Estimating the Greeks using Monte Carlo Methods

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***Abstract*— The Greeks are the sensitivities of the price of derivative with respect to various underlying parameters. They plays crucial roles in understanding the movement of derivative price, hedging and risk management. These papers show how to use Monte Carlo method in estimating the Greeks. We provide an examples in the case of Barrier option and Asian option, where we give a comparison of numerical result of each techniques and analyze their efficiencies.**

I. GREEKS

A. *What is Greeks?*

Greeks are quantities used to measure the sensitives of the price of derivatives with respect to change in underlying parameters. The name is used because the most common quantities are denoted by Greek letters. From the Black-Scholes formula, we see that the price of derivatives depends on various parameters including current underlying asset prices and volatility, time to maturity, and risk free rate. Thus, there are many types of Greeks. The most commonly used Greeks are *Delta, Gamma, Vega* and *Theta*.

B. *Type of Greeks*

We will discuss here the definition of the four most commonly used Greeks in the financial market. We let $C = C(S, r, \sigma, \tau)$ denote the price of a derivative, where S is a current underlying asset price, r is a riskfree rate, σ is a volatility of the asset, and τ is a time to maturity.

1) Delta: Delta or Δ is defined as

$$\Delta = \frac{\partial C}{\partial S}$$

From the definition, we see that Δ is measuring the rate the change of derivative with respect to the price of underlying asset. Delta also determines the number of underlying assets we need to hold in our replicating portfolio. Hence, it plays a crucial role in hedging and risk management.

2) Gamma: Gamma or Γ is defined as

$$\Gamma = \frac{\partial^2 C}{\partial S^2}$$

Gamma represents the convexity of the price of derivative. Gamma is also measuring the change in Delta with respect to change in price. Thus, it determines how much we need to change our position in the replicating portfolio.

3) Vega: Vega or ν is defined as

$$\nu = \frac{\partial C}{\partial \sigma}$$

Vega measures the change in price of derivative with respect to the change in volatility. Gamma is also important in hedging and risk management since the volatility directly affects the movement of underlying assets.

4) Theta: Theta or Θ is defined as

$$\Theta = -\frac{\partial C}{\partial \tau} = \frac{\partial C}{\partial t}$$

where t is a current time. Theta measures the decline in price of derivative over the passage of time.

II. MONTE CARLO METHOD

In many statistical problems, we are interested in finding

$$\mathbb{E}[f(X)] \tag{1}$$

where X is a random variable and f is a function. In most cases, the function f is very complicated that it is impossible to compute (1) explicitly. The Monte Carlo method is a very powerful tool that can be used to estimate (1).

A. *Basic idea*

The Monte Carlo method is based on a basic probability theory - the Strong Law of Large Numbers (SLLN). It states that for any independent identically distributed random variable Y_n with mean $\mu = \mathbb{E}[Y_i]$,

$$\frac{Y_1 + Y_2 + \ldots + Y_n}{n} \to \mu \quad \text{almost surely} \tag{2}$$

Since X_i are i.i.d, $f(X_i)$ are also i.i.d. Thus, we can apply the SLLN to get

$$\frac{f(X_1) + f(X_2) + \ldots + f(X_n)}{n} \to \mathbb{E}[f(X)] \quad \text{almost surely} \tag{3}$$

This fact tells us that we can estimate (1) by drawing an i.i.d sample $X_1, X_2, \ldots, X_n$ and compute

$$\mu_n = \frac{f(X_1) + f(X_2) + \ldots + f(X_n)}{n} \tag{4}$$

as our estimate.

B. *Rate of Convergence*

To analyze how good an estimate is, we need to know its rate of convergence. Equivalently, we need to know how much approximately is the error in term of number of samples. To answer this question, we resort to the most famous theorem in probability - the Central Limit Theorem (CLT). It states that for any sequence of i.i.d random variables X_i with mean $\mu = \mathbb{E}[X_i]$ and variance $\sigma^2 = \text{Var}\ [X_i] < \infty$,

Returning to our estimating problem, the CLT simply asserts that

$$\sqrt{n}(\mu_n - \mu) \overset{D}{\Rightarrow} \sigma N(0,1)$$

or equivalently,

$$\mu_n \overset{D}{\approx} \mu + \frac{\sigma}{\sqrt{n}} N(0,1) \tag{5}$$

The result above tells us that the Monte Carlo estimate converge in the order the $\sqrt{n}$. We can also use (5) to construct a confidence interval.

