data analysis with

Markov chain Monte Carlo

Dan Foreman-Mackey
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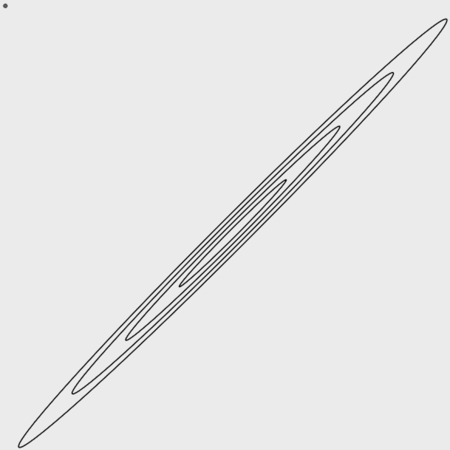
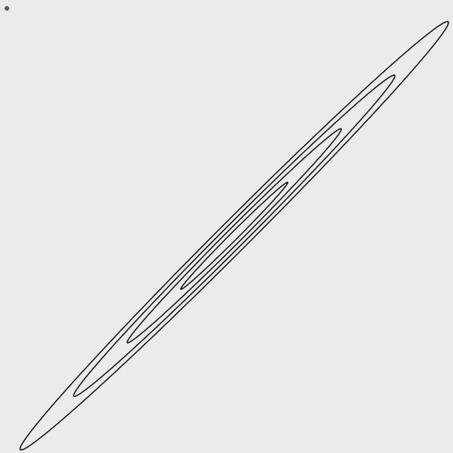
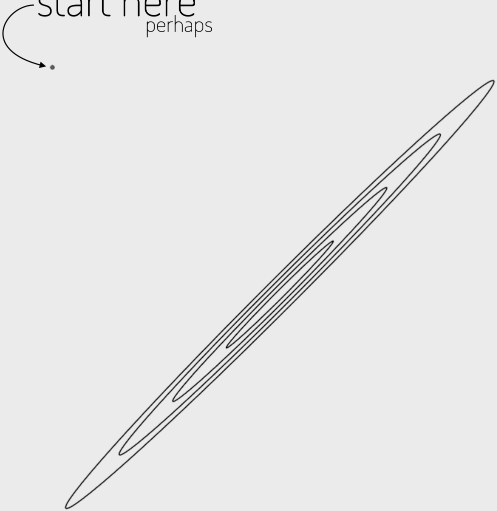


