Affine-Invariant Samplers

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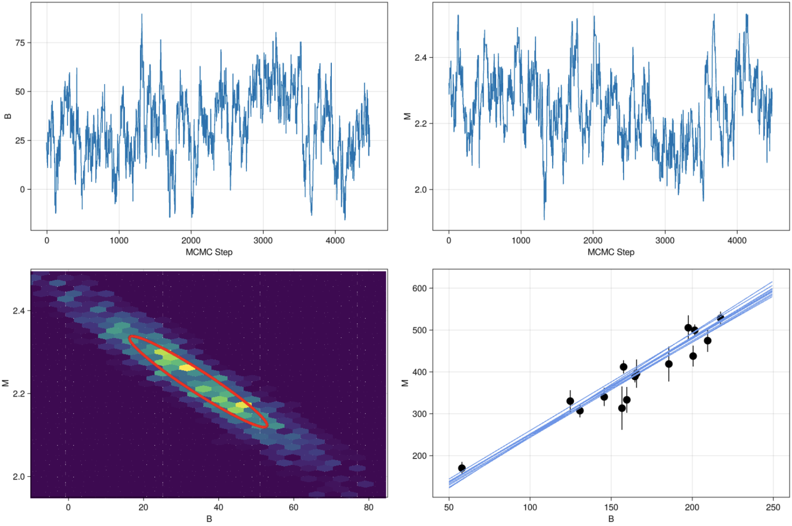
Borrowing heavily from Dan Foreman-Mackey's slides
https://speakerdeck.com/dfm/data-analysis-with-mcmc1
And Jonathan Pritchard's slides
https://www.imperial.ac.uk/media/imperial-college/research-centres-

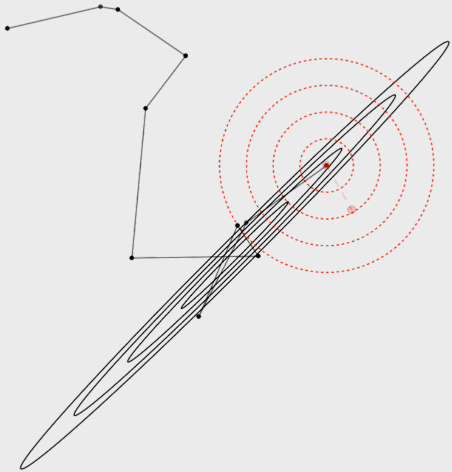
These slides are available at
https://github.com/dstndstn/MCMC-talk

The MCMC Algorithm

```julia
function mcmc(logprob_func, logprob_args,
              propose_func, propose_args,
              initial_pos, nsteps)
    p = initial_pos
    logprob = logprob_func(p, logprob_args)
    chain = zeros(Float64, (nsteps, length(p)))
    naccept = 0
    for i in 1:nsteps
        # propose a new position in parameter space
        p_new = propose_func(p, propose_args)
        # compute probability at new position
        logprob_new = logprob_func(p_new, logprob_args)
        # decide whether to jump to the new position
        if exp(logprob_new - logprob) > rand()
            p = p_new
            logprob = logprob_new
            naccept += 1
        end
        # save the position
        chain[i,:] = p
    end
    return chain, naccept/nsteps
end
```

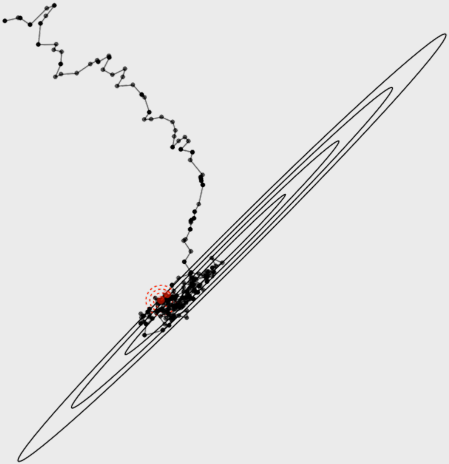
MCMC for model parameter inference





Metropolis–Hastings

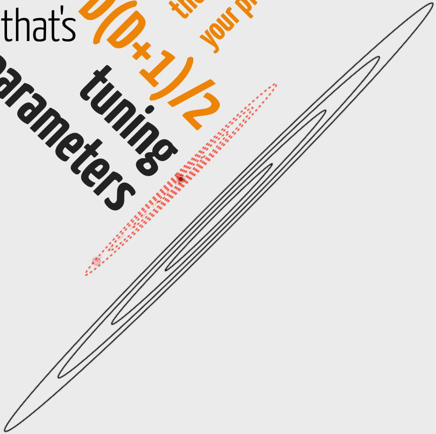
in the real world



Metropolis–Hastings
in the real world

that's D(D+1)/2 tuning parameters

the dimension of your problem



Metropolis–Hastings
in the real world

that's

the dimension of
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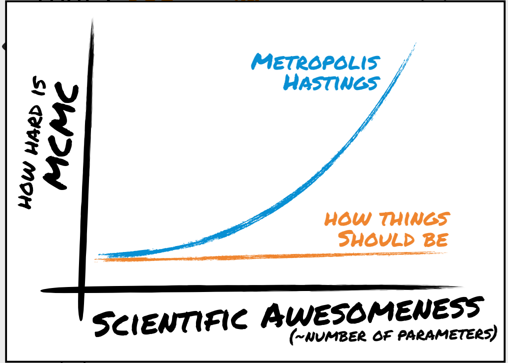


Metropolis–Hastings
in the real world

Jonathan Goodman

Jonathan Weare

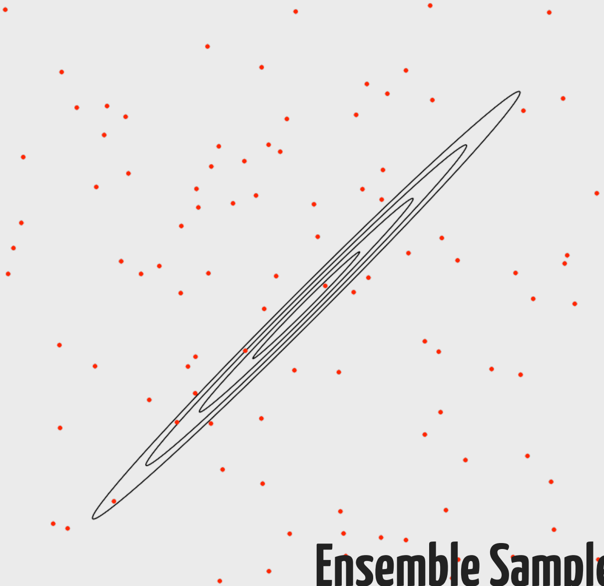
"Ensemble samplers with affine invariance"

(dfm.io/mcmc-gw10)

