Characterising linear optical circuits via phaseless estimation techniques

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Linear-optical circuits may become important elementary building blocks of quantum computers in the future. Especially integrated photonics has the prospect of being scaleable by using technology from chip manufacturing. Here, we present an efficient, robust, and conceptually simple technique for characterising such a chip based on recent advances in low-rank matrix recovery.

Phaseless Estimation via PhaseLift

- *task*: infer signal $x \in \mathbb{C}^n$ from phase insensitive measurements

$$y_i = \left|\langle \alpha^{(i)}, x\rangle\right|^2 \qquad (i = 1, \ldots, m)$$

with $\alpha^{(i)}$ – measurement vectors

- quadratic measurements → linear in tensor product space

$$y_i = \operatorname{Tr}\left[\left(\alpha^{(i)}\alpha^{(i)\dagger}\right)\underbrace{x x^\dagger}_{=Z}\right]$$

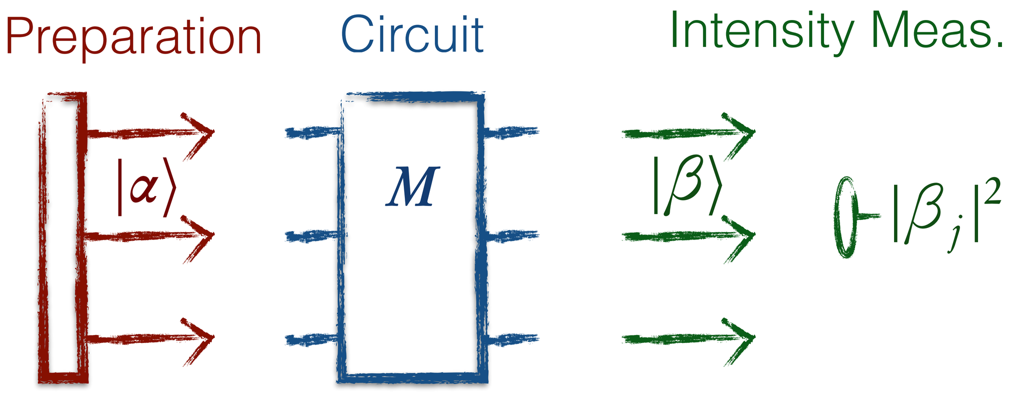
- to find low-rank approximation $Z^\sharp \approx x x^\dagger$ efficiently:

$$Z^\sharp = \operatorname*{argmin}_{Z \in \mathbb{H}^n} \ \cancel{\operatorname{rank}(Z)} \ \operatorname{Tr} Z$$

$$\text{s.t.} \quad \operatorname{Tr}\left[\left(\alpha^{(i)}\alpha^{(i)\dagger}\right) Z\right] = y_i \quad (i = 1, \ldots, m)$$

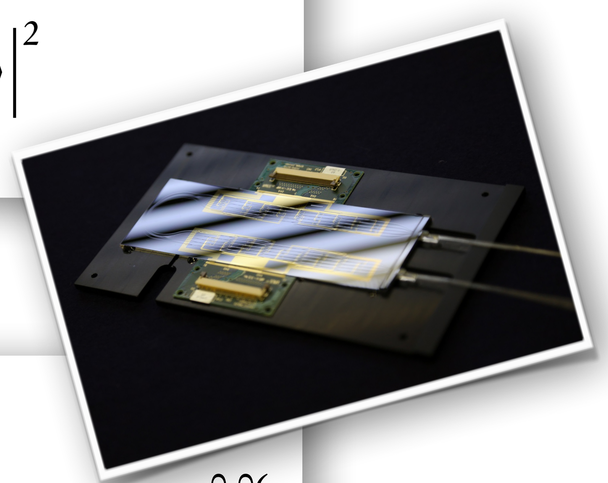
$$Z \succeq 0$$

Characterisation of Linear Optical Chip



- given: optical circuit with n inputs/outputs (modes)
- input laser light characterised by $\alpha \in \mathbb{C}^n$ (so called "coherent state of light" $|\alpha\rangle$)
- output is "coherent state" described by $\beta = M\alpha \in \mathbb{C}^n$
- M is the *transfer matrix* to be determined
- allowed measurements: intensity in j-th output (for $j = 1, \ldots, n$)

$$I_j(\alpha) = |\beta_j|^2 = \left|\sum_{k=1}^n M_{j,k}\alpha_k\right|^2 = \left|\langle \overline{M}_j, \alpha\rangle\right|^2$$



