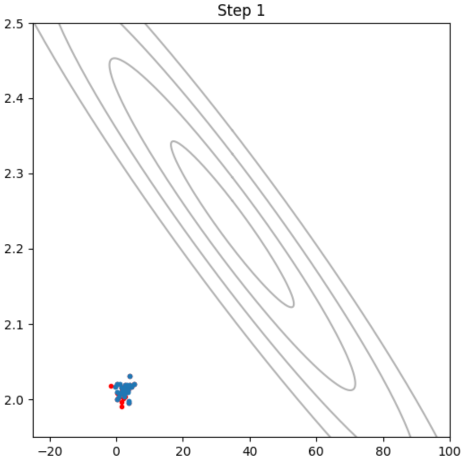

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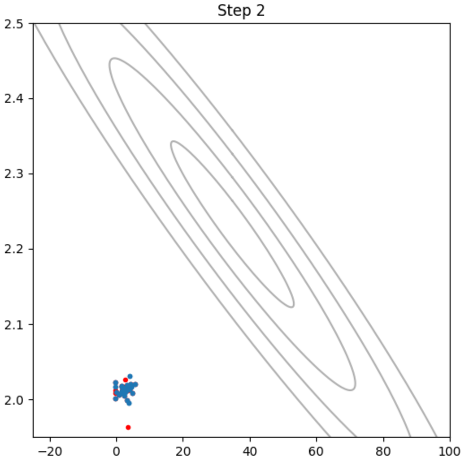

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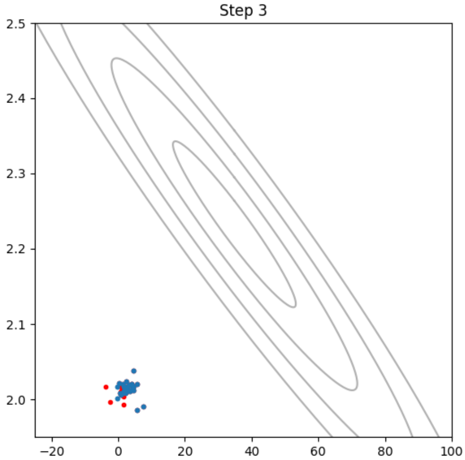

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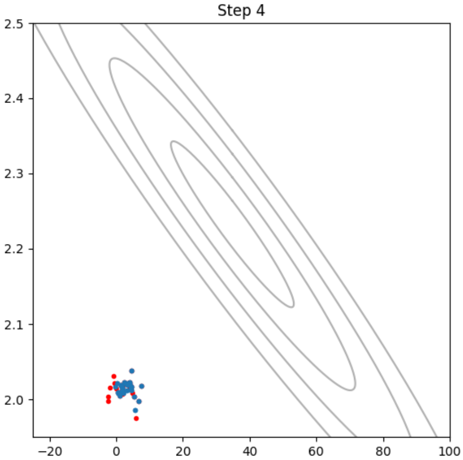

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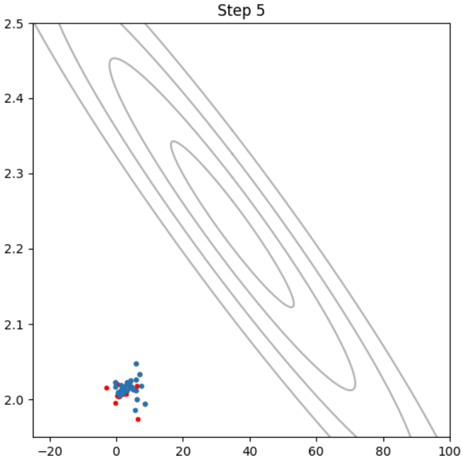

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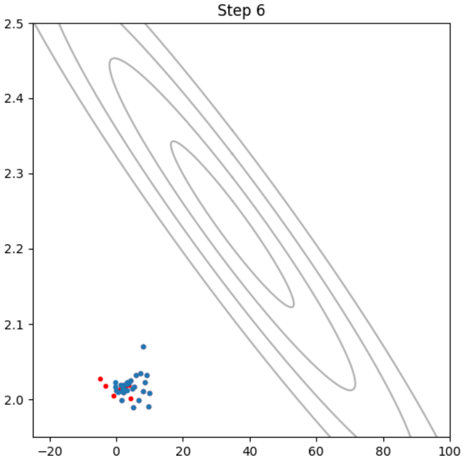

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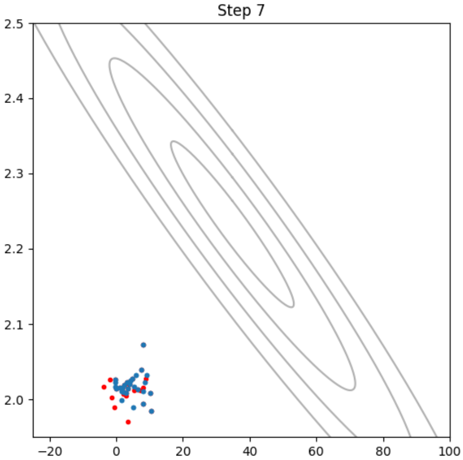

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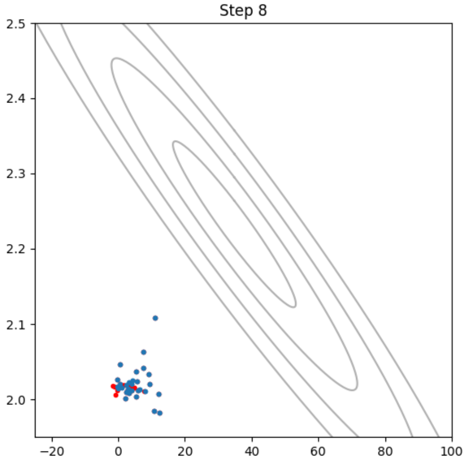

