Characterising linear optical circuits via phaseless estimation techniques

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University of Bristol

The Question is…

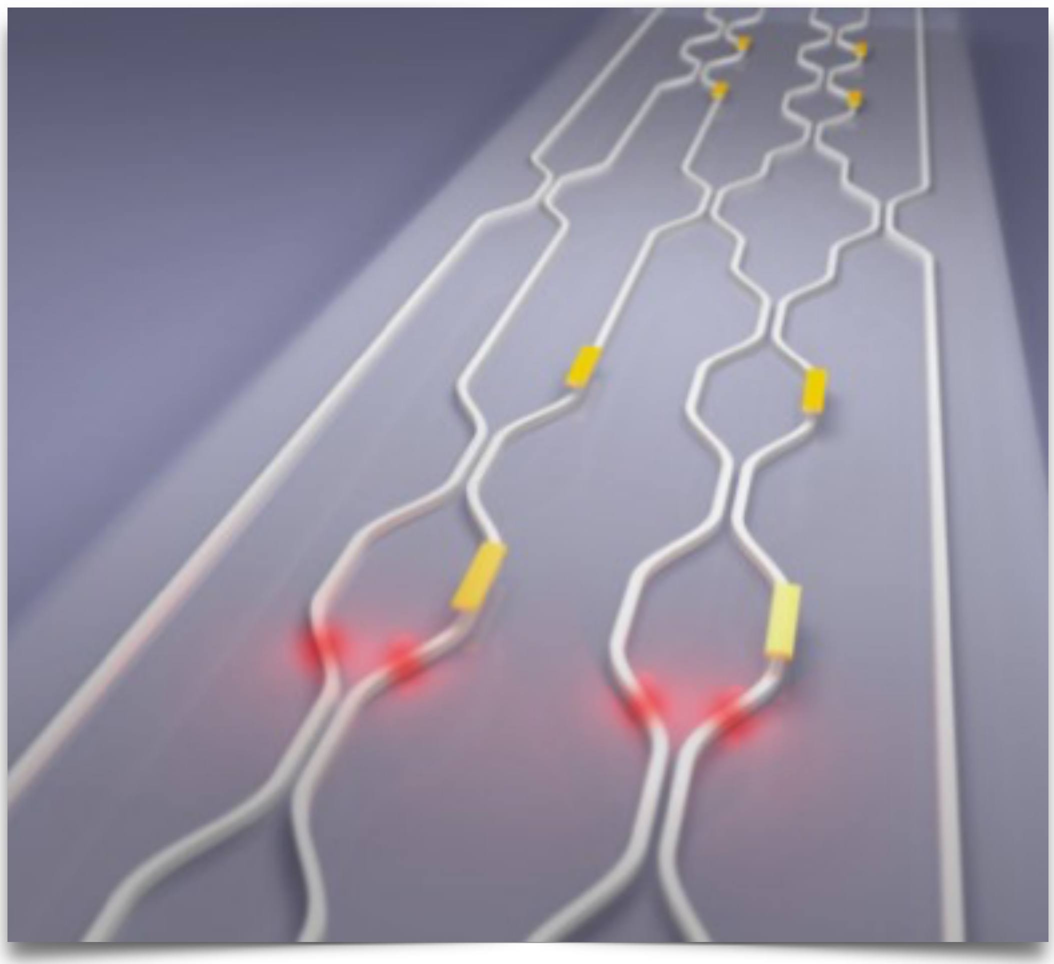

Image: http://www.bris.ac.uk/news/2011/8109.html

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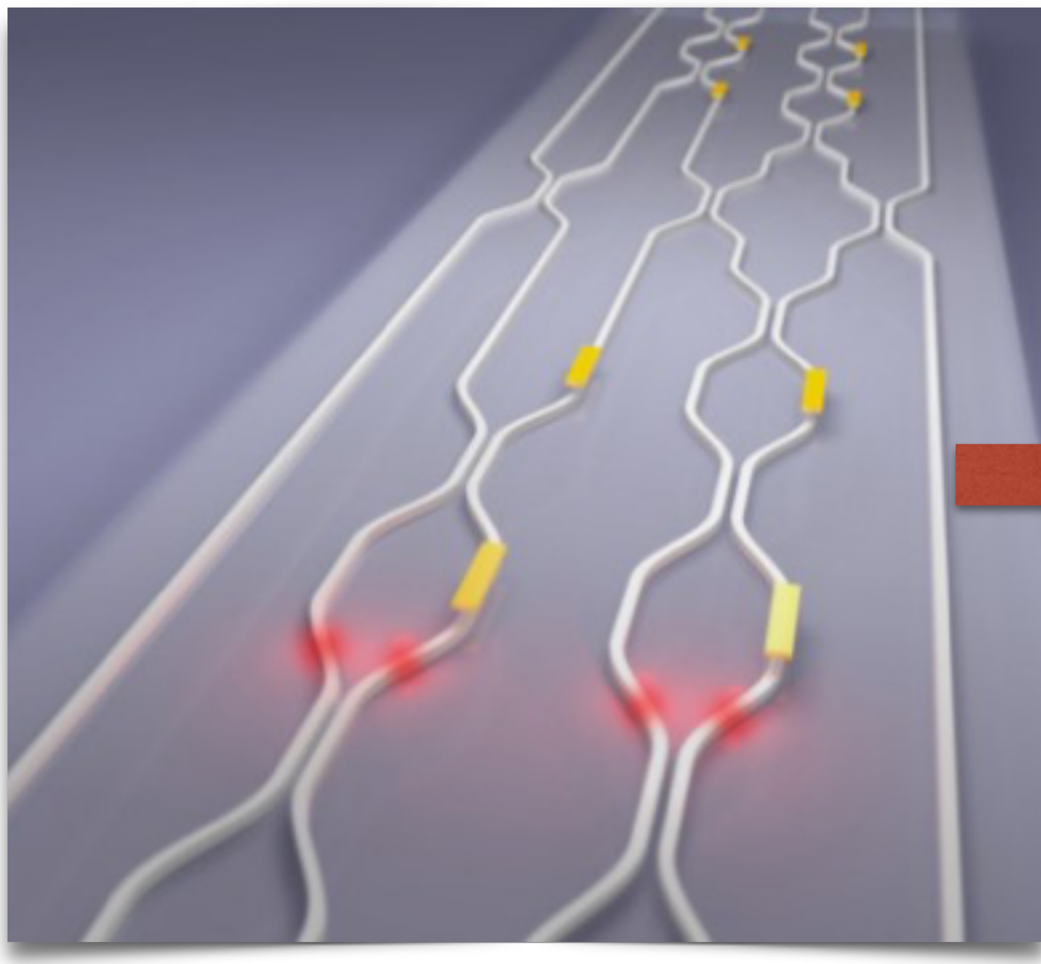

$$\begin{pmatrix} M_{1,1} & \cdots & M_{1,n} \\ M_{2,1} & \cdots & M_{2,n} \\ \vdots & \ddots & \vdots \\ M_{n,1} & \cdots & M_{n,n} \end{pmatrix}$$

