SC42100 Networked and Distributed Control Systems

Assignment 2 - Lectures 3 and 4

Instructions:

- The assignments are individual. You may consult and discuss with your colleagues, but you need to provide independent answers.
- You may use Matlab/Python to solve the assignment. Many questions require you clearly to do so (all practical problems in this set).
- Provide detailed answers, describing the steps you followed.
- Provide clear reports, typed, preferably using LaTex.
- Submit the reports digitally on Brightspace/Peer as indicated on the lectures.
- Include your code, for reproducibility of your results, in your Brightspace submission on a zip file, or through a link on your report.
- The code will *only* be used to verify reproducibility in case of doubts. The grading will be performed based on the results described in the report.

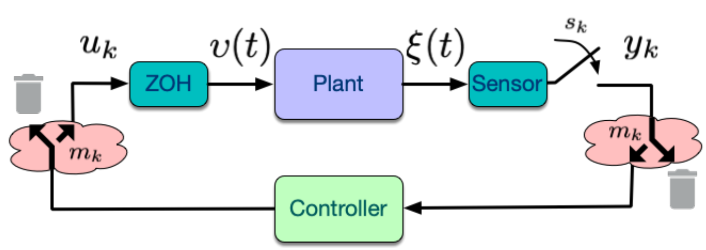

Figure 1: Schematic of a NCS including delays and packet-losses.

Theory

Question 1: (8p)

1. (2 p) Derive the exact switched system resulting from **applying the "to hold" mechanism** to an LTI networked system affected by packet losses (see slide 15 of Lecture 3). Consider to this end a maximum of 2 possible consecutive packet losses.

2. (1 p) **Under the "to zero" mechanism**, derive the expression from slide 22 of Lecture 3 capturing the dynamics of the received samples on a networked LTI control system with a bounded number of consecutive losses.

3. (2 p) Consider again for the remaining questions a controller $u_k = -\bar{K}x_k$ implemented with the **"to hold" mechanism**. Describe a formal approach to analyse the stability of the resulting closed loop. Consider the case of a maximum of 2 possible consecutive packet losses and provide the resulting LMIs one can employ to determine stability of the closed-loop NCS.

4. (3 p) Consider now that the packet-losses are modelled as follows:

   - The channel has a probability of loosing one packet after a successful transmission of p_1, i.e. $\mathbb{P}[m_k = 1\,|, m_{k-1} = 0] = p_1^a$.
   - The channel has a probability of loosing two consecutive packets of p_2, i.e. $\mathbb{P}[m_k = 1\,|, m_{k-1} = 1,\, m_{k-2} = 0] = p_2^a$.
   - The channel never losses more than two consecutive packets.

   Determine the LMI conditions to establish Exponential Mean Square Stability of this system.

Question 2: (2p)

1. (2 p) Consider now a discrete time LTI system $x_{k+1} = Ax_k + Bu_k$ and a given stabilizing state feedback controller $u_k = -Kx_k$. Derive an event-triggered condition guaranteeing global exponential stability of this discrete-time system.

Practice

Consider a sample-data networked control system (NCS), as depicted in Figure 1. Assume that the plant dynamics are linear time-invariant given by:

$$\dot{\xi}(t) = A\xi(t) + Bv(t),$$

where the control signal v is a piece-wise constant signal resulting from the application of a controller in sampled-and-hold fashion, i.e. $v(t) = u_k,\ t \in [s_k,\, s_{k+1})$.

The system matrices are given by:

$$A = \begin{bmatrix} a & b+0.5 \\ 0 & -c \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

where a is given by the first digit, b by the 3rd digit, and c by the last digit of your student ID number.

We start considering the system with a constant sampling interval $h = s_{k+1} - s_k$ for all k, and assuming no delay is present $\tau = \tau^{sc} + \tau^c + \tau^{ca} = 0$.

Question 3: (13p)

1. (2p) Select from assignment 1 one of your static state feedback controllers and sampling time h. Assume a "to zero" mechanism is employed to deal with packet dropouts. For the selected controller compute the maximum number of consecutive packet dropouts $\bar{\delta}$ under which the NCS remains globally asymptotically stable.

2. (2p) Consider now a Bernoulli process determining the packet losses behaviour. Compute numerically an upper bound p^* for the value of $p := \mathbb{P}[m_k = 1]$ such that for every $p < p^*$ mean square stability (MSS) is guaranteed.

3. (1p) Verify with numerical simulations the previous result.

4. (3p) Still under a Bernoulli process, compute an upper bound p^*_{AS} so that almost sure stability (ASS) is guaranteed for any value of $p < p^*_{AS}$.

5. (3p) Consider now a Gilbert-Elliot process, and provide a plot showing the combinations of $p_{00} = \mathbb{P}[m_k = 0 | m_{k-1} = 0$ and $p_{11} = \mathbb{P}[m_k = 1 | m_{k-1} = 1]$ for which MSS is guaranteed.

6. (2p) Compare the results obtained for MSS and ASS under a Bernoulli process, and for MSS under Bernoulli and Gilbert-Elliot models. Discuss the observed differences and similarities, and any possible connections between the different results.

Next we consider the case when delays are present in the networked system. Assume that the system is affected by a **constant** small delay $\tau \in [0, h)$, and it is controlled with the same static controller you designed.

Question 4: (7p)

1. (2p) Employ the so-called Jordan form approach (as in Lecture 4) to construct a polytopic discrete-time model over-approximating the uncertain exact discrete-time closed-loop NCS dynamics.

2. (2p) Perform stability analyses, solving the relevant LMIs resulting from the previous question when employing only 4 vertices in the polytopic over-approximation. Employing this analysis produce a plot illustrating combinations of (h, τ) as in Question 2 of Assignment 1.

3. (2p) Refine the plot of the previous question, by employing more vertices in your polytopic approximation (thus requiring the solution of more LMIs).

4. (1p) Compare the resulting plot with the one from Assignment 1 and discuss any possible differences and the possible causes for such differences.

Finally we go back to the case with no-delays to design an event-triggered controller.

Question 5: (10p)

1. (1p) Consider your system being controlled by the first static controller you designed for your continuous-time system. Design a quadratic event-triggered condition for the system guaranteeing global exponential stability of the closed-loop.
2. (4p) Simulate the resulting closed-loop for various values of the σ parameter controlling the guaranteed performance of the closed-loop. Simulating for multiple initial conditions for each value of σ, compare the average amount of communications/sampling produced by each of the system on a predefined fixed time-length of the simulations.
3. (2p) For $\sigma = 0.5$ compute (via simulations) the average inter-sample time h^{avg} in a time-window *guaranteeing* a 90% decrease of the Lyapunov function, i.e. $V(T) \leq 0.1V(0)$. Compute such average inter-sample time by employing at least 20 different initial conditions.
4. (3p) Compare the event-triggered controller with $\sigma = 0.5$ with a periodic time-triggered controller with constant sampling period h^{avg}. Discuss the observed differences in performance.