Linear and Quadratic Programming Assignment

SC42056 Optimization for Systems and Control

E_1, E_2, and E_3 are parameters ranging from 0 to 18 for each group according to the sum of the last three numbers of the student IDs:

$$E_1 = D_{a,1} + D_{b,1}, \quad E_2 = D_{a,2} + D_{b,2}, \quad E_3 = D_{a,3} + D_{b,3}$$

with $D_{a,3}$ the right-most digit of the student ID of the first student, $D_{b,3}$ the right-most digit of the student ID of the other student, $D_{a,2}$ the one but last digit of the student ID of the first student, etc.

Important: Please note that all questions regarding this assignment should be asked via the Brightspace Discussion forum.

Increasing indoor occupancy has definitely been a lifestyle trend for the current generation, which most recently has been boosted by the increase in home office activities in lock-down times. Among the basic actions taken to make the interior ambient more pleasant (e.g., having air renovation and natural lightning), thermal comfort is also an indoor condition that must be satisfied. Using HVAC (Heating, Ventilation and Air-Conditioning) systems is an artificial, effective way to meet thermal environment requirements. However, such devices are generally powered by traditional energy sources (e.g., electricity or gas) which, despite being technically reliable, are directly related to high economical and environmental expenditures, making their use over the time somehow limited. In addition, design parameters and operational strategies are a critical decision aspect to guarantee the efficiency of the air-conditioning system. Therefore, the development and implementation of effective control techniques for HVAC systems is of primary importance.

The system schematics is shown in Figure 1, where $T_\mathrm{b}(t)$ is the air temperature indoor the building as a function of time. Moreover, $\dot{q}_\mathrm{solar}(t)$ and $\dot{q}_\mathrm{occ}(t)$ are heat inputs from solar energy and people occupancy (i.e., internal heat generated by people, lights, and equipment), respectively. On the other hand, $\dot{q}_\mathrm{ven}(t)$ refers to the heat output by air renovation and infiltration. There is also an external disturbance because the outside ambient temperature, $T_\mathrm{amb}(t)$, is in general different from the indoor temperature. Finally, $\dot{q}_\mathrm{ac}(t)$ is the power input deployed for air-conditioning.

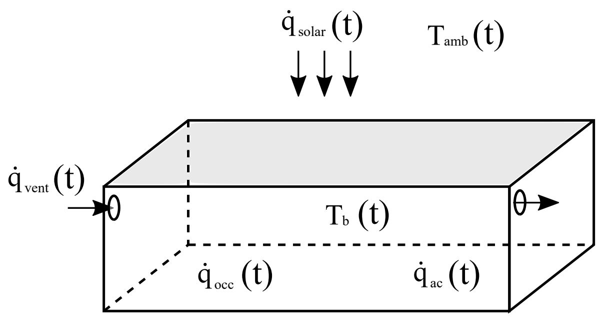

Figure 1: Building scheme.

The dynamic model for the temperature inside the building, $T_\mathrm{b}(t)$, is represented by the following differential equation:

$$\frac{\mathrm{d}T_\mathrm{b}(t)}{\mathrm{d}t} = a_1\dot{q}_\mathrm{solar}(t) + a_2[\dot{q}_\mathrm{occ}(t) + \dot{q}_\mathrm{ac}(t) - \dot{q}_\mathrm{vent}(t)] + a_3[T_\mathrm{amb}(t) - T_\mathrm{b}(t)] \tag{1}$$

where a_1, a_2, and a_3 are model parameters.

Tasks:

1. **Selecting the air-conditioner.** You have to select the HVAC system, as we have two kinds of available products, which are central air conditioner X whose maximum power is 4 kW, and a smaller split-type air conditioner Y, whose maximum power is 2.5 kW. The maximum power denotes the ability of the air conditioner to control the temperature. The prices to install one unit of type X and Y are € 3000 and € 1500, respectively, and the total budget for installing air conditioners is € 24000+300E_1. The total number of the air conditioners is limited to 12 units because of the space restriction.

