Module - IV - Testing of Hypothesis

TUTORIAL - 1

1) Test the difference in the means is significant for the following data:

Sample - I	76	68	70	43	94	68	33	--
Sample - II	40	48	92	85	70	76	68	22

We use two sample t test to determine if the means are significantly different

1) calculate the sample means:

$$\bar{X}_1 = \frac{\Sigma X_1}{n_1} = \frac{76 + 68 + 70 + 43 + 94 + 68 + 33}{7} = 64.57$$

$$\bar{X}_2 = \frac{\Sigma X_2}{n_2} = \frac{40 + 48 + 92 + 85 + 70 + 76 + 68 + 22}{8} = 62.63$$

2) calculate the sample variances:

$$S_1^2 = \frac{\Sigma(X_1 - \bar{X}_1)^2}{n_1 - 1} \qquad S_2^2 = \frac{\Sigma(X_2 - \bar{X}_2)^2}{n_2 - 1}$$

We need to compute each deviation and sum of squares for the variances

$$S_1^2 = \frac{(76-64.57)^2 + (68-64.57)^2 + (70-64.57)^2 + (43-64.57)^2 + (94-64.57)^2 + (68-64.57)^2 + (33-64.57)^2}{7-1}$$

$$= 78.38$$

$$S_2^2 = \frac{(40-62.63)^2 + (48-62.63)^2 + (92-62.63)^2 + (85-62.63)^2 + (70-62.63)^2 + (76-62.63)^2 + (68-62.63)^2 + (22-62.63)^2}{8-1}$$

$$= 149.84$$

3) calculate the test statistic :

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{64.57 - 62.63}{\sqrt{\frac{78.38}{7} + \frac{149.84}{8}}} = \frac{1.94}{5.47}$$

$$= 0.354$$

4) Determine the degrees of freedom :

$$df = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{S_2^2}{n_2}\right)^2}{n_2 - 1}}$$

$$= \frac{29.927^2}{\frac{(11.197)^2}{6} + \frac{(18.73)^2}{7}} = \frac{894.54}{71.08} = 12.59$$

≈ 13

5) Determine the critical value from the t-distribution table at $\alpha = 0.05$ for a two tailed test $df = 13$, the critical value is ± 2.160

6) compare the test statistic

since $t = 0.354$ and $|0.354| < 2.160$, we fail to reject the null hypothesis.

Therefore there is no significant difference in means of two sample

2) Two random samples give following results

samples	size	sample means	sum of the squares of deviation from mean
1	10	15	90
2	12	14	108

Examine whether the sample come from the same normal population.

Given: sample 1 : $n_1 = 10$, $\bar{x}_1 = 15$, $\Sigma(x_1 - \bar{x}_1)^2 = 90$

sample 2 : $n_2 = 12$, $\bar{x}_2 = 14$, $\Sigma(x_2 - \bar{x}_2)^2 = 108$

1) calculate the sample variances :

$$S_1^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2}{n_1 - 1} = \frac{90}{10-1} = 10$$

$$S_2^2 = \frac{\Sigma(x_2 - \bar{x}_2)^2}{n_2 - 1} = \frac{108}{12-1} = 9.82$$

2) F statistic

$$F = \frac{S_1^2}{S_2^2} = \frac{10}{9.82} = 1.02$$

3) For $\alpha = 0.05$, $df_1 = 9$ and $df_2 = 11$

4) compare F-statistic :

If $F < F_{critical}$, we fail to reject the null hypothesis. Assuming the critical value from the F-table, for these degrees of freedom is about 3.18, since $1.02 < 3.18$, we fail to reject the null hypothesis therefore the samples come from the same normal population

3) Before an increase in an excise duty 90 persons out of a sample of 1100 persons were found to be tea drinkers. State whether there is a significant decrease in the consumption of tea after the increase of exise duty ?

