Characterising integrated linear optical circuits via PhaseLift

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University of Cologne

C. Sparrow, N. Maraviglia, and A. Laing

University of Bristol

Slides & Code @ https://github.com/dseuss/presentations

The Question is…

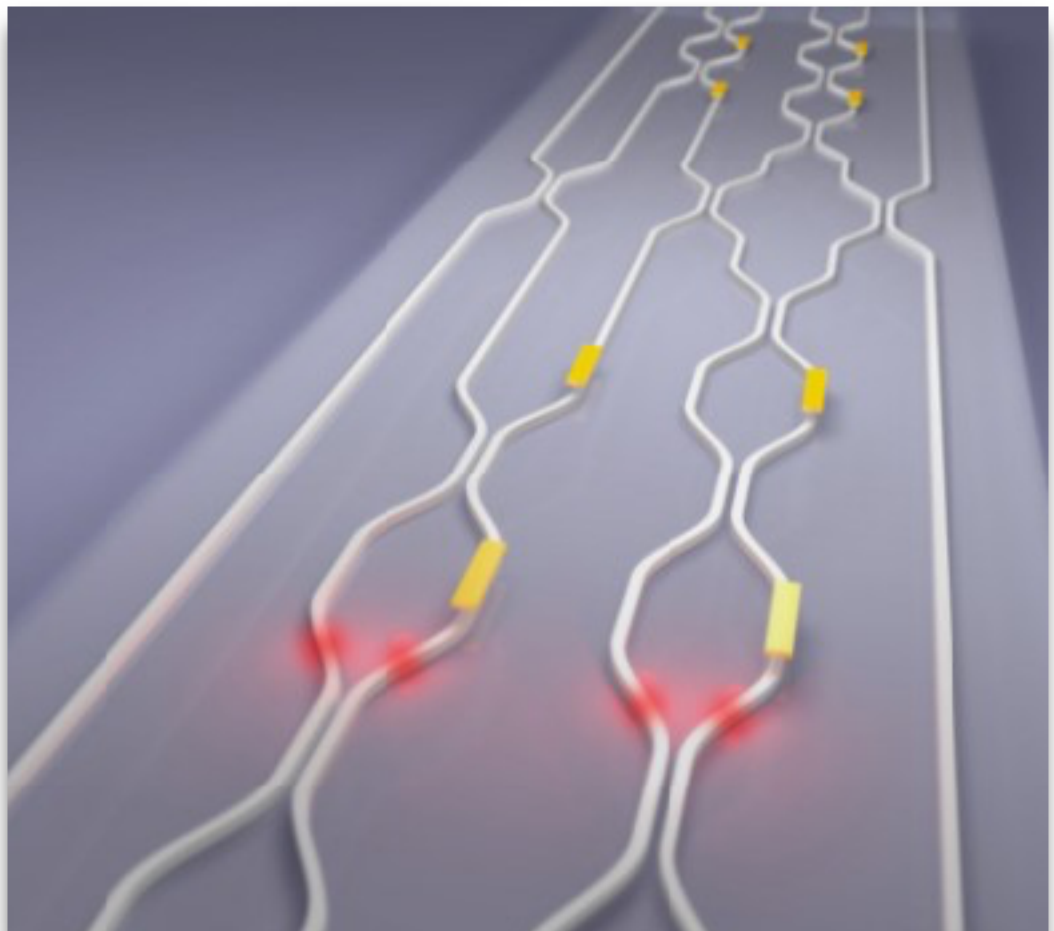

Image: http://www.bris.ac.uk/news/2011/8109.html

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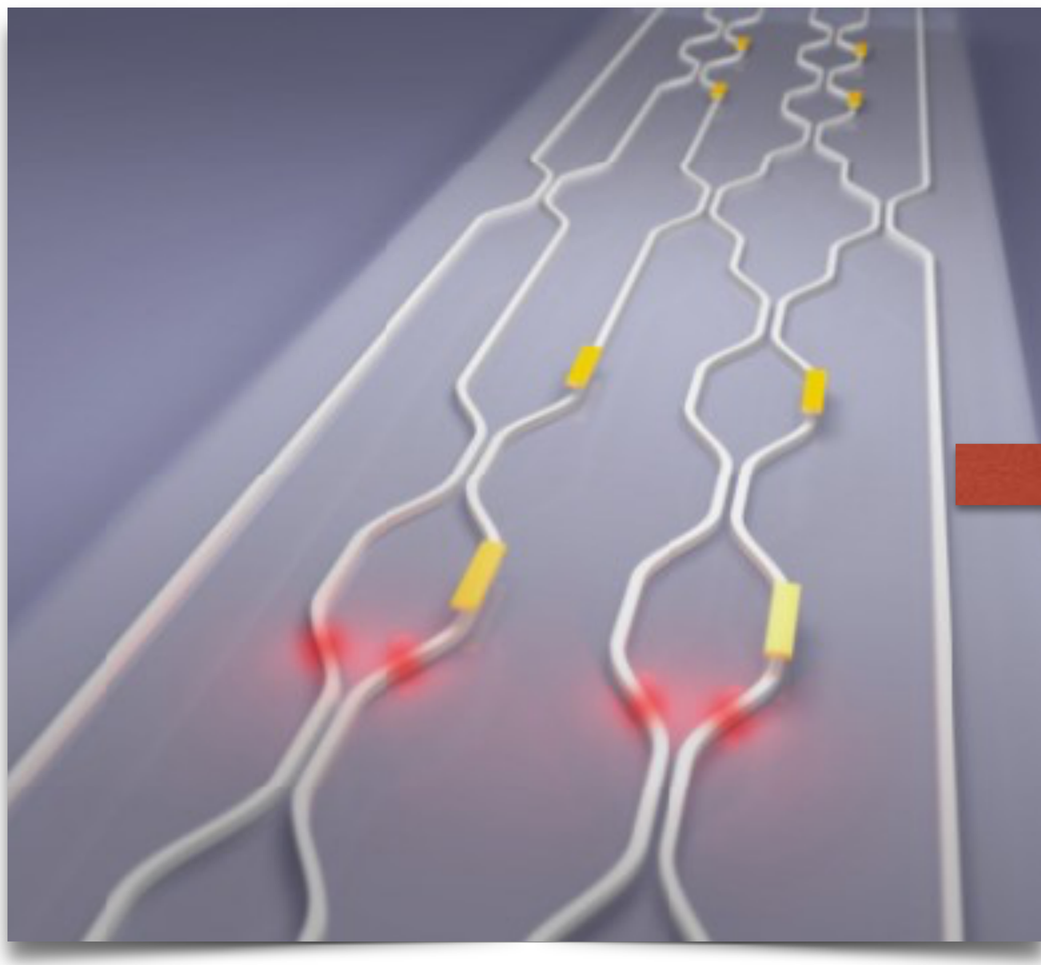

$$\begin{pmatrix} M_{1,1} & \cdots & M_{1,n} \\ M_{2,1} & \cdots & M_{2,n} \\ \vdots & \ddots & \vdots \\ M_{n,1} & \cdots & M_{n,n} \end{pmatrix}$$

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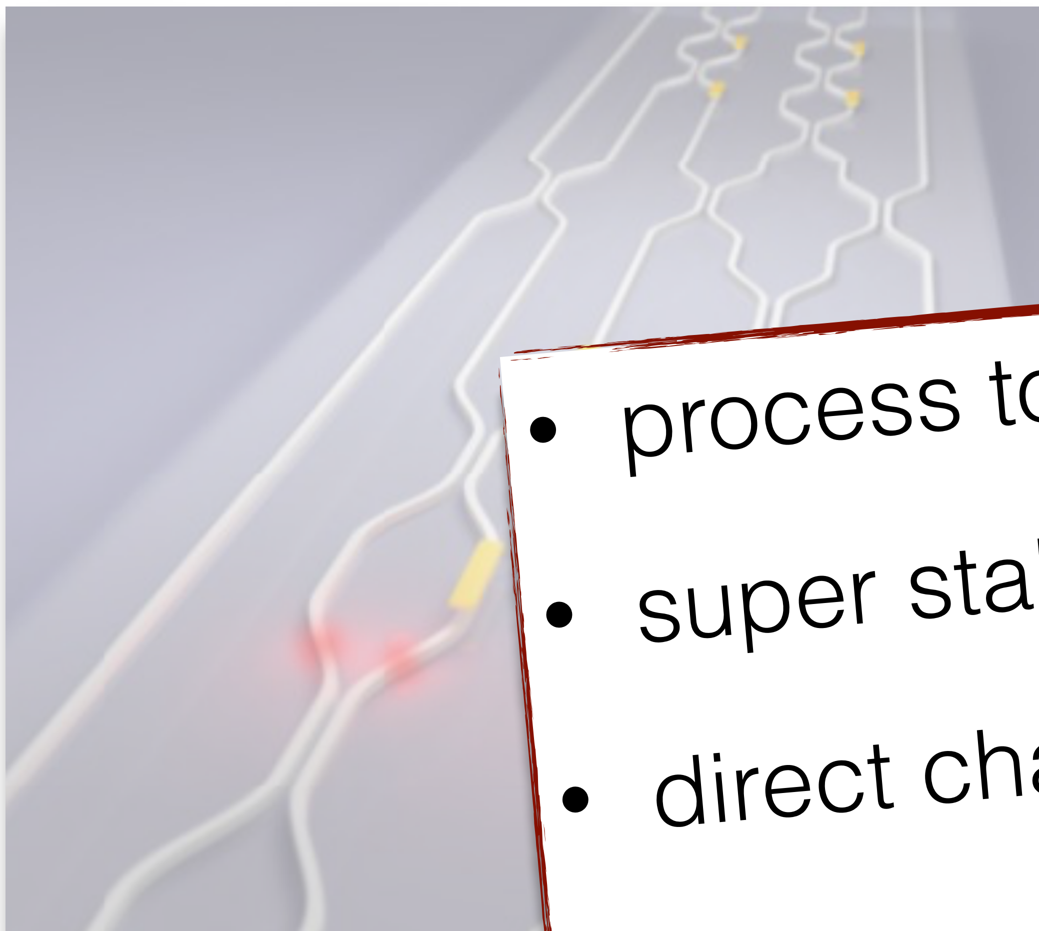