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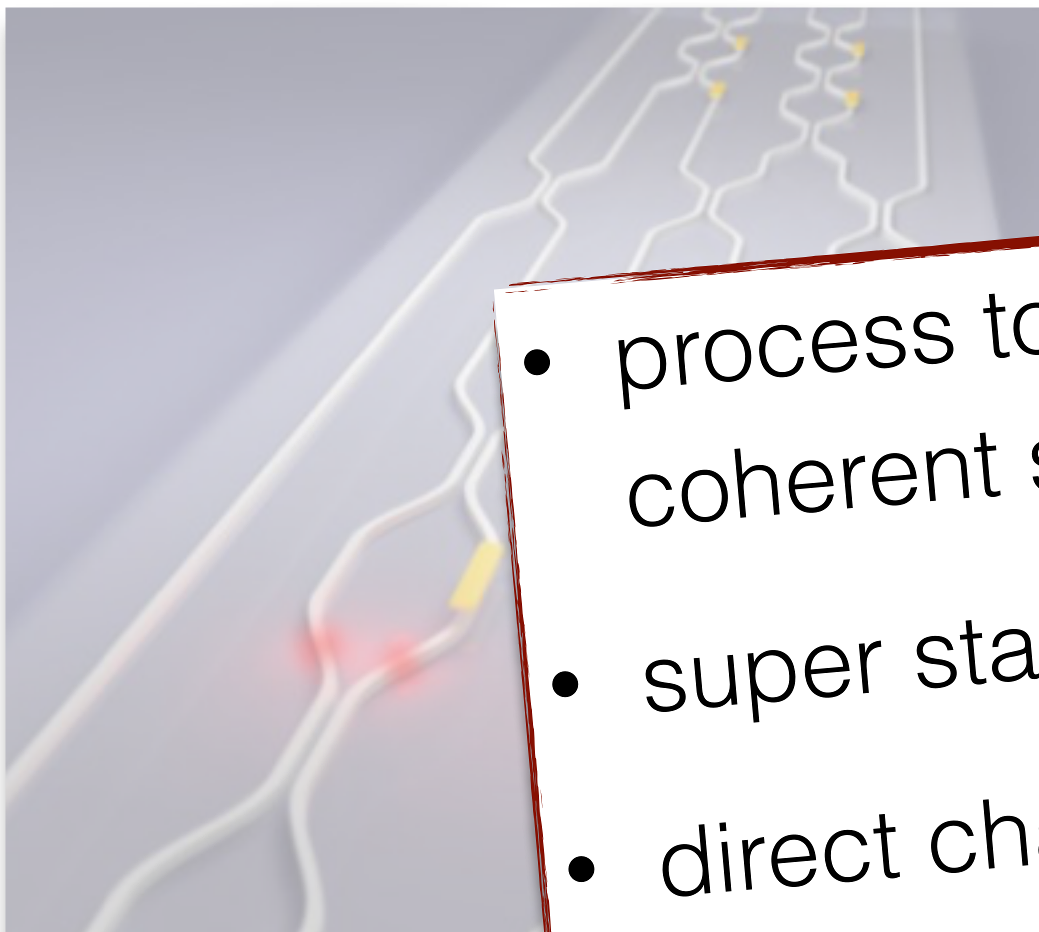

- process tomography with coherent states
- super stable tomography
- direct characterisation
- …

$$\begin{matrix} \cdots & M_{1,n} \\ \cdots & M_{2,n} \\ \ddots & \vdots \\ \cdots & M_{n,n} \end{matrix}\Bigg)$$

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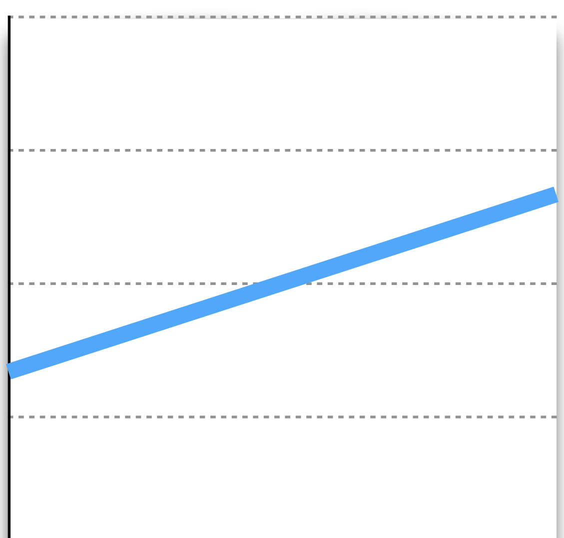
PhaseLift Characterisation