introducing **emcee** the MCMC Hammer

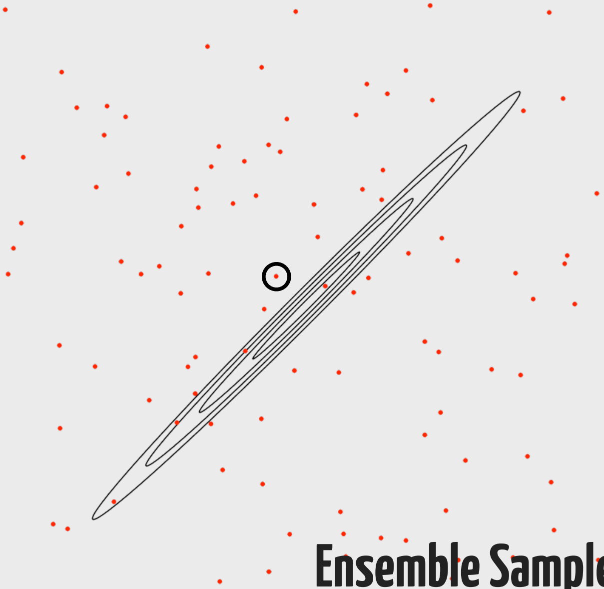
arxiv.org/abs/1202.3665

dan.iel.fm/emcee



Ensemble Samplers

in the **real** world



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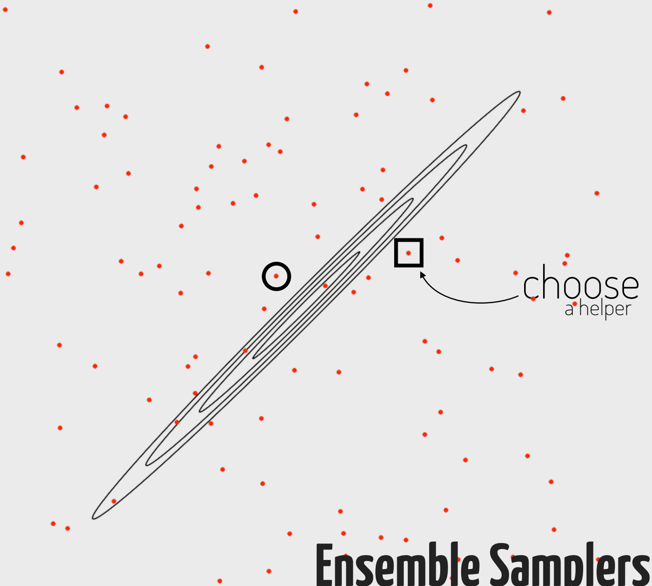
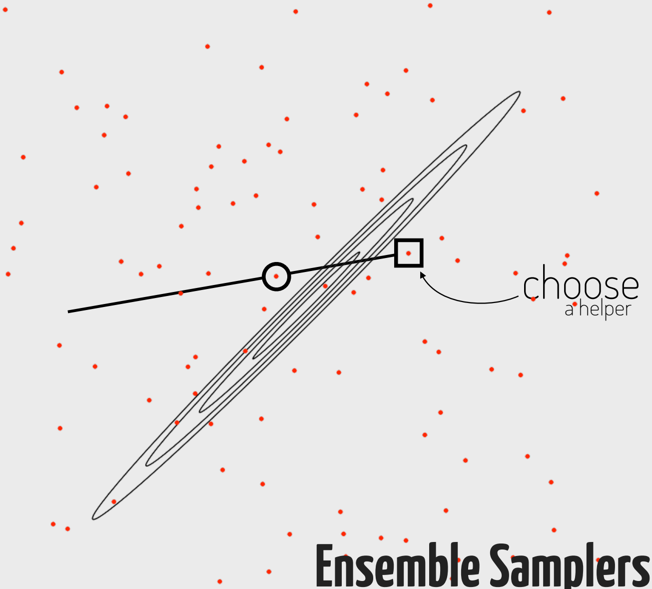


Ensemble Samplers
in the **real** world



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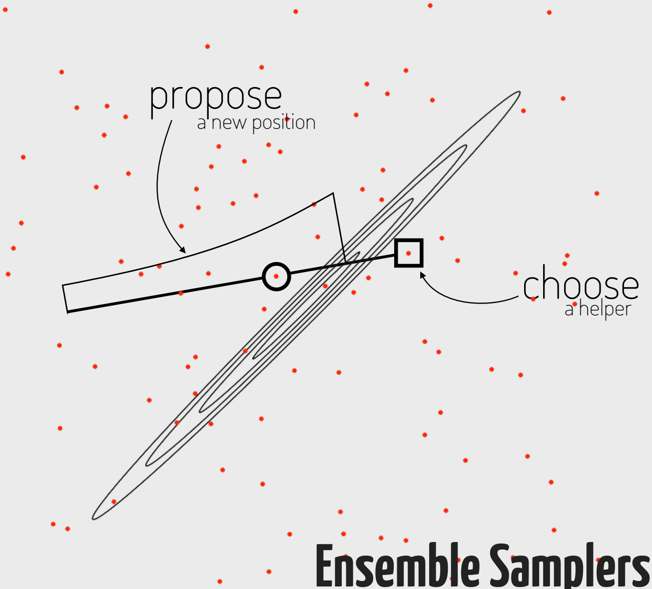
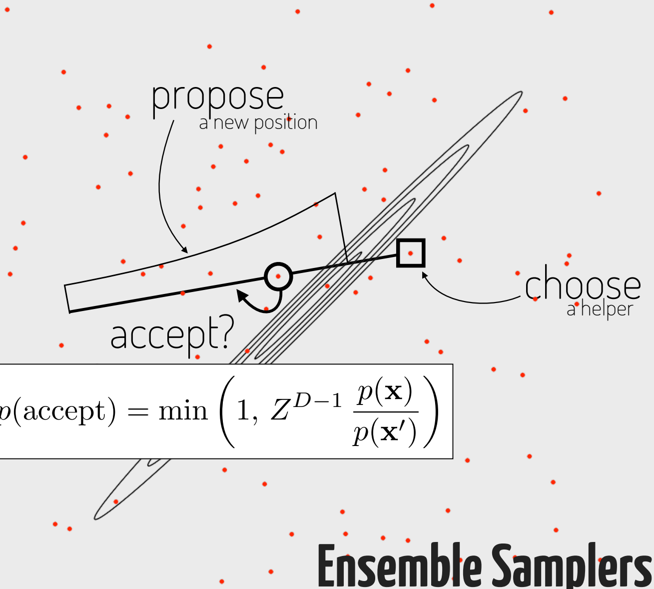


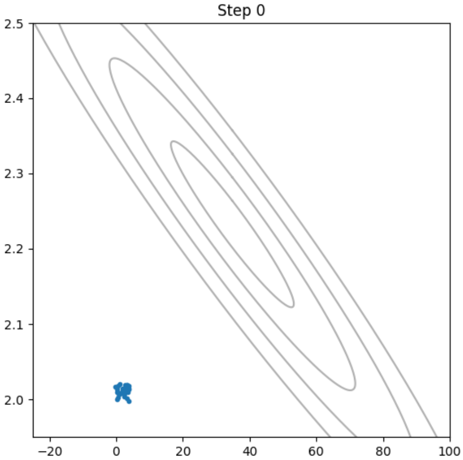
Ensemble Samplers
in the **real** world



$$p(\text{accept}) = \min\left(1,\ Z^{D-1}\,\frac{p(\mathbf{x})}{p(\mathbf{x}')}\right)$$

Ensemble Samplers
in the **real** world

Emcee demo



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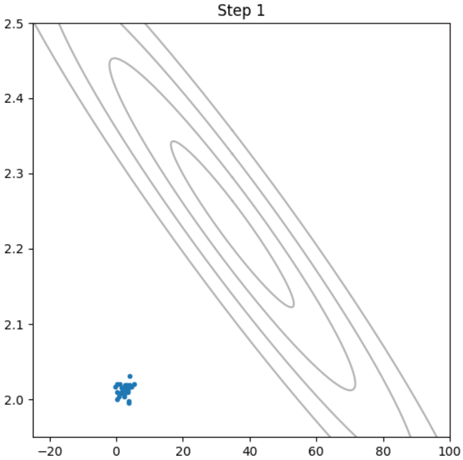

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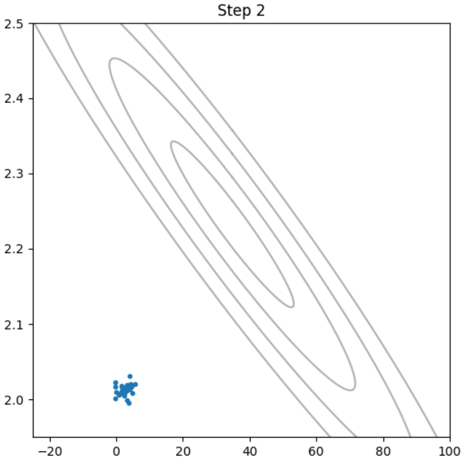

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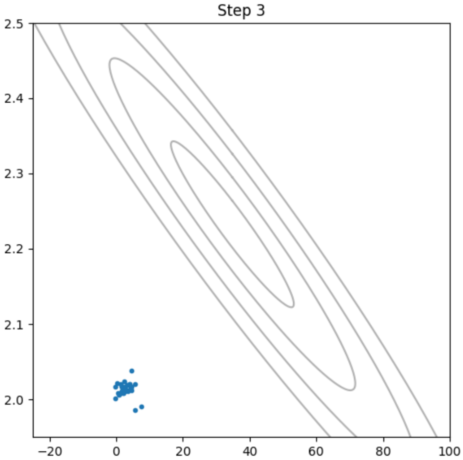

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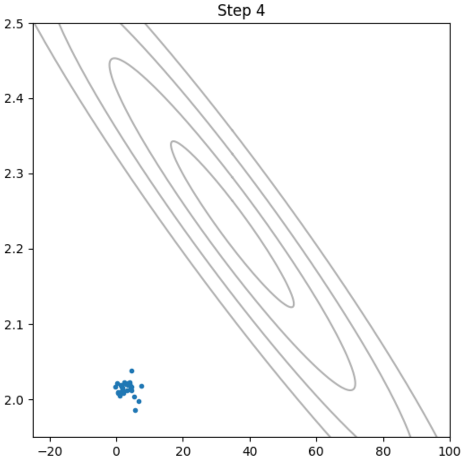