- process tomography
- super stable tomography
- direct characterisation
- …

$$\begin{matrix} \cdots & M_{1,n} \\ \cdots & M_{2,n} \\ \ddots & \vdots \\ \cdots & M_{n,n} \end{matrix}$$

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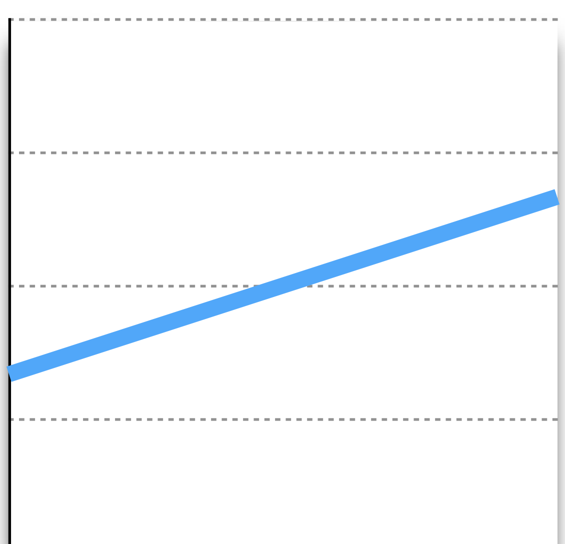
PhaseLift Characterisation

laser source &
photo diodes



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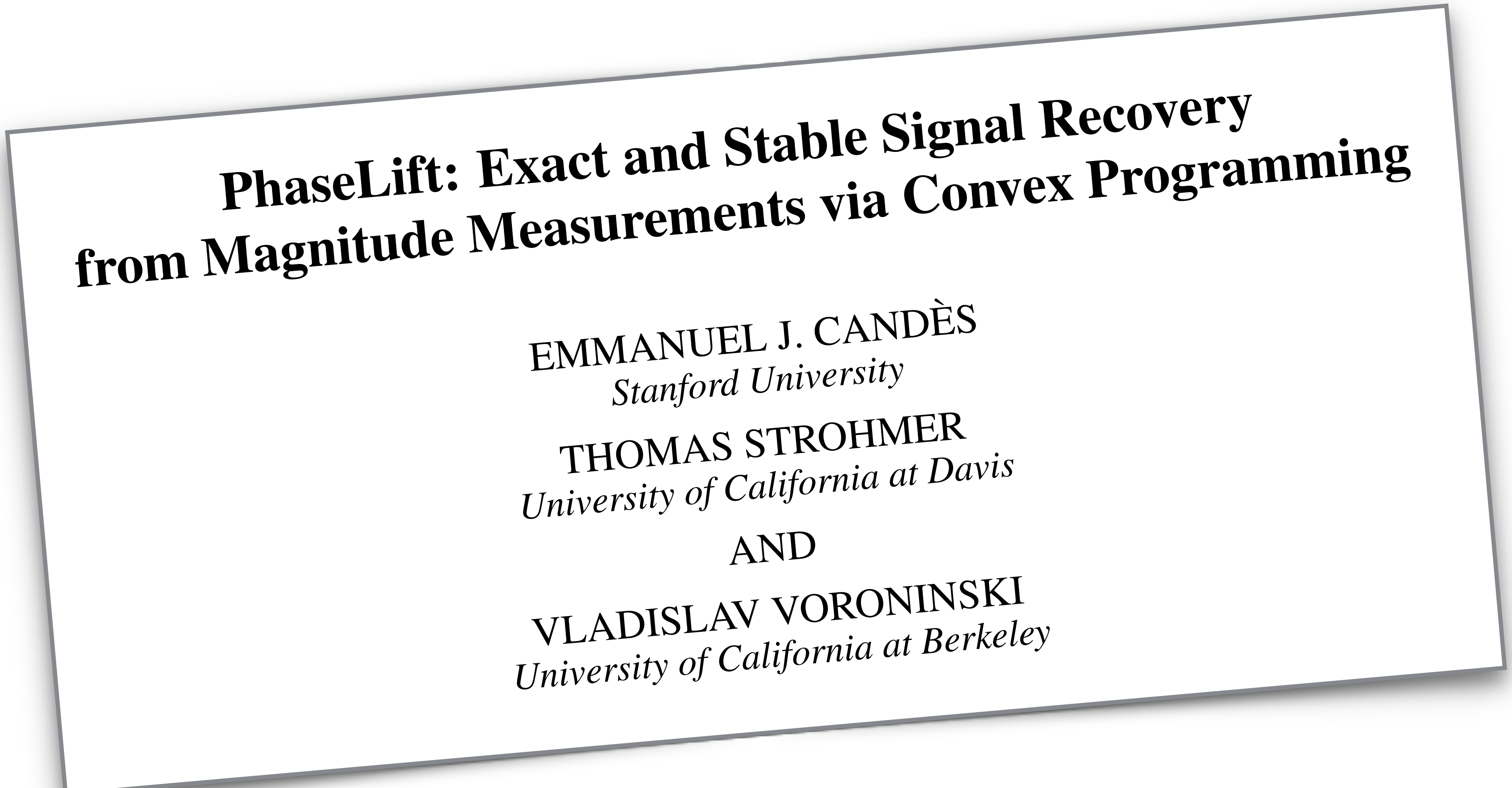
Outline

- What is PhaseLift?
- Characterisation via PhaseLift
- Numerical & experimental results
- Improved sampling scheme
- Conclusions & outlook

What is PhaseLift?



PhaseLift: Exact and Stable Signal Recovery from Magnitude Measurements via Convex Programming

EMMANUEL J. CANDÈS
Stanford University

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$$\boxed{I(a) = |\langle x, a\rangle|^2 \quad \longrightarrow \quad x = ?}$$

What is PhaseLift?

$$y_i = |\langle x, a^{(i)} \rangle| = \text{tr}\left(|x\rangle\langle x| \, |a^{(i)}\rangle\langle a^{(i)}| \right)$$

D. Gross: "Recovering Low-Rank Matrices From Few Coefficients in Any Basis."
R. Kueng, H. Rauhut, U. Terstiege: "Low Rank Matrix Recovery from Rank One Measurements."

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$$= Z$$

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$$\boxed{\begin{array}{ll} \underset{Z \in \mathbb{H}^n}{\text{minimize}} & \operatorname{rank}(Z) \\ \text{subject to} & \operatorname{tr}\left(Z \, |a^{(i)}\rangle\langle a^{(i)}| \right) = y_i \quad (1 \leq i \leq m) \end{array}}$$

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$$Z^\sharp = \sum_i \lambda_i |x_i\rangle\langle x_i| \quad (\lambda_i \ge \lambda_{i+1}) \quad \Longrightarrow \quad x^\sharp := \sqrt{\lambda_1}x_1$$

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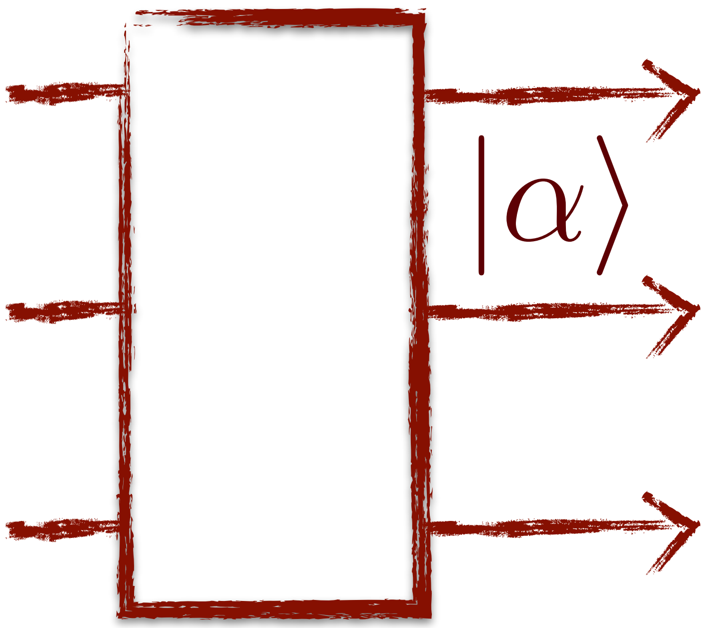
Device Characterisation

