



NATIONAL RESEARCH
UNIVERSITY

Department of Sociology
Laboratory for Comparative Social Research

QUANTITATIVE DATA ANALYSIS

Linear Regression III

Violetta Korsunova

St. Petersburg, 2021

ASSUMPTIONS OF LINEAR REGRESSION

- Independent variables are unrelated
- Effects are unrelated
- The link between the IV and DV is linear
- Dependent variable is normally distributed

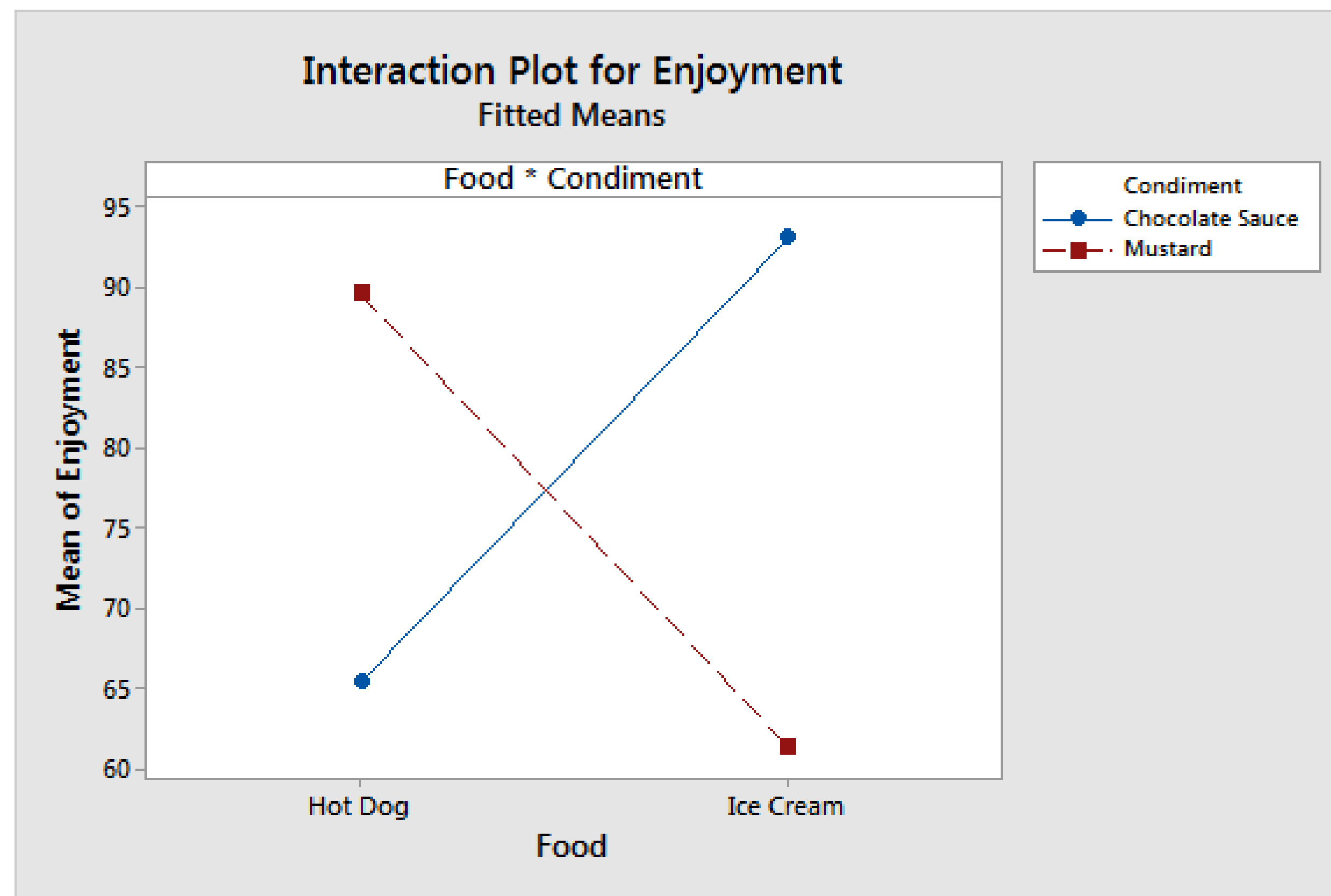


PROBLEMS YOU CAN ENCOUNTER

- **Effect of an IV varies across groups**
- **The relationship between IV and DV is non-linear (i.e. quadratic)**
- **Variables may have non-normal distribution**
- **Two or more IVs are correlated (multicollinearity)**

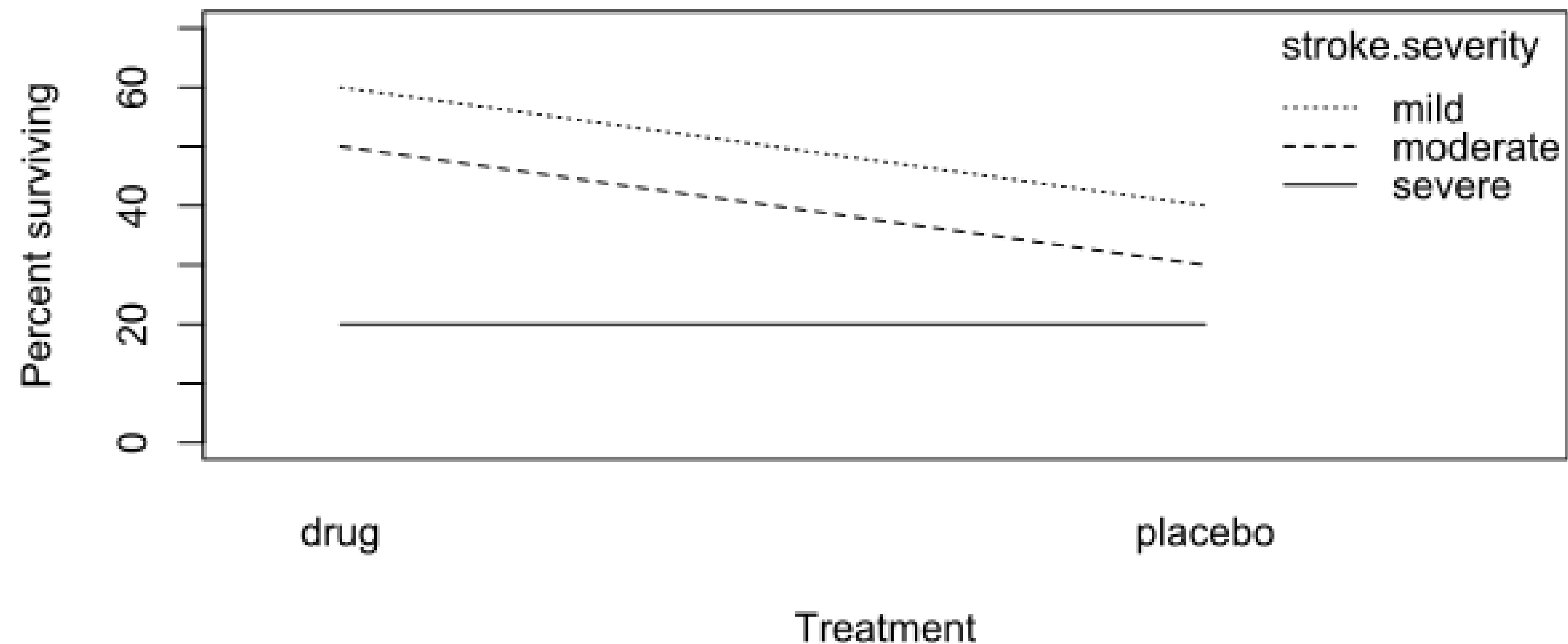
INTERACTION TERMS

Effect of one IV depends on the other IV:



INTERACTION TERMS

Interaction plot for stroke survival
versus treatment and stroke severity





INTERACTION TERMS

Interpretation

- **The effect is not on DV but on the link between DV and another IV**
- **Also depends on the scales of the IVs (categorical or continuous)**
- **3 types of IT:**
 - categorical*categorical,**
 - categorical*continuous,**
 - continuous*continuous**



HOW TO ESTIMATE INTERACTION TERMS IT IN R

Data from WVS wave 7 in Russia (ruswvs7 in LM3.RData)

Variables:

- age1 = age in years
- income = income level (self-placement, 1 – lowest level, 10 – highest level)
- sex = gender (female, male)
- edu = level of education (categorical, primary, sec&prof, tertiary)
- eduf = level of education (numeric, 1-9)
- H_URBRURAL = area of residence (urban, rural)
- sat = life satisfaction (1 – completely dissatisfied, 10 – completely satisfied)



HOW TO ESTIMATE INTERACTION TERMS IT IN R

```
> model0 = lm(sat~age1+sex+eduf+sex+H_URBRURAL+eduf+income, data = ruswvs7)
> summary(model0)
```

Call:

```
lm(formula = sat ~ age1 + sex + eduf + sex + H_URBRURAL + eduf +
    income, data = ruswvs7)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-6.9823	-1.3082	0.0367	1.2911	4.8177

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	5.562435	0.269179	20.664	< 2e-16	***
age1	-0.009518	0.002937	-3.241	0.00121	**
sexMale	0.059918	0.098067	0.611	0.54129	
eduf	-0.030218	0.028300	-1.068	0.28577	
H_URBRURALRural	-0.054334	0.109268	-0.497	0.61907	
income	0.327980	0.026454	12.398	< 2e-16	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.965 on 1720 degrees of freedom
(84 observations deleted due to missingness)

Multiple R-squared: 0.1099, Adjusted R-squared: 0.1073

F-statistic: 42.48 on 5 and 1720 DF, p-value: < 2.2e-16



HOW TO ESTIMATE INTERACTION TERMS IT IN R