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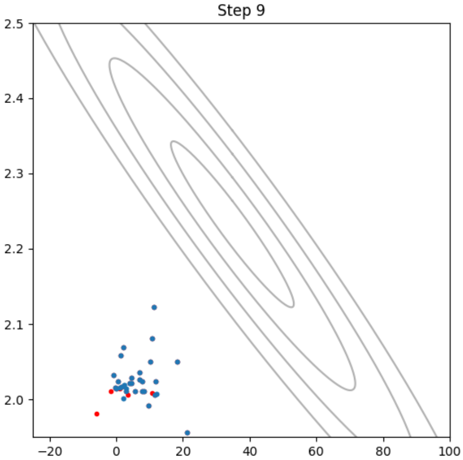
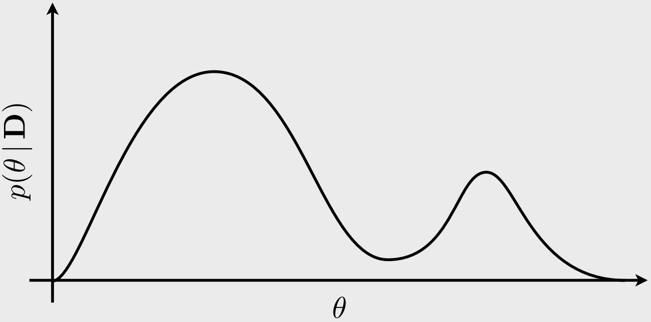


what about multimodal densities?



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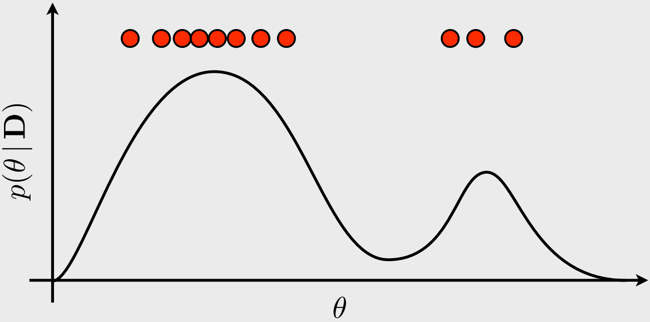

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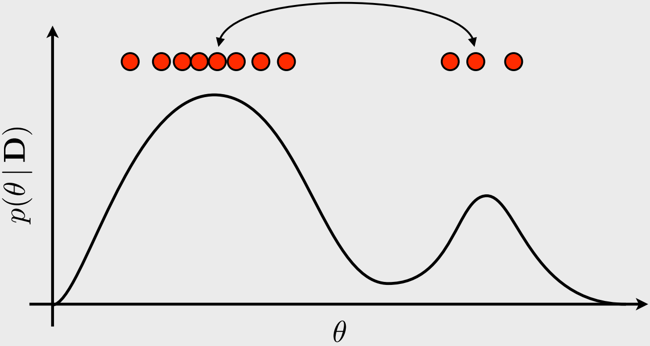

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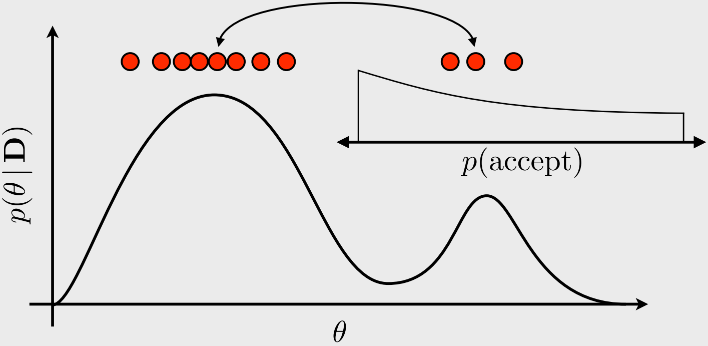
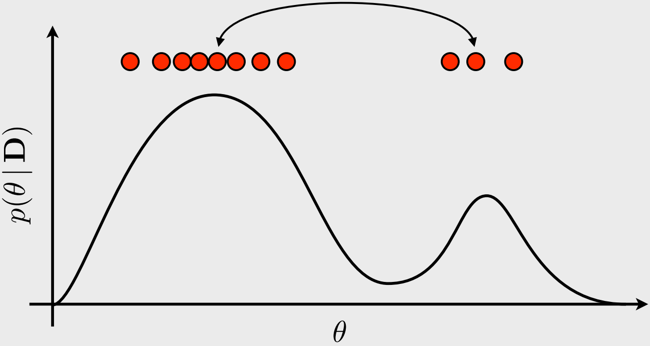


Differential Evolution move

- emcee allows us to use different *move* types (different *proposal* functions)
- The Differential Evolution (DE) move can improve the sampling for multi-modal distributions
- DE move: randomly select *two* “helpers”
- Propose moving by their vector difference
- (If they are from different modes, this proposes *jumping between modes*)
- Mixing in a fraction of DE moves with the regular “Stretch” move works well!

what about multimodal densities?



Summary

- Traditional Metropolis–Hastings MCMC suffers from a *lack of affine invariance* – requires *tuning parameters* that change for each specific probability function
- *Ensemble samplers* like emcee use the *distribution of the walkers* to achieve *affine invariance*
- $\rightarrow$ much easier to use, and faster sampling
- (Huge side effect: parallelizable!)
- Multi-modal distributions still hard, but *DE Move* can help
- MCMC isn't scary!