laser source &
photo diodes



nr. of measurements
scale linear in size

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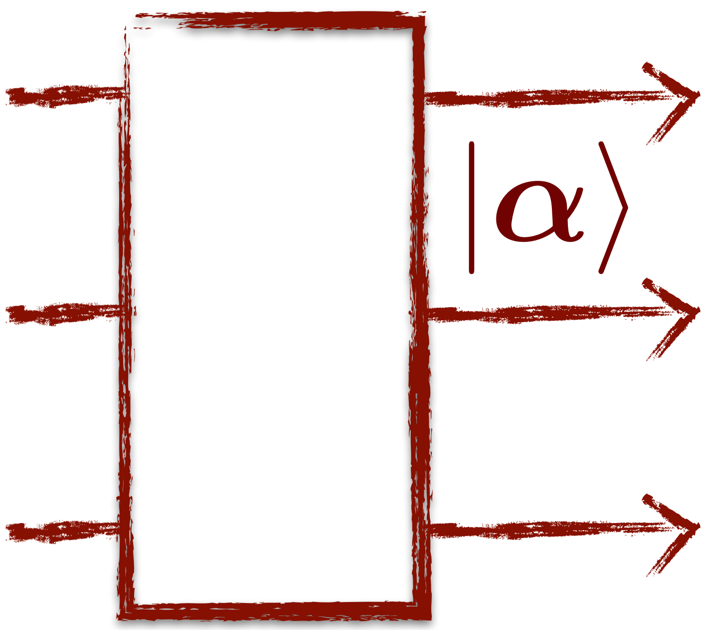
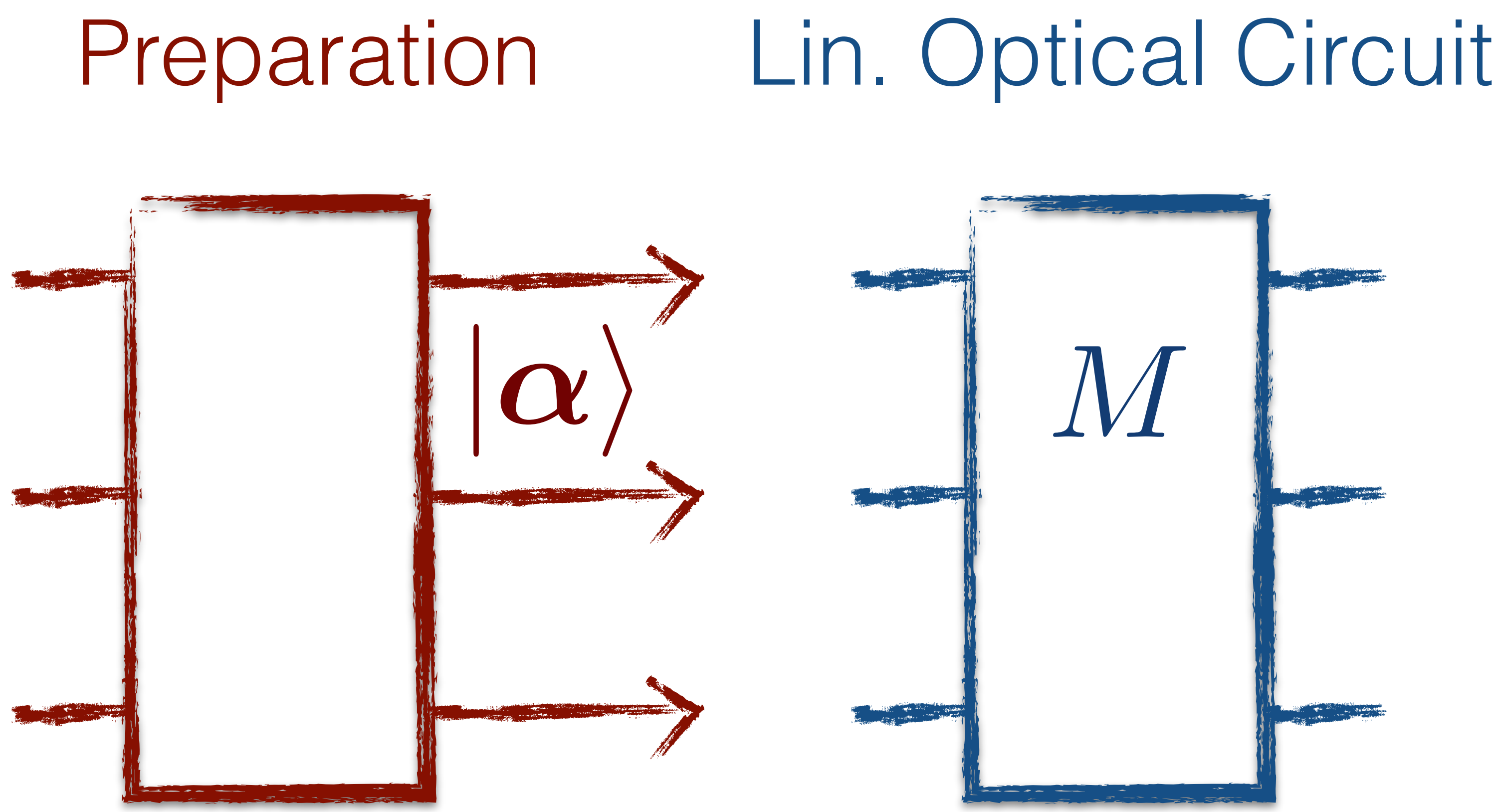
Device Characterisation

Preparation



Device Characterisation



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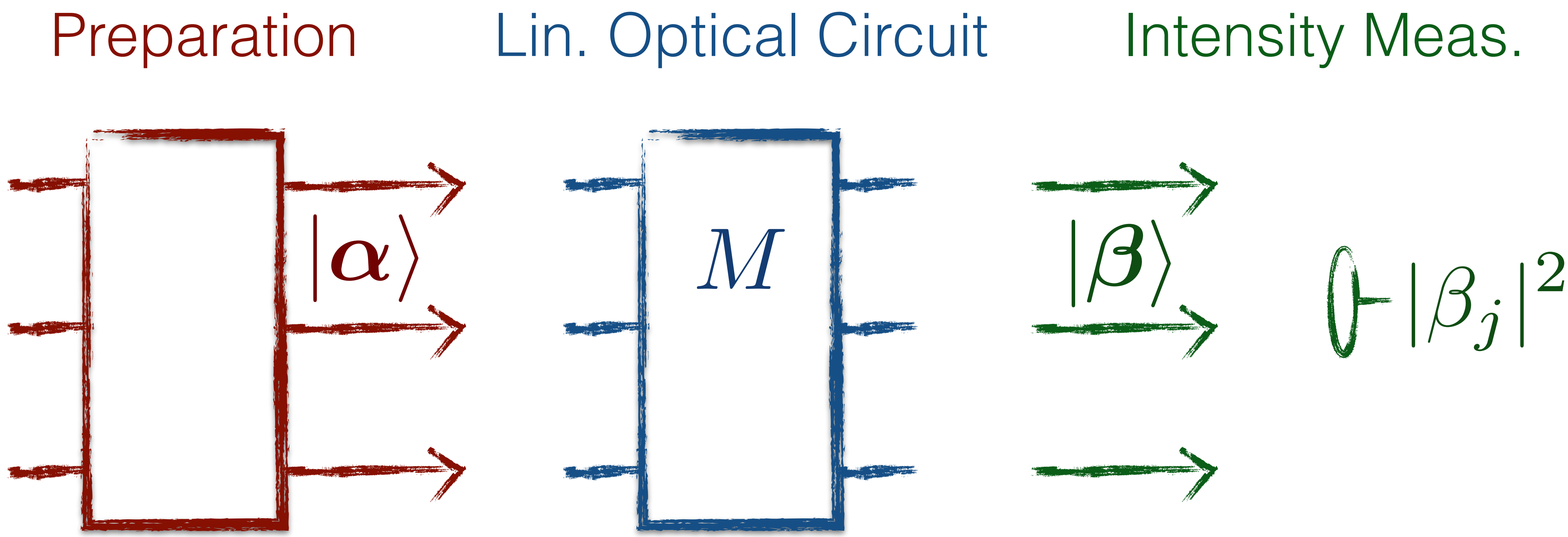