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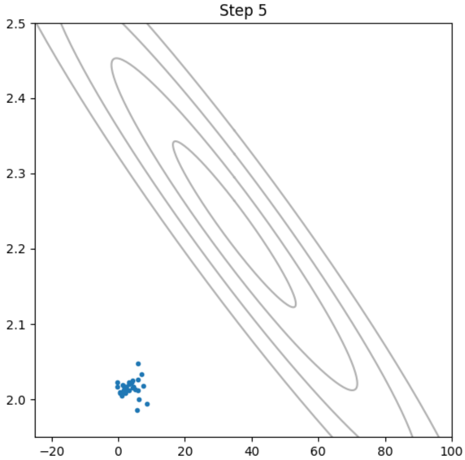

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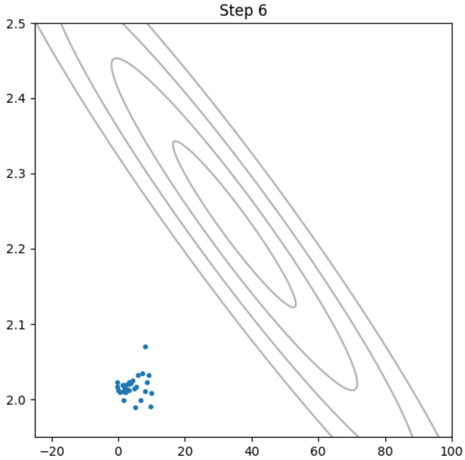

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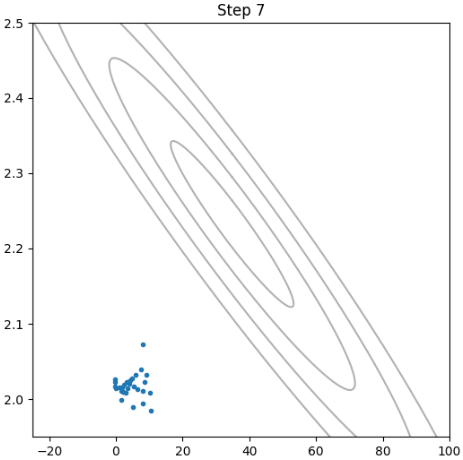

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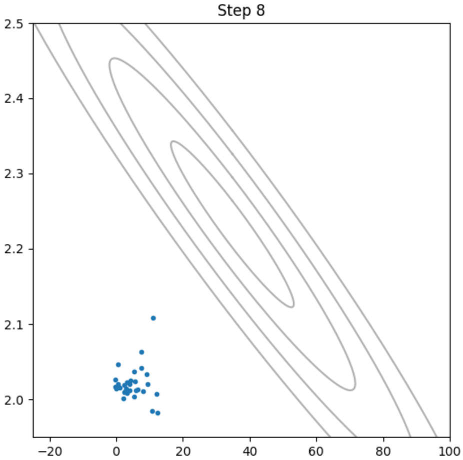

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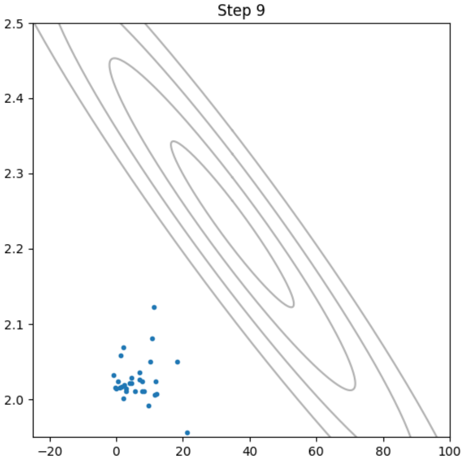

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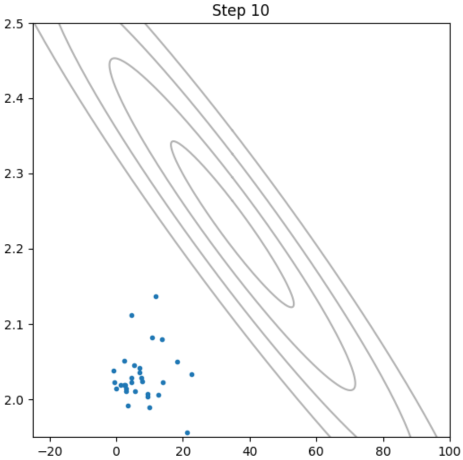

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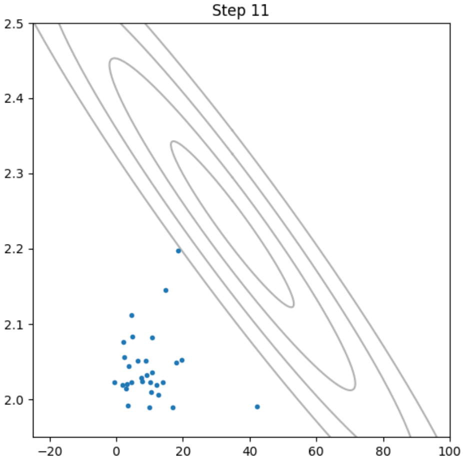

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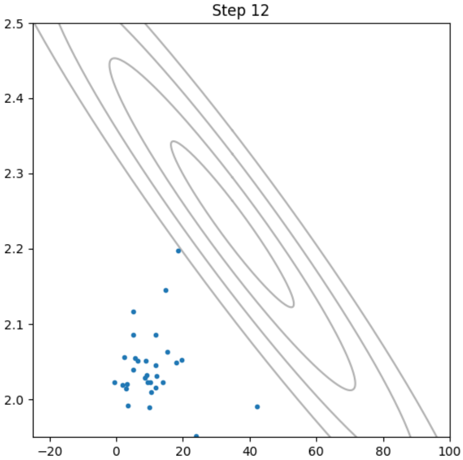

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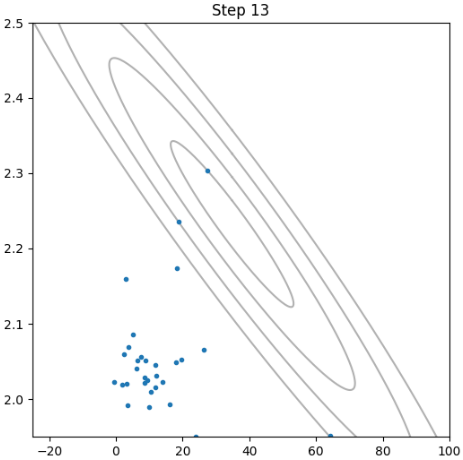

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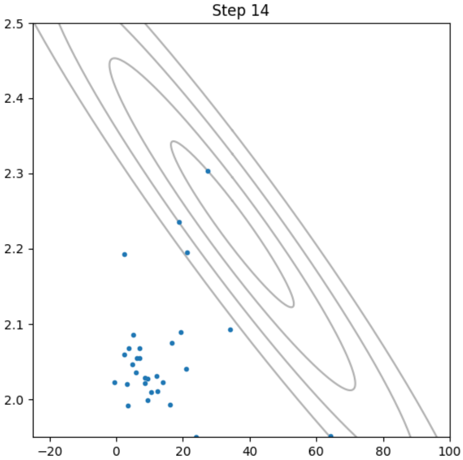

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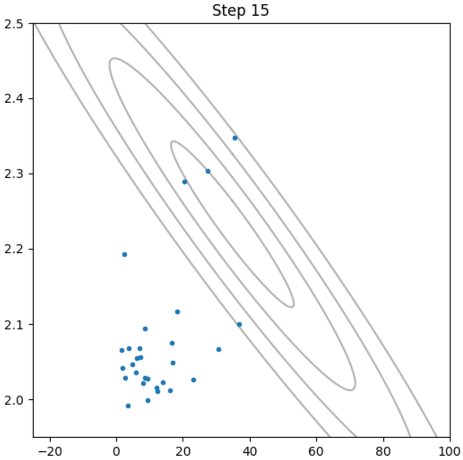

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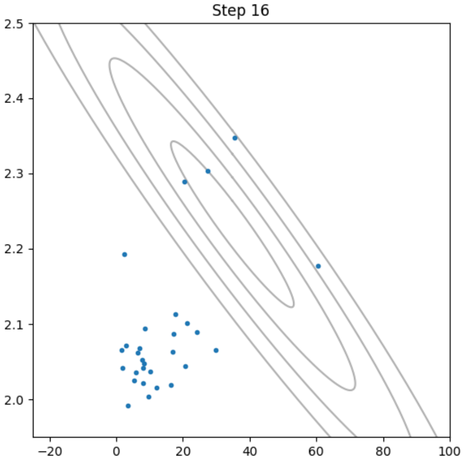

