



A funny glitch: <https://www.youtube.com/watch?v=Nob6kcLod3w&feature=youtu.be>

# **EARLY NON-INTERACTIVE ANIMATION**



## Cave Art and Shadow Play

- Cave art with signs of motion
  - Speculation that flickering fire randomly illuminated different parts of image (e.g. legs appear to move)
- Shadow play developed in many cultures
- Shadow cast by light of puppets



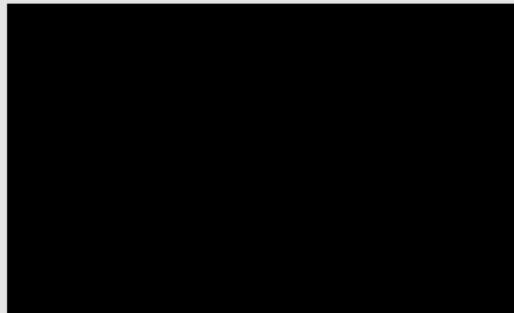
# Puppetry

- Perhaps 4000 years old



# Magic Lantern

- Evidence of basic animation in 1600's



Plans for animation of skeleton in 1659

<https://www.youtube.com/watch?v=t0waEVSXpYA>

Note that video depicts animated magic lantern animations much more recent than 1600's

## Automata

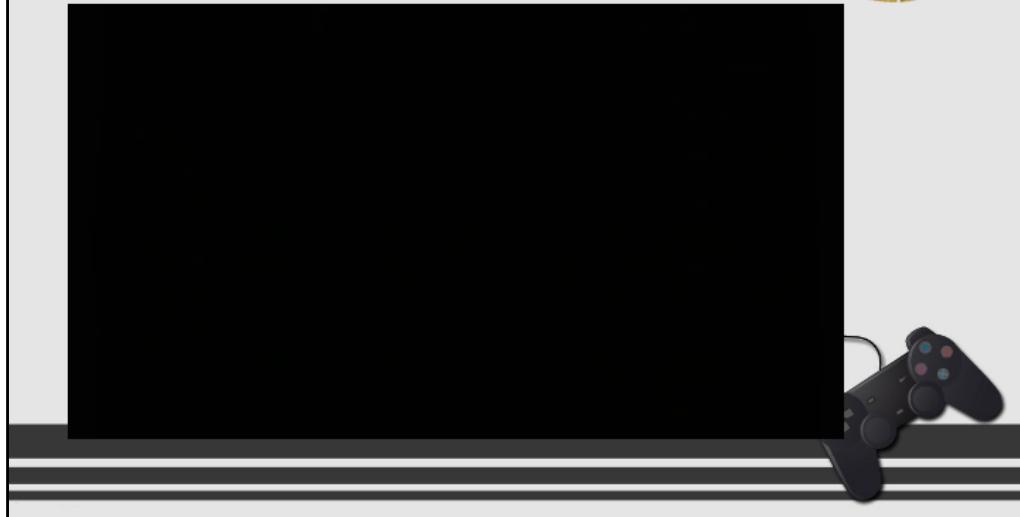
- Likely 1000's of years old techniques
- Machine animation
- Gears, levers, clockwork, etc.
- Spring power, or other mechanical source
- Disney later developed Animatronics



<https://www.youtube.com/watch?v=kd826ZXeGOo>



## Phenakistoscope (1833)



Look through slits in a mirror

<https://www.youtube.com/watch?v=3JeN3uk2CIE>

<https://www.youtube.com/watch?v=UqwkdlwmHig>

# Zoetrope (1833)



<https://www.youtube.com/watch?v=SBg6dAE3mI0>

<https://www.youtube.com/watch?v=-66v1ARI0-Q>

# Kineograph (1868)

- Flipbook



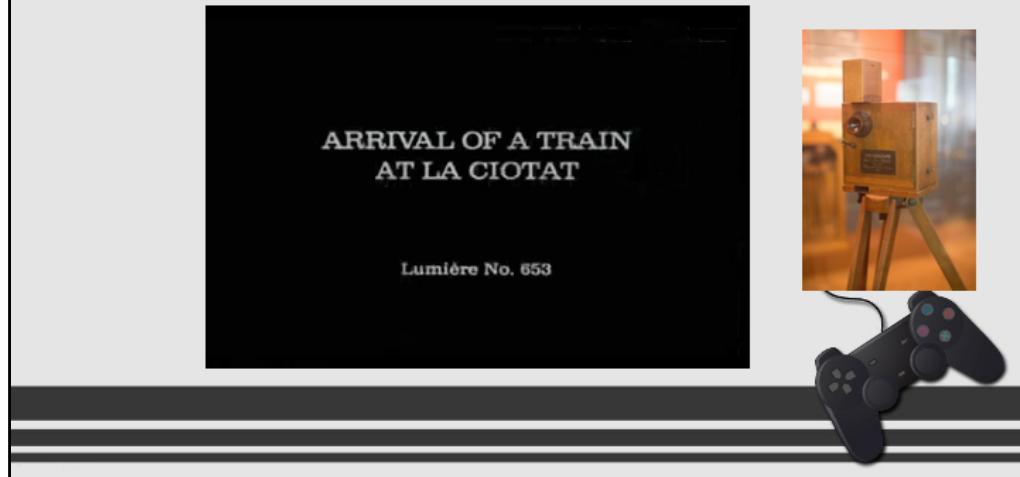
Kineograph, 1868



<https://www.youtube.com/watch?v=7K3aMdnCqXg>

# Early Film

- Arrival of a Train at La Ciotat (1895)



<https://www.youtube.com/watch?v=1dgLEDdFddk>

See-oh-tah ("t" at end silent)

# Early Cartoon

- *Fantasmagorie* (1908)

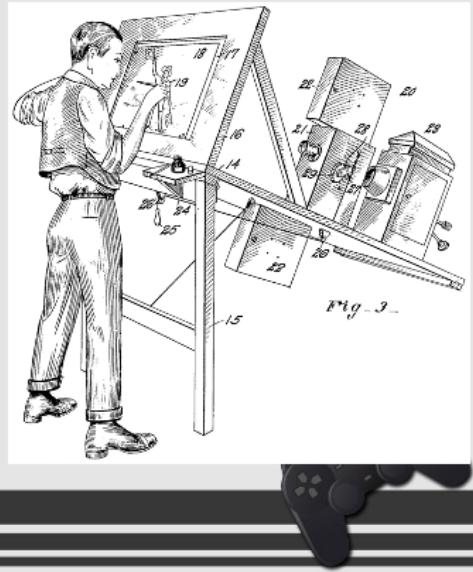


<https://www.youtube.com/watch?v=swh448fLd1g>

1908

# Rotoscope

- Max Fleischer (1915 Patent)
- Video frame projected onto glass
- Sketched over



# Out of the Inkwell

- First use of rotoscoping
- KoKo the Clown



## Disney – Snow White

- Rotoscoped w/ Actress, Marge Champion



<https://www.youtube.com/watch?v=4Wgm2OzDYVo>

Marge Champion - <https://www.youtube.com/watch?v=Hjt13WdEkZc>

## Live Action Reference

- Popularized by Disney



<https://www.youtube.com/watch?v=LWwO-h7ZSlw>

# Rotoscoping Abandoned

- Why did Disney abandon rotoscoping?



