

4) Full conditionals for σ_1^2 and σ_2^2

Prior for $1/\sigma^2$: $\text{Gamma}(V_0/2, V_0 \sigma_0^2/2)$

$$\sigma_1^2 | Y, X, \theta_1 \sim \text{Inverse-Gamma}\left(\frac{V_0 + n_1}{2}, \frac{V_0 \sigma_0^2 + \sum_{i: X_i=1} (Y_i - \theta_1)^2}{2}\right)$$

$$\sigma_2^2 | Y, X, \theta_2 \sim \text{Inverse-Gamma}\left(\frac{V_0 + n_2}{2}, \frac{V_0 \sigma_0^2 + \sum_{i: X_i=2} (Y_i - \theta_2)^2}{2}\right)$$

6.2 - part C

- see code in R

Effective sample size for $\theta/\theta_{\min} = 514.52$

Effective sample size for $\theta/\theta_{\max} = 210.99$

- see plot in R on online submission - ACF experiences a gradual decay suggesting that consecutive samples are correlated and the chain does not become independent quickly. This slow decay indicates that the Gibbs Sampler may require more iterations for the samples to adequately explore the distribution

6.2 - part d - While there was mainly one peak in the distribution in part a, there appears to be two distinct peaks in this distribution. This demonstrates that there are two distinct subgroups in the population based on average glucose concentration

- If we are looking to detect subgroups, the two-component model may be highlighting latent structure that could be worth exploring
- If we want to approximate the original data distribution, the single peak of the original histogram in part a suggests a simpler model can capture the overall distribution of glucose levels