June 4th

DEF Let I be an ideal in R (commutative ring).

$$\sqrt{I} = \{r \mid r^n \in I\}$$

EX

$$\begin{aligned} \sqrt{(x^2, y^3)},\ (x,y) &\subseteq \sqrt{I} \\ I &= (x^2,^3) \\ \sqrt{I} &\subseteq (x,y) \end{aligned}$$

So now let's consider the ideal test. J is an ideal of R if:

1) J is closed under addition
2) $rJ \subseteq J$, $\forall r \in R$.
   Essentially says that it's closed under ANY multiplication.
   $\to$ NOTE: this gives also ADDITIVE INVERSES, so the subgroup test works!.

Sasha's work:

WTS: $rJ \subseteq J, \quad \&(Jr \subseteq R)$ (commutative ring), and the additive subgroup.

Proof. Let $r_1, r_2 \in J$. We know that $\exists n, m$ such that $r_1^n, r_2^m \in I$.

Let $a \in R$, WTS $ar_1 \in J$.

$(ar_1)^n = a^n r_1^n \qquad r_1^n \in I$

so $a^n r_1^n \in I_1$ so $(ar_1)^n \in I$.

Thus, $ar_1 \in J$. ☐

Marcus work:

We want to show that $(r_1 + r_2)^{n_1+n_2} \in I \Rightarrow r_1 + r_2 \in J$.

Proof. We have that:

$$(r_1 + r_2)^{n_1+n_2} = r_1^{n_1+n_2} + b_1 r_1^{n_1+n_2-1} + \cdots + bn_2 r_1^{n_1} r_2^{n_2} + \cdots + r_2^{n_1+n_2}$$

What's useful is that we can group our terms such that we can pull out a $r_1^{n_1}$ from all of our terms, which we know that $r_1^{n_1} \in I$.

Furthermore, the second group of terms will at least have a $r_2^{n_2}$ term, and we know that $r_2^{n_2} \in I$.

So that means every term in the binomial sum is in I, and because I is closed under addition, the sum must be in I, and therefore the sum must be in J.

Thus, J is an ideal. ☐

Gandini Notes

We'll start off with some preliminaries.

DEF Let $A \in GL_n(\mathbb{K})$.
We let A "act" on $\underline{x} = (x_1, \cdots, x_n)$. by $A \cdot \underline{x} = A\underline{x} \leftarrow$ MATRIX MULTIPLICATION.

DEF (Invariant Polynomial)
$f(x_1, \cdots x_n)$ is invariant under A
if $f(A\underline{x}) = f(\underline{x})$ (so f stays the same after ACTION)

DEF Let G be a group of marticies. If $f(A\underline{x}) = f(\underline{x}), \forall A \in G$,
f is an INVARIANT under G, on $f \in R^G$

$$R^G = \{f \mid f \text{ is invariant under } G\}$$

SUBRING TEST S

1. A constant in S
2. Closed under $+$
3. Closed under $\cdot$

Sam's Work:

1. $f(\underline{x}) = C$
   $\rightarrow f(A\underline{x}) = C$
2. $f_1, f_@ \in R^G$.
   WTS $f_1 + f_@ \in R^G$
   $f_2(A\underline{x}) + f_2(A\underline{x}) = f(\underline{x}) + f_2(\underline{x}) \in R^G$
3. $f_1, f_2 \in R^G$
   WTS $f_2 f_2 \in R^G$
   $f_1(A\underline{x}) f_2(A\underline{x})$
   $= f_1(\underline{x}) f_2(\underline{x})$ which is in R^G.

final gandini send-off

$\mathbb{K}[x^2]$ is the subring generated by x^2
(x^2) is the ideal generated by x^2
$1 \in$ SUBRING, $\quad 1 \notin (x^2)$ as $(x^2) \neq R$.
$x^3 \notin \mathbb{K}[x^2], \quad x^2 \in (x^2)$