Snow white versus the 7 dwarfs

Snow white well song

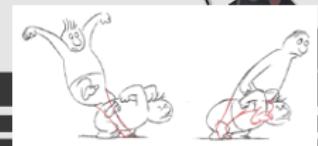
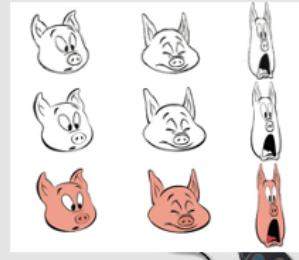
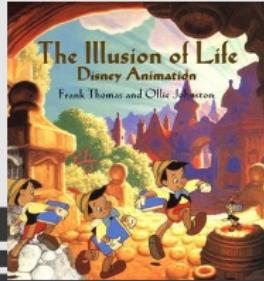
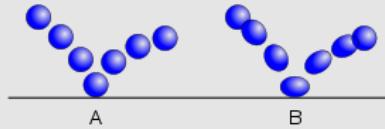
<https://www.youtube.com/watch?v=54QeNL5ih6A>

The washing song

<https://www.youtube.com/watch?v=ziyudbAiWrl>

# Disney's 12 Basic Principles of Animation

- Disney Animation: The Illusion of Life, Frank Thomas, Ollie Johnston (1981)
- **Squash and Stretch**
- Anticipation
- Staging
- Straight Ahead Action and Pose to Pose
- Follow Through and Overlapping Action
- Slow In and Slow Out
- Arc
- Secondary Action
- Timing
- **Exaggeration**
- Solid Drawing
- Appeal



<https://ohmy.disney.com/movies/2016/07/20/twelve-principles-animation-disney/>

[https://en.wikipedia.org/wiki/12\\_basic\\_principles\\_of\\_animation](https://en.wikipedia.org/wiki/12_basic_principles_of_animation)

Making Of Disney's Snowwhite or How Disney Cartoons are made:

<https://www.youtube.com/watch?v=OebUzEhSLBI>

<https://ohmy.disney.com/movies/2016/07/20/twelve-principles-animation-disney/>

Animators

Onion skin

Moviola – scrub through early animations

Inking on celluloid

Stage Settings

Foley Sound effects

Soundtrack

# **EARLY INTERACTIVE ANIMATION**



## Interactive Automata

- Penny Arcades
- Early Simulators



# Interactive Automata Games

- Penny Arcade
  - Bowling, baseball, etc.
- Board Games
  - Hungry Hungry Hippos
  - Rock 'em Sock 'em Robots



<https://www.youtube.com/watch?v=owwJ9Y-Wbzw>

## Simulators

- Link Trainer (WWII)
- Dehmel Flight Simulator
- Aerostructor
- Aetna Drivotrainer
- Etc.



# Aetna Drivotrainer (1951)

Controls coordinated with film projection  
Some configurations could pause the film reel advance until correct controls received  
(8:20)



<https://www.youtube.com/watch?v=jTcCNQ7T4rY>

Link trainer (flight) at beginning

1:00 look at the Drivotrainer classroom setup

2:48 scoring machine

8:20 interactive video playback

# **INTERACTIVE ANIMATION IN VIDEO GAMES**



# Vector Graphics

- Tennis for Two (1958)
- Spacewar! (1962)
- Geometric transformations of wireframe



Tennis for Two – first analog video game

Spacewar! – first digital video game

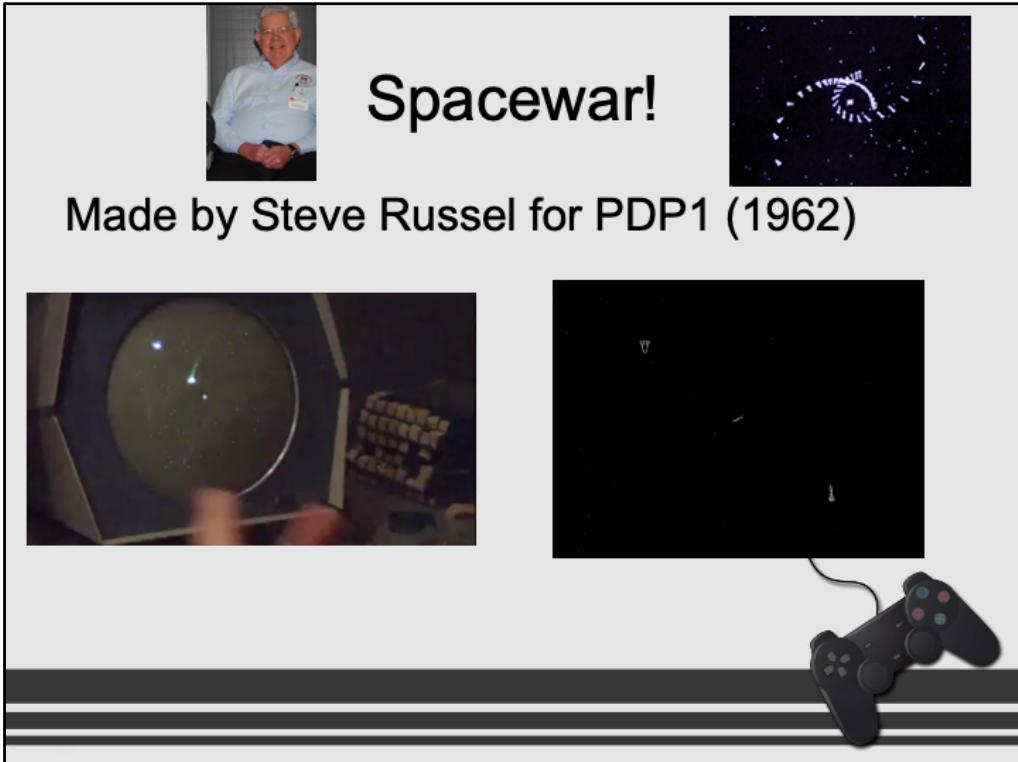
First video game

<https://www.youtube.com/watch?v=Rmvb4Hktv7U>

# Tennis for Two

Made by William Higinbotham with Analog logic! (1958)

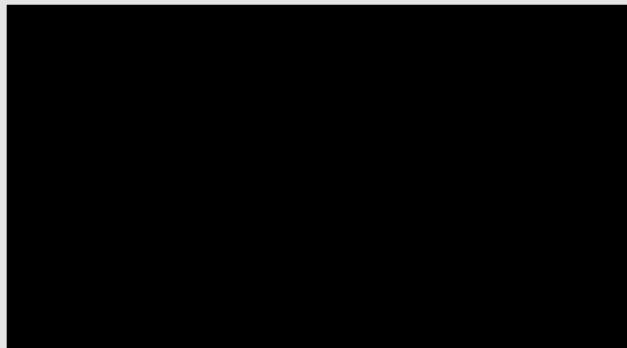




<https://www.youtube.com/watch?v=7bzWnaH-0sg>

# Raster Display

- Pong (1972)
  - Discrete logic!
- Addressable pixels for output
- Procedural line drawing/ poly fills