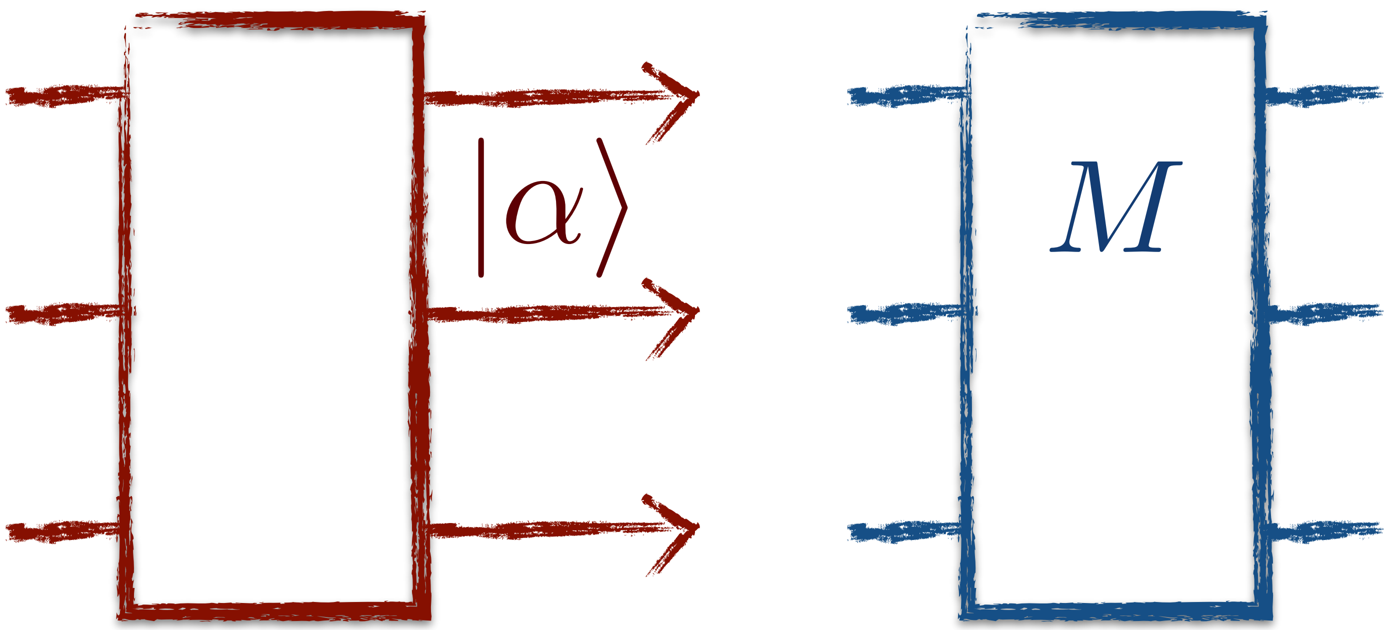
Preparation



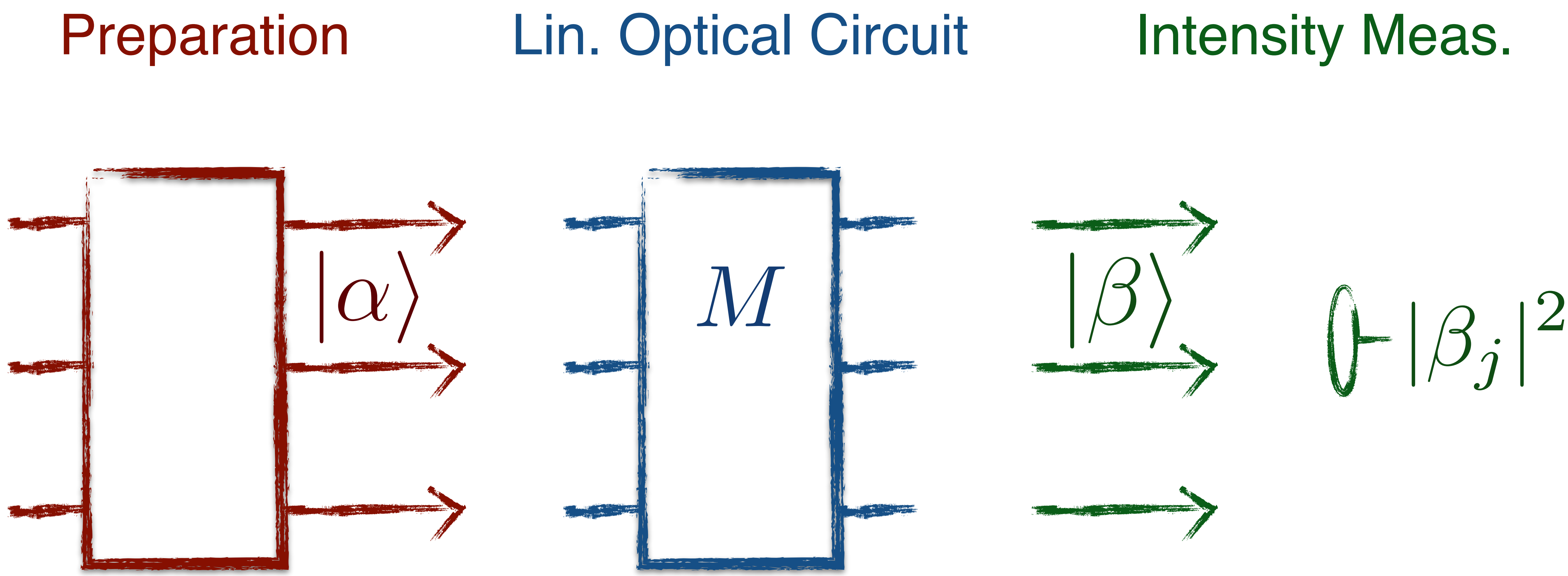
Device Characterisation

Preparation

Lin. Optical Circuit



Device Characterisation



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Characterisation via PL

$$I_j(\alpha) = \left| \sum_k M_{j,k} \alpha_k \right|^2 = \left| \langle M_j, \alpha \rangle \right|^2 ,$$

where $M_j = (M^*_{j,1}, ..., M^*_{j,n})$

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1. Input coherent states sampled uniformly from complex unit sphere

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1. Input coherent states sampled uniformly from complex unit sphere
2. Measure intensities on each output mode
3. Run PhaseLift to recover rows of M up to global phases

Characterisation via PL

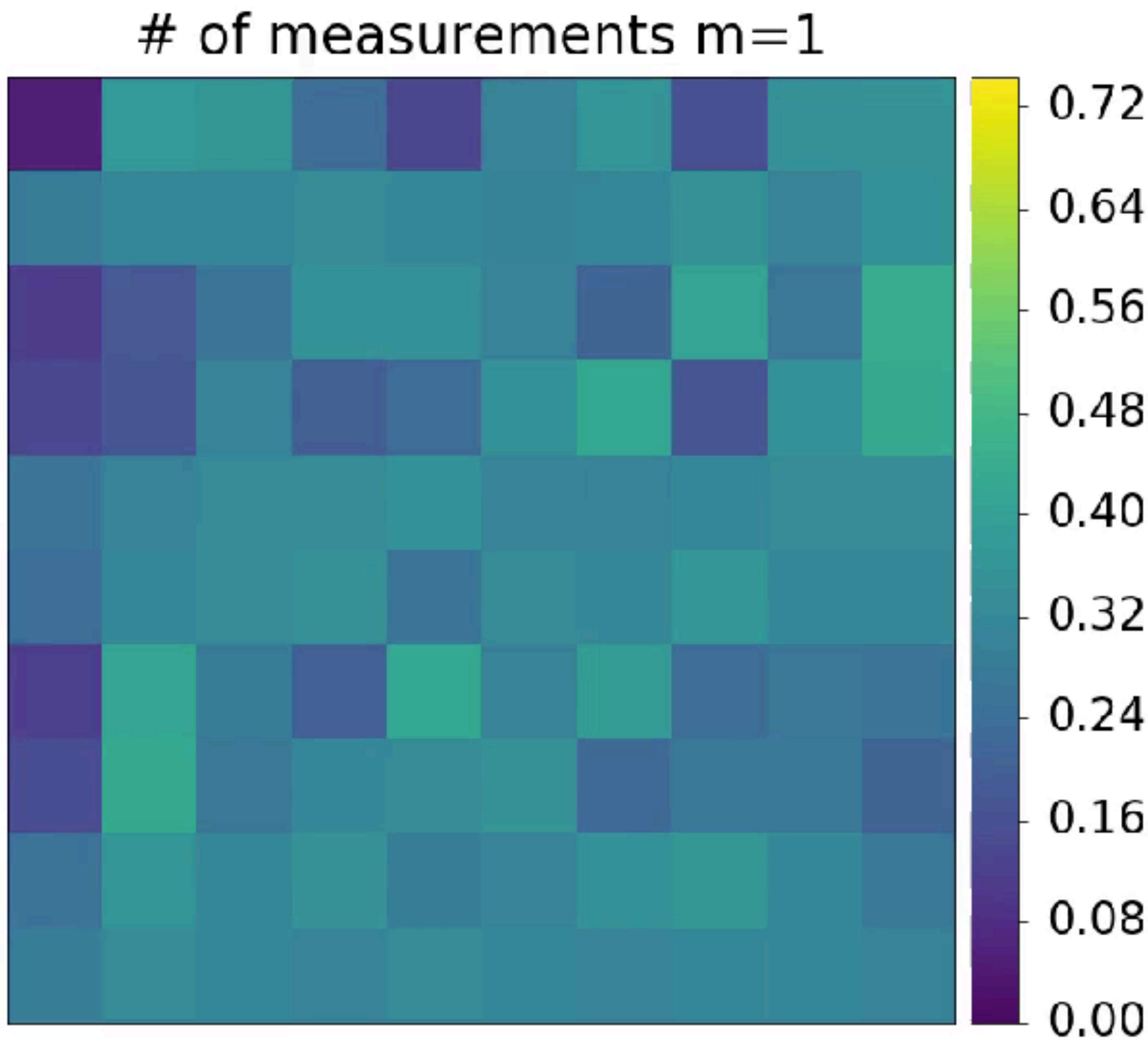
$$I_j(\alpha) = \left|\sum M_{j,k}\alpha_k\right|^2 = |\langle M_j, \alpha\rangle|^2$$