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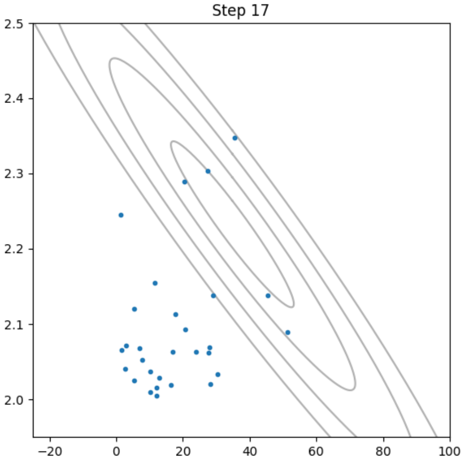

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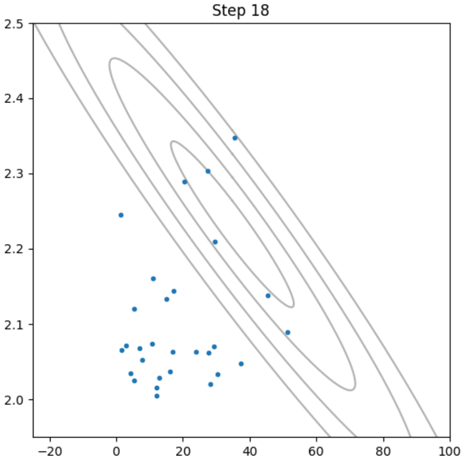

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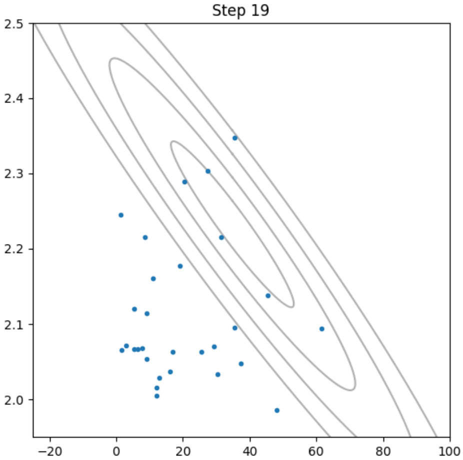

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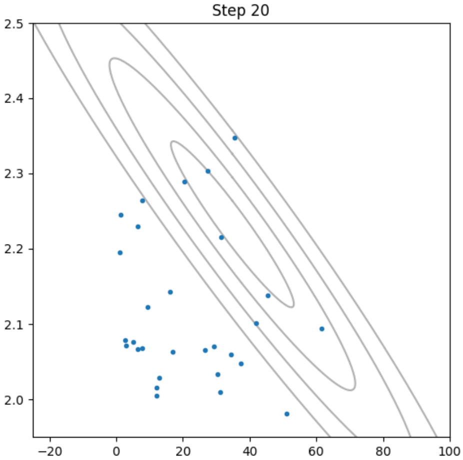

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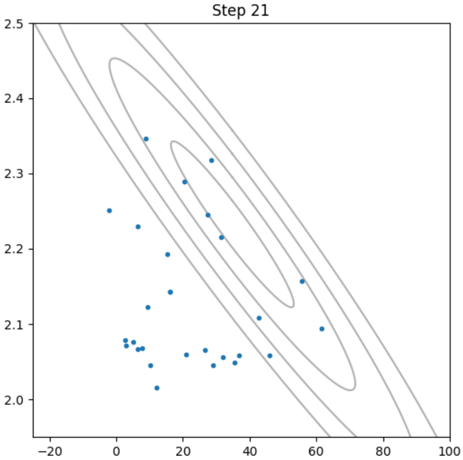

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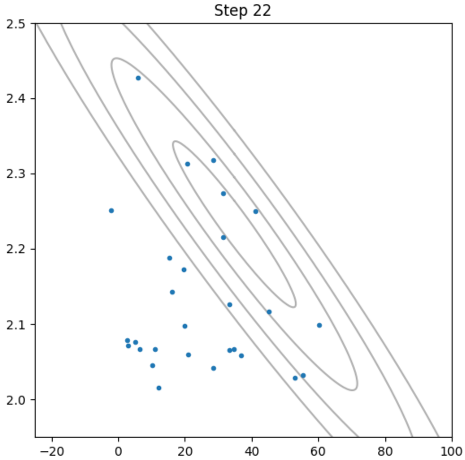

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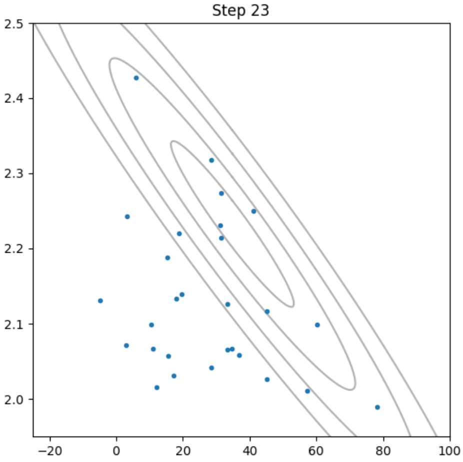

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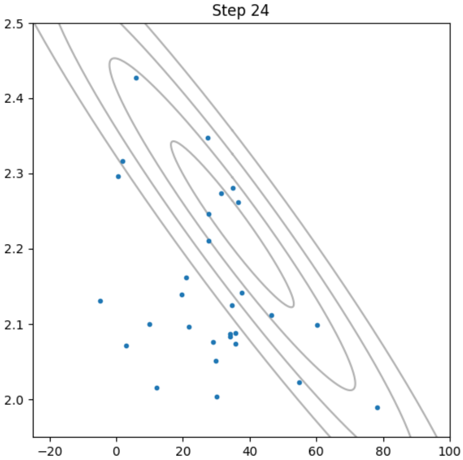

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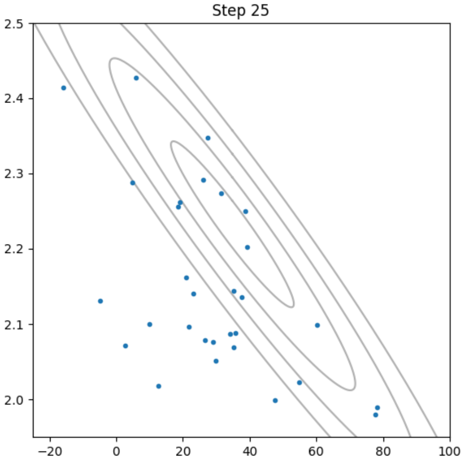

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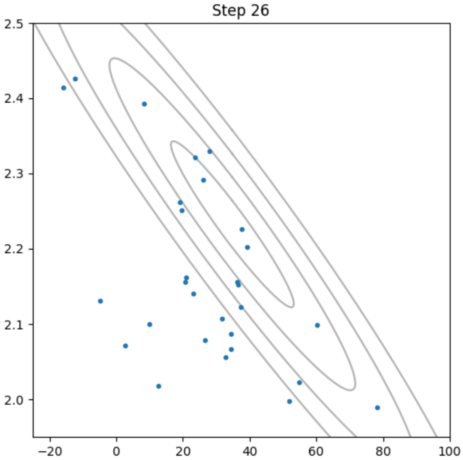
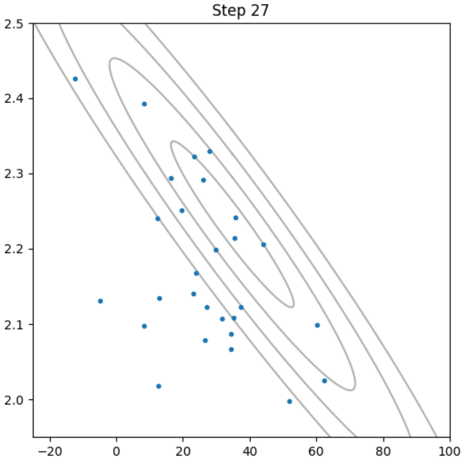