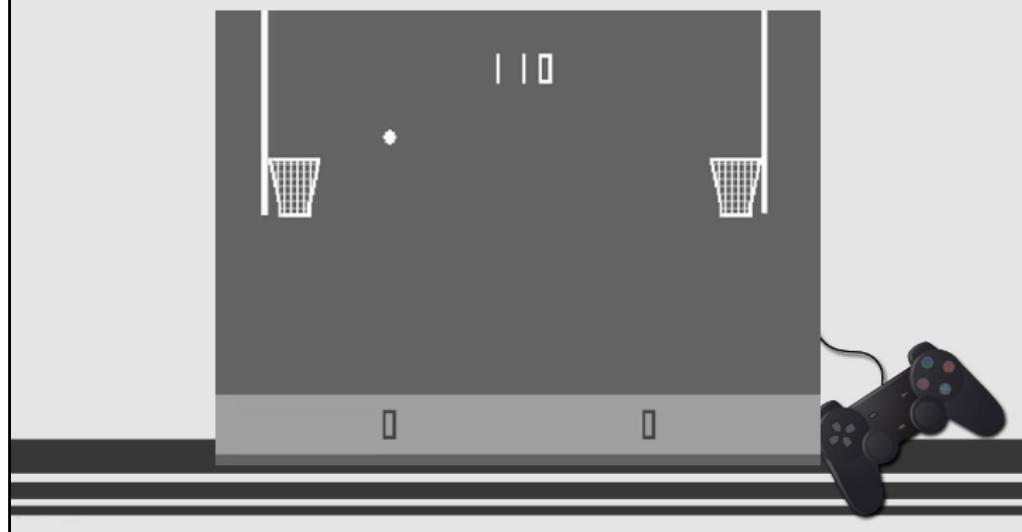
<https://www.youtube.com/watch?v=fiShX2pTz9A>

Discrete logic – not a central processing unit. Individual logic gates physically distributed

Discrete logic used for a number of arcade machines as much cheaper at the time compared to using a CPU

# Sprites

- Taito Basketball (licensed by Midway in US) (1974)



Arrays of pixel data that are copied to output pixel locations

[https://www.youtube.com/watch?v=BMV\\_meVa7m8](https://www.youtube.com/watch?v=BMV_meVa7m8)

# Text Mode

- Castle Adventure (1984)



Castle adventure: (1984)

[https://www.youtube.com/watch?v=OkPr\\_Xz6Ftk](https://www.youtube.com/watch?v=OkPr_Xz6Ftk)



## Sprite Animation

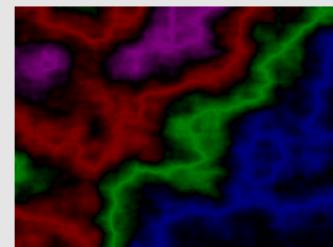


- Small raster frames of animation
- Superimposed onto game scene via Bit blit (or Blit bit block transfer)
- Or Hardware Sprites (game systems)
- No computer support for in-betweens (pixel art limitation)



# Color Cycling

- Animated Color Palette
- Graphics modes that use palette (color lookup table)



[http://www.effectgames.com/effect/article-Old\\_School\\_Color\\_Cycling\\_with\\_HTML5.html](http://www.effectgames.com/effect/article-Old_School_Color_Cycling_with_HTML5.html)

<http://www.effectgames.com/demos/canvascycle/>

## Dragon's Lair

- 1983 Arcade
- Laserdisc game
- Analog media with digital indexing
- Avoided some limitations of sprites and other techniques of the era but sacrificed interactivity



A game that is really closely tied to fundamentals of hand drawn animation, but introduces interactivity

What are the drawbacks of the approach taken by Dragon's Lair?

Only “Quick Time Events”

Laserdisc dump of dragon's lair

[https://www.youtube.com/watch?v=znO\\_m00s8II](https://www.youtube.com/watch?v=znO_m00s8II)

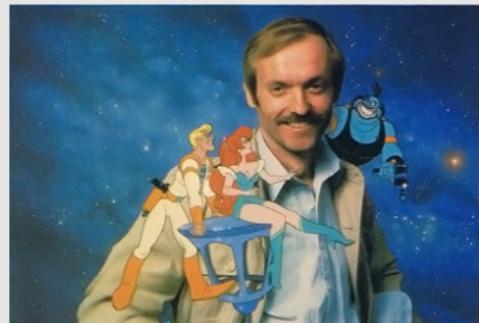
# Don Bluth

- 80's animator
- Game Designer
- The Secret of NIMH
- An American Tale
- The Land Before Time
- Dragon's Lair
- Space Ace
- Etc.



# Don Bluth

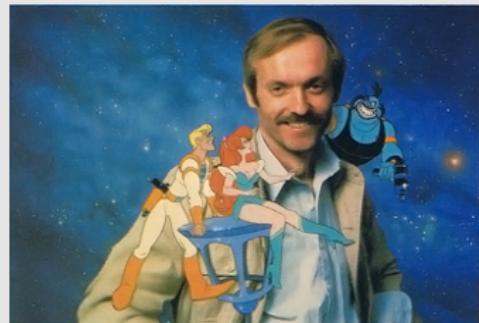
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Donald Knuth – Famous CS Professor from Standford

# Don Bluth

- 80's animator
- Game Designer
- The Secret of NIMH
- An American Tale
- The Land Before Time
- Dragon's Lair
- Space Ace
- Etc.
- (Completely different guy than **Donald Knuth**)



Donald Knuth – Famous CS Professor from Standford

## Rotoscoped Gaming

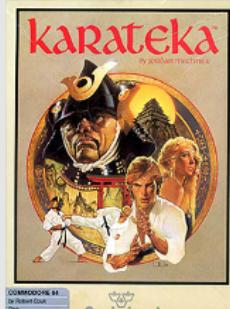
- Karateka (1984)
- Prince of Persia (1989)
- Dragon's Lair (1989 Amiga port of arcade)
- Another World (aka Out of this World) (1991)
- Flashback (1992)
- The Last Express (1997)