Perfect reconstruction – i.e. $M^\sharp = M$ – by means of this protocol holds with probability at least $1 - \mathcal{O}(\mathrm{e}^{-\gamma m})$, provided that the number of randomly chosen input states obeys $m \geq Cn$. Here, C, γ are absolute constants.

2

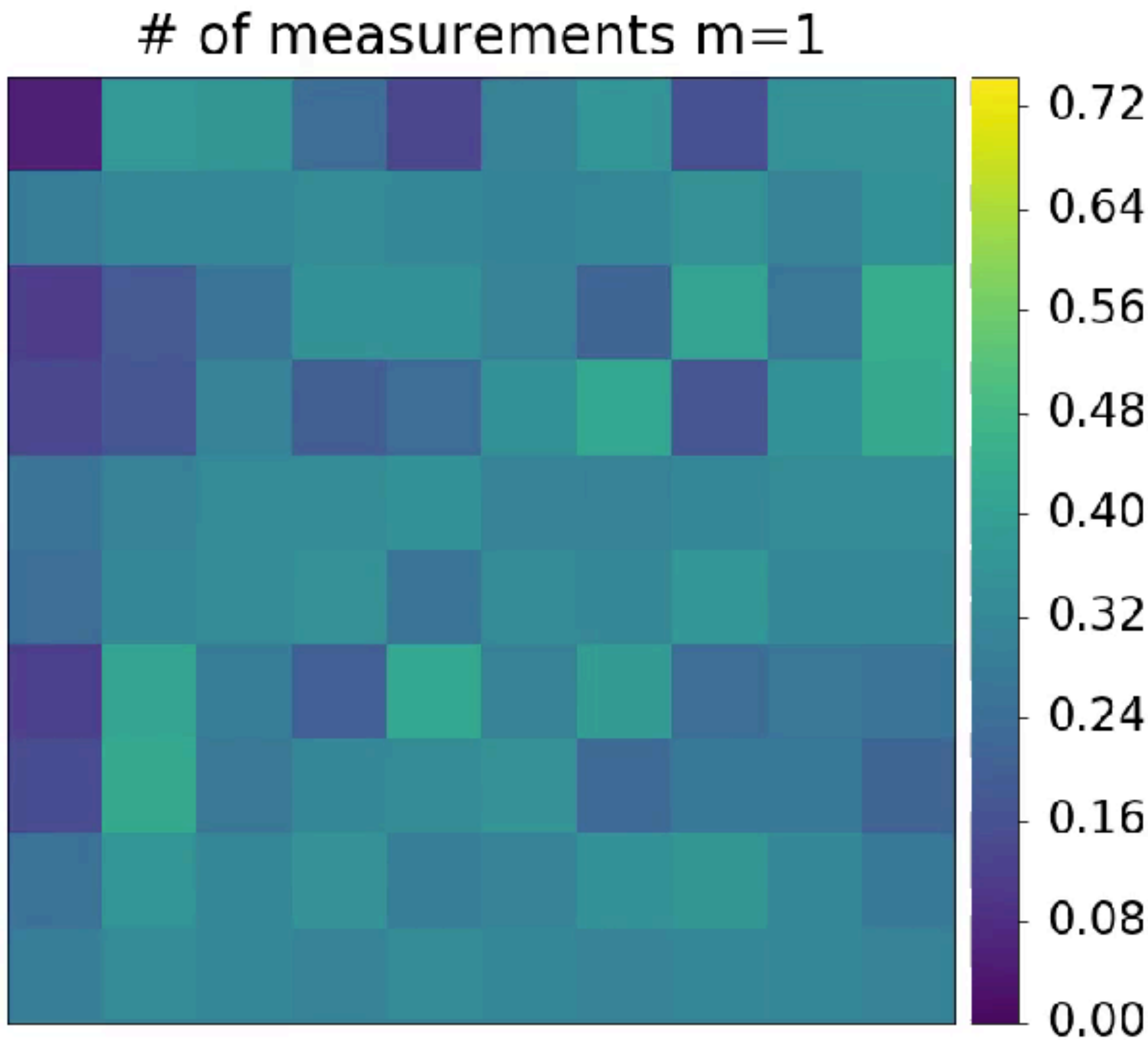
3 haseLift to recover rows of M up to global phases