Before : 90 / 1100 = 0.0818

After : 100 / 300 = 0.3333

sample proportions ;

$$\hat{P}_1 = \frac{90}{1100} = 0.0818 \qquad \hat{P}_2 = \frac{100}{300} = 0.3333$$

pooled proportion :

$$\hat{P} = \frac{90+100}{1100+300} = \frac{190}{1400} = 0.1357$$

calculate the standard error ;

$$SE = \sqrt{\hat{P}(1-\hat{P})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} = \sqrt{0.1357 \cdot 0.8643\left(\frac{1}{1100}+\frac{1}{1300}\right)}$$

$$= 0.0224$$

Z - statistic :

$$z = \frac{\hat{P}_1 - \hat{P}_2}{SE} = \frac{0.0818 - 0.3333}{0.0224} = -11.22$$

Determine the critical value

$\alpha = 0.05$, z-value is ± 1.96

compare z - statistic :

since $|z| > 1.96$, we reject the null hypothesis.

There is a significant decrease in the consumption of tea after the increase of excise duty

TUTORIAL - II

1) pumpkins were grown under two experimental conditions. Two random samples 11 and 9 pumpkins show sample standard of deviations of their weights as 0.8 and 0.5 Test the hypothesis that two variances are equal

sample variances

$S_1^2 = (0.8)^2 = 0.64 \qquad S_2^2 = (0.5)^2 = 0.25$

. F-statistic :

$$F = \frac{S_1^2}{S_2^2} = \frac{0.64}{0.25} = 2.56$$

critical value :

$\alpha = 0.05$, $df_1 = 10$ and $df_2 = 8$

Fstatistcs to critical point :

If $F < F_{critical}$, we fail to reject the null hypothesis . Assuming the critical value from the f-table , $2.56 < 3.44$ We fail to reject the null hypothesis . Therefore the variances are equal

2) Theory predicts that the proportion of beans in four groups A, B, C, D should be 9:3:3:1. In an experiment among 1600 beans, the numbers in the four groups were 882, 313, 287, 118. Does the experiment support the theory.

Observed : 882, 313, 287, 118

Expected ratio : 9:3:3:1

calculate the expected frequencies :

Total = 1600

$E_1 = 1600 \times \frac{9}{16} = 900$ $\quad E_2 = 1600 \times \frac{3}{16} = 300$

$E_3 = 1600 \times \frac{3}{16} = 300$ $\quad E_4 = 1600 \times \frac{1}{16} = 100$

chi-square statistic :

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

$$\chi^2 = \frac{(882-900)^2}{900} + \frac{(313-300)^2}{300} + \frac{(287-300)^2}{300} + \frac{(118-100)^2}{110}$$

$$\chi^2 = \frac{(18)^2}{900} + \frac{(13)^2}{300} + \frac{(13)^2}{300} + \frac{(18)^2}{100}$$

$$\chi^2 = \frac{324}{900} + \frac{169}{300} + \frac{169}{300} + \frac{324}{100}$$

$$\chi^2 = 4.7266$$

For $\alpha = 0.05$ and $df = 3$, the critical value is 7.815

Since $4.7266 < 7.815$, we fail to reject the null hypothesis. Therefore, the observed frequencies fit the expected ratio.

3) In an experiment on immunization of cattle from tuberculosis the following research were obtained. Calculate χ^2 and disscuss the effect of vaccine in controlling susceptibility to tuberculosis

	Affected	Not affected
Inocculated	12	26
Not Inoculated	16	6

Total affected : $12 + 16 = 28$

Total not affected : $26 + 6 = 32$

Total inoculated : $12 + 26 = 38$

Total non inoculated : $16 + 6 = 22$

Overall total : $28 + 32 = 60$

$$E_{11} = \frac{38 \times 28}{60} = 17.73$$

$$E_{12} = \frac{38 \times 32}{60} = 20.27$$

$$E_{21} = \frac{22 \times 28}{60} = 10.27$$

$$E_{22} = \frac{22 \times 32}{60} = 11.73$$

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

$$\chi^2 = \frac{(12 - 17.73)^2}{17.73} + \frac{(26 - 20.73)^2}{20.73} + \frac{(16 - 10.27)^2}{10.27}$$

$$+ \frac{(6 - 11.73)^2}{11.73}$$

$$\chi^2 = \frac{32.83}{17.73} + \frac{32.83}{20.27} + \frac{32.83}{10.27} + \frac{32.83}{11.73}$$

$$\chi^2 = 9.47$$

For $\alpha = 0.05$ and df = 1, the critical value is 3.841

Since 9.47 > 3.841, we reject the null hypothesis

Therefore, there is a significant effect of the vaccine in controuing susceptibility to tubercrosis