# Karateka

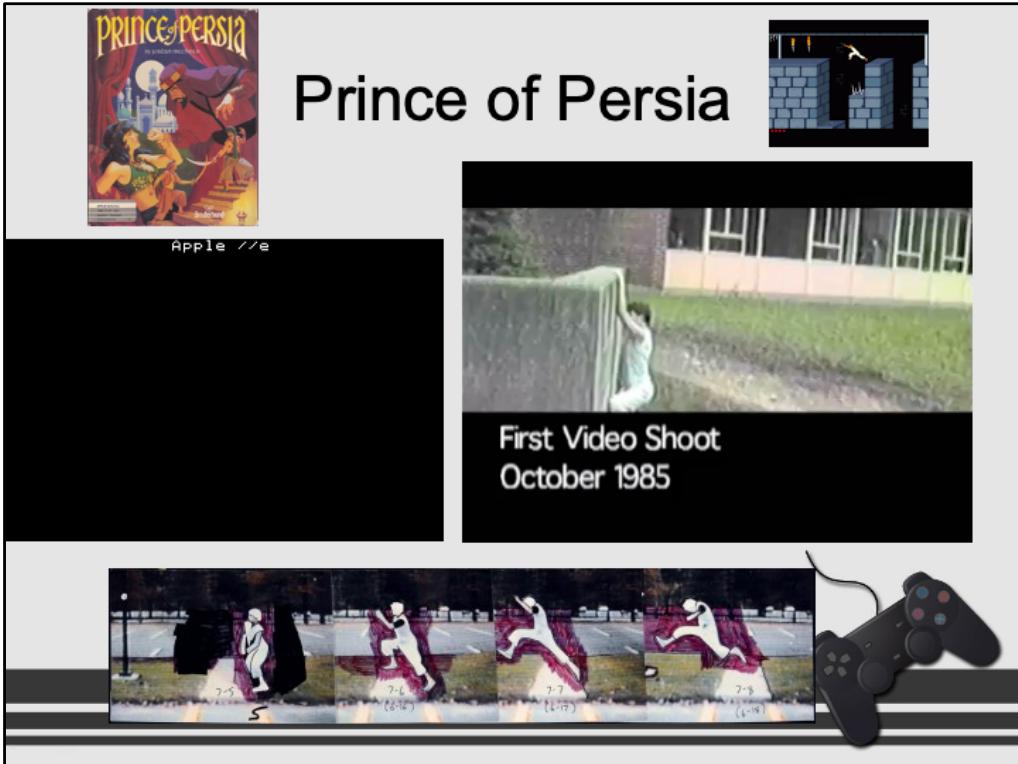
- Jordan Mechner



Mechner says: CARE – ah – TEK – ah  
But thinks any pronunciation is fine

<https://www.youtube.com/watch?v=wKqk9kosCs4>

1984



1989 Apple II version

2:44 level starts

Erol Flynn – The Adventures of Robin Hood

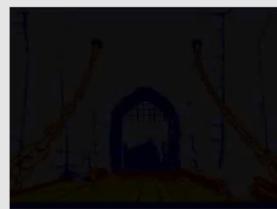
<http://www.museumofplay.org/blog/chegeads/2014/10/jordan-mechner-collection-documents-revolution-in-game-graphics/>

<https://www.youtube.com/watch?v=fggouSd3dr4>

## Dragon's Lair (Amiga)



- Computer-supported rotoscoping for Amiga (and other platforms)
- Fit on ~8 floppies
- Rasterized Vector (polygon fills) instead of sprites

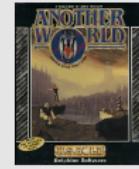


Amiga Version

<https://www.youtube.com/watch?v=dSE5LFdOtQI>



## Another World



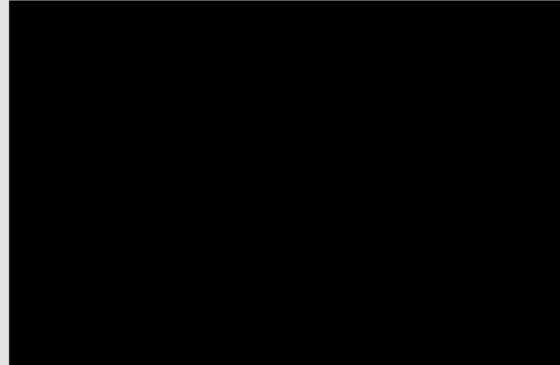
- Eric Chahi – inspired by Amiga Dragon's Lair port for cinematic cut scenes



<https://www.youtube.com/watch?v=GTb6zX7Hd5I>

3D

- Battlezone (1980)



Rendered on a Vector CRT Display (not raster)

## 2D Sprites with Scaling and Rotation

- Super Nintendo Mode 7
- Modern Retro Games
- 3D Effects



Super Mario world

<https://www.youtube.com/watch?v=UToiJXdycog>

Mode 7 is a special background texture rendering pipeline

Terraria Example vid: (warning commentary that may be inappropriate)

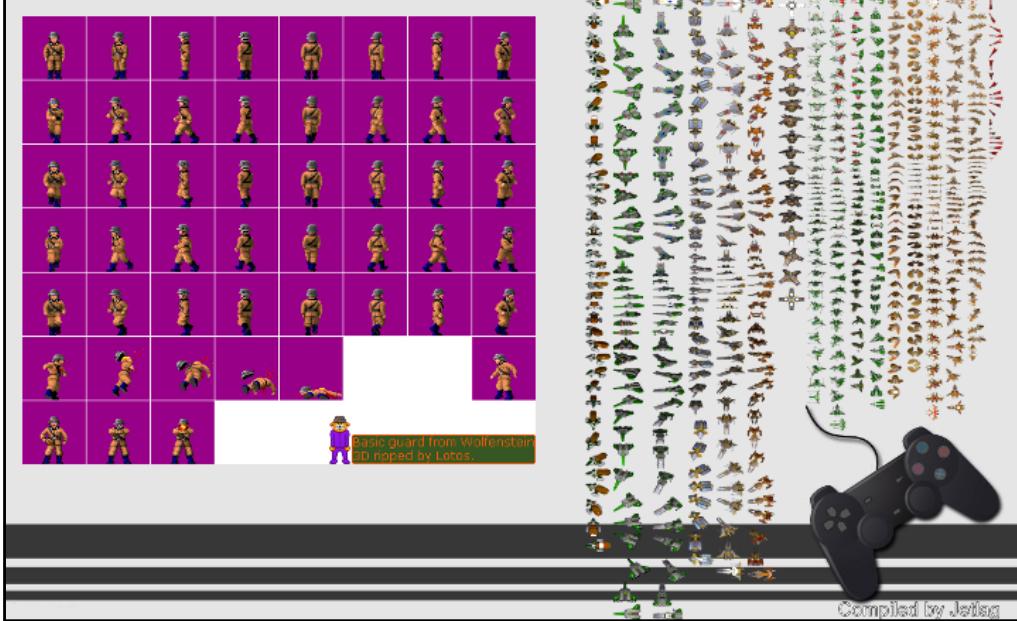
<https://www.youtube.com/watch?v=jsxILQvZcr4>

## 3D with 2D Billboards

- Wolfenstein, Doom
- Wing Commander



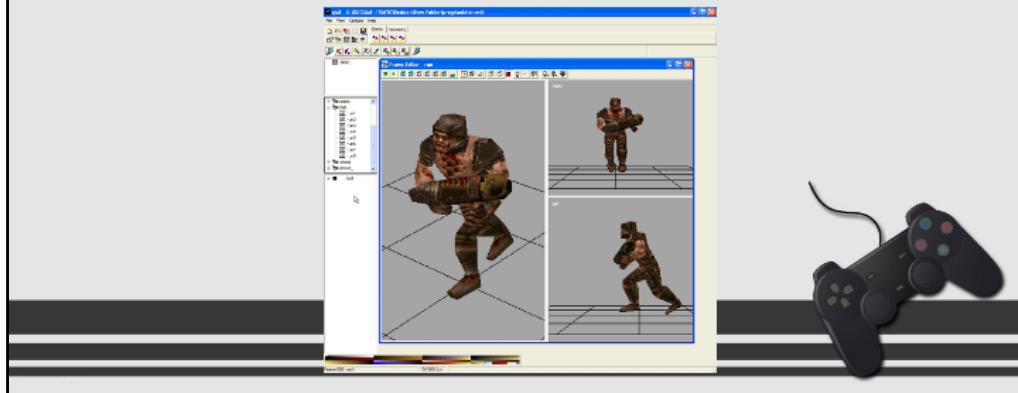
## Billboard Examples



Wolfenstein3d and Wing Commander

## Animated 3D Mesh - Quake

- Keyframes (no interp) at 10 FPS
- Each frame: array of ordered vertexes applied to model triangle mesh



4D Sports Boxing – 1991 (mo-cap, unclear if interpolated)

Alone in the Dark – 1992 (had interpolation)

I captured from Quake Model Explorer (qME) run on a VM. Not hosted online, sorry

# Animated 3D Mesh Rendering

- Identify curr frame
- Use model's triangles to obtain vertex indexes of curr frame's vertices

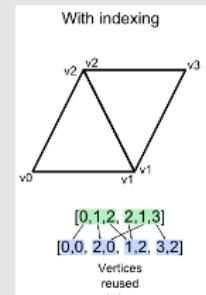
```
struct mdl_vertex_t *pvert1, *pvert2;  
vec3_t v;  
  
for /* ... */  
{  
    pvert1 = &mdl->frames[current].frame.verts[mdl->triangles[i].vertex[j]];
```



# Animated 3D Mesh Storage

- Model consists of vertex ( $x, y, z$ ) indexed triangles ( $v1, v2, v3$ ) (or perhaps triangle strips w/ shared vertex indexes) and texture coordinate refs ( $u, v$ )
- Animation frames contain vertex tables with differing position vectors
- Animation played by parsing model tris with ref to curr frame verts
- Animation storage *increases* as model mesh detail *increases*!
- Increasing frame rate introduces an entire new vertex table per additional frame!