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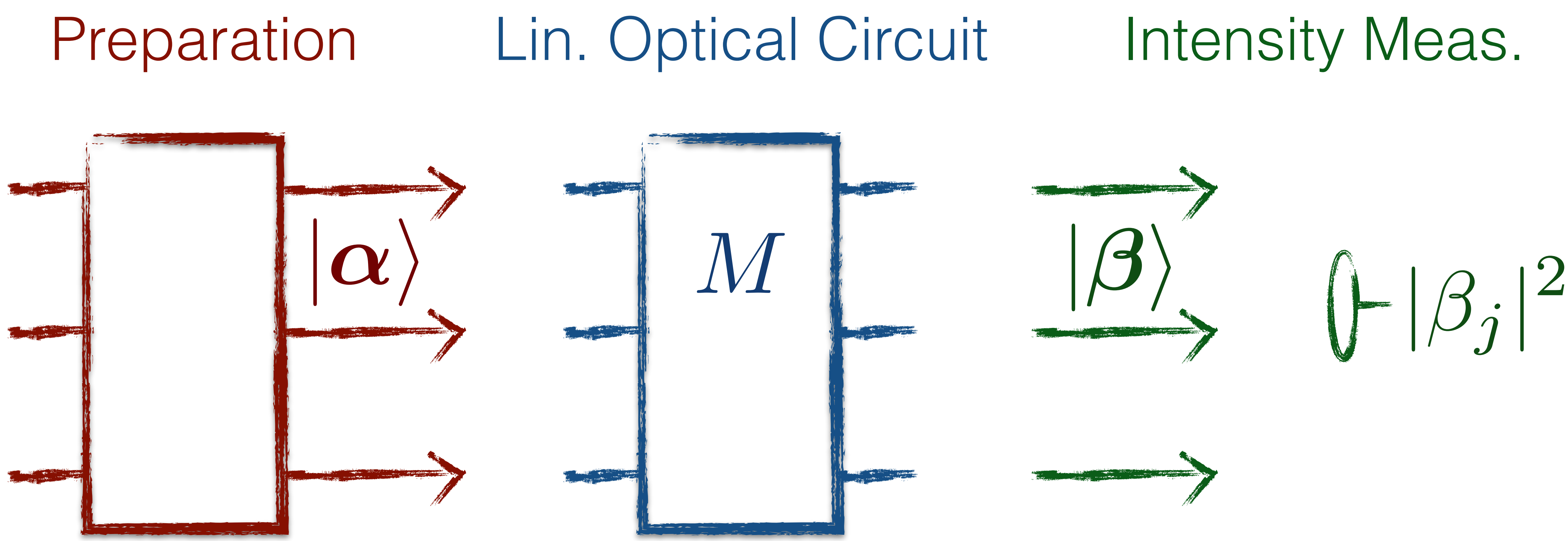

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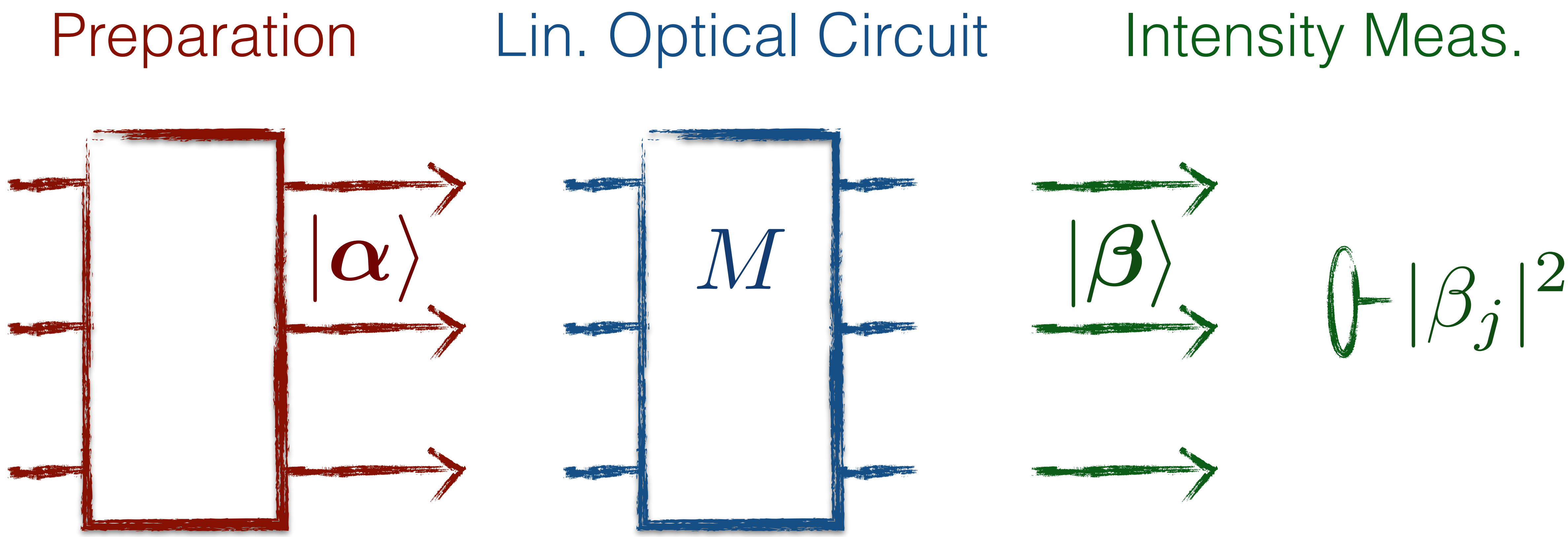

$$I_j(\boldsymbol{\alpha}) = |\beta_j|^2 = \left|\sum_k M_{j,k}\,\alpha_k\right|^2 = |\langle \boldsymbol{M}_j, \boldsymbol{\alpha}\rangle|^2$$

where $\boldsymbol{M}_j = (M^*_{j,1}, \ldots, M^*_{j,n})$

What is PhaseLift?

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$$\boldsymbol{Z}^\sharp = \sum_i \lambda_i\, |\boldsymbol{x}_i\rangle\langle \boldsymbol{x}_i| \quad (\lambda_i \geq \lambda_{i+1}) \quad \Rightarrow \quad \boldsymbol{M}_j^\sharp := \sqrt{\lambda_1}\boldsymbol{x}_1$$