Numerical Results



Ready-to-use library @ http://github.com/dseuss/pypllon

Numerical Results



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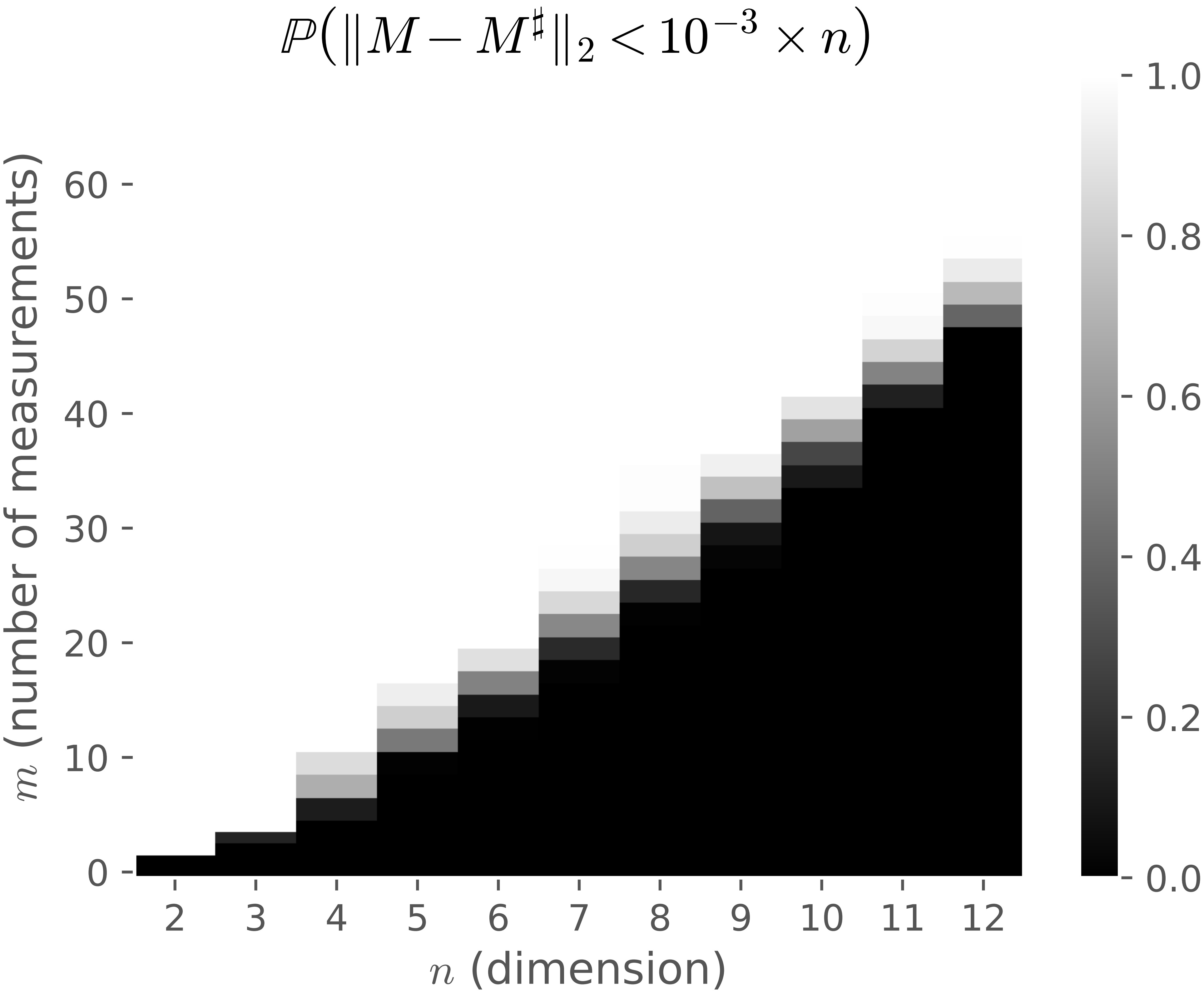
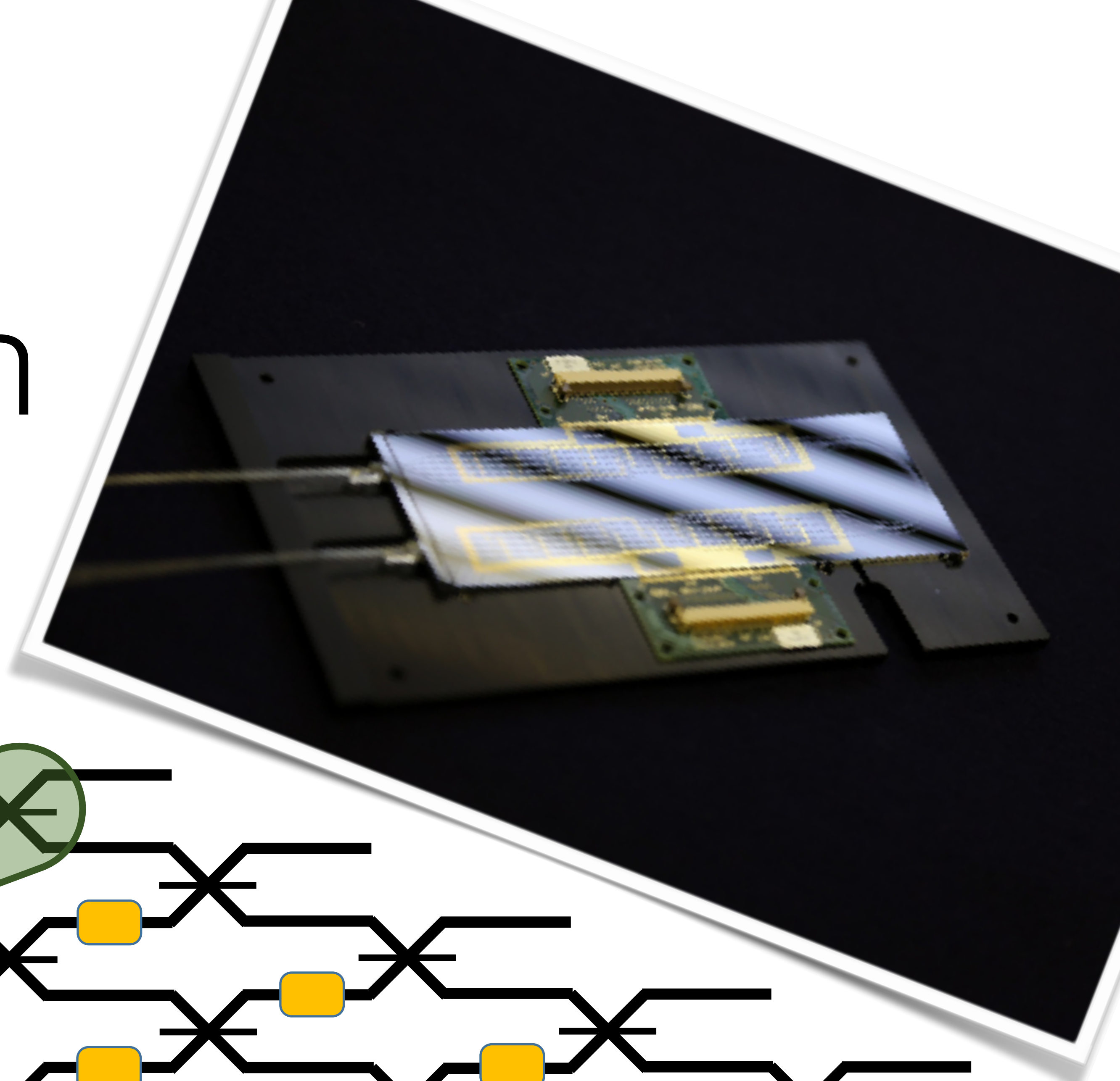
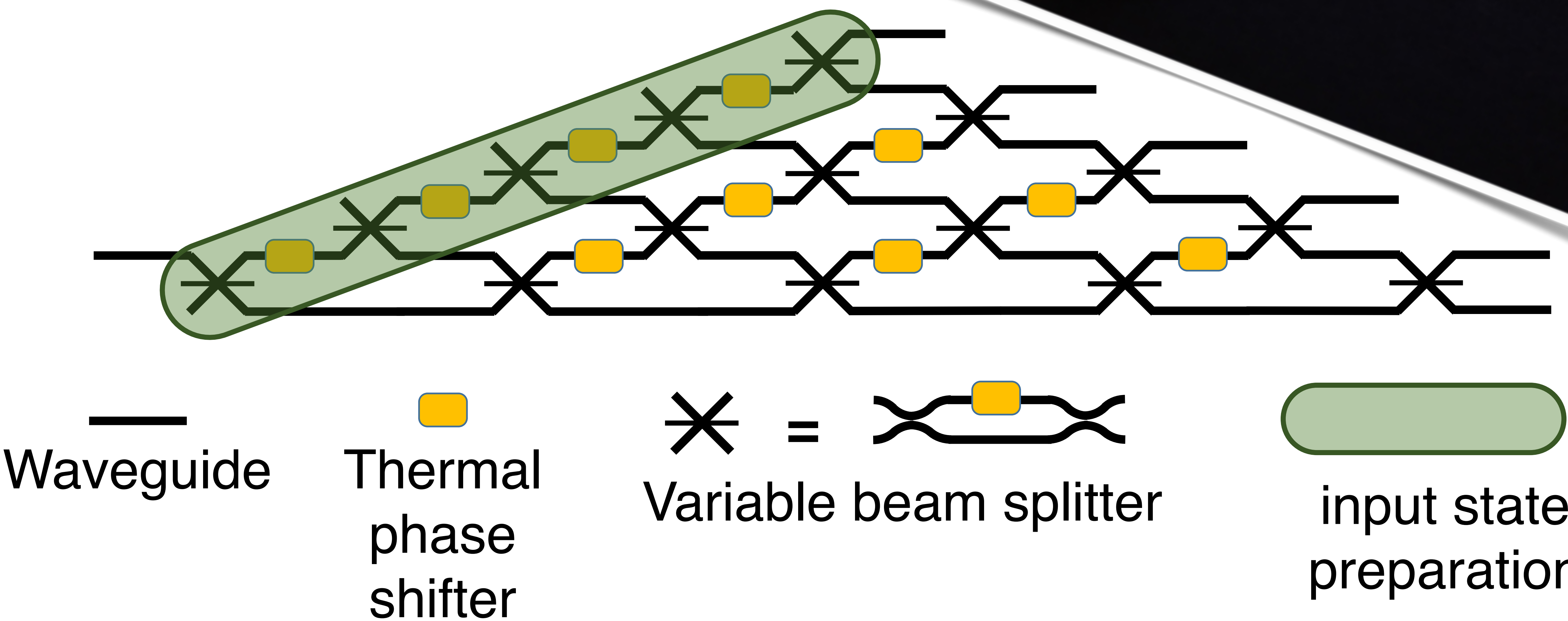


Experimental Implementation





Universal Linear Optics. **Science** 349 (6249): 711–16.
Image: *http://www.bristol.ac.uk/physics/news/2015/science-15.html*