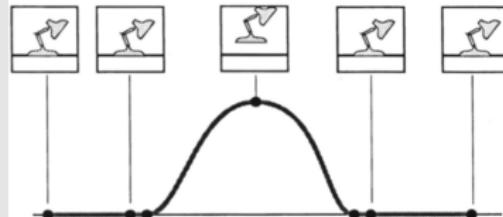
VERTEX TABLE	
$V_1:$	$x_1, y_1, z_1$
$V_2:$	$x_2, y_2, z_2$
$V_3:$	$x_3, y_3, z_3$
$V_4:$	$x_4, y_4, z_4$
$V_5:$	$x_5, y_5, z_5$



# Computer-Assisted Animation

## Keyframing

- automate the inbetweening
- good control
- less tedious
- creating a good animation still requires considerable skill and talent

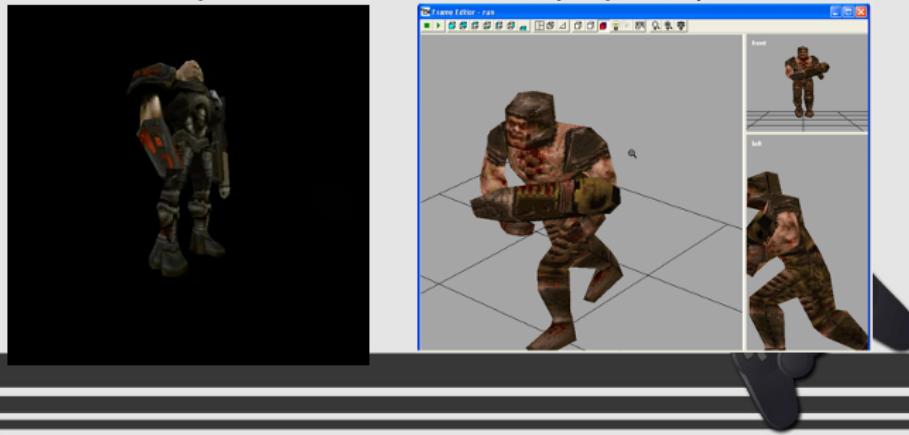


ACM © 1987 "Principles of traditional animation applied to 3D computer animation"



## Anim. 3D Mesh Interp - Quake 2

- Keyframes (linear interp) at 10 fps
- Q2 left; Q1 right (both slowed to 2 fps keyframe animation speed for demonstration purposes)



Quake 1 did not have inbetween frames (linear interp'ed) originally. Quake 2 however, did.

Not actually in-game animation, but appropriate for demonstation purposes  
<https://www.youtube.com/watch?v=7WYSS5Zz2xs>

# Interpolation

- Like before but obtain curr frame and next frame
- Linear Interpolate (lerp) between the two

```
struct mdl_vertex_t *pvert1, *pvert2;
vec3_t v;

for /* ... */
{
    pvert1 = &mdl->frames[current].frame.verts[mdl->triangles[i].vertex[j]];
    pvert2 = &mdl->frames[current + 1].frame.verts[mdl->triangles[i].vertex[j]];

    /* ... */

    v[0] = mdl->header.scale[0] * (pvert1->v[0] + interp * (pvert2->v[0] - pvert1->v[0])) + mdl->header.translate[0];
    v[1] = mdl->header.scale[1] * (pvert1->v[1] + interp * (pvert2->v[1] - pvert1->v[1])) + mdl->header.translate[1];
    v[2] = mdl->header.scale[2] * (pvert1->v[2] + interp * (pvert2->v[2] - pvert1->v[2])) + mdl->header.translate[2];
}
/* ... */
```

## Alone in the Dark (1992)

- One of the first games with interpolated keyframe animation



H. P. Lovecraft inspired game. Cthulhu

Gameplay a bit like resident evil

Different video that embedded but close enough:  
<https://www.youtube.com/watch?v=iSwYY2eoKhQ>

## Computer-Assisted Animation

- Procedural animation
  - describes the motion algorithmically
  - express animation as a function of small number of parameters
  - Example: a clock with second, minute and hour hands
    - Programmatically apply rotation to transforms



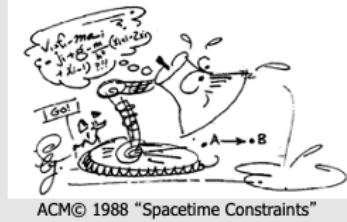
Uncharted 4 – Clock Tower

Probably different video, but close enough:

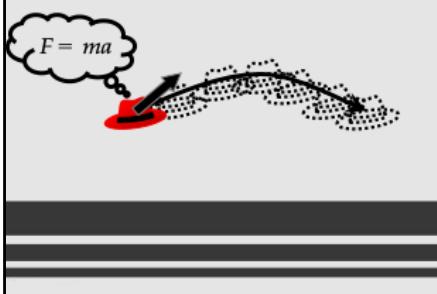
[https://www.youtube.com/watch?v=oIWkkn\\_y\\_t8](https://www.youtube.com/watch?v=oIWkkn_y_t8)

# Computer-Assisted Animation

- Physically Based Animation
  - Assign physical properties to objects (masses, forces, inertial properties)
  - Simulate physics by solving equations
  - Realistic but difficult to control



ACM© 1988 "Spacetime Constraints"



Just Cause 3

Embedded video is heavily edited montage by me. Not posted online. Just search for "Just Cause 3" in youtube and watch several to get an idea of the physics

## Computer-Assisted Animation

### Motion Capture

- Captures style, subtle nuances and realism
- You must observe someone do something
- Difficult to edit

Naughty Dog Mo-cap

[https://www.youtube.com/watch?v=mn5\\_-aZIVIU](https://www.youtube.com/watch?v=mn5_-aZIVIU)

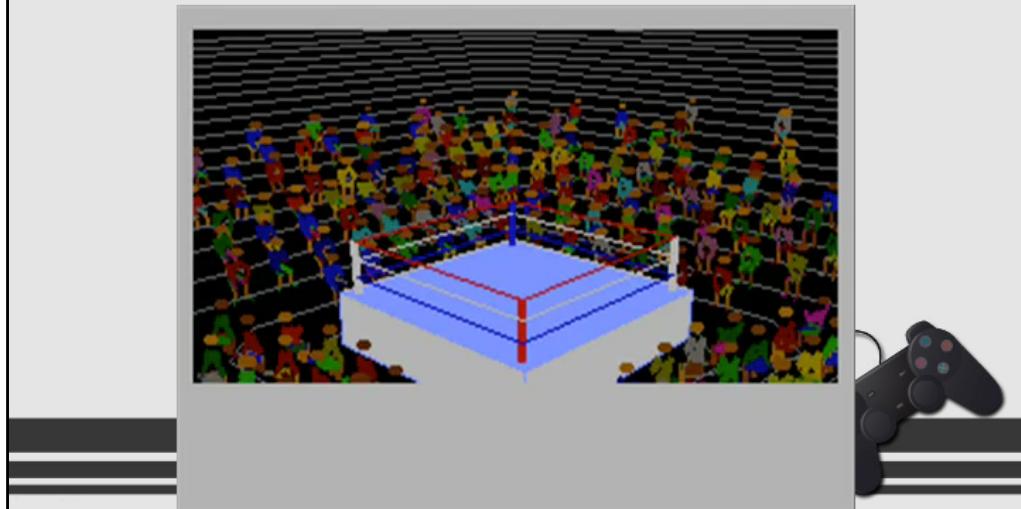
Not same as embed: [https://www.youtube.com/watch?v=7\\_mjLSzW13g](https://www.youtube.com/watch?v=7_mjLSzW13g)