Emcee demo



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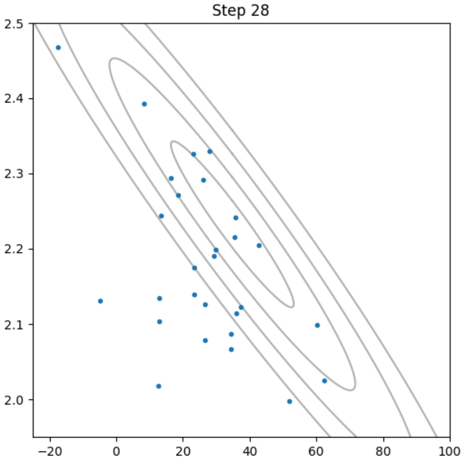

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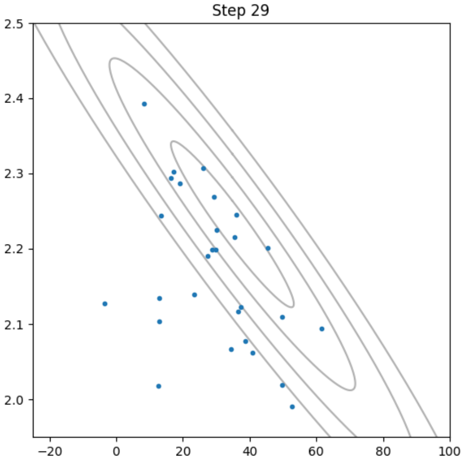

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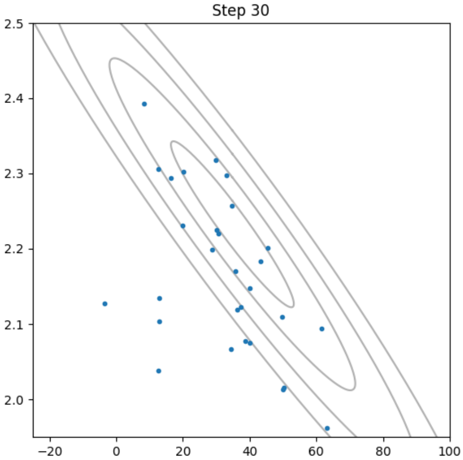

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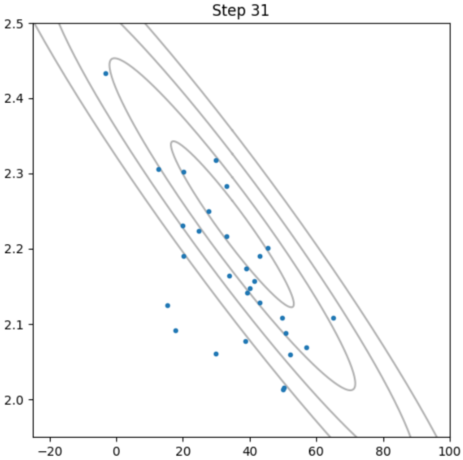

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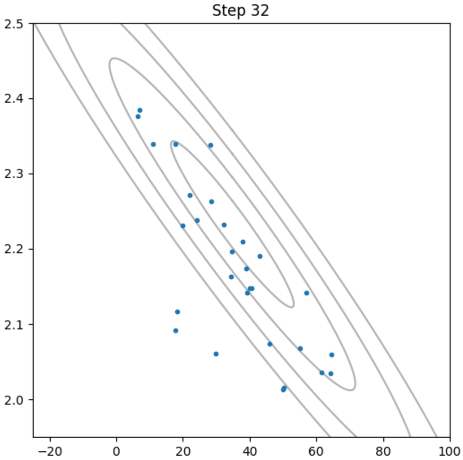

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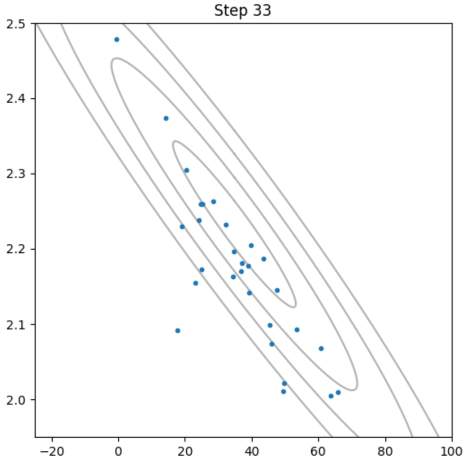

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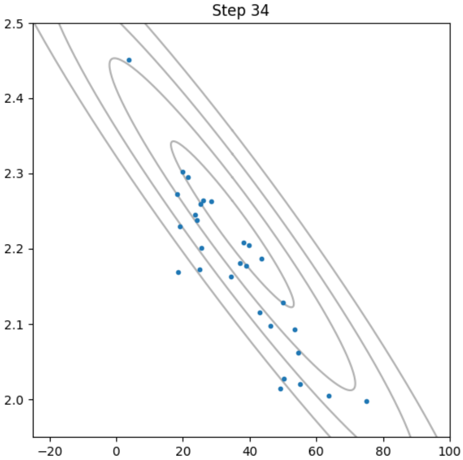

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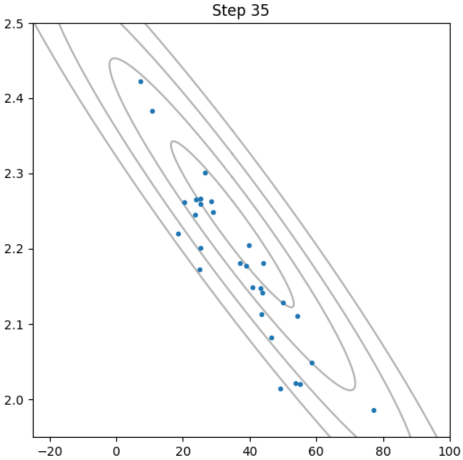

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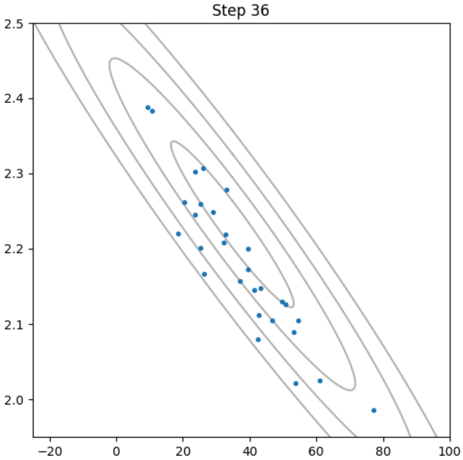

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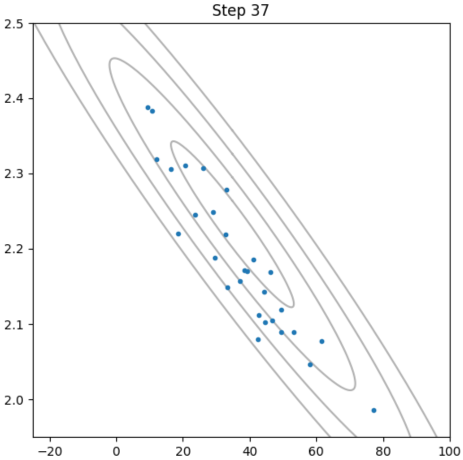

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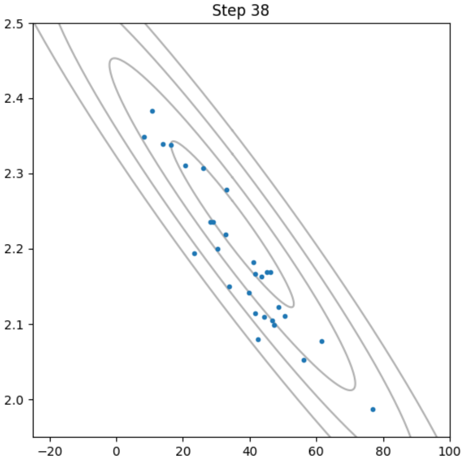

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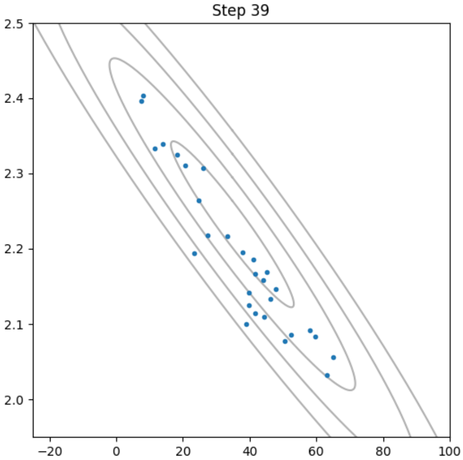