```
> model1 = lm(sat~age1+sex*eduf+H_URBRURAL+income, data = ruswvs7)
> summary(model1)
```

```
Call:
lm(formula = sat ~ age1 + sex * eduf + sex + H_URBRURAL + eduf +
    income, data = ruswvs7)
```

Residuals:

Min	1Q	Median	3Q	Max
-7.1583	-1.3183	0.0406	1.3042	4.9896

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	5.889871	0.299913	19.639	< 2e-16	***
age1	-0.010049	0.002941	-3.417	0.000647	***
sexMale	-0.718625	0.331166	-2.170	0.030145	*
eduf	-0.081644	0.035145	-2.323	0.020294	*
H_URBRURALrural	-0.042483	0.109214	-0.389	0.697330	
income	0.327590	0.026416	12.401	< 2e-16	***
sexMale:eduf	0.133205	0.054127	2.461	0.013954	*

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 1.962 on 1719 degrees of freedom
(84 observations deleted due to missingness)

Multiple R-squared: 0.113, Adjusted R-squared: 0.1099

F-statistic: 36.51 on 6 and 1719 DF, p-value: < 2.2e-16



HOW TO ESTIMATE INTERACTION TERMS IT IN R

```
> model1 = lm(sat~age1+sex*eduf+H_URBRURAL+income, data = ruswvs7)
> summary(model1)
```

Interaction term

```
call:
lm(formula = sat ~ age1 + sex * eduf + sex + H_URBRURAL + eduf +
    income, data = ruswvs7)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-7.1583	-1.3183	0.0406	1.3042	4.9896

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eduf	-0.081644	0.035145	-2.323	0.020294	*
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HOW TO ESTIMATE INTERACTION TERMS IT IN R

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> model1 = lm(sat~age1+sex*eduf+H_URBRURAL+income, data = ruswvs7)
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Call:
lm(formula = sat ~ age1 + sex * eduf + sex + H_URBRURAL + eduf +
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```

```
Residuals:
    Min       1Q   Median       3Q      Max
-7.1583 -1.3183  0.0406  1.3042  4.9896
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```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   5.889871   0.299913   19.639 < 2e-16 ***
age1          -0.010049   0.002941   -3.417 0.00066 **
sexMale       -0.718625   0.331166   -2.170 0.03011 *
eduf          -0.081644   0.035145   -2.323 0.02094 *
H_URBRURALrural -0.042483   0.109214   -0.389 0.697330
income         0.327590   0.026416   12.401 < 2e-16 ***
sexMale:eduf    0.133205   0.054127    2.461 0.013954 *
```

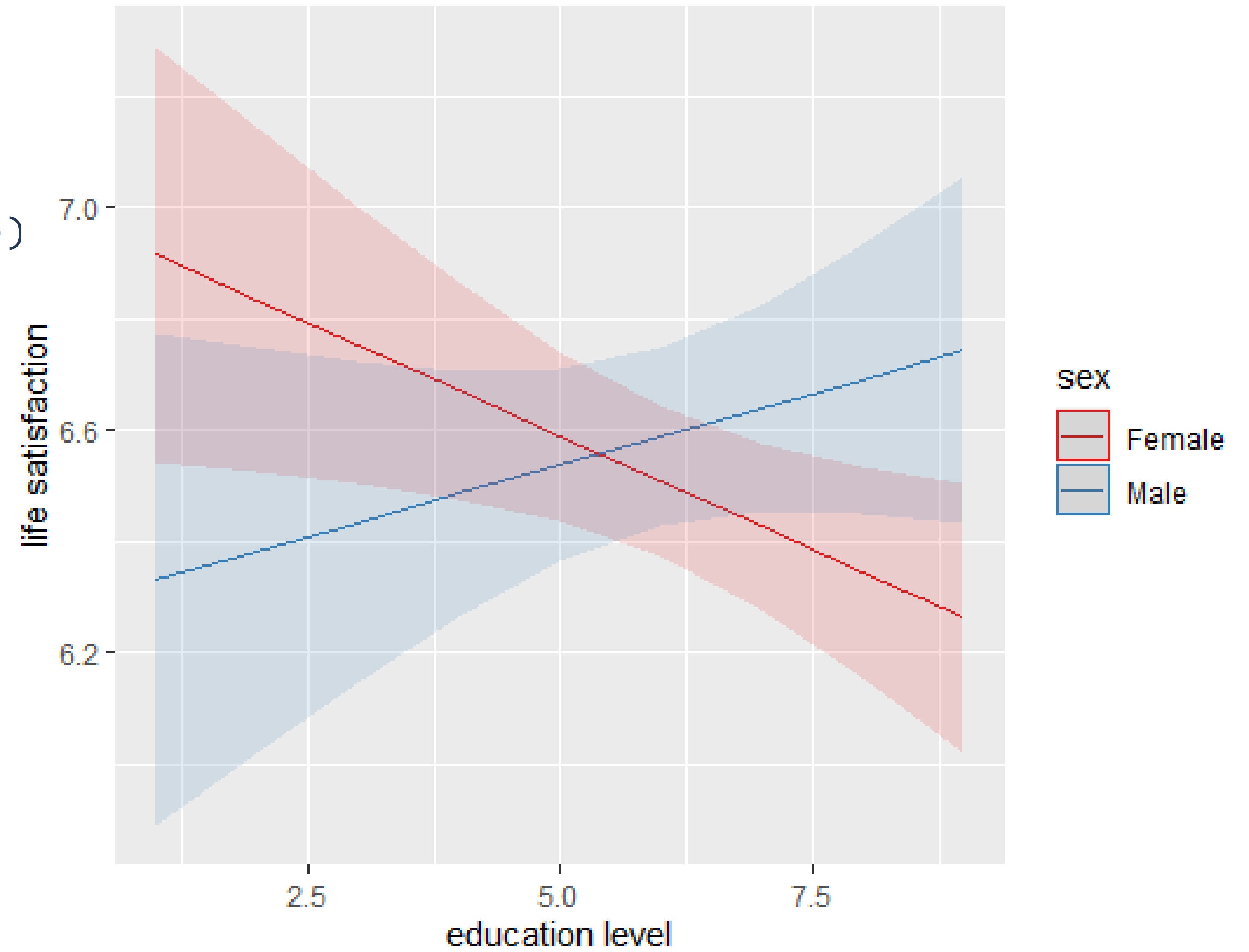
```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 1.962 on 1719 degrees of freedom
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Multiple R-squared:  0.113,    Adjusted R-squared:  0.1099
F-statistic: 36.51 on 6 and 1719 DF,  p-value: < 2.2e-16
```

Education works
differently for men and
women



```
plot_model(model1, type = "pred",  
  terms=c("eduf", "sex"),  
  title = "",  
  axis.title = c("education level",  
    "life satisfaction"))
```

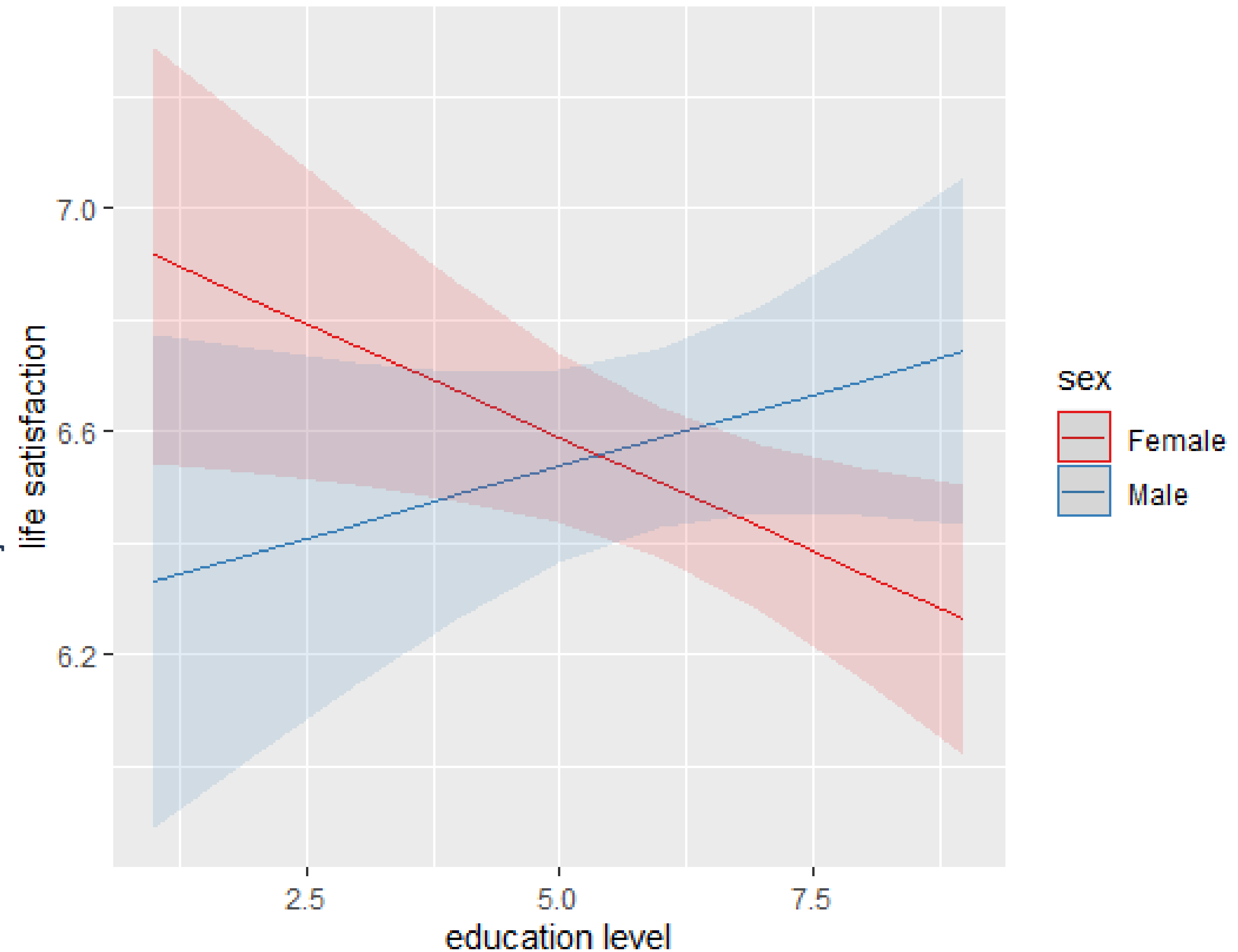




```
plot_model(model1, type = "pred",  
           terms=c("eduf", "sex"),  
           title = "",  
           axis.title = c("education level",  
                          "life satisfaction"))
```