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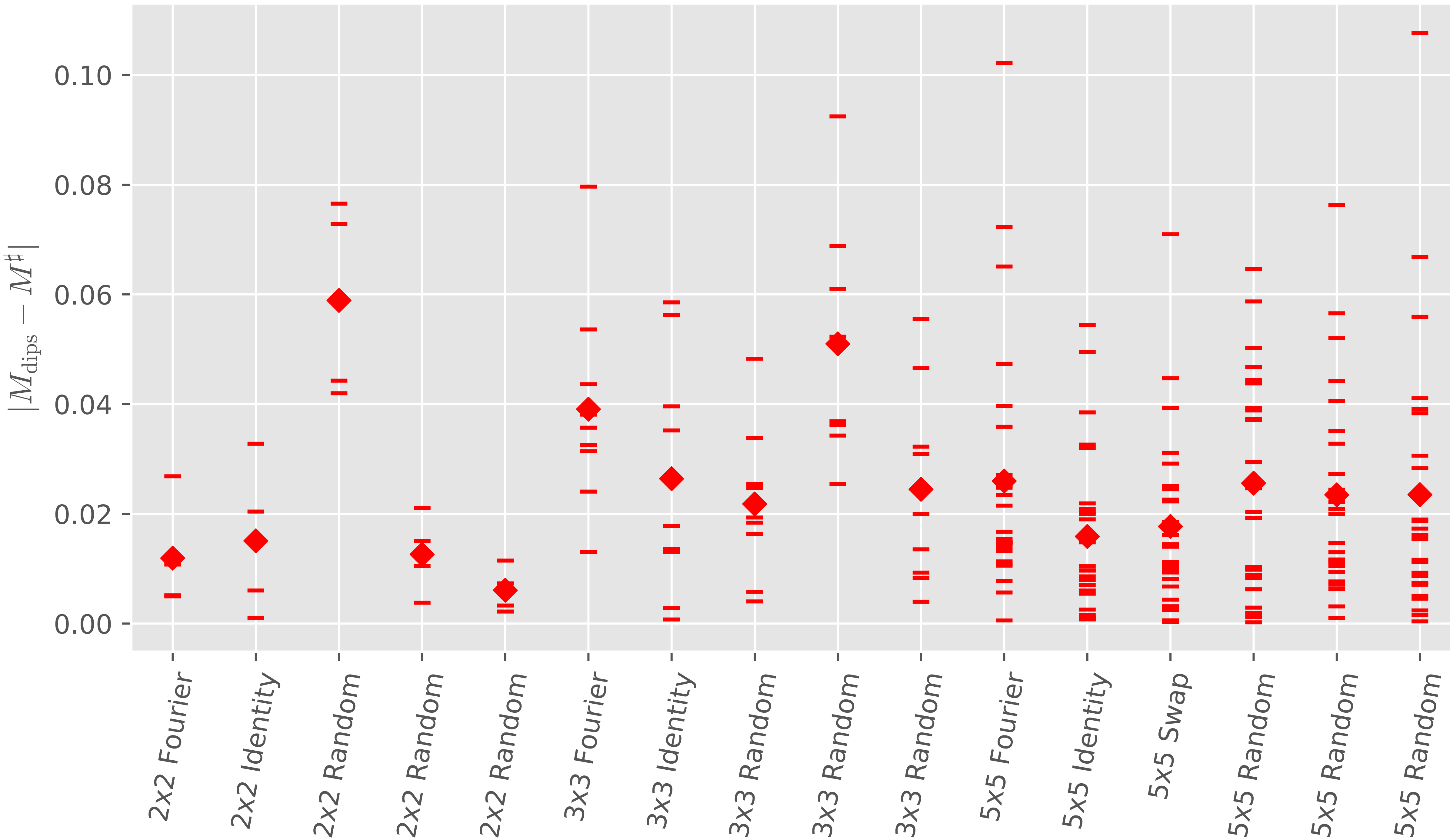


Randomly erased complex Radermacher sampling scheme

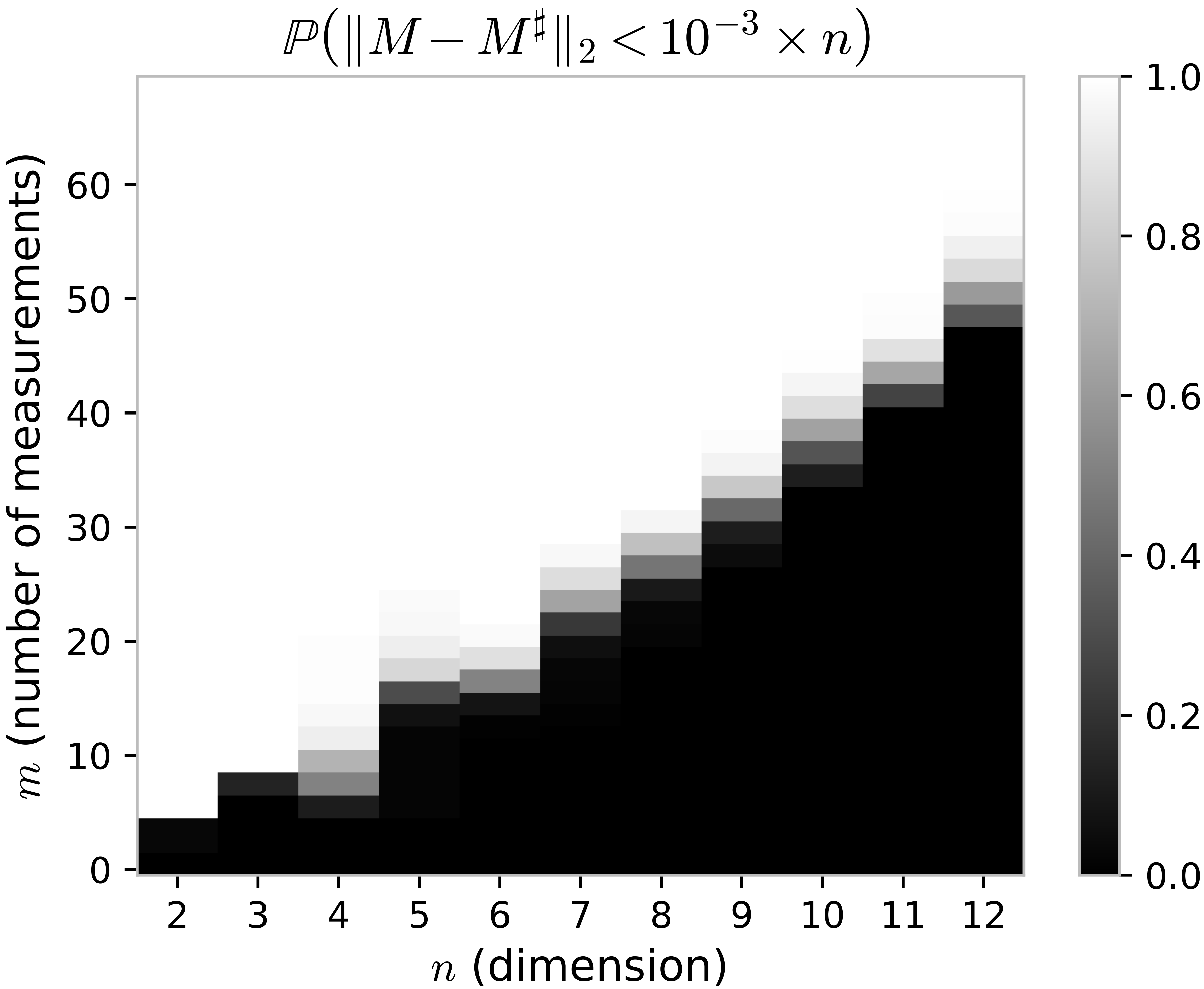
$$\alpha_j^{(k)} \sim \begin{cases} 0 & \text{with prob. } 1-p \\ +1 & \text{with prob. } p/4 \\ +i & \text{with prob. } p/4 \\ -i & \text{with prob. } p/4 \\ -1 & \text{with prob. } p/4 \end{cases}$$

such that $\|\alpha^{(k)}\|_2 = 1$

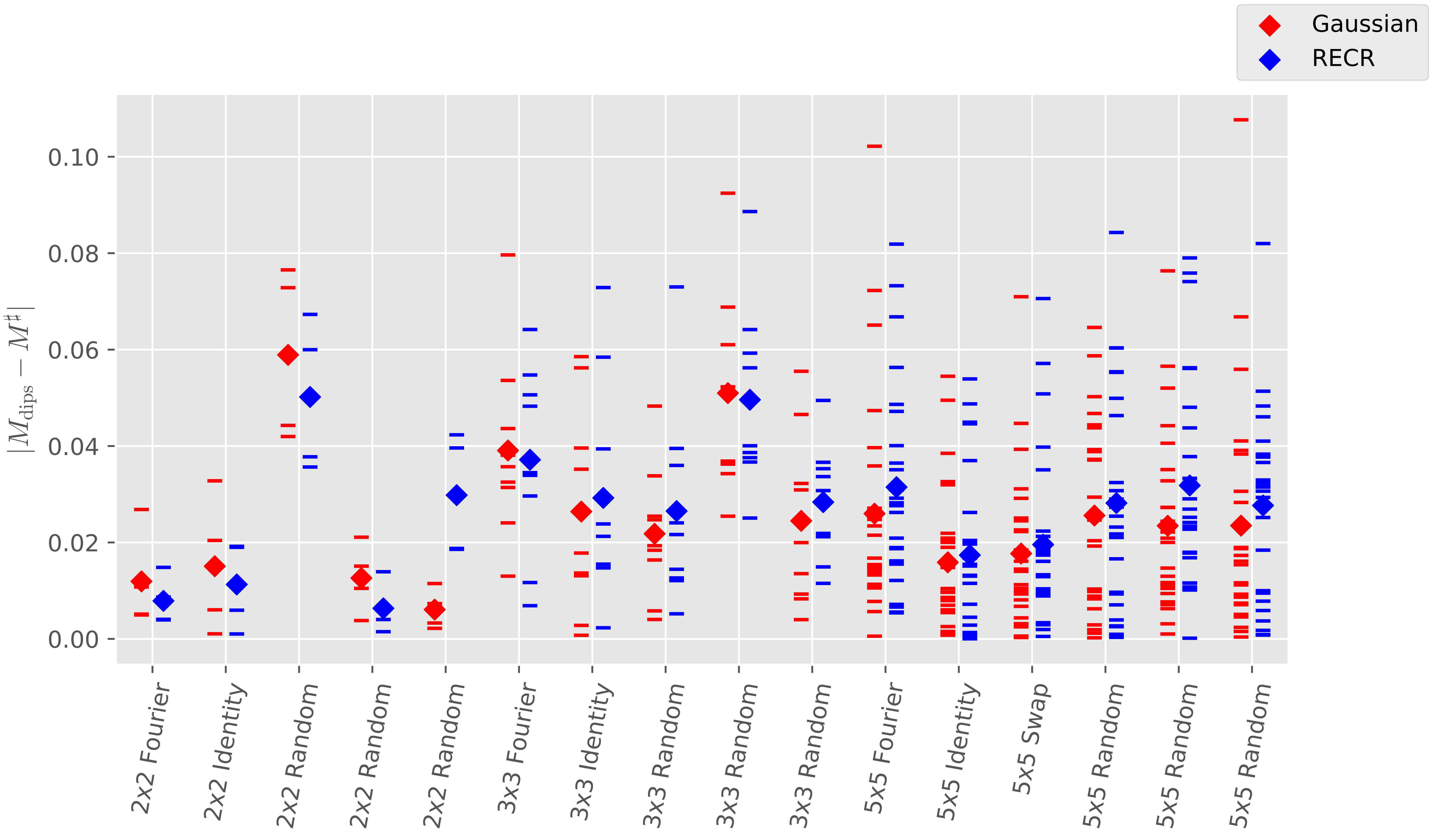
Randomly erased complex Radermacher sampling scheme

