Control Systems

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3.1 Phase

3.1. The Transfer function of Phase Lead Compensator is given by

$$D(s) = \frac{3(s + \frac{1}{3T})}{(s + \frac{1}{T})} \tag{3.1.1}$$

Find out the frequency (in rad/sec), at which $\angle D(j\omega)$ is maximum?

Solution: The basic requirement of the phase lead network is that all poles and zeros of the transfer function of the network must lie on negative real axis interlacing each other with a zero located as the nearest point to origin. Substituting $s = j\omega$ in D(s), we get

$$D(j\omega) = \frac{3(j\omega + \frac{1}{3T})}{(j\omega + \frac{1}{T})} \tag{3.1.2}$$

The phase of this transfer function $\phi(\omega)$ is given by,

$$\phi(\omega) = \tan^{-1}(3\omega T) - \tan^{-1}(\omega T) \tag{3.1.3}$$

$\phi(\omega)$ has its maximum at ω_c Where $\phi'(\omega_c) = 0$,

$$\phi'(\omega_c) = 0 = \frac{3T}{1 + (3\omega_c T)^2} - \frac{T}{1 + (\omega_c T)^2} \tag{3.1.4}$$

After solving and Simplification , we have

$$\omega_c^2 T^2 = \frac{1}{3} \tag{3.1.5}$$

$$\omega_c = \sqrt{\frac{1}{3T^2}} \tag{3.1.6}$$

3.2. Verify your result through a plot.

Solution: The following plots the Phase value of the transfer function,

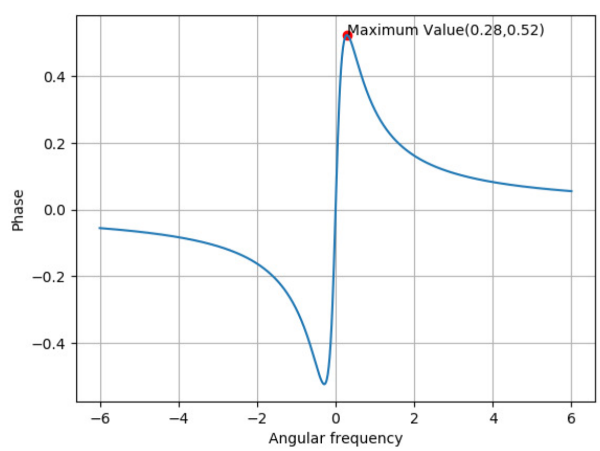

Fig. 3.2

Applications:

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a) Phase lead Compensators can be used as High pass filters,Differentiators.
b) They are used to reduce steady state errors.
c) Increases Phase Margin , relative stability.

3.3. What is purpose of of a Phase Lead Compensator?

Solution:

a) To increase K_v of a system.
b) The slope of the magnitude plot reduces at the gain crossover frequency so that relative stability improves and error decrease due to error is directly proportional to the slope.
c) To increase Phase margin.
d) To increase the response/speed become as it shifts the gain crossover frequency to a higher value.
e) To decrease the maximum overshoot of the system

3.4. Through an example, show how the compensator in Problem 3.1 can be used in a control system.

Solution:
Consider the following :-

Open loop system $G(s) = \frac{4}{(s)(s+2)}$. Our aim is to cascade this with a phase lead compensator $G_c(s)$ such that we can meet the following performance requirements-

Performance requirements for the system:

Steady State: $K_v =$

$$\lim_{s \to 0} sG(s) = 20$$

Transient Response: $PhaseMargin > 50°$
$GainMargin > 10\text{dB}$

Analysis of the system with $G_c(s)$ = K yields the following

For $K_v = 20$, $K = 10$
This leads to: $Phasemargin \approx 17°$
$Gainmargin \approx +\infty$ dB

Design of the lead Compensator:

Let the Phase lead Compensator be of the form

$$G_c(s) = (K_c)(a)\frac{(1 + Ts)}{(1 + aTS)} \tag{3.4.1}$$

a) From $K_v = 20$,$K_v = \frac{4aK_c}{2} \to aK_c = 10$

b) From the Analysis of the system with $G_c(s) = aK_c$ we obtain the additional phase-lead required is: 50°− 17°≈ 33°. We choose lead as 38°(≈ 15% more than 33°)

c) Let $\phi(\omega_m)$ be the maximum phase of $G_c(s)$. After some calculations we arive at the relation
$sin\phi(\omega_m) = \frac{(1-a)}{(1+a)}$
Substituting $\phi(\omega_m)$ = 38°in the above relation , we get the value of $a = 0.24$

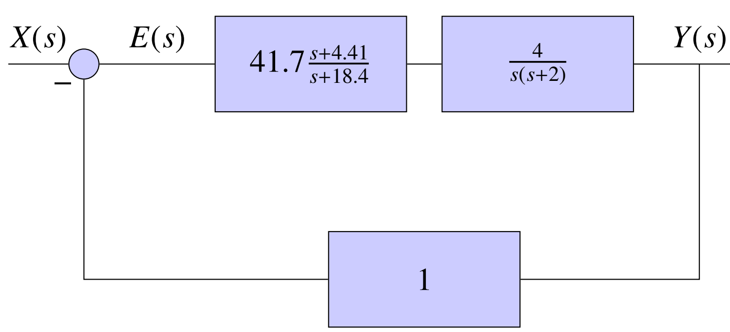
d) Substituting ω_m , the frequency with the maximum phase lead angle and calculating the Magnitude of the Feed Forward Transfer function , we get

$|Gj\omega_m| = aK_c \left|\frac{1+jT\omega_m}{1+jaT\omega_m}\right| = aK_c\frac{1}{\sqrt{a}}$
We choose ω_c , the new gain crossover frequency so that $\omega_c = \omega_m$ and
$|G(j\omega_c)G_c(j\omega_c)| = 1$
Substituting $a = 24$, $aK_c = 10$ in the above equation and solving for ω_c , we get
$\omega_c = 9rad/sec$

e) This implies For T, $\omega_c = \frac{1}{T\sqrt{a}} = 9$ rad/sec which implies $\frac{1}{T} = 4.41$,and

$K_c = \frac{20}{2a} = 41.7$
$G_c(s) = 41.7\frac{s+4.41}{s+18.4}$

The compensated system is given by:



The effect of the lead compensator is :

- Phase margin : from 17°to 50°which implies better transient response with less overshoot.

- ω_c : from 6.3rad/sec to 9 rad/sec which implies the system response is faster.
- Gain margin remains ∞
- $K_v = 20$ as required , which is our acceptable steady-state response.

4 Nyquist Plot