Probably the video I used for Last of Us:

<https://www.youtube.com/watch?v=aDVSnawC5HU>

## 4D Sports Boxing (1991)

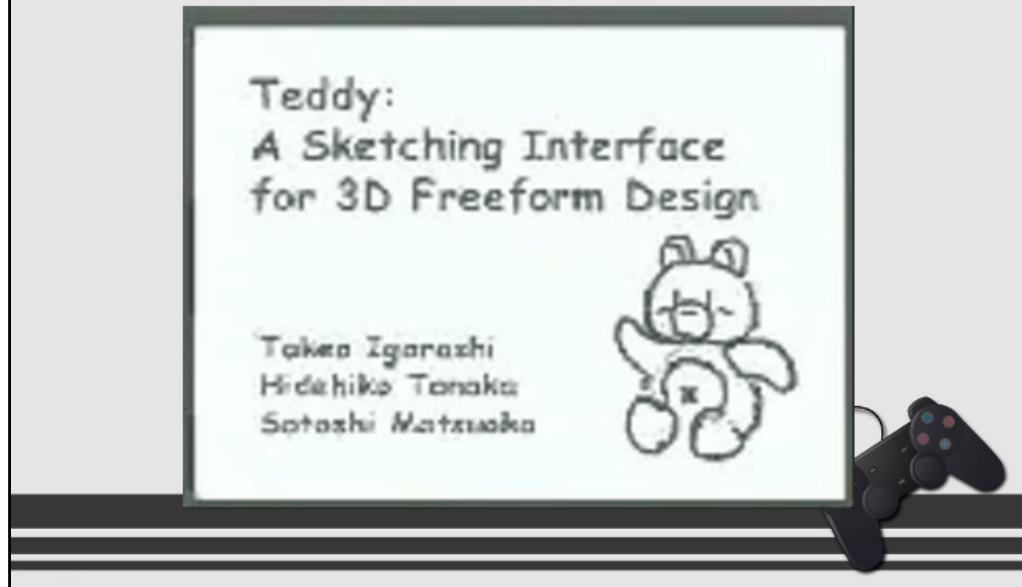
- One of the first mo-cap games



4D sports boxing

[https://www.youtube.com/watch?v=Jt5FrBFXk\\_8](https://www.youtube.com/watch?v=Jt5FrBFXk_8)

## Obtain 3D animations from 2D drawings?



<https://www.youtube.com/watch?v=e2H35SILmUA>

## Skeletal Animation

- Abstract skeleton
- Skeleton deforms mesh as it moves
- Keyframes only for skeleton
- Root: 3 or 6DOF; other bones 1/2/3DOF rotation (depending on joint)
- Mesh vertices assigned to bones by weights

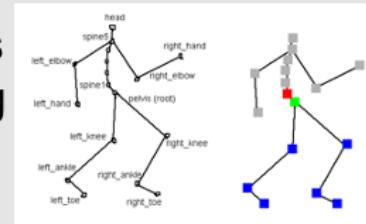
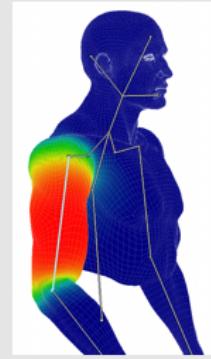


Image from: <http://rodolphe-vaillant.fr/?e=29>

# Skeletal Animation - Rigging



[https://www.youtube.com/watch?v=mhQY2\\_gVoVg](https://www.youtube.com/watch?v=mhQY2_gVoVg)

Image from: <http://rodolphe-vaillant.fr/?e=29>

# Skeletal Animation

- **Advantages**

- Memory Savings relative to mesh animation
- Animation blending (multiple animations, and/or IK)
- Animation portability/reuse
- Animation authoring is simplified

- **Disadvantages**

- Implementation difficulty
- Computational overhead (but can be done in shader!)
- Problems with realistic mesh deformations (e.g.  
preserving mesh volume, muscle flex, etc.)

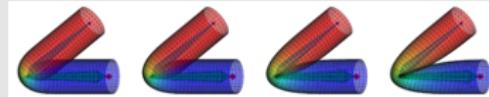
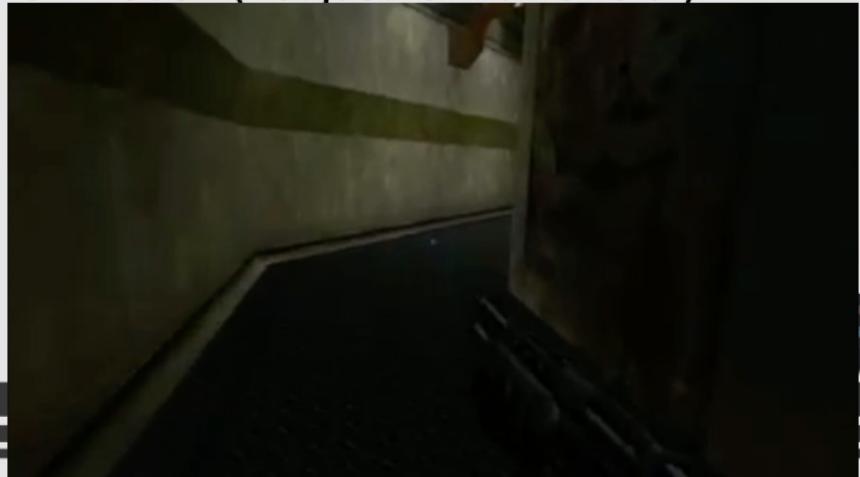


Image from: <http://rodolphe-vaillant.fr/?e=29>

## Half-Life (1998)

- One of the first games with skeletal animation (not just for humanoid!)



<https://www.youtube.com/watch?v=rdNB6lDEp-k>

# Principles of Animation Applied to 3D Games



<https://www.youtube.com/watch?v=BbP6Jsh8M6Y>

Looks like url changed for some reason

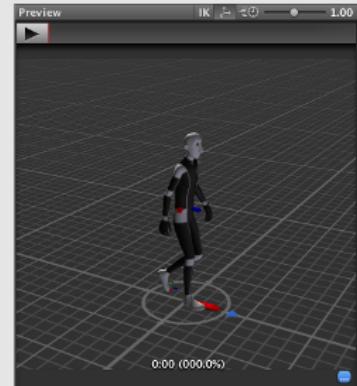
Old:

<https://www.youtube.com/watch?v=HYP7rJni1vc>

# Root Motion

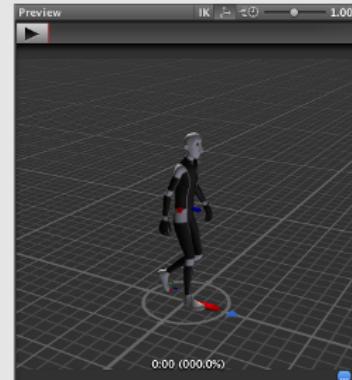


- The root bone (top of bone hierarchy; usually hips/pelvis) has 6DOF
- Root motion refers to the X,Y,Z translation and pitch, yaw, roll rotation of the root bone as a component of a skeletal animation
- This motion can be applied accumulatively to the game object, resulting in locomotion



# Root Motion

- In Unity, the Root Transform is a projection on the Y plane of the Body Transform and is computed at runtime. At every frame, a change in the Root Transform is computed. This change in transform is then applied to the Game Object to make it move.