Perfect reconstruction – i.e. $M^\sharp = M$ – by means of this protocol holds with probability at least $1 - \mathcal{O}(\mathrm{e}^{-\gamma m})$, provided that the number of randomly chosen input states obeys $m \geq Cn$. Here, C, γ are absolute constants.

Numerical Result (RECR)

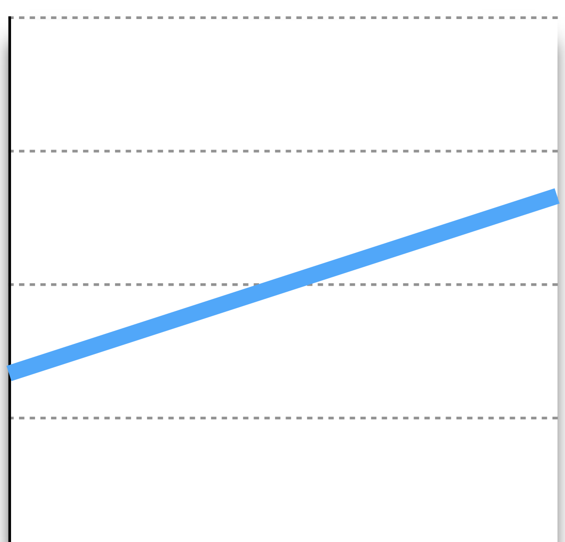


Experimental Results



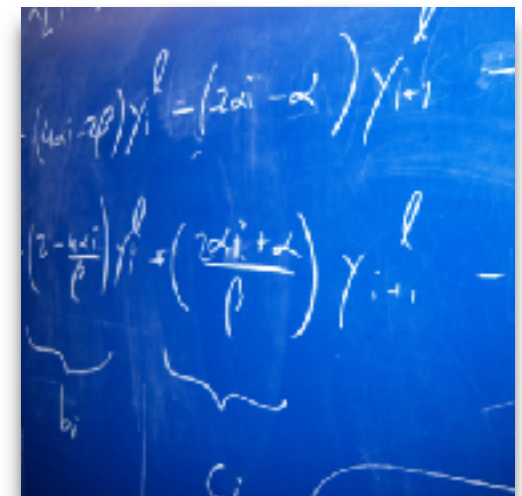
Conclusion

laser source & photo diodes



nr. of measurements scale linear in size

conceptually simple



rigorous recovery guarantees

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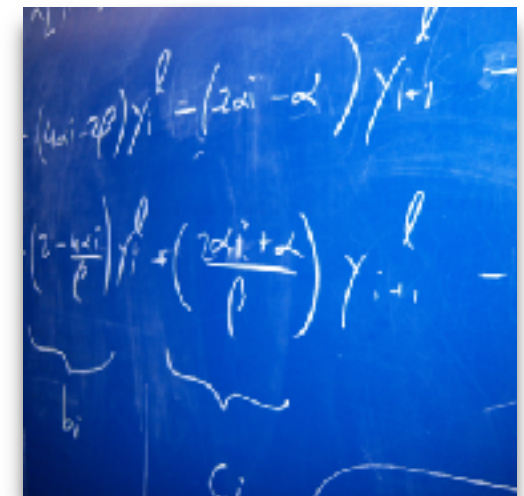

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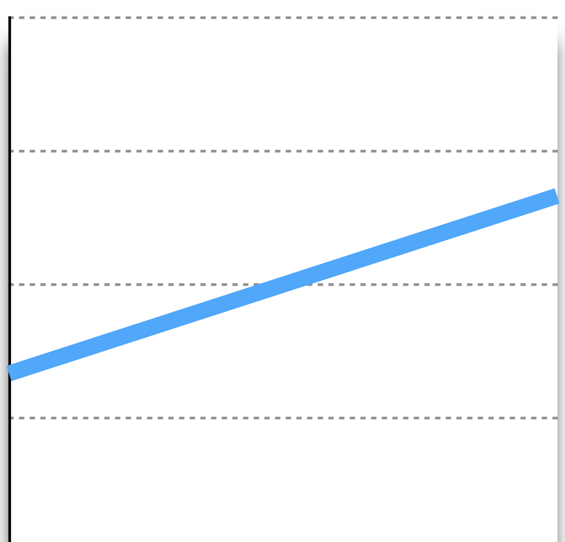

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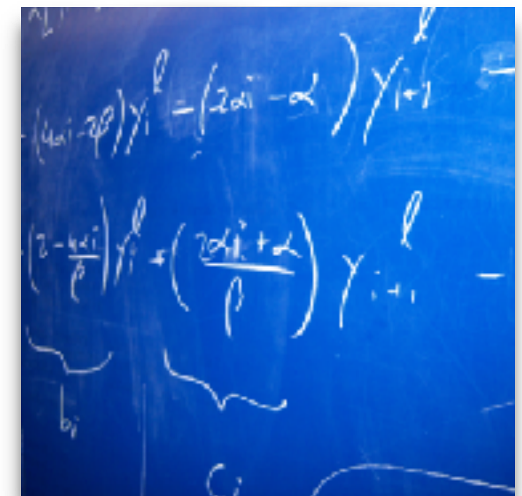

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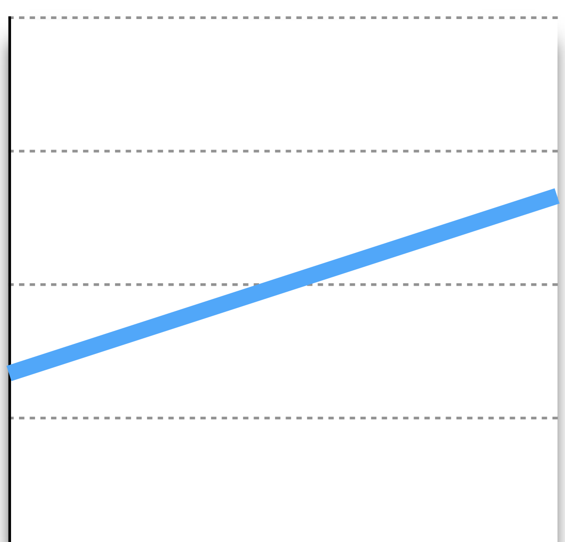

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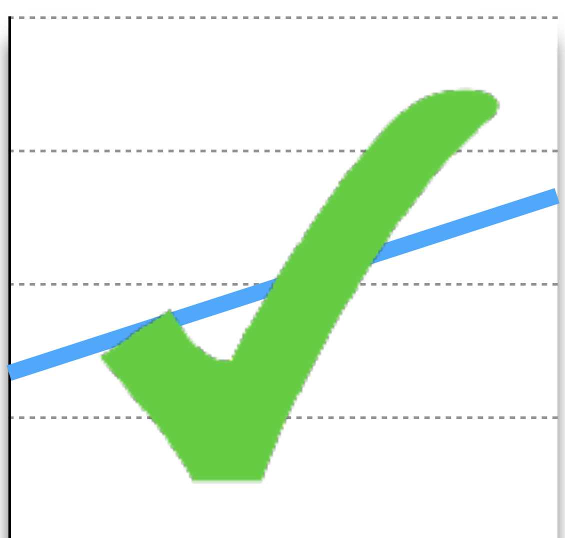

