CS5330: Hints for Week 11 Assignment

Assignment Due: Monday, 20th Apr 2020.

These are some hints for the Week 11 assignment.

1. Some people seem to be confused about what $\phi|_{x_1=0}$ means. For example, if $\phi = x_1 \vee x_2 \vee x_3$, then $\phi|_{x_1=0} = x_2 \vee x_3$ while $\phi|_{x_1=1} = 1$.

   (a) In this problem, you can sample from the satisfying assignments of any boolean formula perfectly uniformly. Let p be the fraction of satisfying assignments of ϕ that set $x_1 = 0$. First, suppose $p \geq 1/2$; in other words, if $N_0 = \#\phi|_{x_1=0}$ and $N = \#\phi$, then $N \geq \frac{1}{2}N_0$. Using Chernoff bounds, you can show that the number p can be estimated to within multiplicative error $(1 \pm \varepsilon)$ by generating $O(1/\varepsilon^2)$ random satisfying assignments and seeing how many of these have x_1 set to 0. If $p < 1/2$, you can estimate the fraction p' of satisfying assignments that set $x_1 = 1$ and get a $(1 \pm \varepsilon)$-approximation of p' instead of p. There is a small technical puzzle that I want you to solve. We do not know whether $p \geq 1/2$ or $p < 1/2$ a priori. So, how do we know whether to estimate p or p'? Hint is that you can check whether there are more satisfying assignments with $x_1 = 0$ or $x_1 = 1$ in your samples. If there are more with $x_1 = 0$ than $x_1 = 1$, find a multiplicative approximation of p; otherwise, find a multiplicative approximation of p'. Why does this work?

   (b) In this problem, the assumption is that you can exactly count the number of satisfying assignments for any boolean formula. Compute $p = \frac{\#\phi|_{x_1=0}}{\#\phi}$. Sample $x_1 = 0$ with probability p and $x_1 = 1$ with probability $1 - p$. If x_1 is sampled to be 0, recursively sample a solution to $\phi|_{x_1=0}$. If x_1 is sampled to be 1, recursively sample a solution to $\phi|_{x_1=1}$.

3. For $i = 1, \ldots, N$, let X_i be the vertex pair corresponding to where the i'th edge is placed. Each X_i is uniformly chosen among the $\binom{n}{2} - i$ vertex pairs that do not already have an edge. Let Z be the number of isolated edges, and let $Y_i = \mathbb{E}[Z \mid X_1, \ldots, X_i]$. Even though the X_i's are not independent, $Y_0, \ldots, Y_n$ form a Doob martingale. Apply Azuma-Hoeffding.