Continuous Sampling Protocol

- sample $\alpha^{(i)}$ independently, uniformly from complex unit sphere
- recover j-th row of M from intensity measurements $I_j(\alpha^{(i)})$:

$$\overline{M}_j^\sharp := \text{eigenvector of } Z_j^\sharp \text{ with largest eigenvalue}$$

with $\left\|\overline{M}_j^\sharp\right\|_2^2 = \left\|Z_j^\sharp\right\|_\infty$ and $Z_j^\sharp$ solution of SDP

- performance guarantees below are uniform → same inputs $\alpha^{(i)}$ for each row possible

Theorem Perfect reconstruction – i.e. $M^\sharp = \operatorname{diag}(\mu)M$ – holds with probability at least $1 - \mathcal{O}(e^{-\gamma m})$ provided $m \geq Cn$. Here, C and γ denote absolute constants and $|\mu_i| = 1$.

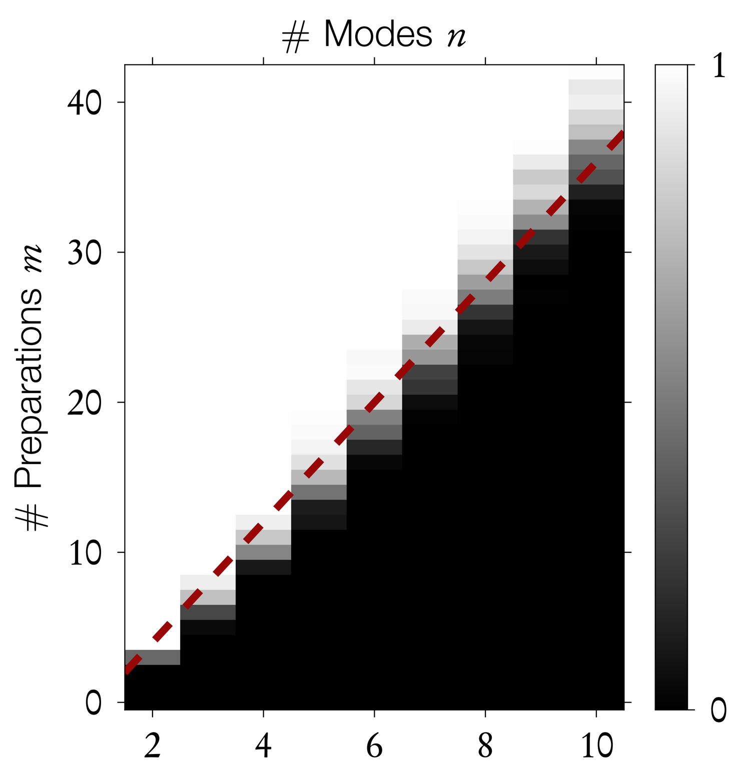


Fig. 1 Recovery Probabilities for 100 Haar-random transfer matrices with varying size n and number of measurements m. Red line: $m = 4n - 4$

