CS5330: Assignment for Week 5

Due: Tuesday, 25th Feb 2020.

Please submit your solutions to the "Assignments/Week 5/Submissions" folder on LumiNUS by 28th February, 6:29 pm. I strongly encourage you to write your solutions using LaTeX.

You may discuss the problems with your classmates, though you should write up your solutions on your own. Please note the names of your collaborators in your submission.

1. Let V be the set of unit vectors in d dimensions whose each coordinate is either $1/\sqrt{d}$ or $-1/\sqrt{d}$. Call a pair of vectors ϵ-*orthogonal* if $|u^T v| \leq \epsilon$. How large (in cardinality) a set of pairwise ϵ-orthogonal unit vectors from V can you construct?

2. ~~Suppose you can draw independent samples of a real random variable X that has expectation μ and standard deviation σ. Explain how to use only $O(\log n)$ samples from this source to generate a random variable Y with expectation μ such that $\Pr[|Y - \mu| > 2\sigma] < 1/n$.~~

3. Show that the randomized quicksort algorithm runs in time $O(n \log n)$ with high probability (i.e., with probability $1 - 1/n^c$ for some constant $c > 0$).