For women the link is negative, for men the link is positive

women with higher level of education have lower life satisfaction, whereas men with higher level of education are more satisfied with their lives





HOW TO ESTIMATE INTERACTION TERMS IT IN R

```
> model2 = lm(sat~age1+sex*eduf+H_URBRURAL+eduf*income, data = ruswvs7)
> summary(model2)
```

```
Call:
lm(formula = sat ~ age1 + sex * eduf + H_URBRURAL + eduf * income,
    data = ruswvs7)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-7.6656	-1.3293	0.0332	1.2873	5.2106

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	5.280658	0.460452	11.468	< 2e-16	***
age1	-0.009558	0.002952	-3.238	0.00123	**
sexMale	-0.772821	0.332427	-2.325	0.02020	*
eduf	0.025036	0.070570	0.355	0.72280	
H_URBRURALrural	-0.040269	0.109157	-0.369	0.71224	
income	0.460069	0.080464	5.718	1.27e-08	***
sexMale:eduf	0.142514	0.054358	2.622	0.00883	**
eduf:income	-0.023144	0.013279	-1.743	0.08153	.

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 1.961 on 1718 degrees of freedom
(84 observations deleted due to missingness)
```

```
Multiple R-squared:  0.1146,    Adjusted R-squared:  0.111
F-statistic: 31.77 on 7 and 1718 DF,  p-value: < 2.2e-16
```



HOW TO ESTIMATE INTERACTION TERMS IT IN R

```
> model2 = lm(sat~age1+sex*eduf+H_URBRURAL+eduf*income, data = ruswvs7)
> summary(model2)
```

```
Call:
lm(formula = sat ~ age1 + sex * eduf + H_URBRURAL + eduf * income,
    data = ruswvs7)
```

```
Residuals:
    Min       1Q   Median       3Q      Max
-7.6656 -1.3293  0.0332  1.2873  5.2106
```

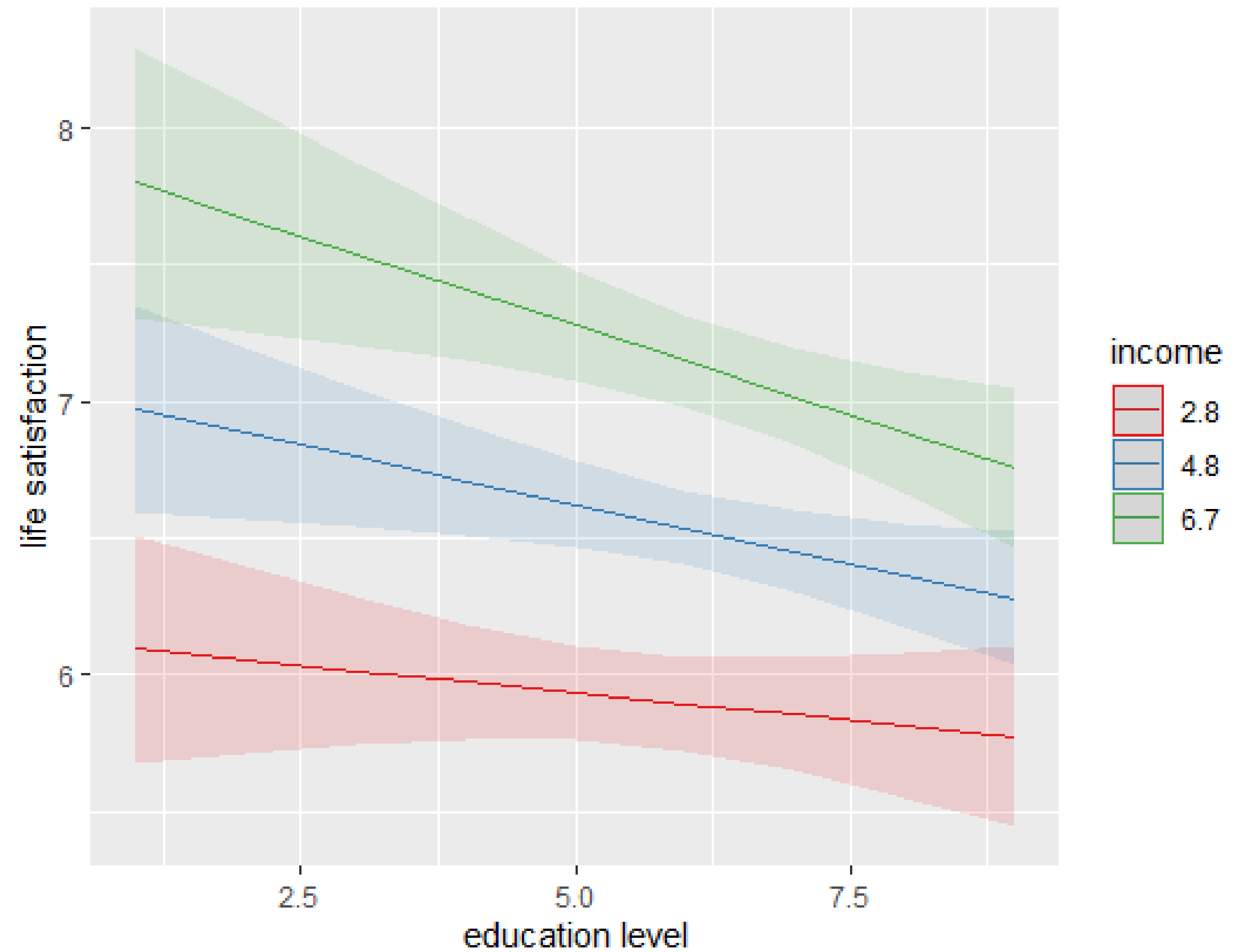
```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   5.280658   0.460452   11.468  <.001
age1          -0.009558   0.002952   -3.238  0.002
sexMale       -0.772821   0.332427   -2.325  0.020
eduf           0.025036   0.070570    0.355  0.722
H_URBRURAlRural -0.040269   0.109157   -0.369  0.712
income         0.460069   0.080464    5.718 1.27e-05 ***
sexMale:eduf    0.142514   0.054358    2.622  0.00883 **
eduf:income    -0.023144   0.013279   -1.743  0.08153 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Marginally significant, let's have a closer look