<https://docs.unity3d.com/Manual/RootMotion.html>

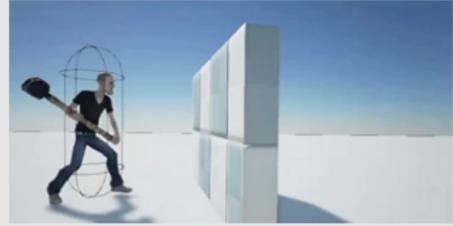
The Body Transform is the mass center of the character. It is used in Mecanim's retargeting engine and provides the most stable displacement model. The Body Orientation is an average of the lower and upper body orientation relative to the Avatar T-Pose.

## Generic Root Motion and Loop Pose

This works in essentially the same as Humanoid Root Motion, but instead of using the Body Transform to compute/project a Root Transform, the transform set in **Root Node** (e.g. the hips) is used. The Pose (all the bones which transform below the Root Motion bone) is made relative to the Root Transform

## Root Motion - Benefits

- Better cohesion between artist vision and world simulation (avoid capsule separation, movement aligned with foot-fall)
- Declarative (movement spec independent of code)



<https://docs.unrealengine.com/latest/INT/Engine/Animation/RootMotion/>

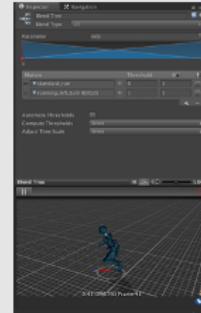
## Root Motion Constraints (Unity)

- Limit root transforms to only certain dimensions: rotation, Y-axis (up), XZ
- Green light indicates candidate for bake into pose
- Y-axis: special case. If baked, allows falling according to physics

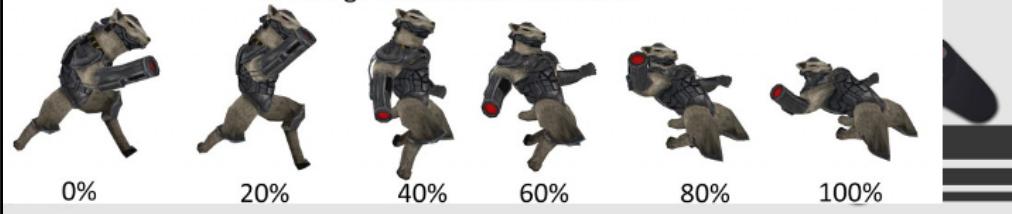


# Animation Blending

- Achieve variations between different extremes of similar animations
- **This means just a few animations can define a whole range of movement possibilities!**
- Basic case, usually must be "similar" animations
- Skeletal animation highly beneficial
- **Root motion can be blended too!**



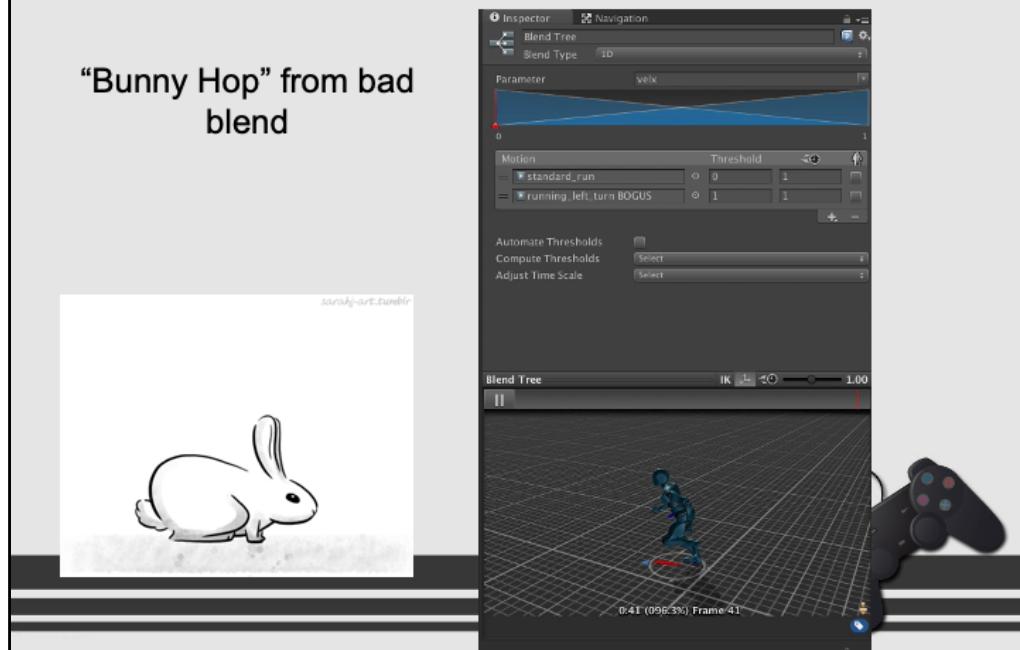
blending run and slide animation



<https://markobl.com/2014/12/03/animation-blending-in-monogame-xna/>

# Can only blend similar animations

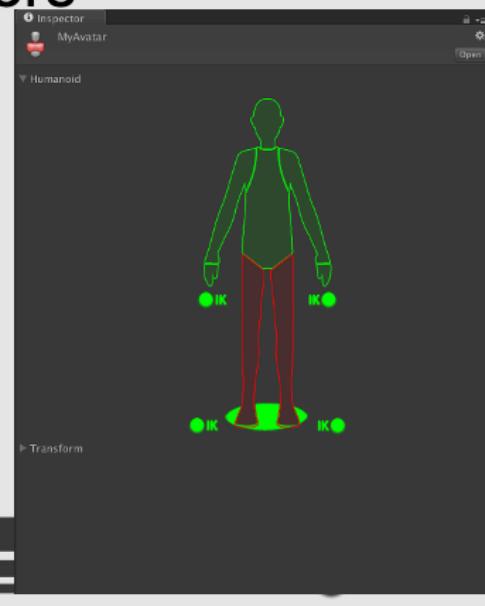
“Bunny Hop” from bad blend



Bunny Hop animation example – made by me but not hosted

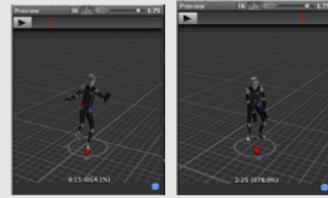
# Avatar Masks and Animation Layers

- Layers good for dissimilar animation blending
- Masks allow more control of blending/layering of animations
- Animations that would negatively interfere w/o mask
- Ex: running while shooting
- Works well with IK