Continuous Sampling Protocol

- $\boldsymbol{\alpha}_i \sim$ independently, uniformly from complex unit sphere
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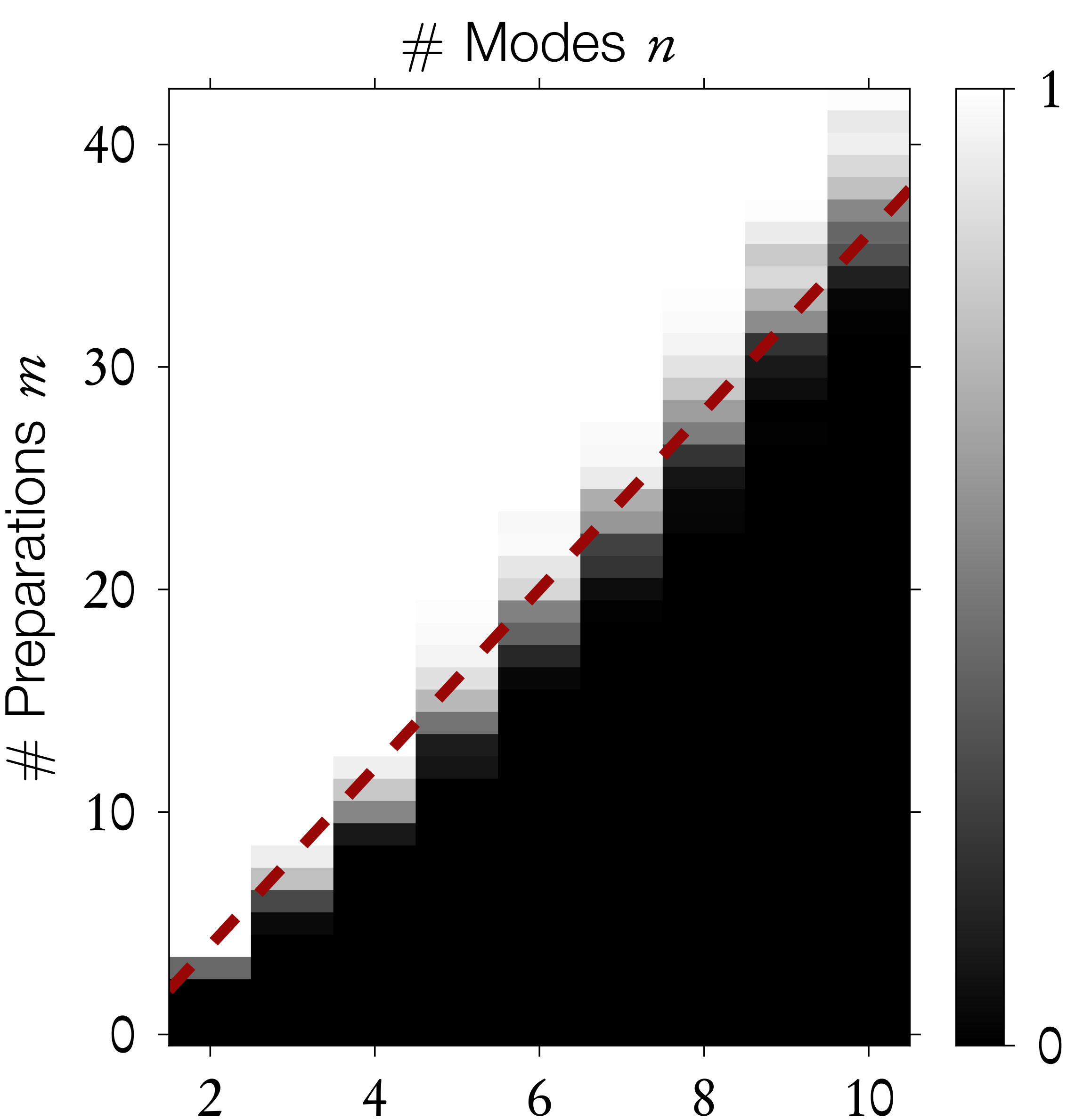