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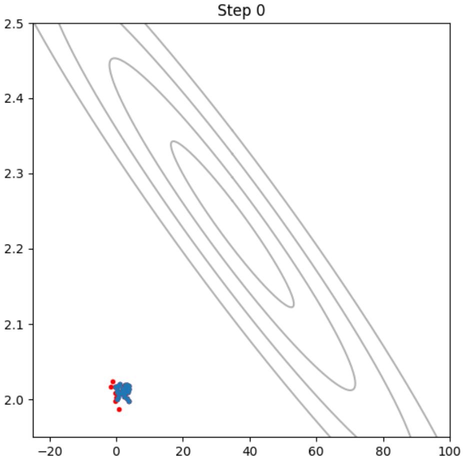

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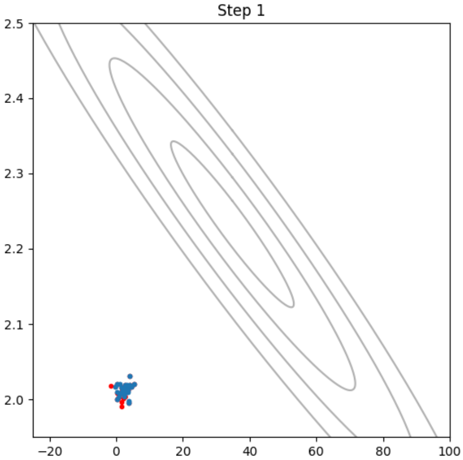

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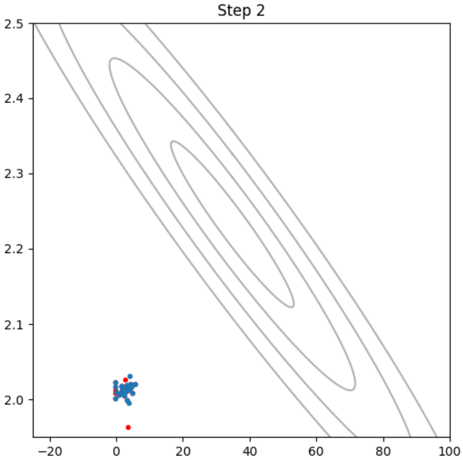

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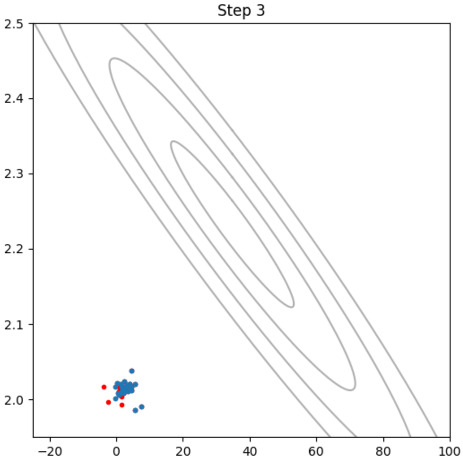

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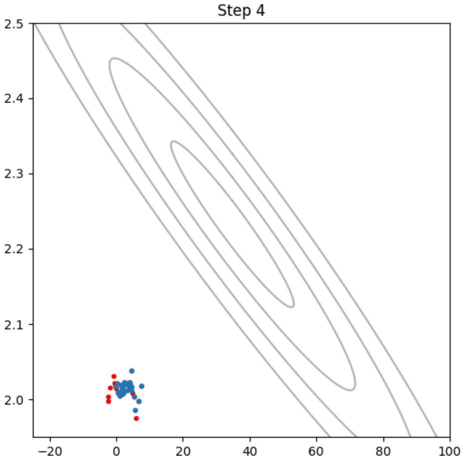

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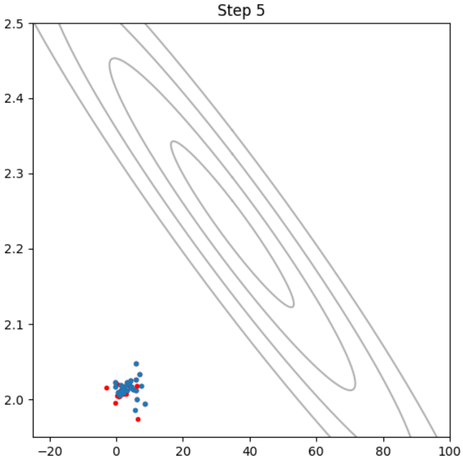

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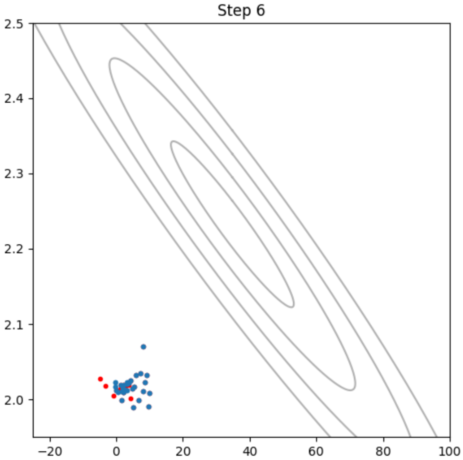

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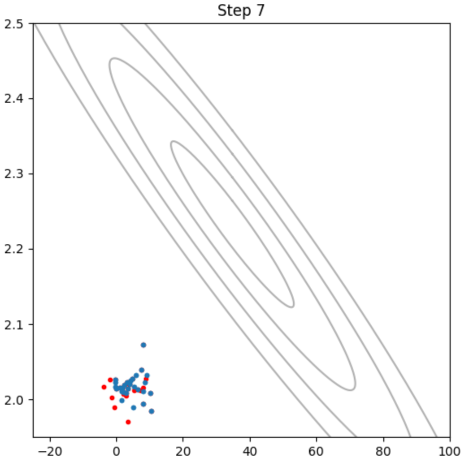

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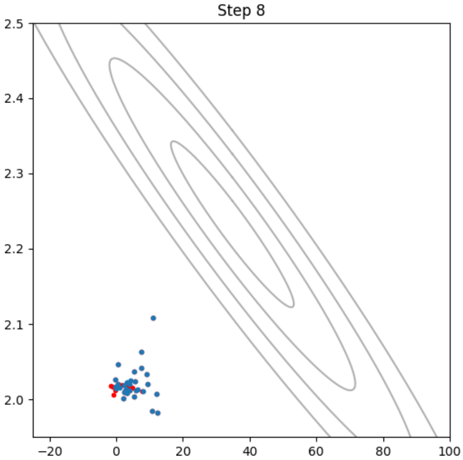

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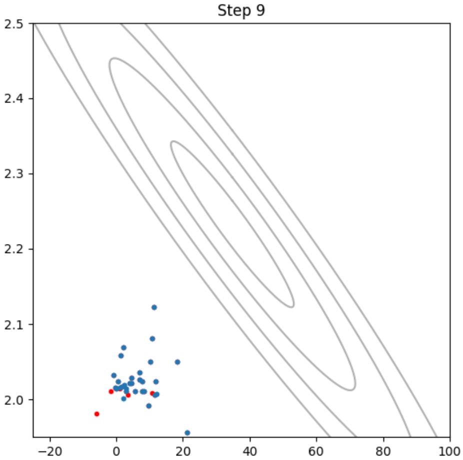
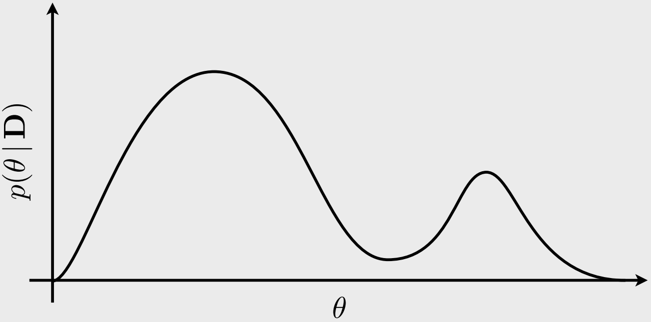


Remember:

emcee isn't always The Right Choice™

Brendon Brewer's words of wisdom…

what about multimodal densities?