```
Residual standard error: 1.961 on 1718 degrees of freedom
(84 observations deleted due to missingness)
Multiple R-squared:  0.1146,    Adjusted R-squared:  0.111
F-statistic: 31.77 on 7 and 1718 DF,  p-value: < 2.2e-16
```



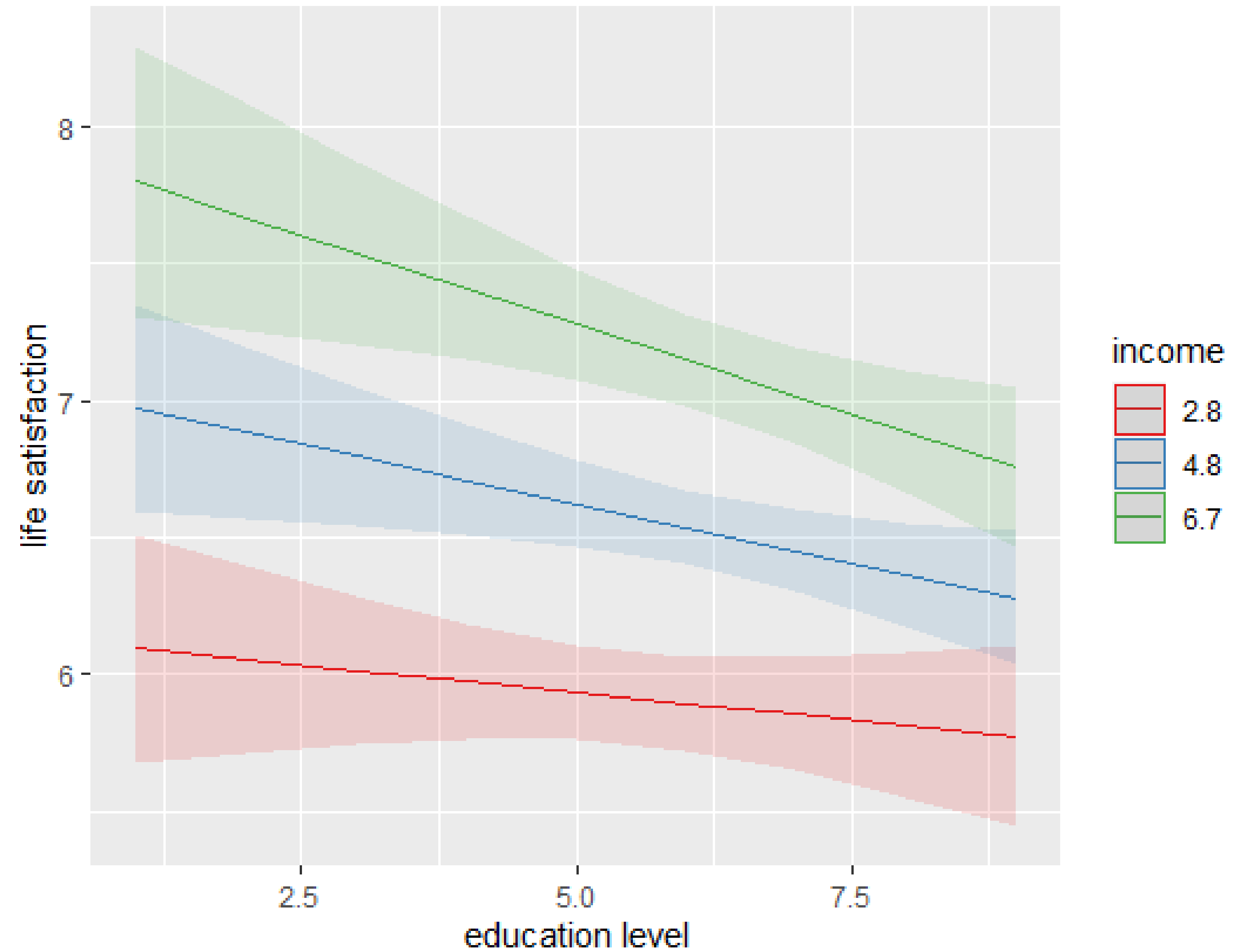
```
plot_model(model2, type = "pred",  
           terms=c("eduf", "income"),  
           title = "",  
           axis.title = c("education level",  
                          "life satisfaction"))
```





```
plot_model(model2, type = "pred",  
           terms=c("eduf", "income"),  
           title = "",  
           axis.title = c("education level",  
                          "life satisfaction"))
```

The impact of income on life satisfaction weakens as education level grows





HOW TO ESTIMATE INTERACTION TERMS IT IN R

```
> model2.1 = lm(sat~age1+sex*edu+H_URBRURAL+edu*income, data = ruswvs7)
```

```
> summary(model2.1)
```

Call:

```
lm(formula = sat ~ age1 + sex * edu + H_URBRURAL + edu * income,  
    data = ruswvs7)
```

Residuals:

Min	1Q	Median	3Q	Max
-7.8292	-1.3326	0.0337	1.2708	5.2986

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	5.331836	0.353593	15.079	< 2e-16	***
age1	-0.009963	0.002985	-3.338	0.000862	***
sexMale	-0.426193	0.270070	-1.578	0.114732	
eduSec&Prof	0.045638	0.339105	0.135	0.892957	
eduTertiary	0.348347	0.402156	0.866	0.386501	
H_URBRURALLrural	-0.039778	0.108626	-0.366	0.714268	
income	0.433427	0.066073	6.560	7.11e-11	***
sexMale:eduSec&Prof	0.366960	0.299204	1.226	0.220195	
sexMale:eduTertiary	0.879246	0.319913	2.748	0.006052	**
eduSec&Prof:income	-0.087184	0.074683	-1.167	0.243214	
eduTertiary:income	-0.192222	0.080978	-2.374	0.017718	*

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.96 on 1715 degrees of freedom

(84 observations deleted due to missingness)

Multiple R-squared: 0.1171, Adjusted R-squared: 0.1119

Let's look at the categories of education



HOW TO ESTIMATE INTERACTION TERMS IT IN R

```
> model2.1 = lm(sat~age1+sex*edu+H_URBRURAL+edu*income, data = ruswvs7)
```

```
> summary(model2.1)
```

Call:

```
lm(formula = sat ~ age1 + sex * edu + H_URBRURAL + edu * income,  
    data = ruswvs7)
```

Residuals:

Min	1Q	Median	3Q	Max
-7.8292	-1.3326	0.0337	1.2708	5.2986

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	5.331836	0.353593	15.079	< 2e-16	***
age1	-0.009963	0.002985	-3.338	0.000862	***
sexMale	-0.426193	0.270070	-1.578	0.114732	
eduSec&Prof	0.045638	0.339105	0.135	0.892957	
eduTertiary	0.348347	0.402156	0.866	0.386501	
H_URBRURALRural	-0.039778	0.108626	-0.366	0.714268	
income	0.433427	0.066073	6.560	7.11e-11	***
sexMale:eduSec&Prof	0.366960	0.299204	1.226	0.220195	
<u>sexMale:eduTertiary</u>	<u>0.879246</u>	<u>0.319913</u>	<u>2.748</u>	<u>0.006052</u>	<u>**</u>
eduSec&Prof:income	-0.087184	0.074683	-1.167	0.243214	
<u>eduTertiary:income</u>	<u>-0.192222</u>	<u>0.080978</u>	<u>-2.374</u>	<u>0.017718</u>	<u>*</u>

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

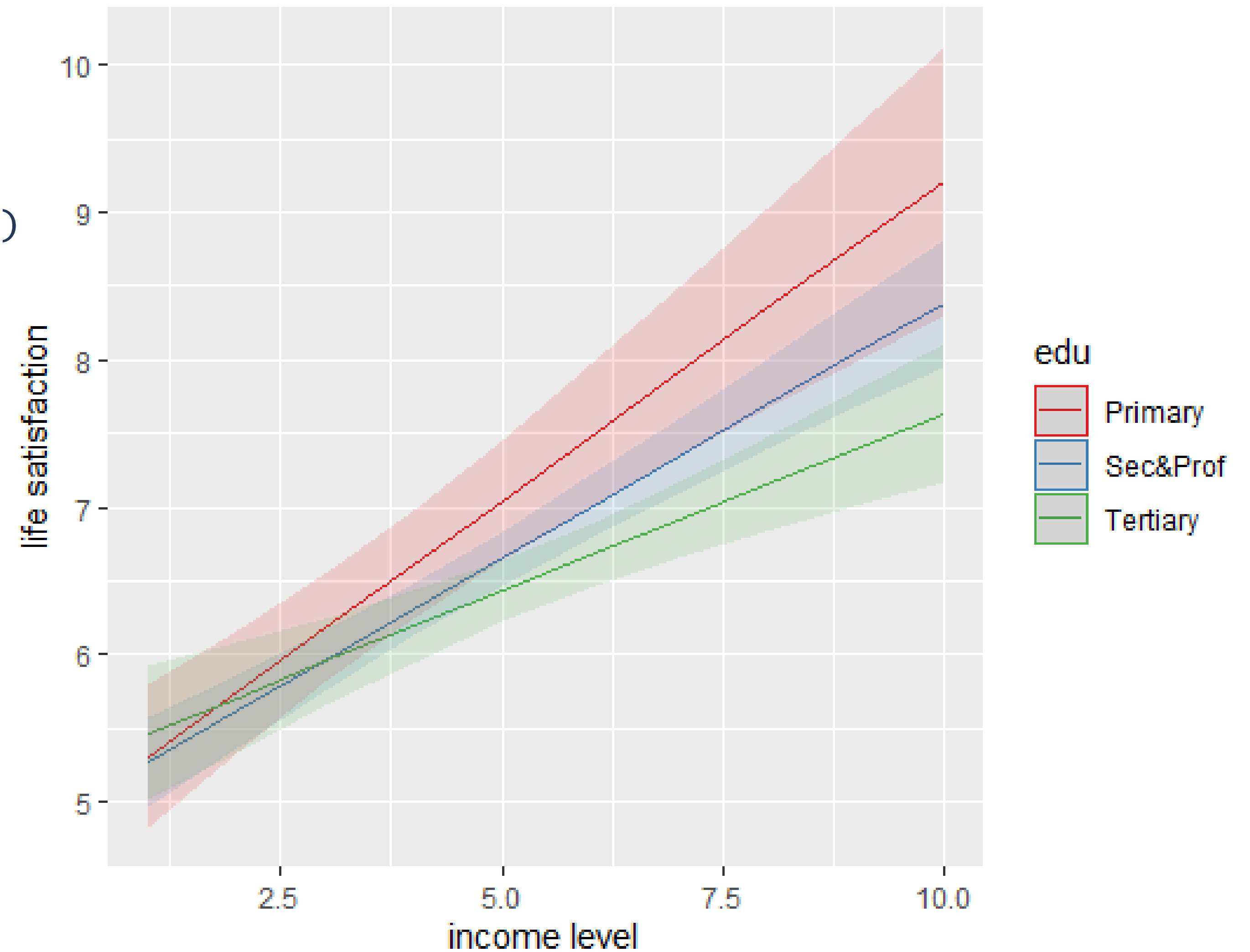
Residual standard error: 1.96 on 1715 degrees of freedom

(84 observations deleted due to missingness)

Multiple R-squared: 0.1171, Adjusted R-squared: 0.1119



