CS5330: Hints for Week 2 Assignment

Assignment Due: Tuesday, 5th Feb 2020.

These are some hints for Assignment 2.

1. No hints :)

2. Let $T(n)$ be the random variable denoting the number of people not sitting in the correct seat. Justify the following recurrence:

$$T(n) = 1 + \frac{1}{n}\sum_{i=2}^{n} T(n-i+1)$$

3. (a) What is the probability that a particular subsequence is increasing? Use the inequalities $\binom{n}{k} \le \left(\frac{n}{k}\right)^k$ and $k! \ge k^k/e^k$ to upperbound $\Pr[L(\pi) \ge k]$ as a function of n and k. Now, use the following reasoning:

$$\begin{aligned}\mathbf{E}[L(\pi)] &= \sum_{k\ge 0} \Pr[L(\pi) \ge k] \\ &= \sum_{k=1}^{10\sqrt{n}} \Pr[L(\pi) \ge k] + \sum_{k>10\sqrt{n}} \Pr[L(\pi) \ge k] \\ &\le 10\sqrt{n} + \sum_{k>10\sqrt{n}} \Pr[L(\pi) \ge k].\end{aligned}$$

Argue that the last summation is $O(1)$, so that $\mathbf{E}[L(\pi)] = O(\sqrt{n})$.

(b) Argue that $\mathbf{E}[L(\pi)] \ge \sum_{i=1}^{\sqrt{n}} \mathbf{E}[X_i]$. Now, bound $\Pr[X_i = 0]$ as $\binom{n-\sqrt{n}}{\sqrt{n}} / \binom{n}{\sqrt{n}} < c$ for some constant $c < 1$.