C. Monte Carlo in Finance

Most often, we are interested in estimating the price of derivative with a discounted payoff function h, where h is a function of asset price at maturity. Assume the underlying asset is log-normally distributed. That is, its dynamic is of the form

$$dS = \alpha S dt + \sigma S dz$$

Then we change the measure to the risk-neutral measure Q and the dynamic of S_t becomes

$$dS = rSdt + \sigma S d\tilde{z}$$

where $\tilde{z}$ is a $Q-$Brownian motion. The derivative price is then given by

$$\mathbb{E}_Q[h(S_T)]$$

Note that h already include a discounted term e^{-rT}. From the well known property of log-normal random variable, we know that

$$S_T = S_0 e^{\left(r - \frac{\sigma^2}{2}\right)T + \sigma \tilde{z_T}}$$

The derivative price can be written as

$$\mathbb{E}_Q[f(X)], \quad f(x) = h\left(S_0 e^{\left(r-\frac{\sigma^2}{2}\right)T + \sigma x}\right)$$

where $X \sim N(0,T)$, which we can estimate directly using estimate (4).

However, many interesting derivative are path-dependent. In other words, h is not a function of S_T, but a function of path γ_T of S_t. To estimate the price of derivative, we then need to construct a sample path of S_t. The simplest way is what is called *incremental path construction* , where we approximate how S_t changes in the small time step. That is, let $0 = t_0 < t_1 < \ldots < t_N = T$, then we construct S_{t_i} recursively by the dynamic

$$S_{t_{i+1}} = S_{t_i} + rS_{t_i}\Delta t + \sigma S_{t_i} z_{\Delta t}, \quad z_{\Delta t} \sim N(0, \Delta t) \tag{6}$$

We then construct a path by linear interpolation. We need to redo the process above to get n paths $\gamma_1, \gamma_2, \ldots, \gamma_n$, then we compute

$$\mu_n = \frac{h(\gamma_1) + h(\gamma_2) + \ldots + h(\gamma_n)}{n}$$

to get the Monte Carlo estimate.

D. Important Sampling

Suppose we would like to estimate $\mathbb{P}[X > b]$, where X is normally distributed. A straightforward approach would be to generate sample $X_1, \ldots, X_m$, then use

$$\hat{p} = \frac{\sum_{i=1}^m \mathbf{1}(X_i > b)}{m}$$

as our estimate. However, if b is extremely large, then most of our samples will gives zero value, or even worse all of our samples might gives zero yielding an uninformative estimate. To fix this problem, we change our measure so that it concentrates more on the region that contributes to our estimate. That is, suppose we would like to estimate (1) where X has a density ϕ, then

$$\begin{aligned}
\mathbb{E}_\phi[f(X)] &= \int f(x)\phi(x)dx \\
&= \int \frac{f(x)\phi(x)}{\psi(x)} \psi(x) dx \\
&= \mathbb{E}_\psi\left[\frac{f(X)\phi(X)}{\psi(X)}\right]
\end{aligned}$$

For instance, we can choose ψ to be a density of $N(b,1)$, then most of our samples will be *important*, hence the name of the method. One thing to keep in mind is that we are dividing by ψ in our estimate, so our choice of ψ must satisfy

$$f(x)\phi(x) \neq 0 \text{ implies } \psi(x) \neq 0$$

In other words, ψ does not vanish at any point in the support of $f\phi$. We refer to [] for more detailed discussion on a choice of ψ.

Consider a general random variable X, with cumulative distribution function Φ. Then we have

$$\mathbb{E}[f(X)] = \mathbb{E}[f \circ \Phi^{-1}(U)]$$

where U is uniformly distributed on [0,1]. Suppose we would like to change our measure so that our samples U lie in $[a, a+h] \subset [0,1]$. Denote the original and new measure by P, Q respectively, then we have

$$\begin{aligned}
\mathbb{E}_P[f \circ \Phi^{-1}(U)] &= \mathbb{E}_Q[h \cdot f \circ \Phi^{-1}(U)] \\
&= \mathbb{E}_Q[hf \circ \Phi^{-1}(hW + a)] \\
&= h \cdot \mathbb{E}_P[f \circ \Phi^{-1}(hU + a)] \qquad (7)
\end{aligned}$$

since U, W are uniformly distributed under P, Q respectively.