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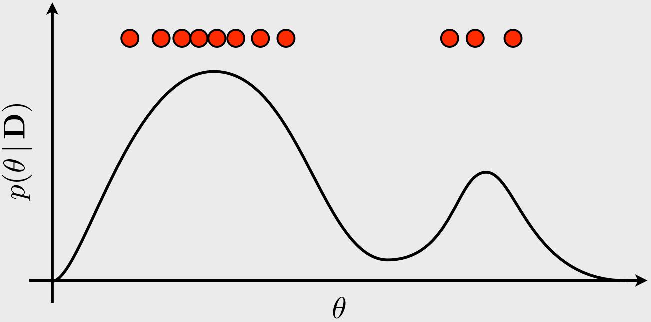

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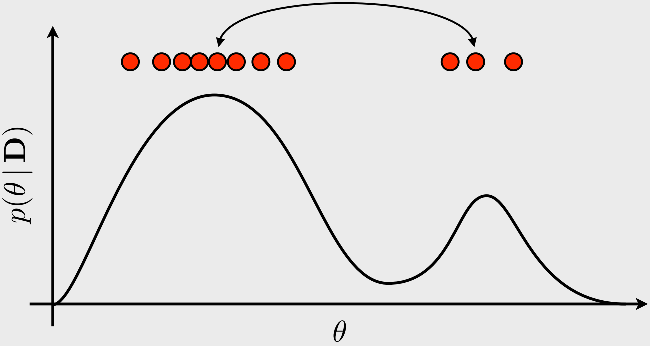

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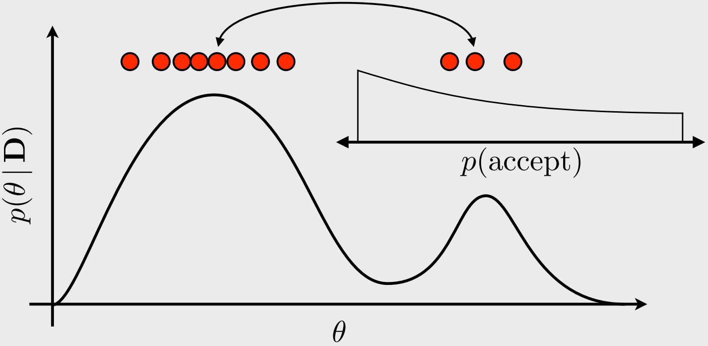
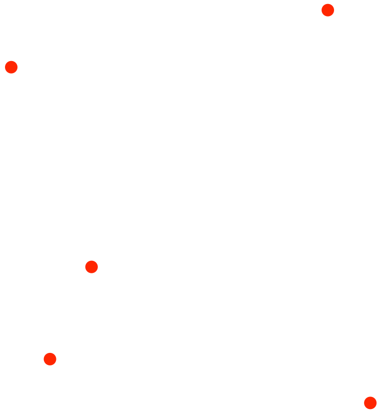


wh… es?

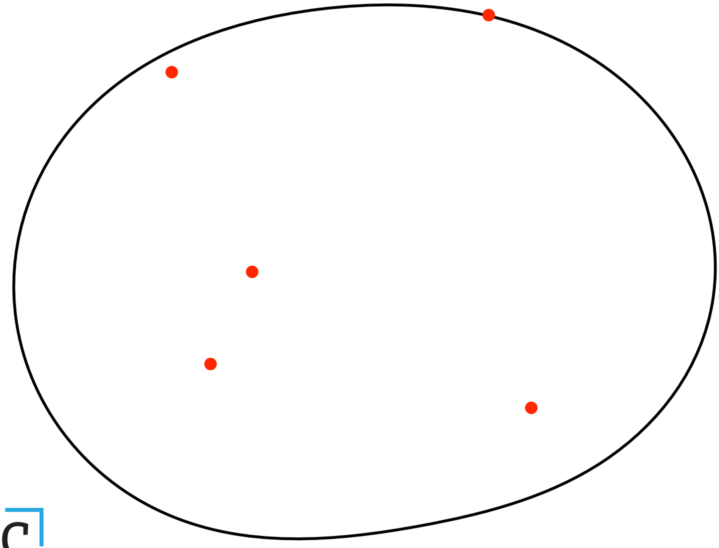
Nested Sampling

- An alternative to MCMC (and emcee)
- (Originally built for integrating the *evidence*)
- Idea: successively resample points, demanding they be above a minimum likelihood level
- Think of slowly raising the water level to find terrain contours

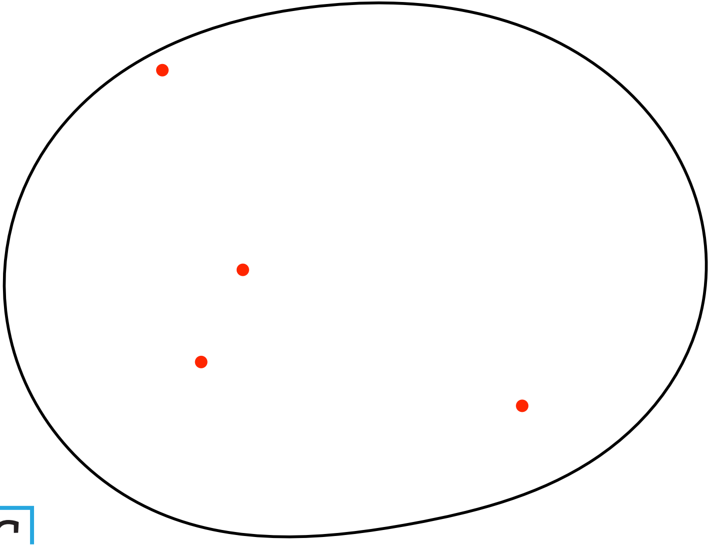
five live points chosen uniformly from prior



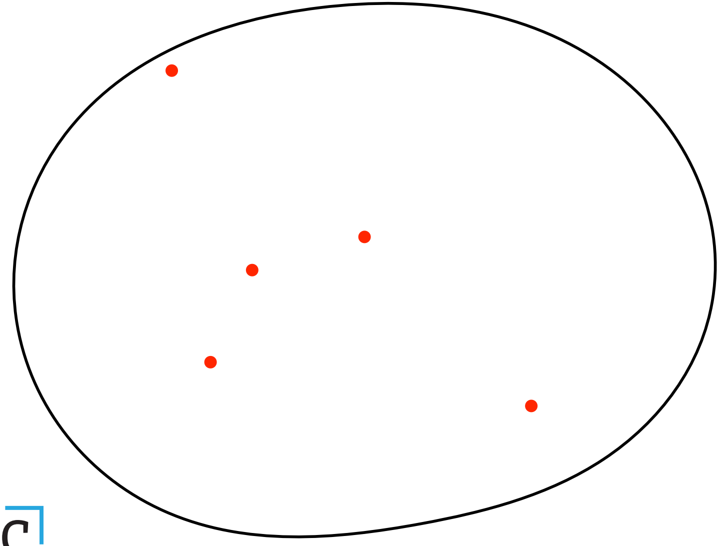
Define likelihood contour from lowest point



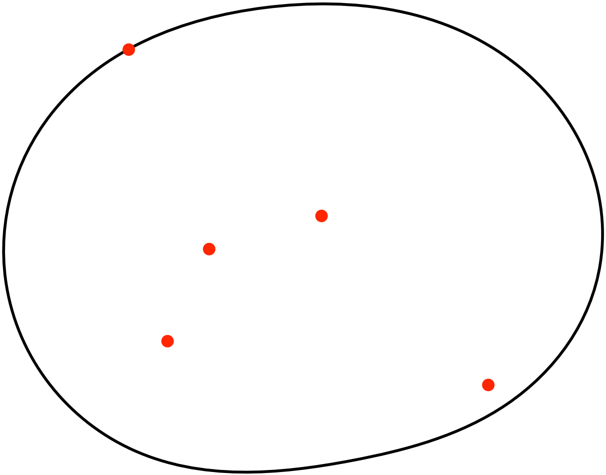
Delete that point (storing it’s value)

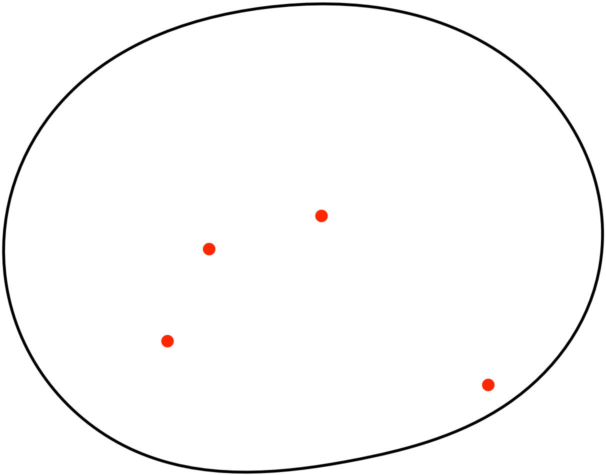


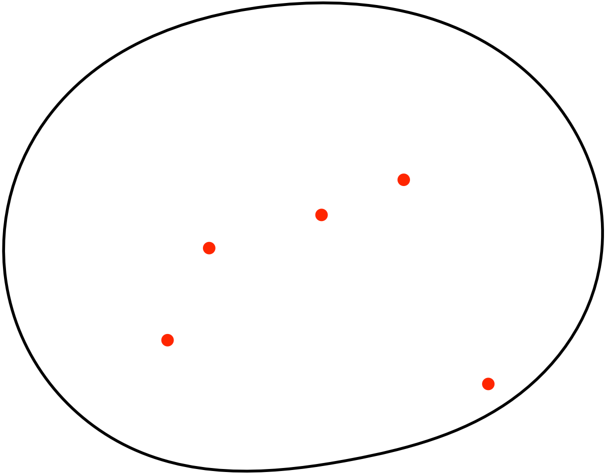
Select a new point uniformly sampled subject to requirement $L(X_{new})>L(X_{old})$

