Midterm

Jinge Bao (e0522068)

P1 (a). True. Use Markov's Inequality.

$$Pr(X \geq 5) \leq \frac{E(X)}{5} = 1$$

(b) True. Because $E(X) = 1$ and X must be non-negative integers. ~~and~~ $Var(X) = 2$, so we must have some values > 1 and some < 1. Thus. $Pr(X > 1) > 0$, which means $Pr(X \geq 2) > 0$

(c) False. If ~~sample space are~~ random process are $P(X = 0) = \frac{2}{3}$ $P(X = 3) = \frac{1}{3}$

(d) False. Because X can only pick $0, 1, 2, 3, \cdots$, which means to keep $E(X) = 1$, there must be ~~more~~ 0's ~~than~~ with bigger probability.

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P2. Let A denotes not lose and X denotes the score.

$$Pr(X=1 | A) = \frac{Pr(\{X=1\} \cap A)}{Pr(A)} = \frac{Pr(X=1)}{Pr(A)} = \frac{\frac{1}{6}}{P(A)}$$

Because $P(A) = \cancel{\frac{1}{2} + \frac{1}{2} \times \frac{1}{2} + \cdots + \frac{1}{2} \times \cdots \times \frac{1}{2}}$

$$= \sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k = 1$$

so $P(X=1|A) = \frac{\frac{1}{6}}{P(A)} = \frac{1}{6}$

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P3 (a) As karger's algorithm will ~~awalys~~ always contract edge between two vertices. If the graph is a tree, there are just (n-1) edges (suppose tree has n nodes), when contract (n-2) edges, there will be one edge remaining. Obviously, this edge can always be an min-cut.

(b) What this algorithm find it's the mini-cut between two cliques. To make algorithm ~~get~~ right answer, we can't choose edge between two cliques. Let E_i denotes successful choose vertices in one of cliques.

$$Pr(E_1) = \cancel{\frac{C\binom{n-k}{2} + C\binom{k}{2}}{C\binom{n}{2}}} = \frac{\binom{n-k}{2} + \binom{k}{2}}{\binom{n}{2}}, \text{ which } k \geq 2$$

$$Pr(E_i | E_{i-1} \cap E_{i-2} \cap E_{i-3} \cap \cdots \cap E_1) =$$

$$= \frac{\binom{n-i-1-k}{2} + \binom{k}{2}}{\binom{n-i-1}{2}}, \text{ which } k \geq 2$$

we can prove $Pr(E_i | E_{i-1} \cap E_{i-2} \cap E_{i-3} \cap \cdots \cap E_1) \leq \frac{2}{3}$

so $Pr(E_1) Pr(E_2|E_1) \cdots Pr(E_{n-2} | E_{n-3} \cap \cdots E_1) \leq (\frac{2}{3})^{n-2}$

$$\leq (\tfrac{1}{2})(\tfrac{4}{3})^{n-2}$$

$$= (\tfrac{1}{2})^{\log_{\frac{1}{2}}(\frac{4}{3})(n-2)} \leq (\tfrac{1}{2})^{\Omega(n)} = 2^{-\Omega(n)}$$

P4. If there are edge between two vertices in I, we ~~sup~~ let two vertices v_i and v_j and $i<j$ ~~be~~. Because v_i is inserted first than v_j, so v_j is v_i's neighbor, However v_i has been inserted ~~yet~~ before v_j will be inserted. So v_j ~~___~~'s all neighbors can't be all after v_j in order because v_i. So False. In other way, I is an independent set

In other way, what vertex inserted into S is a ~~a~~ vertex in a ~~clique~~ graph associated with all other vertices in ~~cliqu~~ graph (like a center)

For each ~~node~~ vertex i, ~~if~~ let I_i ~~denote~~ indicate vertex i is in I

$|I| = \sum_{i=1}^{n} I_i \qquad E(|I|) = \sum_{i=1}^{n} E(I_i)$

~~$Pr(I_i=1) = E(I_i) =$~~ R_1

~~$E(I_i) = Pr(I_i=1) = \frac{(n-1)!}{\binom{n-1}{deg(v_i)} deg(v_i)!}$~~

$E(I_i) = Pr(I_i=1) = \frac{\binom{n-1}{deg(v_i)} deg(v_i)!}{n!} = \frac{1}{deg(v_i)+1}$

correct (n-1)!

use ~~Expe~~ Linearity of expectation

$E(|I|) = \sum_{v \in V} \frac{1}{deg(v)+1}$

correct (n-1)! permutations except vertex i

To illustrate, ~~for each per~~ there are n! permutation, to let $I_i = 1$, we need make all his neighbors, deg(v) ~~neighb~~ neighbors to fall his back in order there are $\binom{n-1}{deg(v_i)}$ choices.