Metropolis–Hastings



Metropolis–Hastings

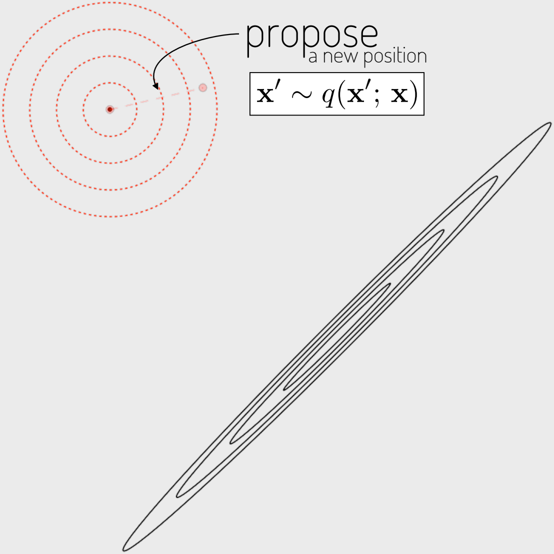
in an ideal world



start here
perhaps

Metropolis–Hastings

in an ideal world



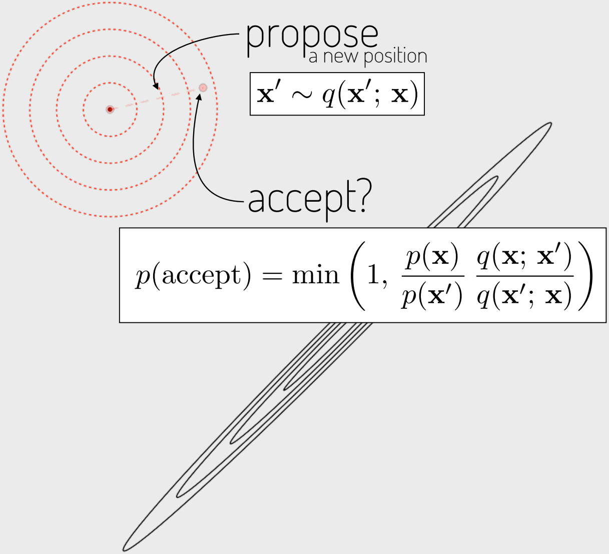
propose

a new position

$$\boxed{\mathbf{x}' \sim q(\mathbf{x}';\, \mathbf{x})}$$

Metropolis–Hastings

in an ideal world



propose
a new position

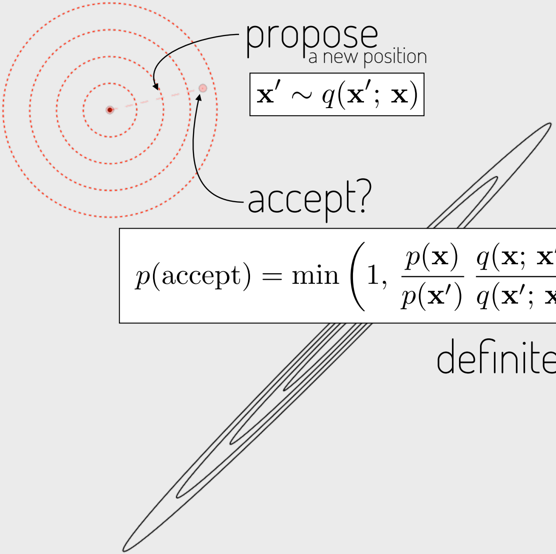
$$\boxed{\mathbf{x}' \sim q(\mathbf{x}';\, \mathbf{x})}$$

accept?

$$\boxed{p(\text{accept}) = \min\left(1,\ \frac{p(\mathbf{x})}{p(\mathbf{x}')}\,\frac{q(\mathbf{x};\,\mathbf{x}')}{q(\mathbf{x}';\,\mathbf{x})}\right)}$$

Metropolis–Hastings

in an ideal world



propose
a new position

$$\mathbf{x}' \sim q(\mathbf{x}'; \mathbf{x})$$

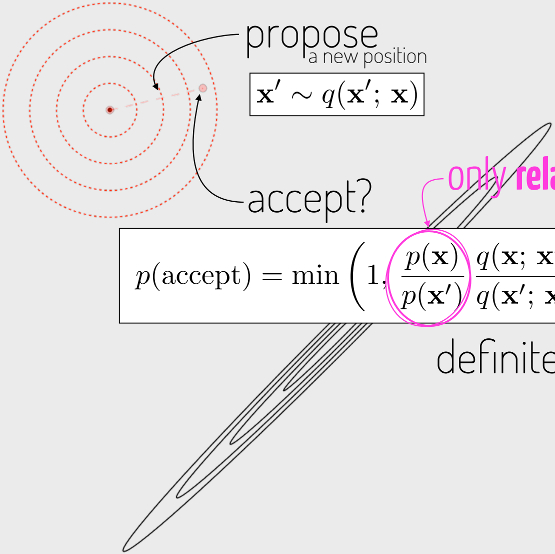
accept?

$$p(\text{accept}) = \min\left(1,\ \frac{p(\mathbf{x})}{p(\mathbf{x}')}\ \frac{q(\mathbf{x}; \mathbf{x}')}{q(\mathbf{x}'; \mathbf{x})}\right)$$

definitely.

