EE2227 - Control Systems

EE18BTECH11010

Gate 2019 - 30(EE section)

D. Shree Ujjwal

February,2020

Question

The transfer function of phase lead compensator is given by

$$D(s) = \frac{3(s + \frac{1}{3T})}{(s + \frac{1}{T})}$$

The frequency (in rad/sec), at which $\angle D(j\omega)$ is maximum, is

(a) $\sqrt{\frac{1}{T^2}}$ **(b)** $\sqrt{\frac{1}{3T^2}}$ **(c)** $\sqrt{3T}$ **(d)** $\sqrt{3T^2}$

Solution

The basic requirement of the phase lead network is that all poles and zeros of the transfer function of the network must lie on (-)ve real axis interlacing each other with a zero located as the nearest point to origin.

The given transfer fuction is

$$D(s) = \frac{3(s + \frac{1}{3T})}{(s + \frac{1}{T})}$$

Solution

Now substituting $s = j\omega$ in D(s), we get

$$D(j\omega) = \frac{3(j\omega + \frac{1}{3T})}{(j\omega + \frac{1}{T})}$$

The phase of this transfer function $\phi(\omega)$ is given by

$$\phi(\omega) = \tan^{-1}(3\omega T) - \tan^{-1}(\omega T)$$

$\phi(\omega)$ has its maximum at ω_c such that $\phi'(\omega_c) = 0$,

$$\phi'(\omega_c) = \frac{3T}{1 + (3\omega_c T)^2} - \frac{T}{1 + (\omega_c T)^2}$$

Solution

After simplification,

$$\omega_c^2 T^2 = \frac{1}{3}$$

$$\omega_c = \sqrt{\frac{1}{3T^2}}$$

Hence **(b)** is the correct option.