```
plot_model(model2.1, type = "pred",  
  terms=c("income", "edu"),  
  title = "",  
  axis.title = c("income level",  
    "life satisfaction"))
```

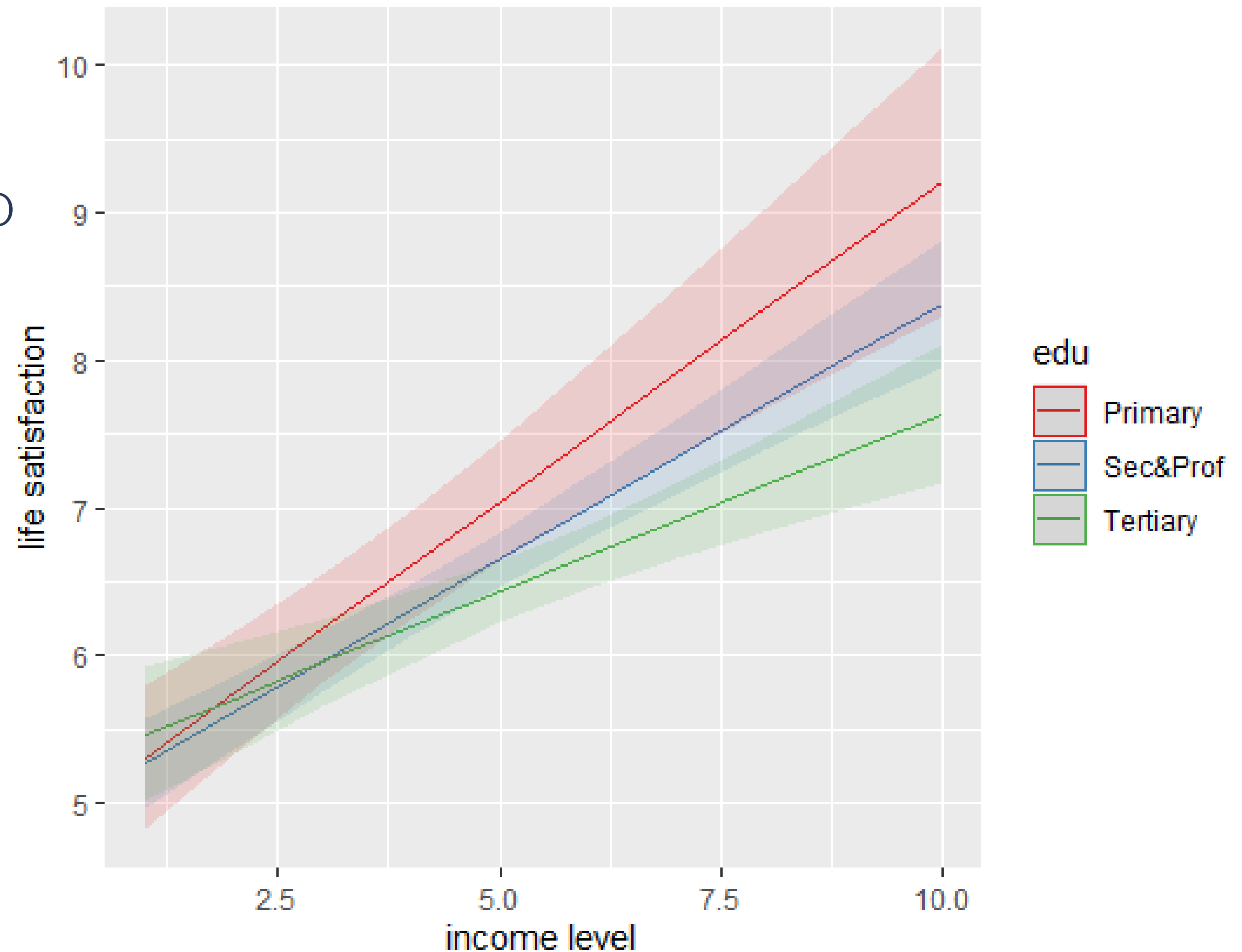




```
plot_model(model2.1, type = "pred",  
           terms=c("income", "edu"),  
           title = "",  
           axis.title = c("income level",  
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```

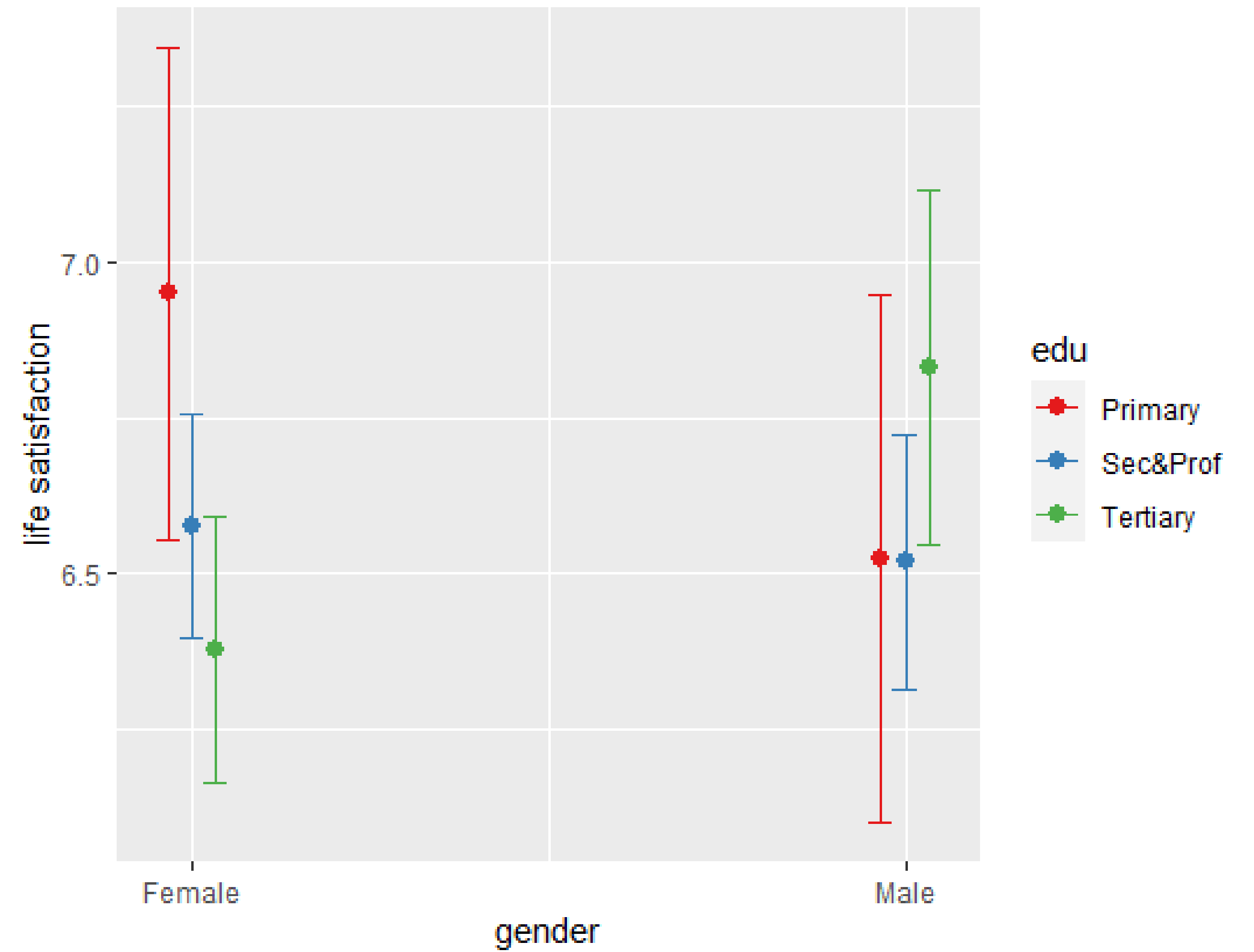
For people with primary education the effect of income on life satisfaction is higher

As income level grows, the difference in life satisfaction among education groups becomes more prominent