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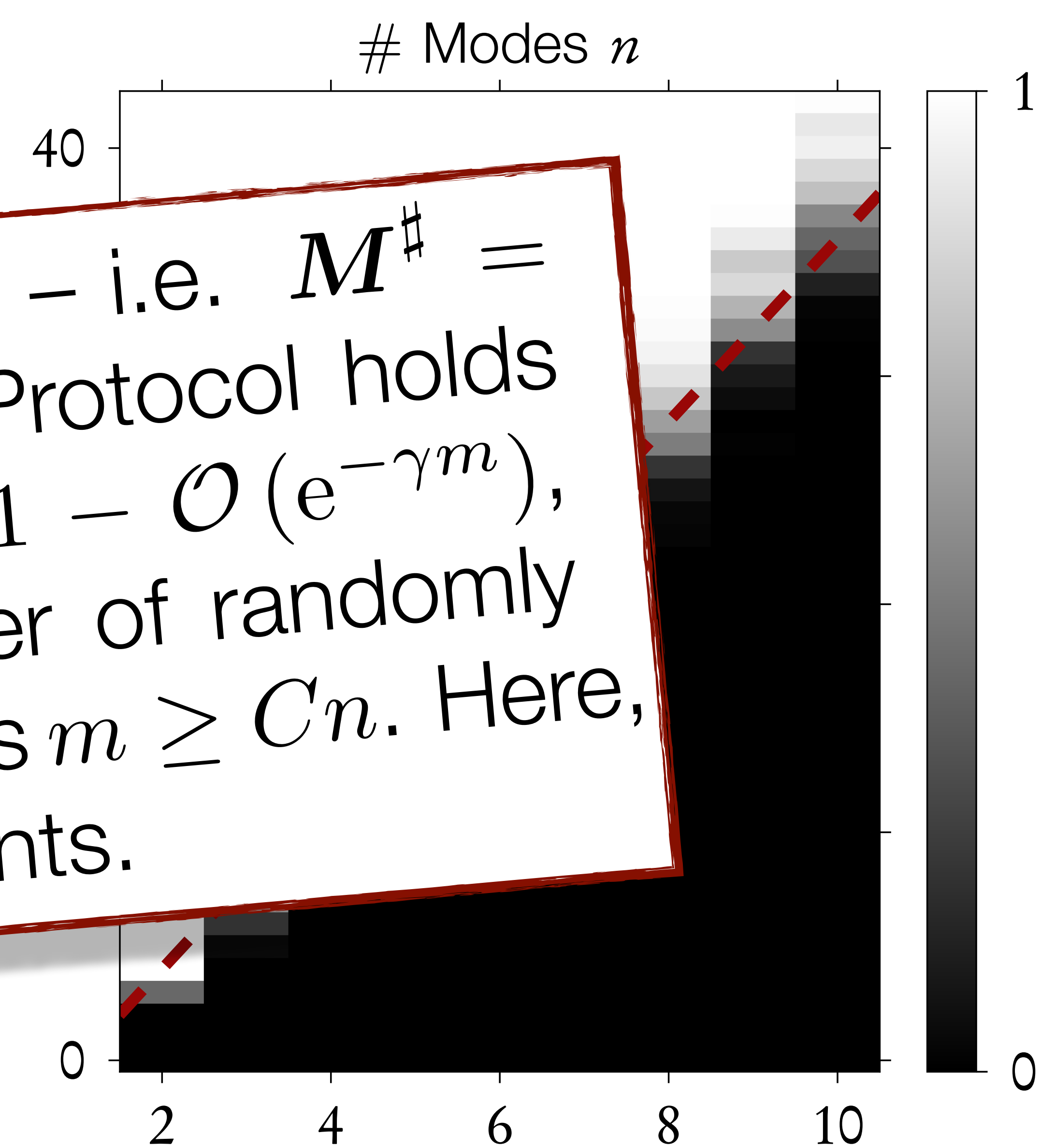
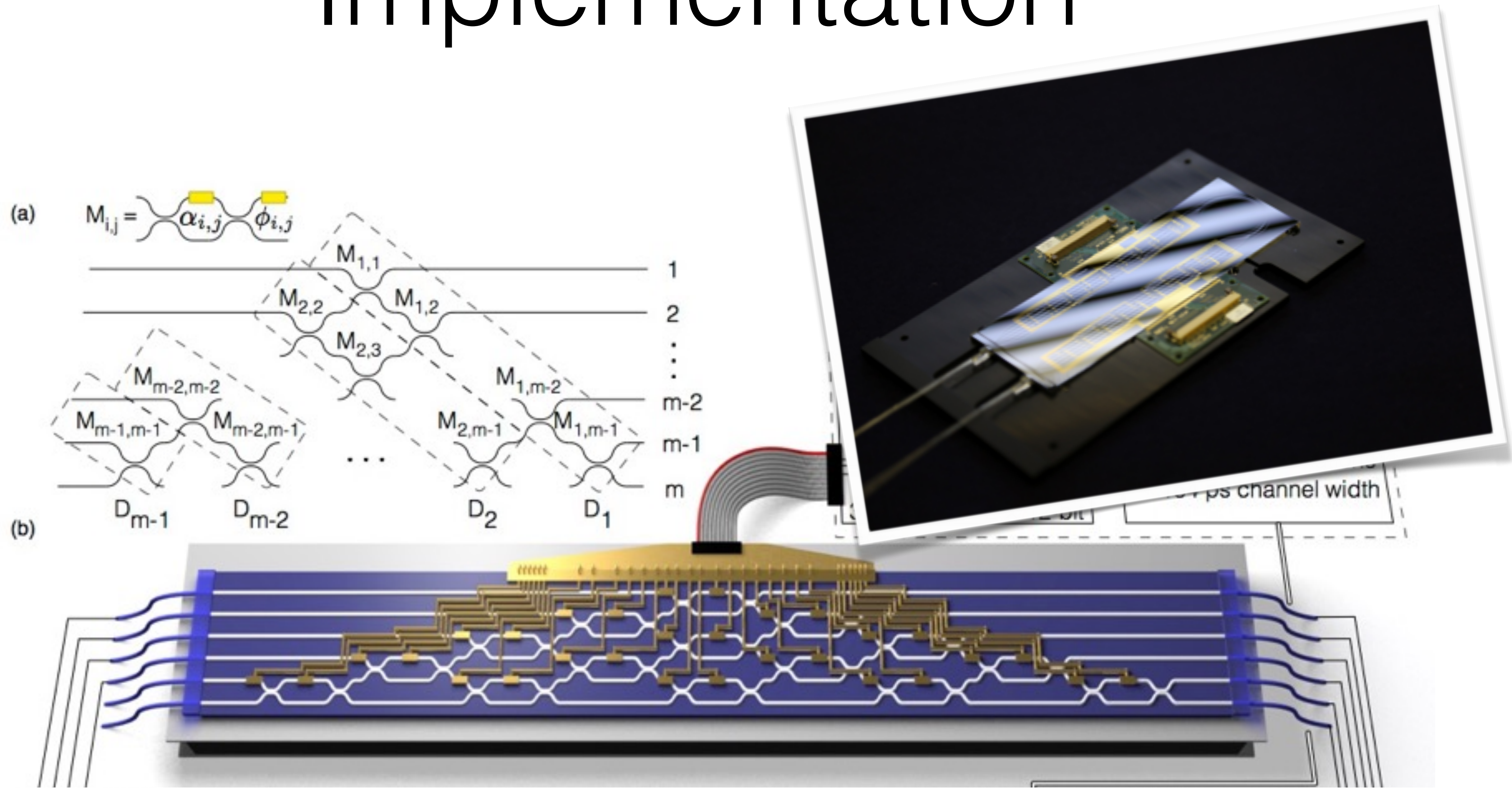
Perfect reconstruction – i.e. $\boldsymbol{M}^\sharp = \boldsymbol{M}$ – by means of this Protocol holds with probability at least $1 - \mathcal{O}\left(\mathrm{e}^{-\gamma m}\right)$, provided that the number of randomly chosen input states obeys $m \geq Cn$. Here, C, γ are absolute constants.