   (a) If we want to achieve the maximum power of the available units of air conditioners without violating the above constraints, and assuming that we only consider the cost for the first 5 years, how should we design the installation plan of X and Y units? Is it a linear programming problem? If so, formulate the optimization problem and transform it into the *standard* form for linear programming problems considering the number of X and Y as continuous variables.

   (b) Find the optimal solution of the LP problem using MATLAB (use the continuous formulation and `linprog`; do not use `intlinprog`). What is the maximum air-conditioning power of whole building, and the optimal number of unit of X and Y to be installed (take into account that the number of unit of X and Y should be integers, but still do not use `intlinprog`). Which of the constraints is limiting the installation?

   (c) We now also consider the cost for the maintenance of the air conditioners, which increases as the years move on. The price for the maintenance of each unit of X and Y per year is given in the following table:

Years	1	2	3	4	5
X (€)	200+E_2	200+2E_2	200+3E_2	300+4E_2	300+5E_2
Y (€)	50+E_3	50+2E_3	100+3E_3	150+4E_3	150+5E_3
Years	6	7	8	9	10
X (€)	400+5E_2	500+5E_2	600+5E_2	700+5E_2	800+5E_2
Y (€)	200+5E_3	250+5E_3	300+5E_3	350+5E_3	400+5E_3

   Consider the usage of the facilities within the first 10 years. An extra budget is provided for the maintenance of the air conditioners, which is € 4000+100E_1per year. Now we need to design the installation plan of the air conditioners, which means we should determine the number of durable years of the facilities. If we want to have the maximum power for use with all the available budget, how many years we should consider as the durable time, without violating any of the constraints mentioned above? Formulate the new optimization problem (use the continuous formulation; do not use `intlinprog`). Is it possible to formulate it as a single LP problem? What are the optimal numbers of unit of X and Y, and the maximum available power?

2. **Obtaining a discrete-time model.** For optimization purposes, data available is provided at discrete time instants $t_k = k\Delta t$ where Δt is the sample interval. Therefore, the first step is obtain a discrete-time model for the indoor temperature. To do so, use the following approximation:

$$\frac{\mathrm{d}T_{\mathrm{b},k}}{\mathrm{d}t} \approx \frac{T_{\mathrm{b},k+1} - T_{\mathrm{b},k}}{\Delta t} \tag{2}$$

where $T_{\mathrm{b},k}$ represents the indoor temperature in the building at time step k. The resulting model has to be of the form:

$$T_{\mathrm{b},k+1} = AT_{\mathrm{b},k} + B[\dot{q}_{\mathrm{solar},k}, \dot{q}_{\mathrm{occ},k}, \dot{q}_{\mathrm{ac},k}, \dot{q}_{\mathrm{vent},k}, T_{\mathrm{amb},k}]^\top \tag{3}$$

Provide values of A and B as a function of a_1, a_2, and a_3.

3. **Identifying the model parameters.** Once you have obtained the discrete-time model, you can rely on historic data regarding the operation for identifying the design parameters. Such historic data is available in the file *measurements.csv* from Brightspace, where there is a record of 2160 real measurements for $\dot{q}_{\mathrm{solar},k}$ (kW), $\dot{q}_{\mathrm{occ},k}$ (kW), $\dot{q}_{\mathrm{ac},k}$ (kW), $\dot{q}_{\mathrm{vent},k}$ (kW), $T_{\mathrm{amb},k}$ (°C), and $T_{\mathrm{b},k}$ (°C). Note that all these data are provided at hourly intervals (i.e., $\Delta t = 3600s$). To read the file, use the Matlab function `readtable`. Then, formulate and solve the following optimization problem:

$$\underset{a_1,a_2,a_3}{\text{minimize}} \quad \sum_{k=1}^{2159} (T_{\mathrm{b},k+1} - (AT_{\mathrm{b},k} + B[\dot{q}_{\mathrm{solar},k}, \dot{q}_{\mathrm{occ},k}, \dot{q}_{\mathrm{ac},k}, \dot{q}_{\mathrm{vent},k}, T_{\mathrm{amb},k}]^\top))^2 \tag{4}$$

What are the estimated values for a_1, a_2, and a_3?