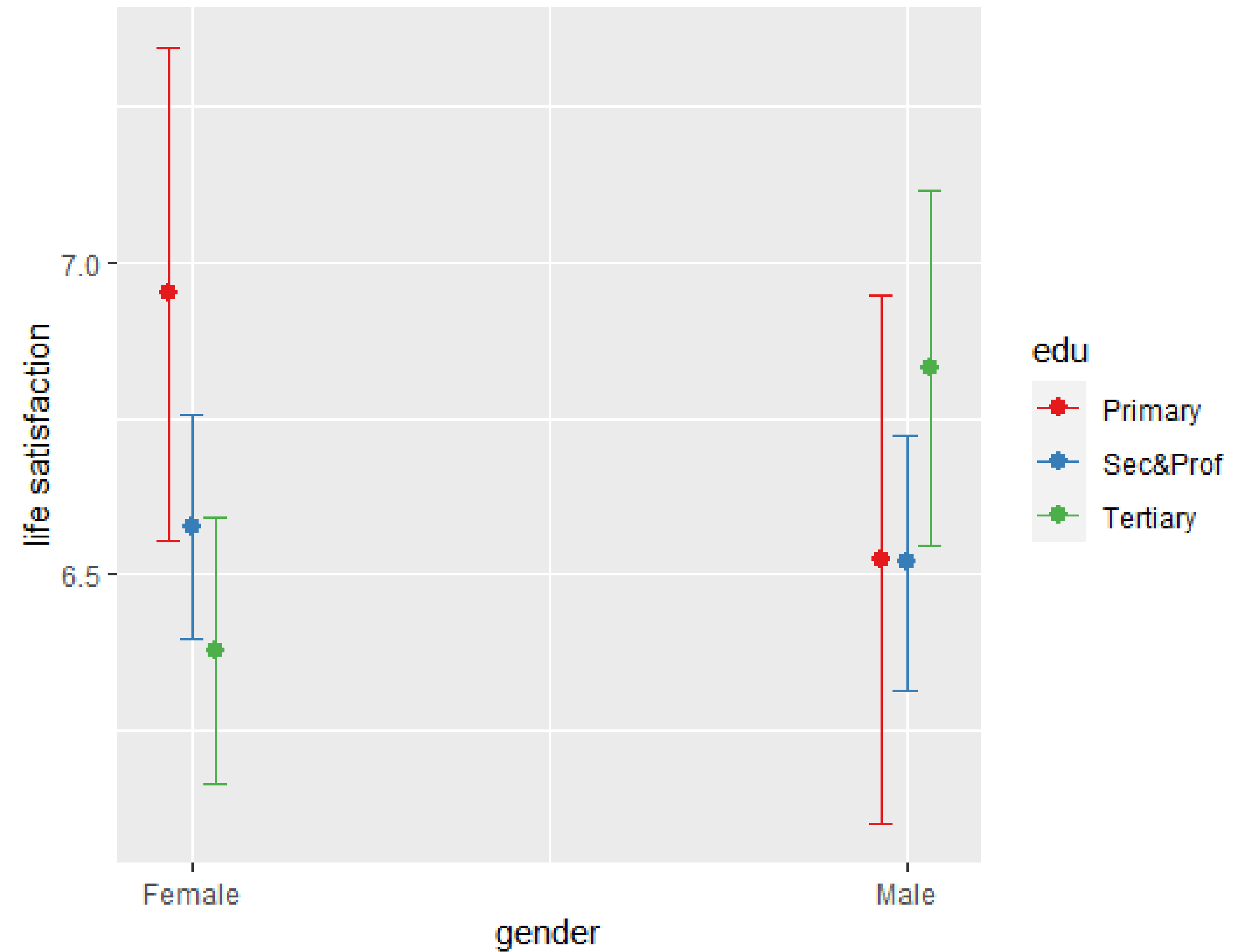
```
plot_model(model2.1, type = "pred",  
           terms=c("sex", "edu"),  
           title = "",  
           axis.title = c("gender",  
                          "life satisfaction"))
```





```
plot_model(model2.1, type = "pred",  
           terms=c("sex", "edu"),  
           title = "",  
           axis.title = c("gender",  
                          "life satisfaction"))
```

Men with tertiary education have higher life satisfaction than women with tertiary education





TRANSFORMATION OF VARIABLES

- **Effects may be non-linear**
- **Variables may be distributed non-normally**

In this case it is possible to transform the variables

COMMON TYPES OF TRANSFORMATIONS

- Raising to the n^{th} power
- Log transformation
- Scaling (if the scales are considerably different)



RAISING TO THE POWER OF 2 (QUADRATIC TERMS)

```
> model3 = lm(sat~age1+I(age1^2)+sex*edu+H_URBRURAL+edu*income, data = ruswvs7)
> summary(model3)
```

```
Call:
lm(formula = sat ~ age1 + I(age1^2) + sex * edu + H_URBRURAL +
    edu * income, data = ruswvs7)
```

Residuals:

Min	1Q	Median	3Q	Max
-8.1594	-1.3675	0.0174	1.2702	5.3721

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	5.9830119	0.4459386	13.417	< 2e-16	***
age1	-0.0474668	0.0159665	-2.973	0.00299	**
I(age1^2)	0.0003926	0.0001642	2.391	0.01691	*
sexMale	-0.3647574	0.2709210	-1.346	0.17836	
eduSec&Prof	0.1887776	0.3438911	0.549	0.58311	
eduTertiary	0.5133028	0.4074868	1.260	0.20796	
H_URBRURALRural	-0.0298525	0.1085565	-0.275	0.78335	
income	0.4393251	0.0660285	6.654	3.84e-11	***
sexMale:eduSec&Prof	0.3136172	0.2996249	1.047	0.29539	
sexMale:eduTertiary	0.8102063	0.3207761	2.526	0.01163	*
eduSec&Prof:income	-0.0984022	0.0747279	-1.317	0.18808	
eduTertiary:income	-0.1985871	0.0809110	-2.454	0.01421	*

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.958 on 1714 degrees of freedom
(84 observations deleted due to missingness)
Multiple R-squared: 0.12, Adjusted R-squared: 0.1144



RAISING TO THE POWER OF 2 (QUADRATIC TERMS)

```
> model3 = lm(sat~age1+I(age1^2)+sex*edu+H_URBRURAL+edu*income, data = ruswvs7)
> summary(model3)
```

```
Call:
lm(formula = sat ~ age1 + I(age1^2)
    edu * income, data = ruswvs7)
```

Quadratic term

Residuals:

Min	1Q	Median	3Q	Max
-8.1594	-1.3675	0.0174	1.2702	5.3721

Coefficients:

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RAISING TO THE POWER OF 2 (QUADRATIC TERMS)

```
> model3 = lm(sat~age1+I(age1^2)+sex*edu+H_URBRURAL+edu*income, data = ruswvs7)
> summary(model3)
```

```
Call:
lm(formula = sat ~ age1 + I(age1^2) + sex * edu + H_URBRURAL +
    edu * income, data = ruswvs7)
```

Residuals:

Min	1Q	Median	3Q	Max
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H_URBRURALRural	-0.0298525	0.1085565	-0.275	0.78335	
income	0.4393251	0.0660285	6.654	3.84e-11	***
sexMale:eduSec&Prof	0.3136172	0.2996249	1.047	0.29539	
sexMale:eduTertiary	0.8102063	0.3207761	2.526	0.01163	*
eduSec&Prof:income	-0.0984022	0.0747279	-1.317	0.18808	
eduTertiary:income	-0.1985871	0.0809110	-2.454	0.01421	*

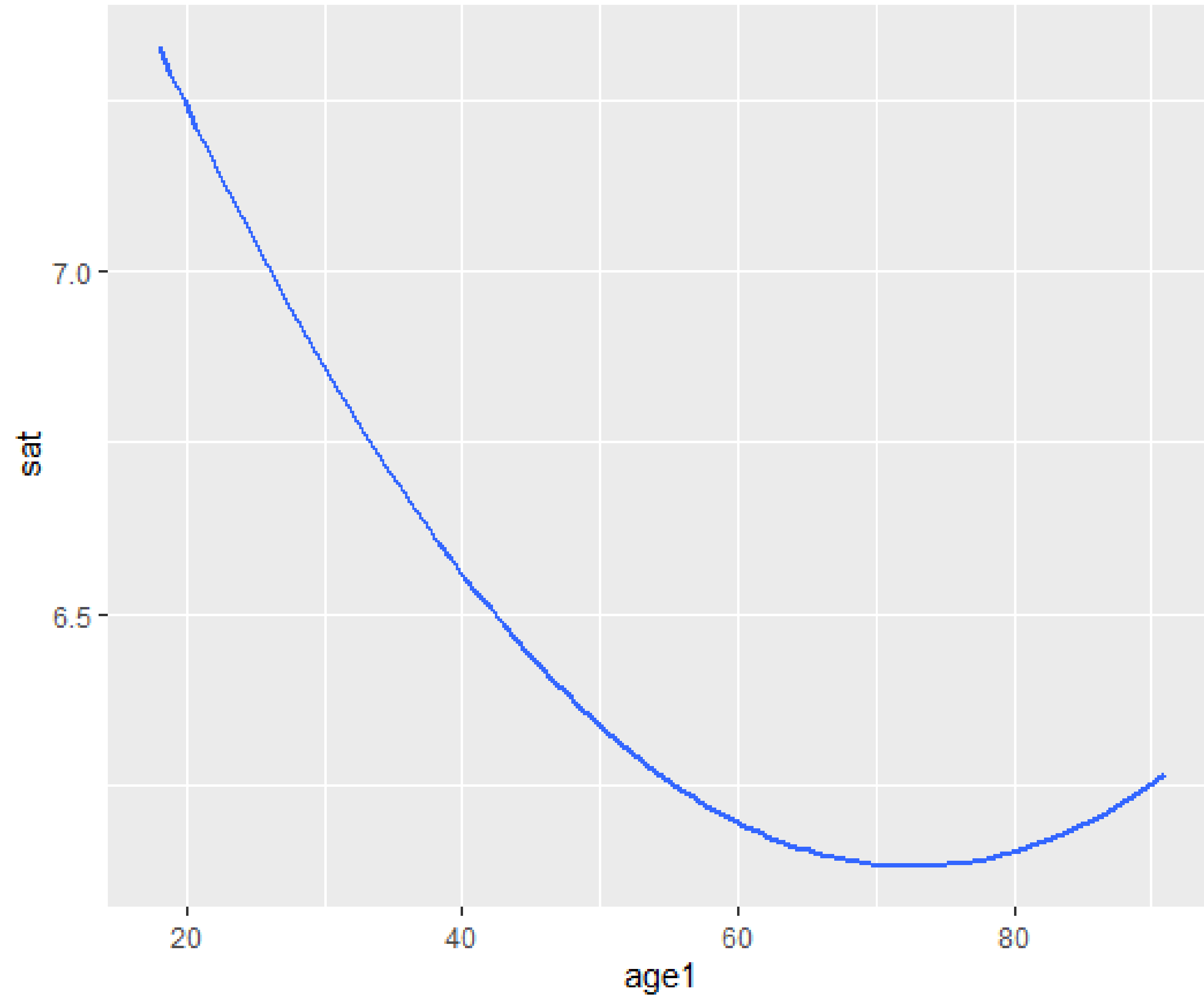
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.958 on 1714 degrees of freedom
(84 observations deleted due to missingness)
Multiple R-squared: 0.12, Adjusted R-squared: 0.1144