Metropolis–Hastings

in an ideal world



propose
a new position

$$\boxed{\mathbf{x}' \sim q(\mathbf{x}';\, \mathbf{x})}$$

accept?

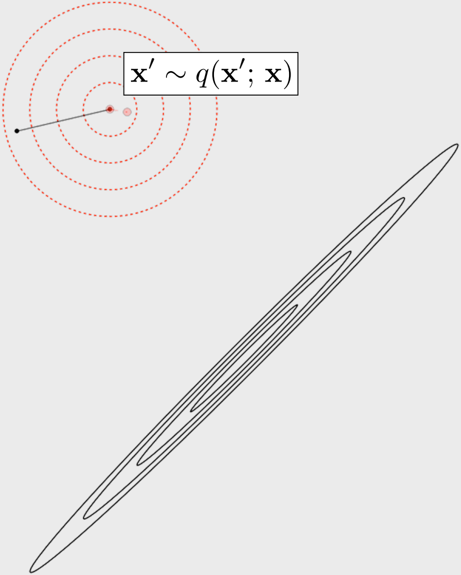
only **relative** probabilities

$$p(\text{accept}) = \min\left(1, \frac{p(\mathbf{x})}{p(\mathbf{x}')}\frac{q(\mathbf{x};\, \mathbf{x}')}{q(\mathbf{x}';\, \mathbf{x})}\right)$$

definitely.

Metropolis–Hastings

in an ideal world



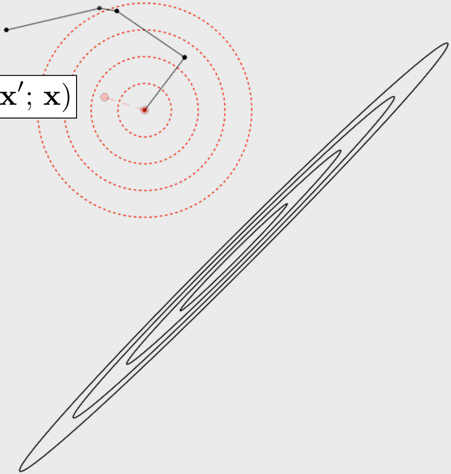
$$\mathbf{x}' \sim q(\mathbf{x}';\, \mathbf{x})$$

Metropolis–Hastings

in an ideal world

Metropolis–Hastings

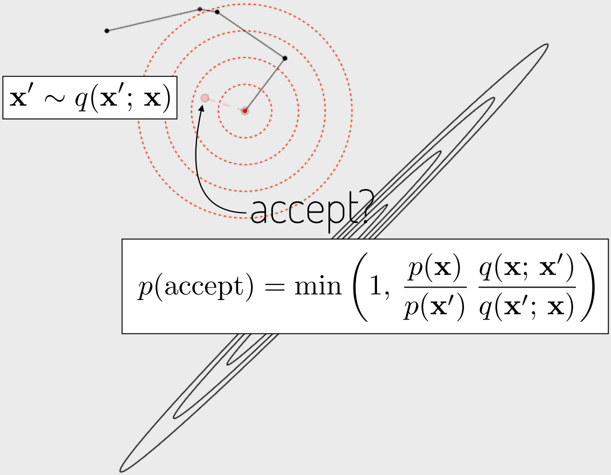
in an ideal world



$$\boxed{\mathbf{x}' \sim q(\mathbf{x}';\, \mathbf{x})}$$

Metropolis–Hastings

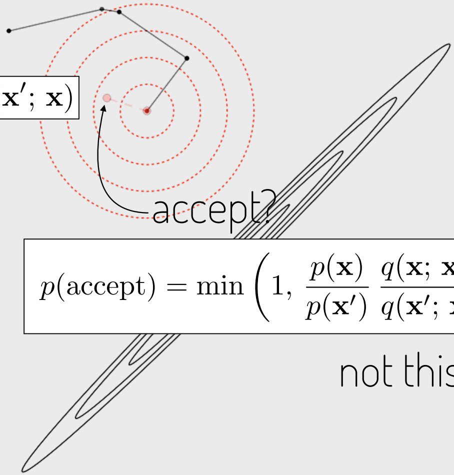
in an ideal world



$$\mathbf{x}' \sim q(\mathbf{x}'; \mathbf{x})$$

accept?