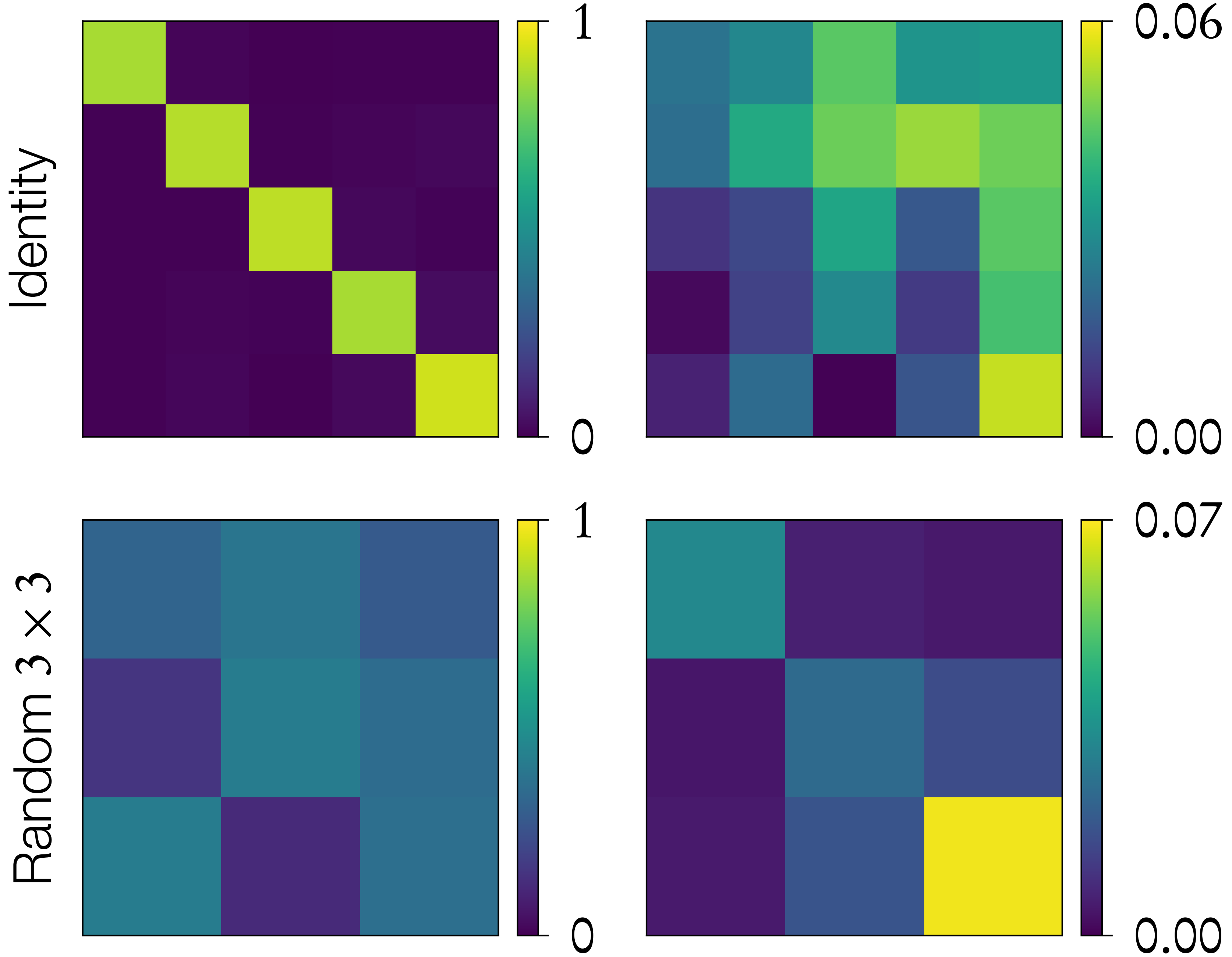


Experimental Implementation



Universal Linear Optics. **Science** 349 (6249): 711–16.
Image: *http://www.bristol.ac.uk/physics/news/2015/science-15.html*

Experimental Results



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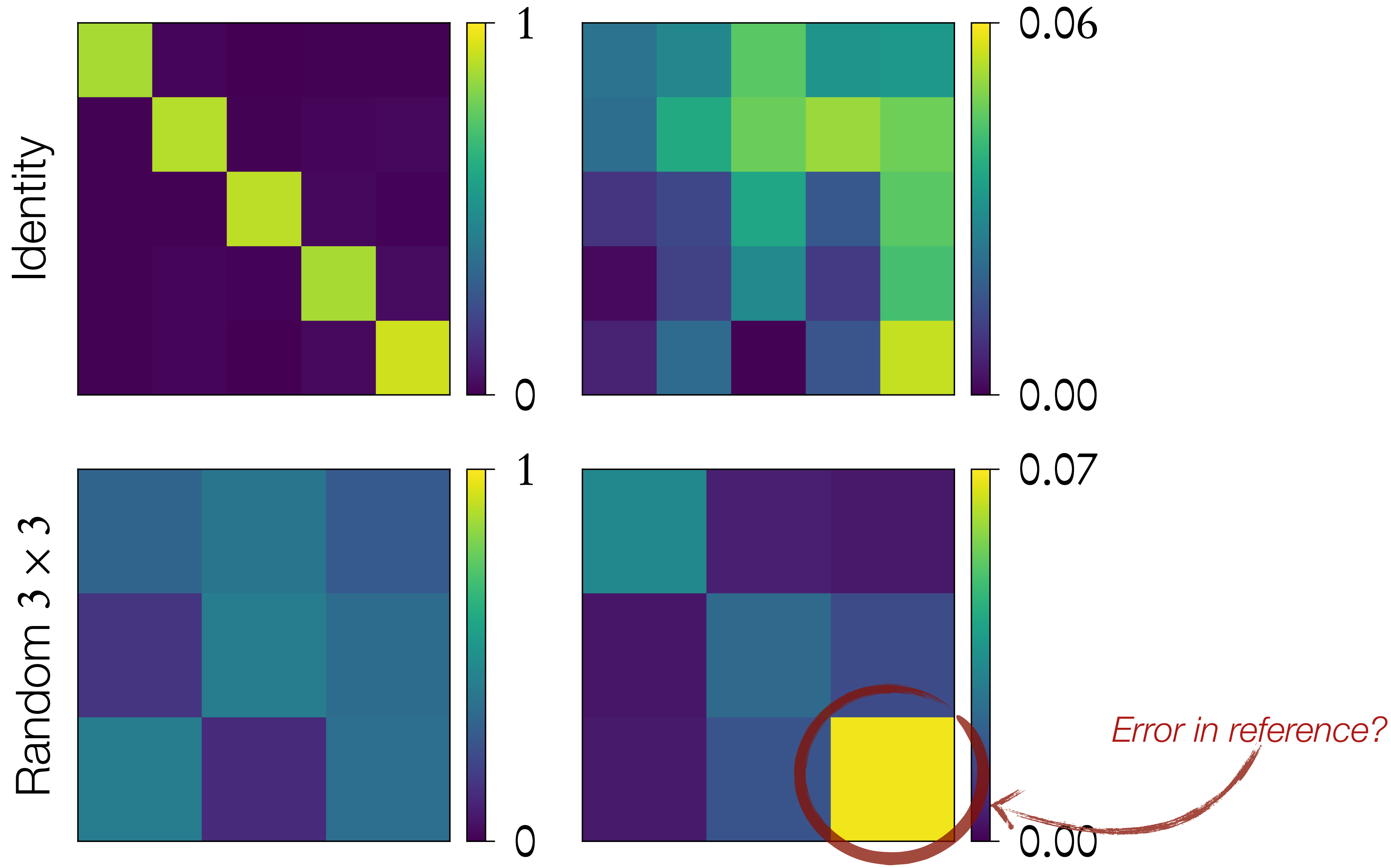

Conclusion

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$$(\boldsymbol{\alpha}_i)_j \sim \begin{cases} +1 & \text{with prob. } p/4 \\ +\mathrm{i} & \text{with prob. } p/4 \\ 0 & \text{with prob. } 1-p \\ -\mathrm{i} & \text{with prob. } p/4 \\ -1 & \text{with prob. } p/4 \end{cases}$$