The link is indeed non-linear

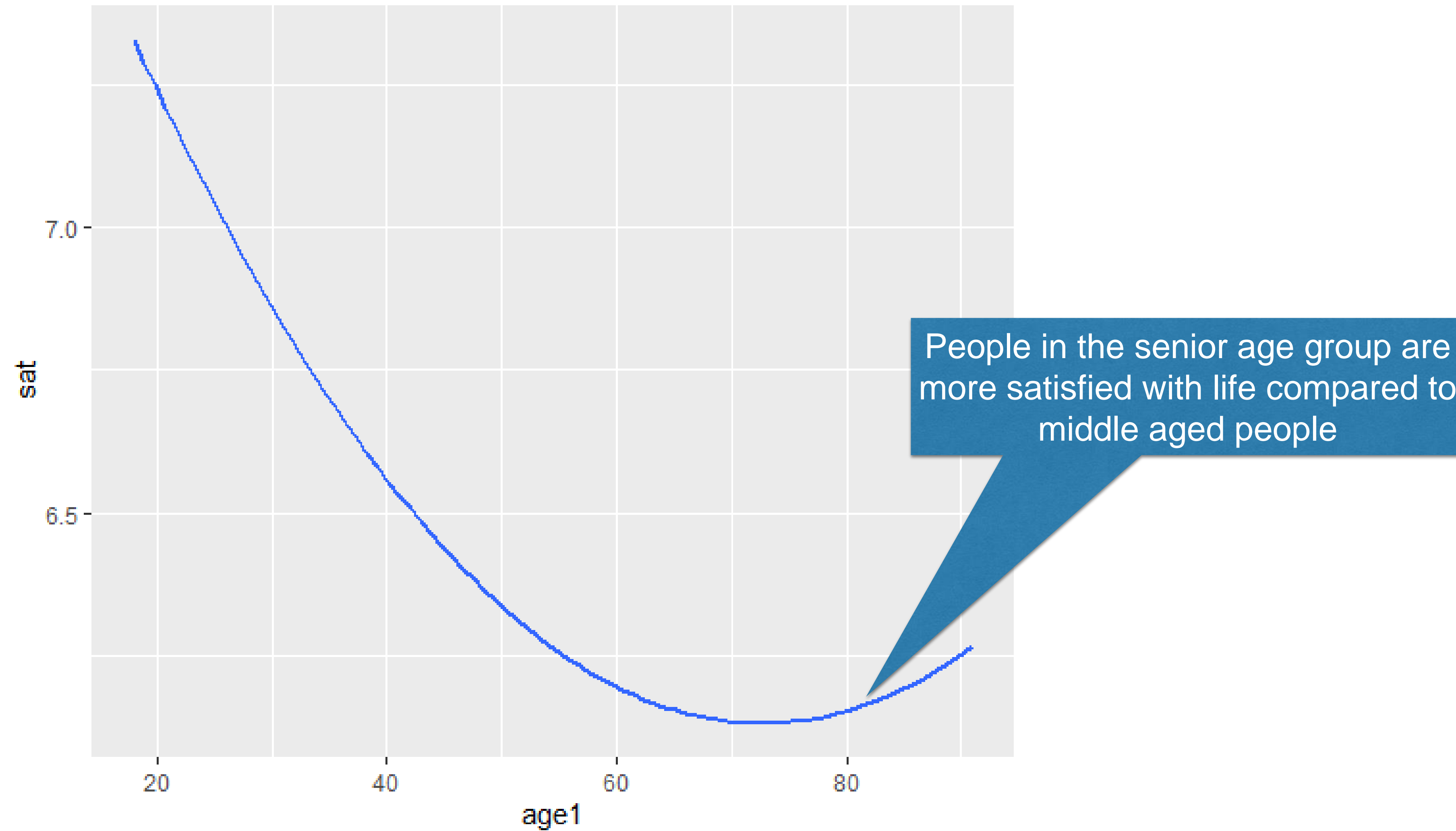


```
ggplot(ruswvs7, aes(x=age1, y=sat)) + stat_smooth(se=F,method='lm', formula=y~x+I(x^2))
```





```
ggplot(ruswvs7, aes(x=age1, y=sat)) + stat_smooth(se=F, method='lm', formula=y~x+I(x^2))
```





SCALING

```
> model4 = lm(sat~scale(age1)+I(scale(age1)^2)+sex*edu+H_URBRURAL+edu*income, data = ruswvs7)
> summary(model4)
```

Scaling (standard deviations instead of values)

```
Call:
lm(formula = sat ~ scale(age1)
    H_URBRURAL + edu * income,
```

Residuals:

Min	1Q	Median	3Q	Max
-8.1594	-1.3675	0.0174	1.2702	5.3721

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	4.63720	0.31538	14.703	< 2e-16	***
scale(age1)	-0.20225	0.05273	-3.836	0.00013	***
I(scale(age1)^2)	0.11512	0.04815	2.391	0.01691	*
sexMale	-0.36476	0.27092	-1.346	0.17836	
eduSec&Prof	0.18878	0.34389	0.549	0.58311	
eduTertiary	0.51330	0.40749	1.260	0.20796	
H_URBRURAlRural	-0.02985	0.10856	-0.275	0.78335	
income	0.43933	0.06603	6.654	3.84e-11	***
sexMale:eduSec&Prof	0.31362	0.29962	1.047	0.29539	
sexMale:eduTertiary	0.81021	0.32078	2.526	0.01163	*
eduSec&Prof:income	-0.09840	0.07473	-1.317	0.18808	
eduTertiary:income	-0.19859	0.08091	-2.454	0.01421	*

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.958 on 1714 degrees of freedom
(84 observations deleted due to missingness)
Multiple R-squared: 0.12, Adjusted R-squared: 0.1144



SCALING

```
> model4 = lm(sat~scale(age1)+I(scale(age1)^2)+sex*edu+H_URBRURAL+edu*income, data = ruswvs7)
> summary(model4)
```

Call:
lm(formula = sat ~ scale(age1) + I(scale(age1)^2) + sex * edu + H_URBRURAL + edu * income, data = ruswvs7)

Scale is in st. dev. Increase in
1 st. dev. of age decreases life satisfaction by 0.2

Residuals:

Min	1Q	Median	Max
-8.1594	-1.3675	0.0174	1.273721

Coefficients:

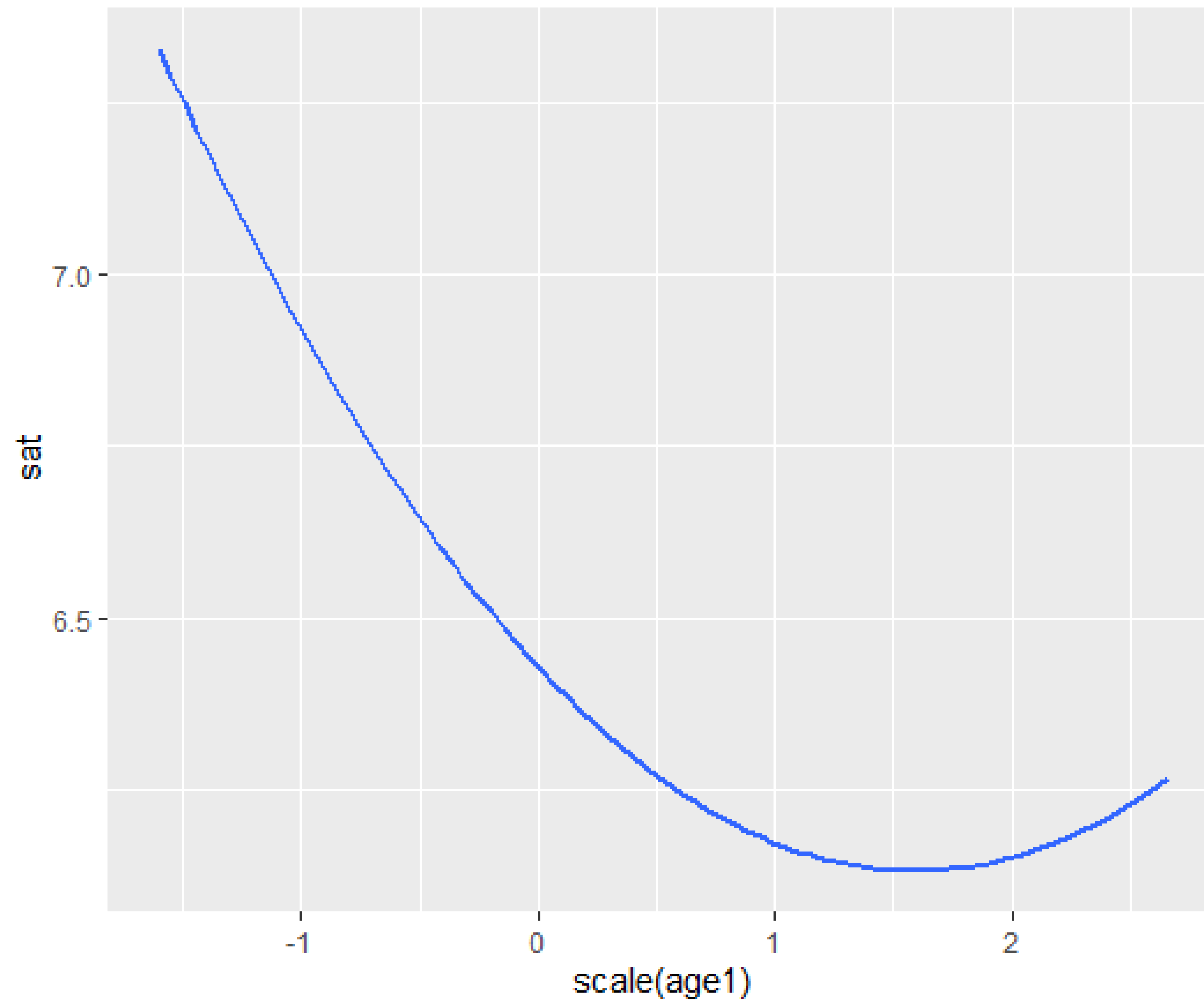
	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	4.63720	0.31538	14.703	< 2e-16 ***
scale(age1)	-0.20225	0.05273	-3.836	0.00013 ***
I(scale(age1)^2)	0.11512	0.04815	2.391	0.01691 *
sexMale	-0.36476	0.27092	-1.346	0.17836
eduSec&Prof	0.18878	0.34389	0.549	0.58311
eduTertiary	0.51330	0.40749	1.260	0.20796
H_URBRURAlRural	-0.02985	0.10856	-0.275	0.78335
income	0.43933	0.06603	6.654	3.84e-11 ***
sexMale:eduSec&Prof	0.31362	0.29962	1.047	0.29539
sexMale:eduTertiary	0.81021	0.32078	2.526	0.01163 *
eduSec&Prof:income	-0.09840	0.07473	-1.317	0.18808
eduTertiary:income	-0.19859	0.08091	-2.454	0.01421 *

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.958 on 1714 degrees of freedom
(84 observations deleted due to missingness)
Multiple R-squared: 0.12, Adjusted R-squared: 0.1144

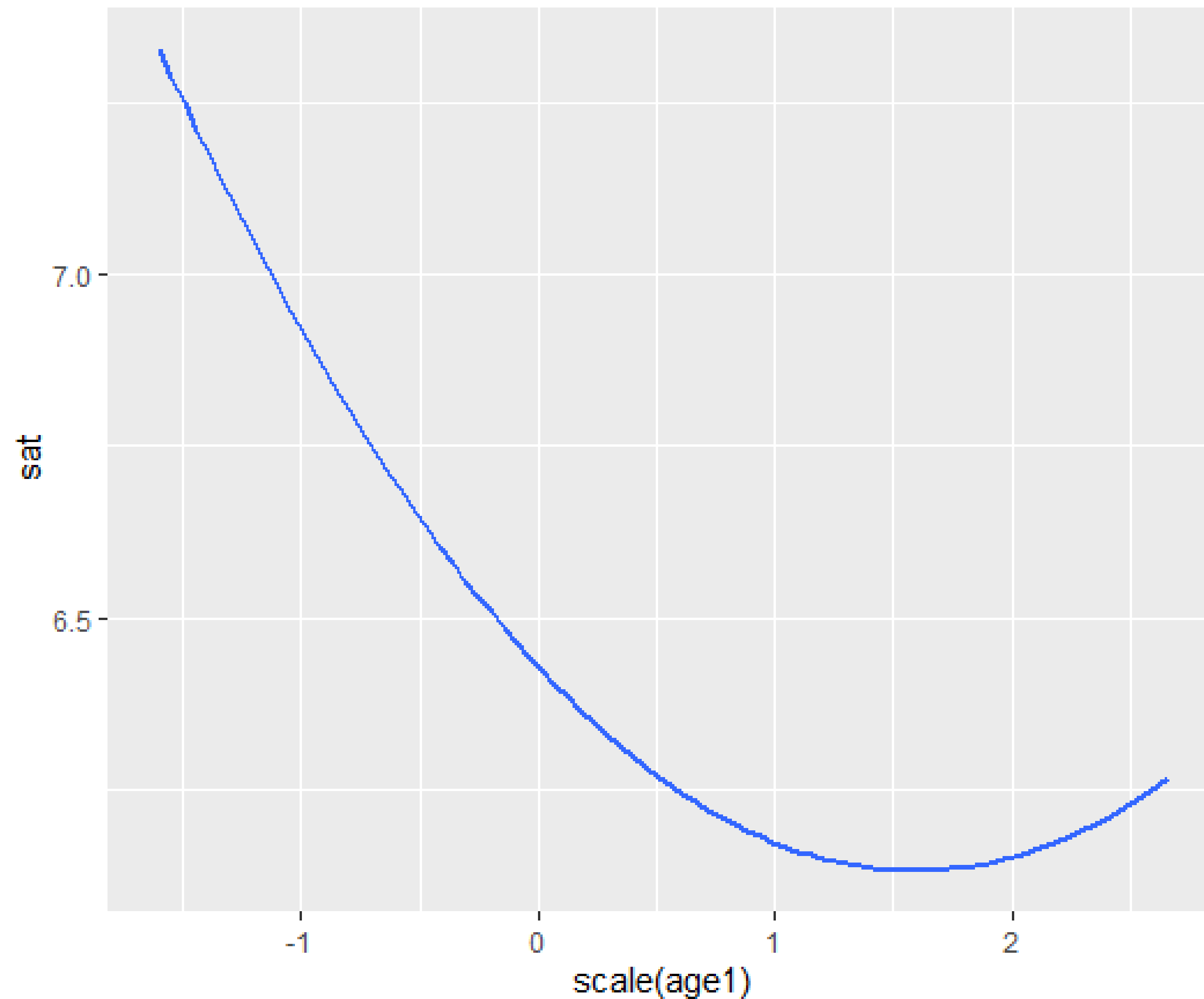


```
ggplot(ruswvs7, aes(x=scale(age1), y=sat)) + stat_smooth(se=F, method='lm', formula=y~x+I(x^2))
```





```
ggplot(ruswvs7, aes(x=scale(age1), y=sat)) + stat_smooth(se=F, method='lm', formula=y~x+I(x^2))
```



Scale of age is not in years but in st.dev. Mean age is now 0, 1 refers to 1 standard deviation from the mean value



LOG TRANSFORMATION

```
> model5 = lm(sat~age1+sex+edu+H_URBRURAL+ log(income), data = ruswvs7)
> summary(model5)
```

```
Call:
lm(formula = sat ~ age1 + sex + edu + H_URBRURAL + log(income),
    data = ruswvs7)
```

Residuals:

Min	1Q	Median	3Q	Max
-6.3725	-1.3768	0.0258	1.2837	5.4056

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	5.290339	0.268732	19.686	< 2e-16 ***
age1	-0.009899	0.002977	-3.325	0.000902 ***
sexMale	0.067232	0.098332	0.684	0.494241
eduSec&Prof	-0.167735	0.153036	-1.096	0.273209
eduTertiary	-0.210854	0.170092	-1.240	0.215276
H_URBRURALRural	-0.089365	0.108672	-0.822	0.410998
log(income)	1.265376	0.102512	12.344	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.966 on 1719 degrees of freedom
(84 observations deleted due to missingness)

Multiple R-squared: 0.1098, Adjusted R-squared: 0.1066

F-statistic: 35.32 on 6 and 1719 DF, p-value: < 2.2e-16

Use the log of income



LOG TRANSFORMATION

```
> model5 = lm(sat~age1+sex+edu+H_URBRURAL+ log(income), data = ruswvs7)
> summary(model5)
```

```
Call:
lm(formula = sat ~ age1 + sex + edu + H_URBRURAL + log(income),
    data = ruswvs7)
```

```
Residuals:
    Min       1Q   Median       3Q      Max
-6.3725 -1.3768  0.0258  1.2837  5.4056
```

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   5.290339   0.268732  19.686 < 2e-16 ***
age1          -0.009899   0.002977  -3.325 0.000902 ***
sexMale        0.067232   0.098332   0.684 0.494241
eduSec&Prof    -0.167735   0.153036  -1.096 0.273209
eduTertiary    -0.210854   0.170092  -1.240 0.215276
H_URBRURALRural -0.089365   0.108672  -0.822 0.410998
log(income)    1.265376   0.102512  12.344 < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 0.976 on 84 observations of
Multiple R-squared:  0.722, Adjusted R-squared:  0.1066
F-statistic: 35.32 on 6 and 84 df, p-value: 2.2e-16
```

The values are now in
log



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