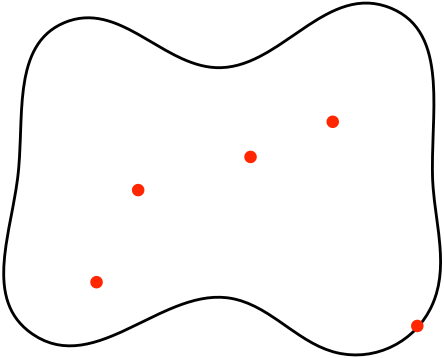


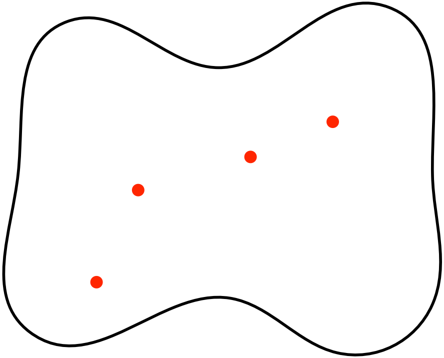
Iterate - contour shrinks by X~exp(1/n)

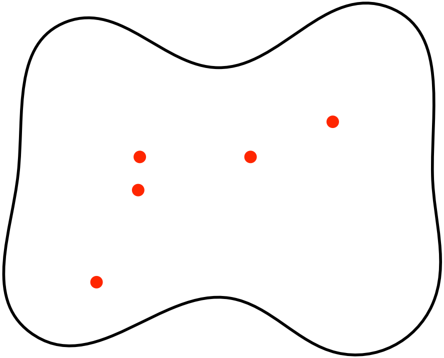


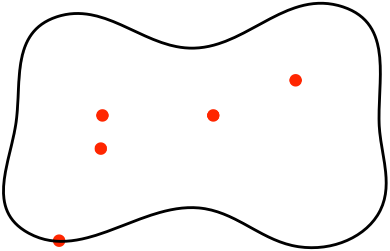


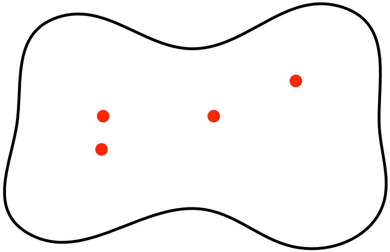


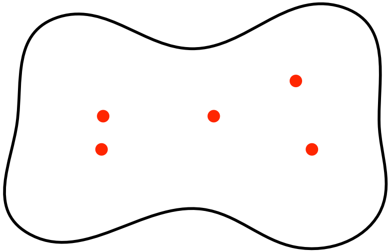


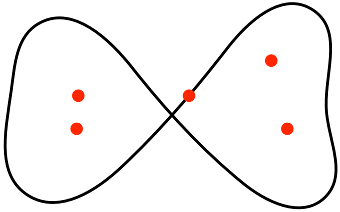


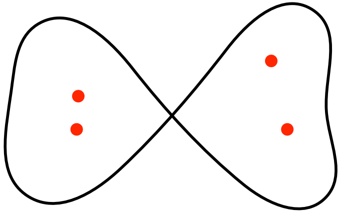


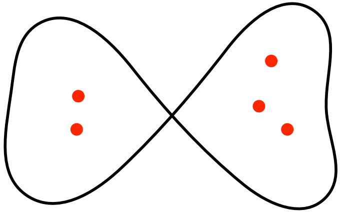


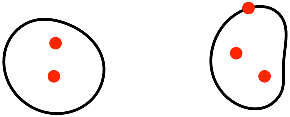


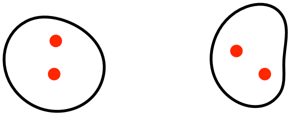


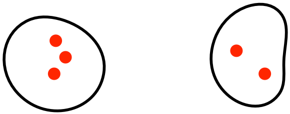




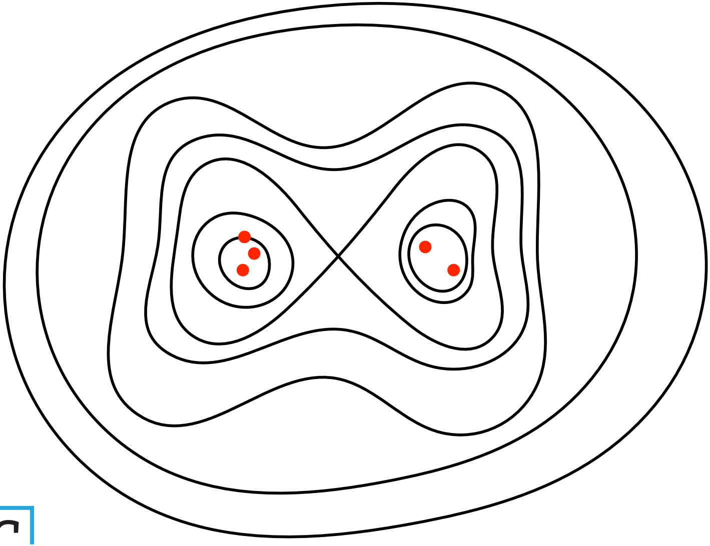




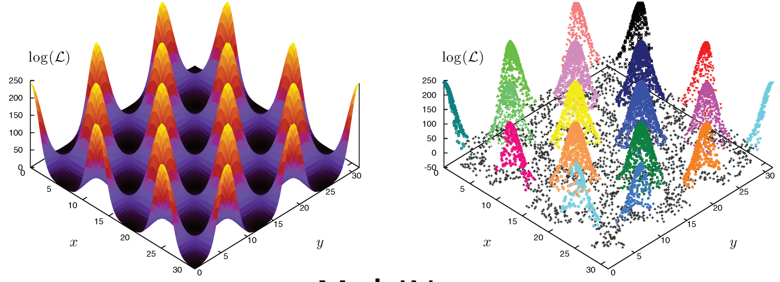




Successive points map likelihood in ordered way

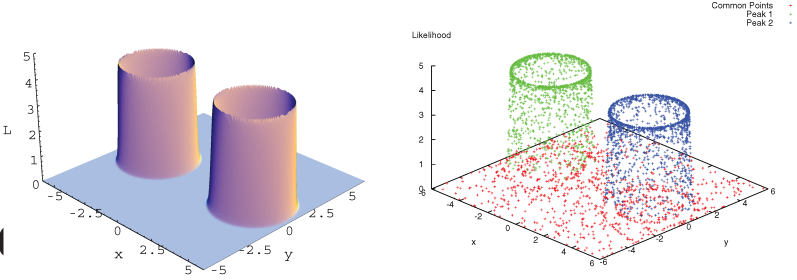


ICIC



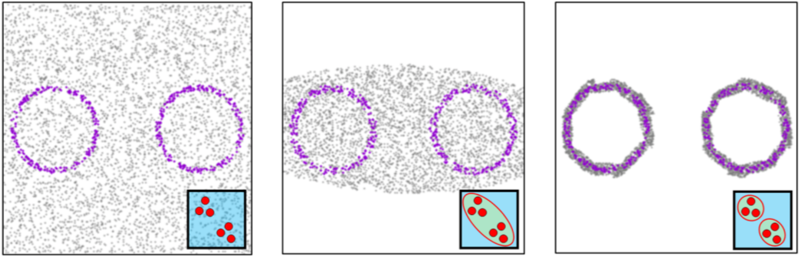
MultiNest

(a) (b)



(a) (b)

Dynamic Nested Sampling



Unit Cube
(no bound)

Single
Ellipsoid

Multiple
Ellipsoids

Dynamic Nested Sampling

https://dynesty.readthedocs.io/en/stable/
https://chi-feng.github.io/mcmc-demo/app.html