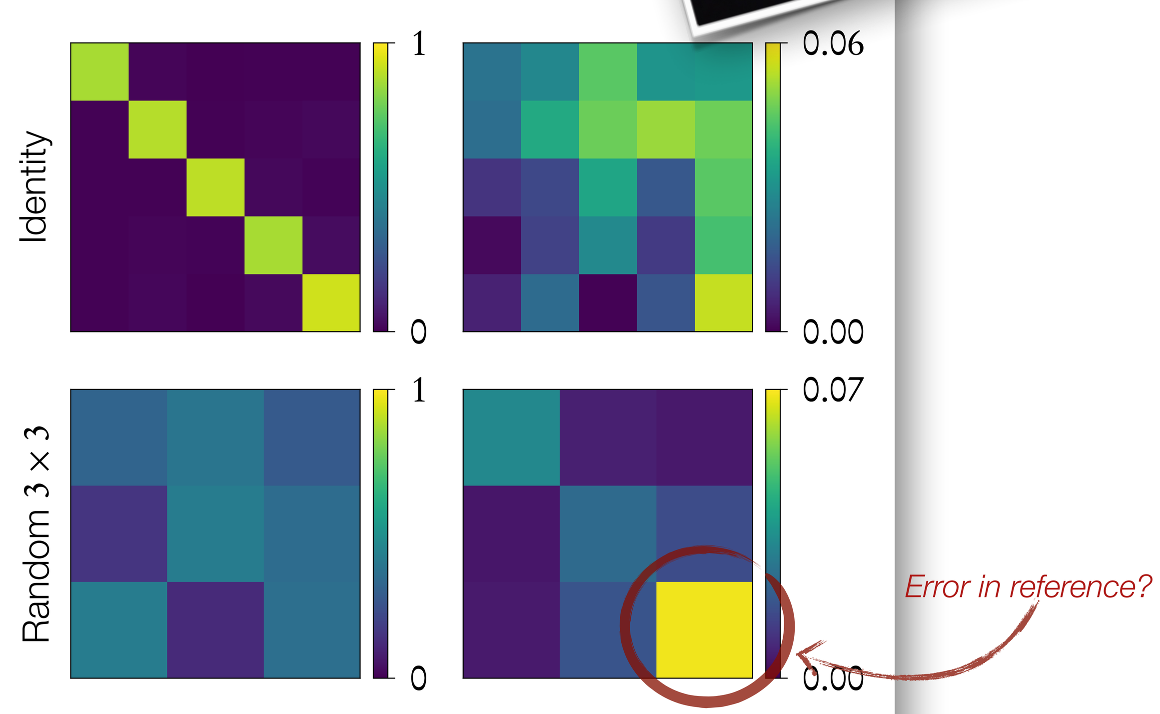


Fig. 2 Experimental reconstructions — absolute values of transfer matrix elements (left) and deviation from reference (right)

RECR Sampling Protocol

- discrete sampling scheme: RECR – randomly erased complex Rademacher sampling

$$\alpha_j^{(i)} \sim \begin{cases} +1 & \text{with prob. } p/4 \\ +i & \text{with prob. } p/4 \\ 0 & \text{with prob. } 1-p \\ -i & \text{with prob. } p/4 \\ -1 & \text{with prob. } p/4 \end{cases}$$

with $p \in [0,1]$ ($1-p$ – erasure probability)

- smaller error in preparation stage on chip since it takes into account the experiment's characteristics

