$$\boxed{P(X_i=2 \mid Y_i,\theta_1,\theta_2,\sigma_1^2,\sigma_2^2,p) = 1-P(X_i=1 \mid Y_i,\theta_1,\theta_2,\sigma_1^2,\sigma_2^2,p)}$$

Homework 4.

6.2a) - See R code and histogram - Skewed to the right suggesting it may not fit to a normal distribution. (Some people have unusually high glucose levels)

6.2b) For each study participant, there is an unobserved group membership variable X_i which is equal to 1 or 2 with probability p and $1-p$. If $X_i=1$, then $Y_i \sim \text{normal}(\theta_1,\sigma_1^2)$, and if $X_i=2$ then $Y_i \sim \text{normal}(\theta_2,\sigma_2^2)$. let $p \sim \text{Beta}(a,b)$, $\theta_j \sim \text{normal}(\mu_0, \tau_0^2)$ and $1/\sigma_j \sim \text{gamma}(\nu_0/2, \nu_0\sigma_0^2/2)$ for both $j=1$ and $j=2$. Obtain the full conditional distributions of $(X_1,\dots X_n)$, p, θ_1, θ_2, σ_1^2, σ_2^2

Priors: $p \sim \text{Beta}(a,b)$, $\theta \sim \text{Normal}(\mu_0,\tau_0^2)$, $1/\sigma_j^2 \sim \text{Gamma}(\nu_0/2, \nu_0\sigma_0^2/2)$ for $j=1,2$

1) Full conditional for X_i (group indicator)

$$p(X_i=1 \mid Y_i,\theta_1,\theta_2,\sigma_1^2,\sigma_2^2,p) \propto p\cdot \text{Normal}(Y_i \mid \theta_1,\sigma_1^2)$$
$$p(X_i=2 \mid Y_i,\theta_1,\theta_2,\sigma_1^2,\sigma_2^2,p) \propto (1-p)\cdot \text{Normal}(Y_i \mid \theta_2,\sigma_2^2)$$

$$\boxed{P(X=1 \mid Y_i,\theta_1,\theta_2,\sigma_1^2,\sigma_2^2,p) = \frac{\overbrace{p\cdot \text{Normal}(Y_i \mid \theta_1,\sigma_1^2)}^{\text{likelihood}}}{p\cdot \text{Normal}(Y_i \mid \theta_1,\sigma_1^2) + (1-p)\cdot \text{Normal}(Y_i \mid \theta_2,\sigma_2^2)}}$$

2) Full conditional for P: prior for p is $\text{Beta}(a,b)$

$$\boxed{p \mid X_1,\dots X_n \sim \text{Beta}\left(a+\sum_{i=1}^{n} I(X_i=1), b+\sum_{i=1}^{n} I(X_i=2)\right)}$$

$$p \mid X \sim \text{Beta}(a+n_1, b+n_2)$$

3) Full conditionals for θ_1 and θ_2

$$\theta_1 \mid Y,X,\sigma_1^2 \sim \text{Normal}\left(\frac{\frac{\mu_0}{\tau_0^2}+\frac{\sum_{i:X_i=1} Y_i}{\sigma_1^2}}{\frac{1}{\tau_0^2}+\frac{n_1}{\sigma_1^2}}, \frac{1}{\frac{1}{\tau_0^2}+\frac{n_1}{\sigma_1^2}}\right)$$

n_1 = # of participants in group 1

n_2 = # of participants in group 2

$$\theta_2 \mid Y,X,\sigma_2^2 \sim \text{Normal}\left(\frac{\frac{\mu_0}{\tau_0^2}+\frac{\sum_{i:X_i=2} Y_i}{\sigma_2^2}}{\frac{1}{\tau_0^2}+\frac{n_2}{\sigma_2^2}}, \frac{1}{\frac{1}{\tau_0^2}+\frac{n_2}{\sigma_2^2}}\right)$$

$n_2 = n - n_1$