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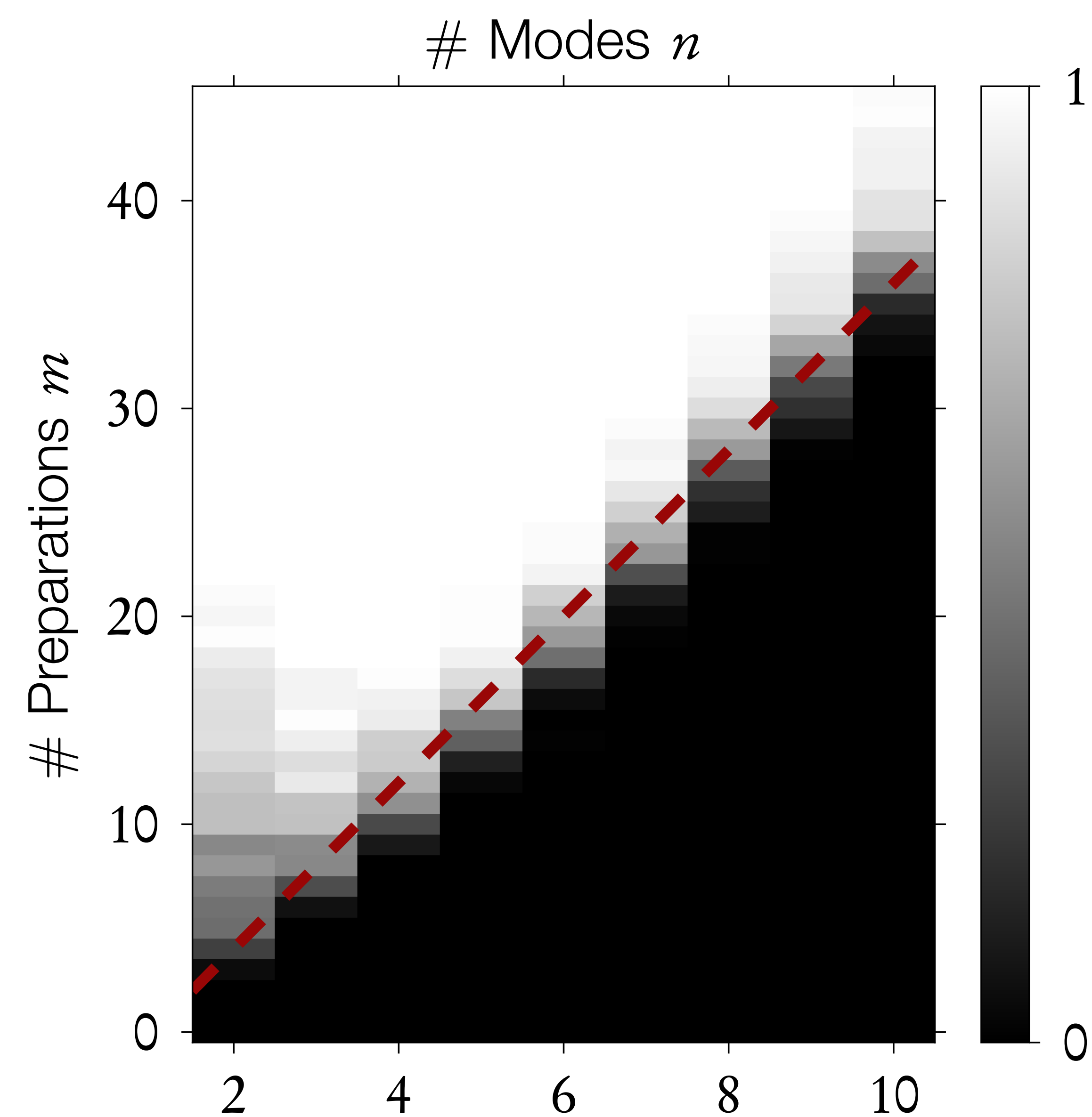

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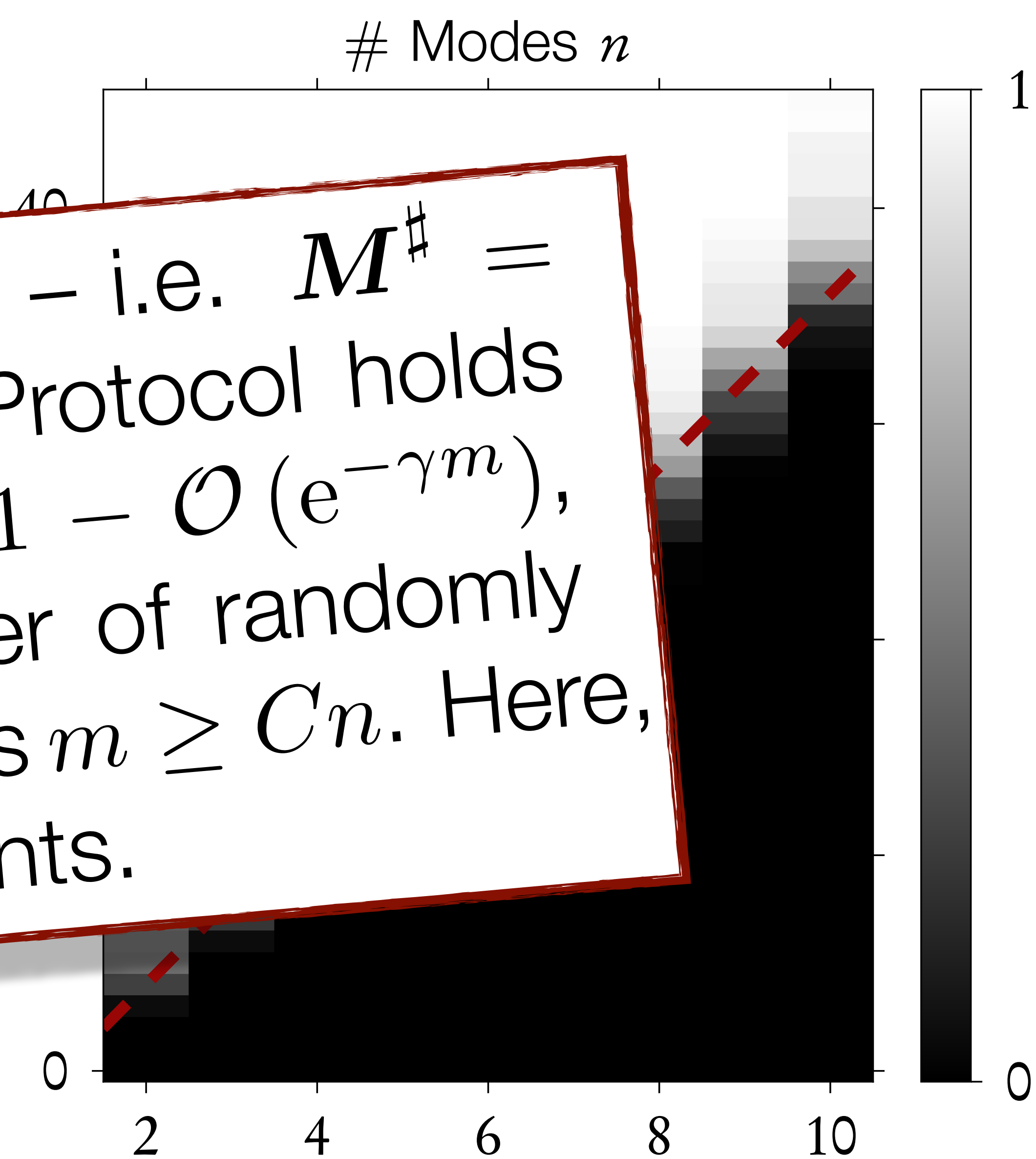

$$\|\boldsymbol{A}\|_p := \left(\sum_k \sigma_k(\boldsymbol{A})^p\right)^{\frac{1}{p}}$$