$$p(\text{accept}) = \min\left(1, \frac{p(\mathbf{x})}{p(\mathbf{x}')} \frac{q(\mathbf{x}; \mathbf{x}')}{q(\mathbf{x}'; \mathbf{x})}\right)$$



$$\mathbf{x}' \sim q(\mathbf{x}';\, \mathbf{x})$$

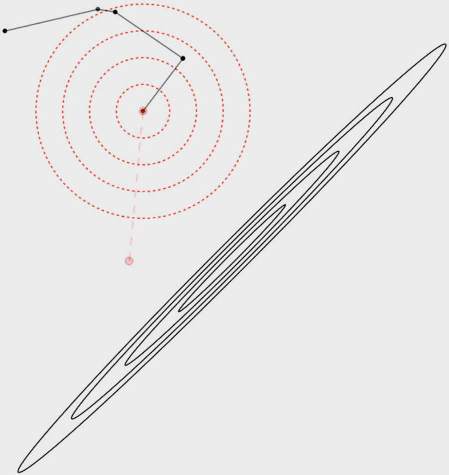
accept?

$$p(\text{accept}) = \min\left(1,\ \frac{p(\mathbf{x})}{p(\mathbf{x}')}\ \frac{q(\mathbf{x};\, \mathbf{x}')}{q(\mathbf{x}';\, \mathbf{x})}\right)$$

not this time.

Metropolis–Hastings

in an ideal world



Metropolis–Hastings

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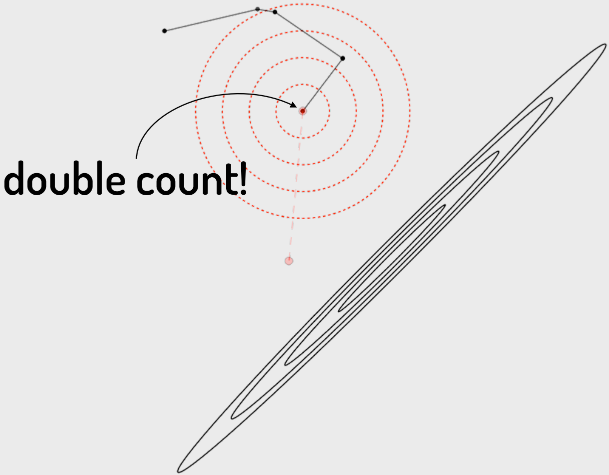
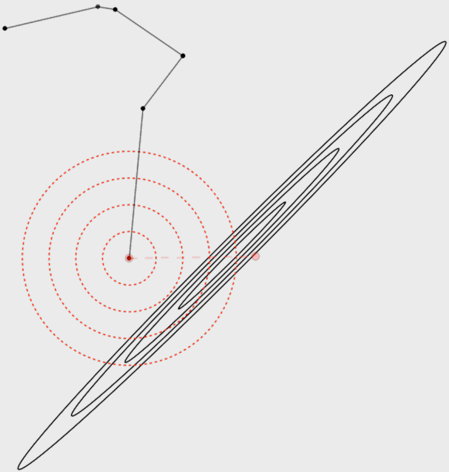
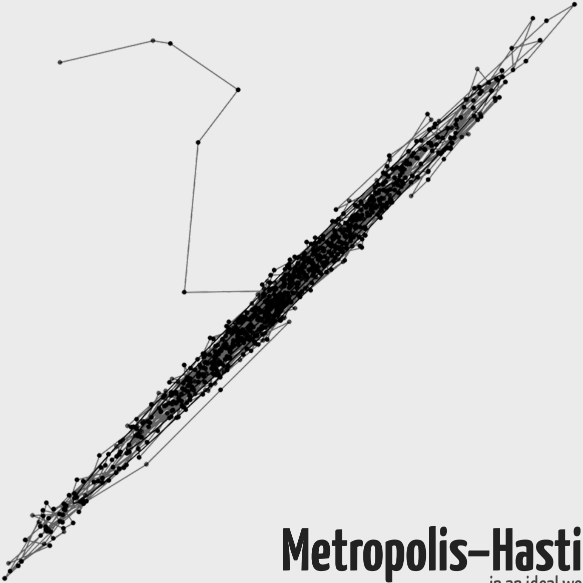