Midterm

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P5 Here we use Yao's minimax principle. If I design a deterministic algorithm to compress any binary string of 10^6. Then the randomized algorithm will get best accuracy than this deterministic algorithm.

Here we choose binary string in uniformly.

Suppose we design algorithm ~~compress~~ keep only first 90% bits of string, then when recovering.

when means 0.9×10^6 will be kept and 0.1×10^6 bits on the tail will be removed.

~~The~~ There are 10^5 bits we don't know when recovering. Thus we can recover right ~~is $\frac{1}{10^5}$~~ w.p. ~~Θ~~ $(\frac{1}{2})^{10^5}$, but we can't recover from this compression is $1-(\frac{1}{2})^{10^5}$

Use Yao's Principle, we know that randomized algorithm can't get better accuracy than deterministic, so ~~this~~ her claims cannot be true.

Midterm

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P6 (a) suppose for each ball-i $X_{i,j}$ indicate color is j

$X_j = \sum_i X_{ij}$ $\quad E(X_1) = E(\sum_i X_{i1}) = m q_1^{(m)} = \sum_i E(X_{i1})$

Use Chernoff Bound

~~$Pr(|X_1 - mp_1|) = Pr(m|q_1^{(m)} - p_1|$~~

$Pr(m|q_1^{(m)} - p_1| \geq m\varepsilon) = Pr(|X_1 - mp_1| \geq m\varepsilon) \leq 2e^{-\frac{mp_1 \cdot (\frac{\varepsilon}{p_1})^2}{3}} = 2e^{-\frac{m\varepsilon^2}{3p_1}}$

Let $m = \frac{3p_1}{\varepsilon^2} \ln \frac{2}{\delta}$, because $m = O(\varepsilon^{-2} \log \delta^{-1})$

we get $Pr(|X_1 - mp_1| \geq m\varepsilon) \leq 2e^{-\frac{m\varepsilon^2}{3p_1}} = \delta$

so proof

(b) Because $\max_{S \subseteq [n]} |\sum_{i \in S} q_i^{(m)} - \sum_{i \in S} p_i| \leq \varepsilon$

which means for every $S \subseteq [n]$ (a) is true

so $Pr(|\sum_{i \in S} q_i^{(m)} - \sum_{i \in S} p_i| \geq \varepsilon) \leq \frac{\delta}{2^n}$

which for each element in $[n]$, S can pick or not pick, so 2^n choices.

let $m = \frac{3p_1}{\varepsilon^2}(n + \log \frac{2}{\delta})$ we get this answer.

(c) ~~$\max_{S \subseteq [n]} |\sum_{i \in S} q_i^{(m)}$~~ we get $Pr(|\sum_{i \in S} q_i^{(m)} - \sum_{i \in S} p_i| \geq \varepsilon) \leq \frac{\delta}{2^n}$

use union Bound ~~max Pr(for~~ e

$Pr(\forall S \subseteq [n] \, |\sum_{i \in S} q_i^{(m)} - \sum_{i \in S} p_i| \geq \varepsilon) \leq \delta$

so we get the answer.

(C) from (a) we get

$$Pr(|X_i - mp_1| \geq m\varepsilon) \leq 2e^{-\frac{m\varepsilon^2}{3p_1}} \leq \cancel{2e^{-\frac{m\varepsilon}{3}}}$$

Here we let $m = \frac{3p_1}{\varepsilon^2}(n\delta^{-1})$

we get $Pr(|X_i - mp_1| \geq m\varepsilon) \leq \frac{\delta}{n}$

use union bound

$$Pr(\max_{i\in[n]} |q_i^{(m)} - p_i| \geq \varepsilon) \leq Pr(\forall i \in [n] \, |m|q_i^{(m)} - p_i| \geq m\varepsilon) \leq \frac{\delta}{n} \cdot n$$

we get proof