# Match Targets

- Utility method for Lerp/Slerp with animation timeline and transforms
- Other game engines might not provide or call it something else
- Adjust a limb position/rotation over the course of time to land in a particular pose
- Applications:
  - Stand in a specific spot
  - Jump and land at specific spots
  - Press a button
  - Grab hold of a ledge
- Unity:Animator.MatchTarget()
- Problem: introduces sliding



<https://docs.unity3d.com/ScriptReference/Animator.MatchTarget.html>

<https://www.youtube.com/watch?v=JaLh7fkCsKY>

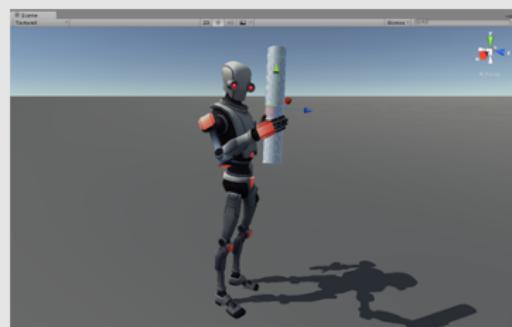
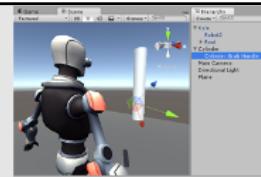
# Inverse Kinematics

- Forward Kinematics
  - Given the skeleton parameters (position of the root and the joint angles  $\mathbf{p}$ ) and the position of the sensor/effecter in local coordinates  $\mathbf{v}_s$ , what is the position of the sensor in the world coordinates  $\mathbf{v}_w$ .
  - Not too hard, we can solve it by evaluating  $\mathbf{S}(\mathbf{p})\mathbf{v}_s$
- Inverse Kinematics
  - Given the the position of the sensor/effecter in local coordinates  $\mathbf{v}_s$  and the position of the sensor in the world coordinates  $\mathbf{v}_w$ , what are the skeleton parameters  $\mathbf{p}$ .
  - Much harder requires solving the inverse of  $\mathbf{S}(\mathbf{p})$  the non-linear function
  - We can solve it by root-finding  $\mathbf{p}?$  such that  $\mathbf{S}(\mathbf{p})\mathbf{v}_s - \mathbf{v}_w = 0$
  - We can solve it by optimization minimize  $\underset{\mathbf{p}}{\mathbf{p}} (\mathbf{S}(\mathbf{p})\mathbf{v}_s - \mathbf{v}_w)^2$



# IK

- Lets you move joint to a target
- E.g. Press button, grab object, adapt leg animation to terrain, look in a direction, etc.



[https://www.youtube.com/watch?v=\\_W\\_uE70YuHQ](https://www.youtube.com/watch?v=_W_uE70YuHQ)

## IK and Authoring

- IK also facilitates manual animation creation (or perhaps puppetry)



# IK in Unity

- public void **SetIKPositionWeight**([AvatarIKGoal](#) goal, float value)
- public void **SetIKRotationWeight**([AvatarIKGoal](#) goal, float value)
- public void **SetIKPosition**([AvatarIKGoal](#) goal, [Vector3](#) goalPosition)
- public void **SetIKRotation**([AvatarIKGoal](#) goal, [Quaternion](#) goalRotation);
- public void **SetLookAtPosition**([Vector3](#) lookAtPosition);
- Weights can be interpolated [0,1], perhaps from an animation's normalized time (for instance, a button press animation)
- AvatarIKGoal: Left/RightFoot, Left/RightHand



AvatarIKGoal refers to an enumeration of skeleton bones

## IK with Locomotion



Uncharted 4

Constraint feet, legs

Pivot foot

Captured by me – not hosted

# Early IK Animation

- Terra Nova Strike Force Centauri ('96)



Animations with IK adjustments of legs

<https://www.youtube.com/watch?v=YdBGaNTBs8U>

# Early IK Animation

- Trespasser ('98)



Animations with IK adjustments of legs

Possibly not the video I embedded, but close. Ignore the commentary

<https://www.youtube.com/watch?v=3CMGxd9nhPw&t=1207s>

## Retargeting Skeletal Animations

- Map animation from one skeleton to another
- Unity's Muscle Space (normalized skeletal movements – rel. to joint limits)
  - Foot IK correction
- Must also worry about interpenetration and joint limits
  - Mixamo Fuse Arm Spacing for bulky characters



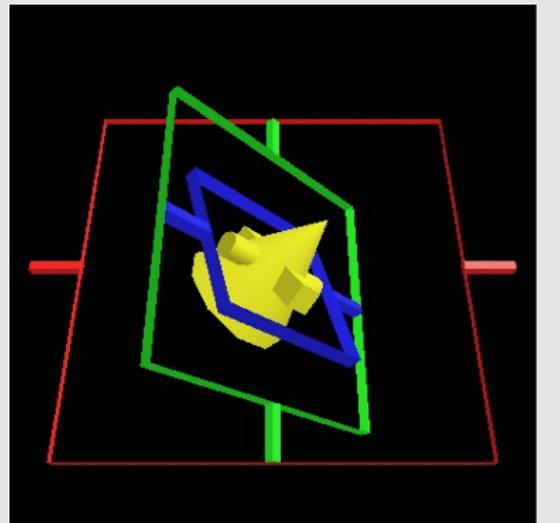
Detailed info on how Unity works

<https://blogs.unity3d.com/2014/05/26/mecanim-humanoids/>

Rufus from Street Fighter IV

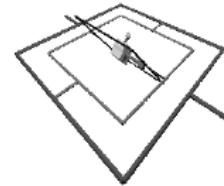
## Euler Angles

An euler angle is a rotation about a single axis. Any orientation can be described composing three rotation around each coordinate axis. We can visualize the action of the Euler angles: each loop is a rotation around one coordinate axis.



## Interpolating Euler Angles

- **Natural orientation representation:** three angles for three degrees of freedom
- **Unnatural interpolation:** A rotation of 90-degrees first around the z-axis and then around the y-axis has the effect of a 120-degree rotation around the axis (1, 1, 1). But rotation of 30-degrees around the z- and y-axis does not have the effect of a 40-degree rotation around the axis (1, 1, 1).
- **Gimbal lock:** two or more axis align resulting in a loss of rotation degrees of freedom. For example, if the green loop in previous slide aligns with the red loop then both the rotation around the blue loop and the rotation around the red loop produces identical rotation.



## Quaternion Interpolation

- Linear interpolation (lerp) of quaternion representation of orientations gives us something better:

$$\text{lerp}(\mathbf{q}_0, \mathbf{q}_1, t) = \mathbf{q}(t) = \mathbf{q}_0(1-t) + \mathbf{q}_1 t$$

- Quaternion Refresher  $\vec{v} = (\vec{v}_1, \vec{v}_2, \vec{v}_3)$   $\mathbf{q} = (s, \vec{v})$ 
  - a *general* quaternion  $\mathbf{q}$  consists of four numbers: a scalar  $s$  and a 3-D vector  $\mathbf{q}_1 \mathbf{q}_2 = (s_1 s_2 - (\vec{v}_1 \cdot \vec{v}_2), s_1 \vec{v}_2 + s_2 \vec{v}_1 + \vec{v}_1 \times \vec{v}_2)$
  - two general quaternions are multiplied by a special rule:  
 $\mathbf{q} = (\cos(\theta/2), \sin(\theta/2) \vec{a})$
  - a *unit* quaternion  
can represent a rotation of  $\theta$  radians around the unit axis vector  $\vec{a}$