4. **Optimizing the cost for air-conditioning.** You must now minimize the cost for air-conditioning over some horizon of N steps, starting at $k = 1$ for same standardized input and output signals, while making sure that the dynamic model is followed throughout the time horizon. Moreover, you should constrain the temperature indoor the building to satisfy thermal comfort requirements. In addition, there is an additional penalty cost if the indoor temperature is above or below a certain reference temperature T_{ref}, where this reference temperature represents the ideal condition to satisfy the thermal comfort. This additional cost is defined as $0.1 + E_2/10$ (€/°C^2) times the square error between the building and reference temperatures. This optimization problem is defined as:

$$\begin{aligned}
\underset{\substack{T_{\mathrm{b},2},\ldots,T_{\mathrm{b},N+1} \\ \dot{q}_{\mathrm{ac},1},\ldots,\dot{q}_{\mathrm{ac},N}}}{\text{minimize}} \quad & \sum_{k=1}^{N} \Phi_k \dot{q}_{\mathrm{ac},k} \Delta t + (0.1 + E_2/10)(T_{\mathrm{b},k} - T_{\mathrm{ref}})^2 \\
\text{subject to} \quad & T_{\mathrm{b},k+1} = AT_{\mathrm{b},k} + B[\dot{q}_{\mathrm{solar},k}, \dot{q}_{\mathrm{occ},k}, \dot{q}_{\mathrm{vent},k}, \dot{q}_{\mathrm{ac},k}, T_{\mathrm{amb},k}]^\top, k = 1, \ldots, N-1 \\
& 0 \le \dot{q}_{\mathrm{ac},k} \le \dot{q}_{\mathrm{ac,max}}, k = 1, \ldots, N \\
& T_{\min} \le T_{\mathrm{b},k} \le T_{\max}, \quad \text{for all time steps } k \text{ such that } \dot{q}_{\mathrm{occ},k} > 0
\end{aligned} \tag{5}$$

where Φ_k is the price of buying energy in €/kWh, $\dot{q}_{\mathrm{ac,max}}$ is the maximum air-conditioner power input, and $T_{\min}$ and $T_{\max}$ are, respectively, the lower and upper temperature bounds to ensure thermal comfort. You can find operational parameters such as $\dot{q}_{\mathrm{solar},k}$, $\dot{q}_{\mathrm{occ},k}$, $\dot{q}_{\mathrm{vent},k}$ and $T_{\mathrm{amb},k}$ in the file *measurements.csv* from Brightspace. In addition, you can find in the same file values for the energy costs Φ_k. Then, considering a horizon of 90 days (i.e., $N = 2160$), $\dot{q}_{\mathrm{ac,max}}$ of 100 kW, $T_{\mathrm{b},1} = 22.43$ °C, $T_{\min} = 15$ °C, $T_{\max} = 28$ °C, $T_{\mathrm{ref}} = 22$ °C and the model parameters estimated in Task 2, solve the above problem. Solve it in Matlab using *quadprog*. What is the optimal cost for air-conditioning along the horizon of N steps?
If (and only if) you find that for your values of E_1, E_2, and E_3, the optimization problem (5) is infeasible, then solve the problem again while minimizing the constraint violation for the temperature (i.e., for the last constraint of (5)). Hint: If needed, you may get some inspiration from the goal attainment method for multi-objective optimization.

The solutions of the assignment should be uploaded to Brightspace before Thursday, October 8, 2020 at 17:00 as two separate files:

1. A written report on the practical exercise as a single .pdf file (no other formats allowed).
2. A single .m file with the Matlab code you used; please make sure that the code is error free.

After uploading, please verify the uploaded files so as make sure that you have uploaded the correct files and that they are not broken.

Please also note that you will lose 0.5 point from your grade for this assignment for each (started) day of delay in case you exceed the deadline.