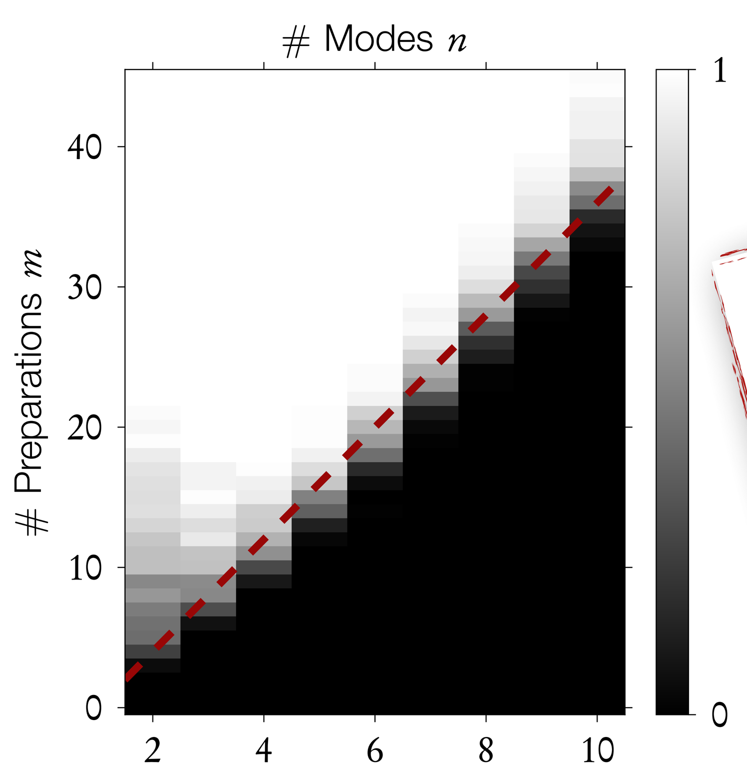
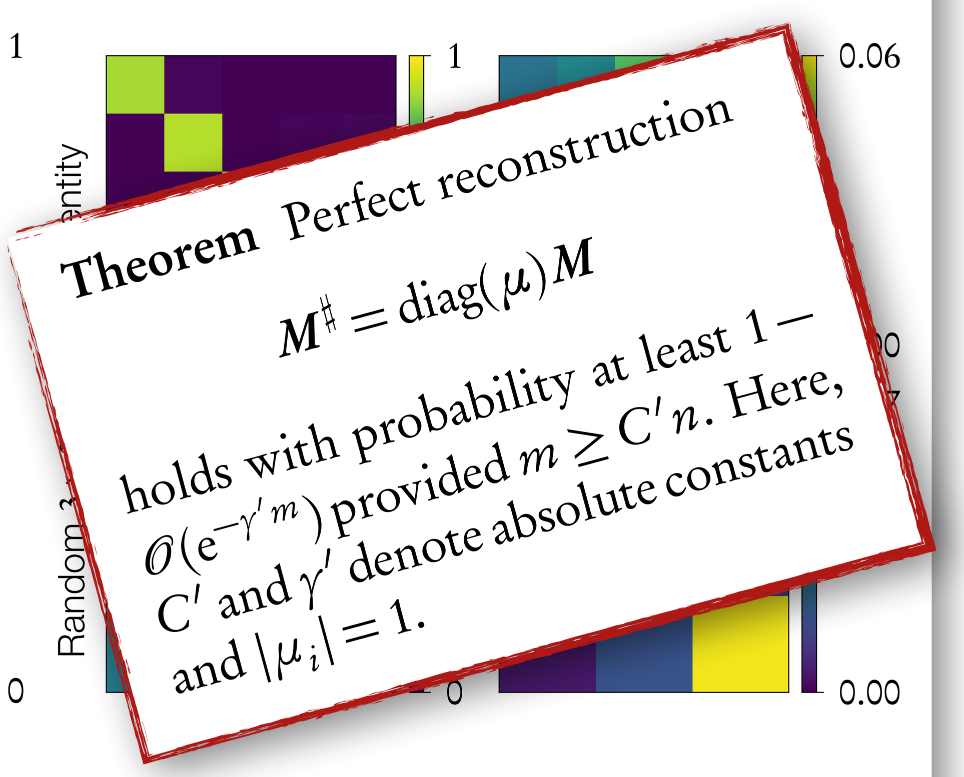


Fig. 3 Recovery Probabilities for 100 Haar-random transfer matrices with varying size n and number of measurements m using RECR with $p = 1/2$. Red line: $m = 4n - 4$



Theorem Perfect reconstruction

$$M^\sharp = \operatorname{diag}(\mu)M$$

holds with probability at least $1 - \mathcal{O}(e^{-\gamma' m})$ provided $m \geq C'n$. Here, C' and γ' denote absolute constants and $|\mu_i| = 1$.

References

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- E. J. Candès, , T. Strohmer, and V. Voroninski. 2013. "PhaseLift: Exact and Stable Signal Recovery from Magnitude Measurements via Convex Programming." *Communications on Pure and Applied Mathematics* 66 (8): 1241–74.
- R. Kueng, H. Rauhut, and U. Terstiege. 2014. "Low Rank Matrix Recovery from Rank One Measurements." *arXiv: 1410.6913 [quant-Ph]*.

Picture *http://www.bristol.ac.uk/physics/news/2015/science-15.html*

There is animated content @
http://thp.uni-koeln.de/~dsuess/phaselift