double count!



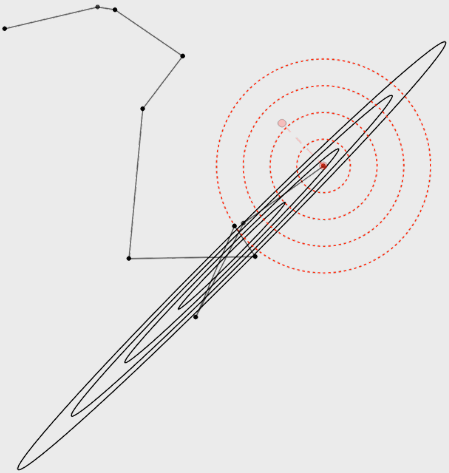
Metropolis–Hastings

in an ideal world



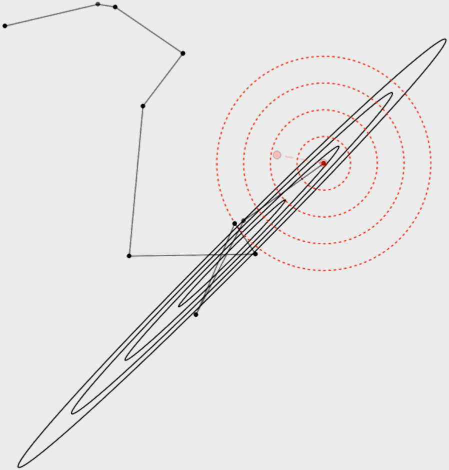
Metropolis–Hastings

in an ideal world



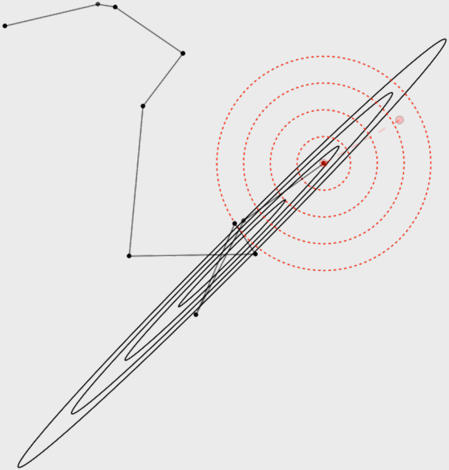
Metropolis–Hastings

in the real world



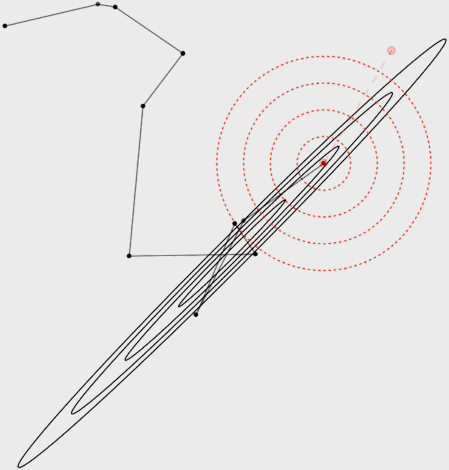
Metropolis–Hastings

in the real world



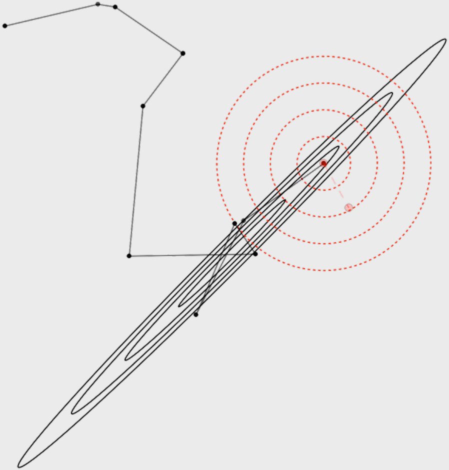
Metropolis–Hastings

in the real world