III. ESTIMATING THE GREEKS

In this section, we will attempt to estimate two of the most commonly used Greeks - Delta (Δ) and Gamma (Γ). We will discuss difficulties and different techniques that can be used to overcome them.

A. Finite Differencing : Straightforward Approach

Recall the definition of Delta and Gamma,

$$\text{Delta} = \frac{\partial C}{\partial S}, \qquad \text{Gamma} = \frac{\partial^2 C}{\partial S^2}$$

Also, by the definition of derivative, we can approximate the derivative by

$$\frac{\partial C}{\partial S} \approx \frac{C(S+\Delta S) - C(S)}{\Delta S} \tag{8}$$

Thus, to find Delta, we can estimate the option price with varied inputs and approximate the derivative by (8). Alternatively, we can use the centre differencing instead of the forward one to approximate the Delta as follows

$$\text{Delta } = \frac{\partial C}{\partial S} \approx \frac{C(S+\Delta S) - C(S-\Delta S)}{2\Delta S} \tag{9}$$

For Gamma, we proceed in the similar manner by first noting that

$$\frac{\partial^2 C}{\partial S^2} = \lim_{\Delta S \to 0} \frac{C(S+\Delta S) + C(S-\Delta S) - 2C(S)}{\Delta S^2}$$

Thus, we can approximate Gamma by,

$$\text{Gamma } = \frac{\partial^2 C}{\partial S^2} \approx \frac{C(S+\Delta S) + C(S-\Delta S) - 2C(S)}{\Delta S^2}$$

We see that the estimation of Delta and Gamma consists of two main approximation. First, we estimate $C(S+\Delta S), C(S), C(S-\Delta S)$ using usual Monte Carlo method. Then we approximate our desired derivative by means of finite difference. To measure the accuracy of the proposed estimate, we calculate the variance and get,

$$\begin{aligned} &\text{Var } [\frac{C(S+\Delta S) - C(S)}{\Delta S}] \\ &= \frac{1}{\Delta S} \text{Var } (C(S))\,(1 - \text{Corr}(C(S+\Delta S), C(S)) \end{aligned} \tag{10}$$

Please see equation (10.5) in [] for the detailed derivation. From (10), we need to maximize the correlation of $C(S+\Delta S)$ and $C(S)$ to minimize the variance. One simple way to achieve that is to use the same sample path in both calculations. The method is called *variate recycling* .

Using finite differences method, one first need to face the question of how to choose an appropriate ΔS. Before we proceed to answer this question, let's discuss what happen if we choose way too large or too small ΔS. Recall that the finite difference is a decent approximation of a derivative since it captures the first order in the Taylor series expansion of $C(S+\Delta S) - C(S)$. Therefore, if we choose a relatively large ΔS, we might see the effect from the higher order term. As a result, our estimate is be much less accurate. What if we choose ΔS to be very small then? From (10), one see that a choice of too small ΔS will result in a higher error. To be able to choose an appropriate value of ΔS, we need to analyze our finite difference approximation in more detail. Consider the Taylor approximation of our estimate

$$\begin{aligned} C(S+\Delta S) \approx C(S) + \partial_S C \Delta S + \frac{1}{2}\partial_S^2 C(\Delta S)^2 + \frac{1}{6}\partial_S^3 C(\Delta S)^3 + \\ \frac{1}{24}\partial_S^4 C(\Delta S)^4 + O((\Delta S)^5) + \varepsilon C \end{aligned} \tag{11}$$

where ε is a measure of machine precision defined as the smallest positive number such that $1, 1+\varepsilon$ are still distinct number. We can write the estimate for Gamma as

$$\hat{\Gamma}(S, \Delta S, \varepsilon) = \partial_S^2 C + \frac{1}{12}\partial_S^3 C(\Delta S)^2 + \frac{\varepsilon C}{(\Delta S)^2} + \varepsilon_2 \partial_S^2 C$$

where ε_2 represents a roundoff error from the machine. We can see more clearly now that a large ΔS will increase the error term $\frac{1}{12}\partial_S^4 C(\Delta S)^2$, while a small ΔS will decrease the error term $\frac{\varepsilon C}{\Delta S^2}$. Minimizing this sum of two error terms gives us a good choice for ΔS, which is

$$\Delta S = \left(12\varepsilon \frac{C}{\partial_S^4 C}\right)^{1/4}$$

However, note that our choice depends on the options price and its fourth derivative which are both unobservable. Thus, we make a reasonable assumption that all terms in the Taylor expansion are of the same magnitude. That is, we assume that $O(C) \approx O(\partial_S^4 C \cdot S^4)$. Therefore,

$$\Delta S \approx \varepsilon^{1/4} \cdot S$$

To sum up, we can estimate Delta and Gamma of an option with payoff h using Finite Difference method by