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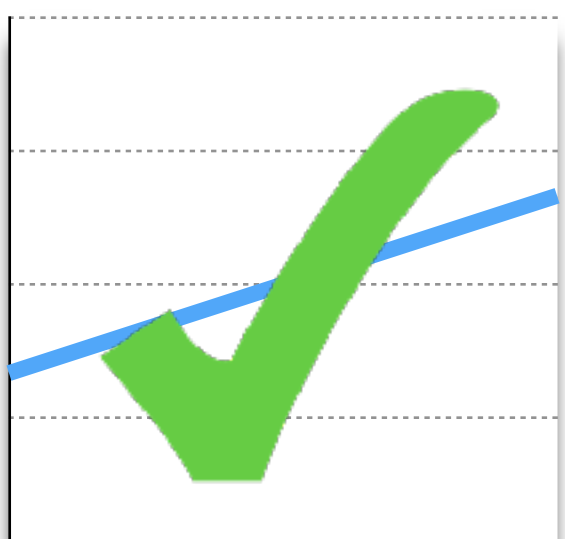

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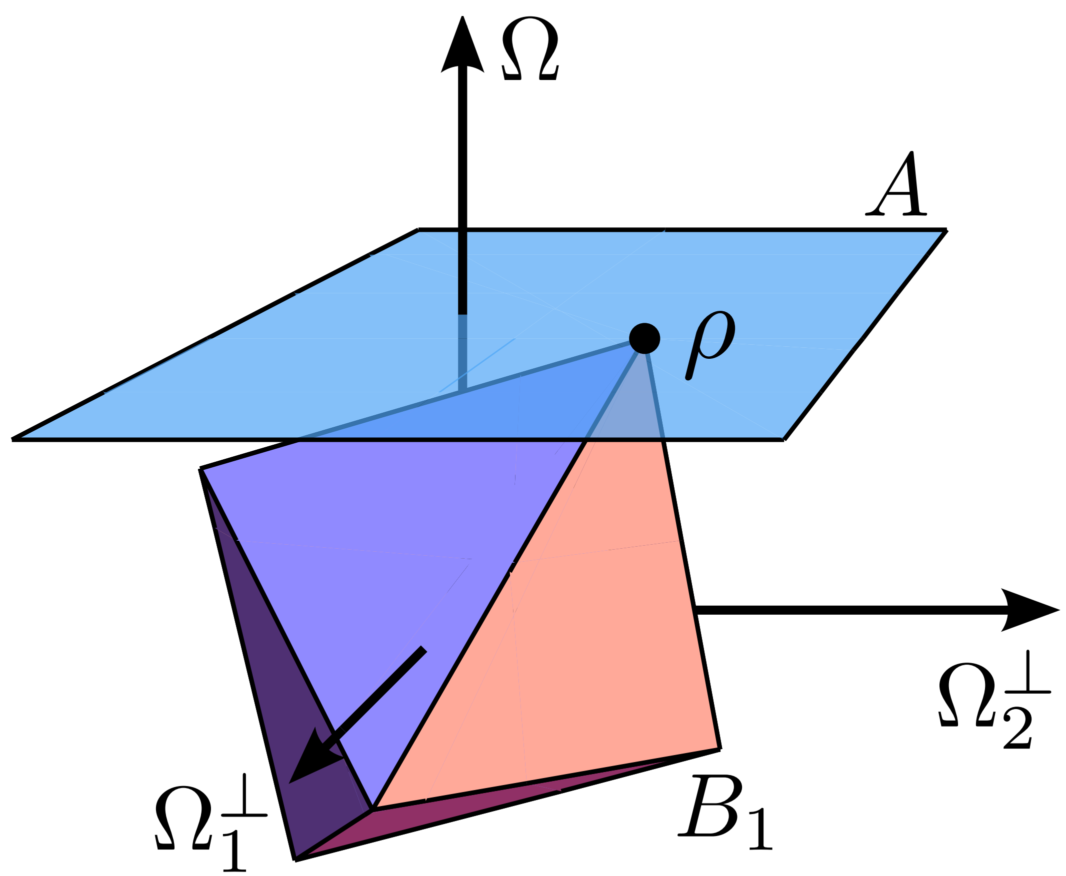
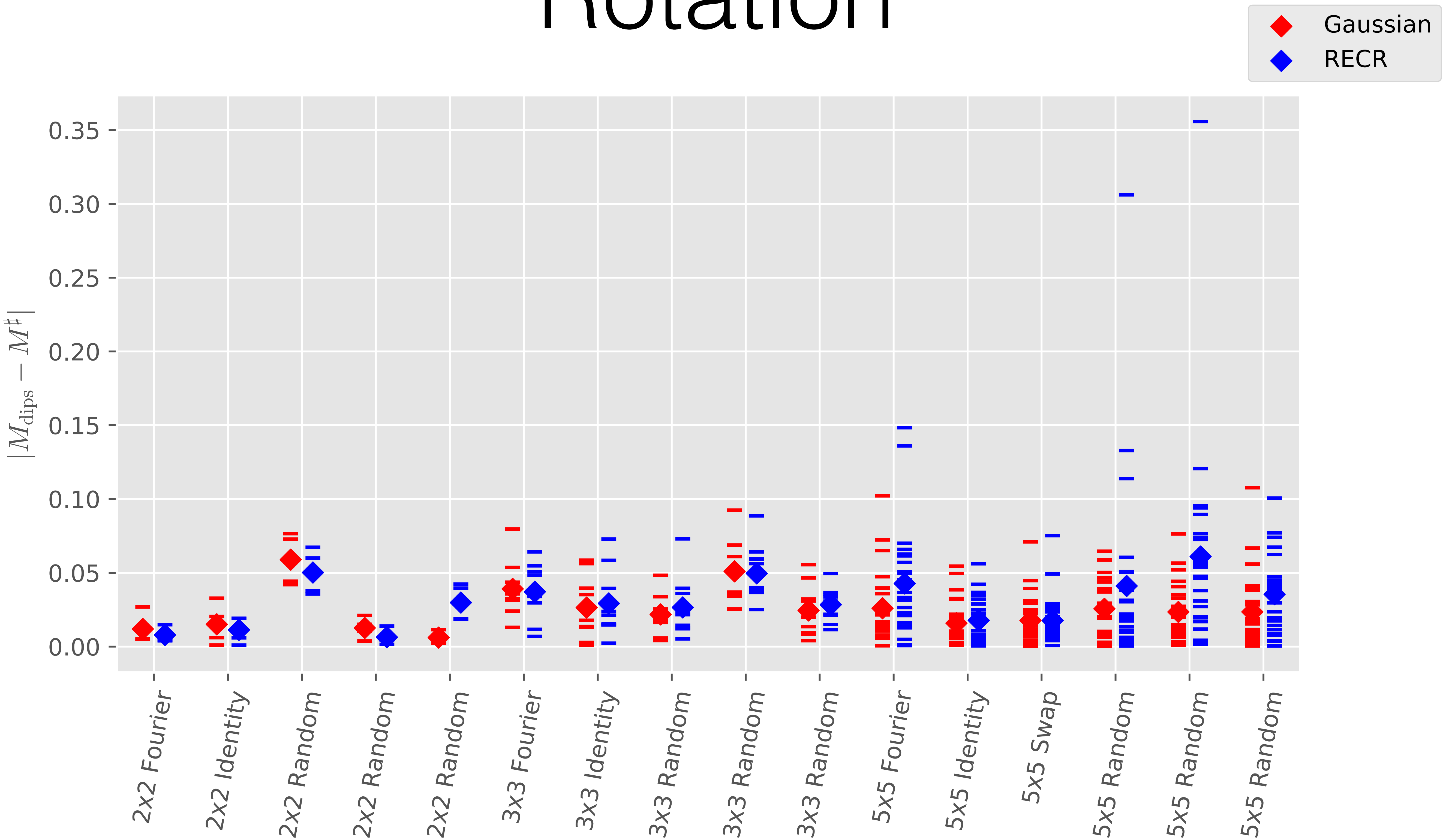
Outlook & Open Questions

- How does PL compare to other benchmarking techniques?
- Exploit benefits of RECR sampling scheme
- Characterisation beyond the single-mode approximation
- Adaption of more efficient algorithms
- Build a debugging device?

$$\|\boldsymbol{A}\|_p := \left(\sum_k \sigma_k(\boldsymbol{A})^p\right)^{\frac{1}{p}}$$



Experimental Result without Rotation



Noisy Measurements

$$y_k = |\langle x, a_k \rangle|^2 + \epsilon_k \quad \text{with} \ \ |\epsilon_k| < \eta$$

$$\begin{aligned} \underset{Z \in \mathbb{H}^n}{\text{minimize}} \quad & \sum_{k=1}^{m} |\mathrm{tr}\left(|a_k\rangle\langle a_k| \, Z \right) - y_k| \\ \text{subject to} \quad & Z \geq 0 \end{aligned}$$

$$\implies \min_{\phi} \|x^{\sharp} - \mathrm{e}^{\mathrm{i}\phi} x\|_2 \leq C \frac{\eta}{m}$$