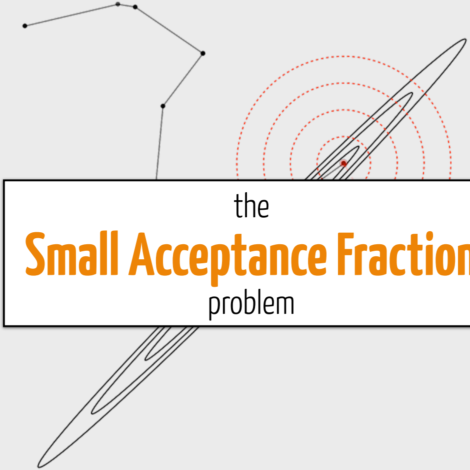
Metropolis–Hastings

in the real world



Metropolis–Hastings

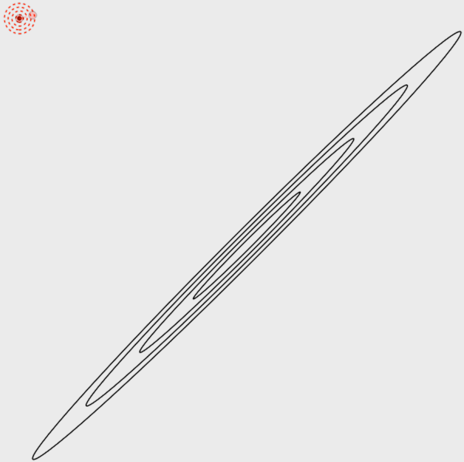
in the real world



the

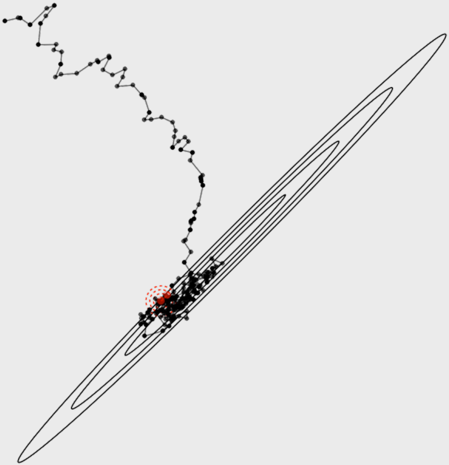
Small Acceptance Fraction

problem



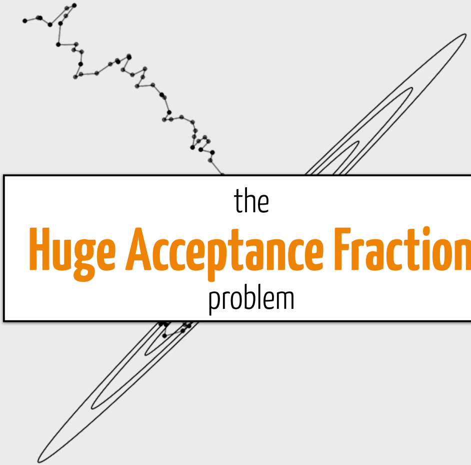
Metropolis–Hastings

in the real world



Metropolis–Hastings

in the real world



the

Huge Acceptance Fraction

problem