- Choose $\Delta S \approx \varepsilon^{1/4} S$.
- Generate N different paths of underlying asset prices starting at S. Denote them by $\gamma_1, ..., \gamma_N$.
- Using the same set of normal random variables that we generated for the asset price process staring at S, we generate the sample path of asset price starting at $S+\Delta S$ and $S-\Delta S$. Denote the resulting path by γ_i' and $\gamma_i'', 1 \le i \le N$ respectively.
- Compute the estimate

$$\hat{\Delta} = \frac{\hat{C}(S+\Delta S) - \hat{C}(S-\Delta S)}{2\Delta S} \quad = \frac{1}{2N\Delta S}\sum_{i=1}^{N}\left[h(\gamma_1') - h(\gamma_1'')\right]$$

and

$$\hat{\Gamma} = \frac{\hat{C}(S+\Delta S) + \hat{C}(S-\Delta S) - 2\hat{C}(S)}{(\Delta S)^2} \tag{12}$$

$$= \frac{1}{N(\Delta S)^2}\sum_{i=1}^{N}\left[h(\gamma_1') + h(\gamma_1'') - 2h(\gamma)\right] \tag{13}$$

Next, we will discuss an alternative method that will greatly improve the estimates in many cases.

B. Finite Differencing : Important Sampling

Suppose we would like to estimate a European call option whose strike price K is much bigger than the current underlying asset price S. In other words, the call option is far out of the money. Let's assume we use the straightforward finite different method. Then most of the generated sample paths will yield the zero payoff, while rarely one of them will finish in the money and give the payoff that will contribute to our estimate. Not only that the sample will be wasteful but the variance of an estimate will be much higher. As addressed in earlier section, one of the best way to overcome this type of difficulty is to use *important sampling* technique. That is, we change the measure so that it concentrates more on a suitable region so that most of our sample path will finish in-the-money.

Let Φ denote the cumulative distribution function of $N(0,\Delta t)$, where Δt is the time step used in (6). Then we create a sample path by generating $z_i = \Phi^{-1}(u_i)$ where u_i is i.i.d uniformly distributed in [0,1]. At each step, after generating S_{i-1}, we then change the uniform measure by an affine transformation $u_i \to \alpha_i u_i + \beta_i$ whose range is in [0,1]. That is, we generate z_i by

$$z_i = \Phi^{-1}(\alpha_i u_i + \beta_i)$$

Similar to (7), we have a correct factor at each time step Δt giving a total correct factor

$$\Lambda = \prod_{i=1}^{n} \alpha_n$$

Note that Λ could depend on the path, since each factors α_i, β_i might depend on S_{i-1} as we shall see in the case of Barrier option. Then our desired important sampling estimate is given by

$$\hat{C}(S) = \frac{1}{N}\sum_{i=1}^{n} \Lambda(\gamma_i) h(\gamma_i)$$

The factor α_i, β_i is not universal and depends very much on the derivative. In fact, there is no need to use affine transformation at all.

Let us now illustrate how to use the above method in up-and-out Barrier call option.Consider an up-and-out Barrier call option with knock out level H and strike price K, our goal is to ensure that all the generated path will contribute to our calculation, that is, not going above H at anytime. This objective leads us to choose

$$\alpha_i = \Phi\left(\frac{\ln(H/S_{i-1}) + \sigma^2\Delta t/2}{\sigma\sqrt{\Delta t}}\right), \quad \beta_i = 0 \tag{14}$$

If we also would like to ensure that the path not only survive the knock out but also finish in the money, we can alter the last factor by let

$$\tilde{\alpha_n} = \alpha_n - k, \quad \tilde{\beta}_n = k \tag{15}$$

where

$$k = \Phi\left(\frac{\ln(K/S_{i-1}) + \sigma^2\Delta t/2}{\sigma\sqrt{\Delta t}}\right)$$

The main part of important sampling method is in choosing an appropriate change of measure. This selection process requires a good understanding of the derivative and it could be very difficult in derivative with complicated structure.

C. Infinitesimal Perturbation Analysis

Recall that the derivative price is given by

$$C(S) = \mathbb{E}_Q[h(\gamma)]$$

where γ is a price path of underlying asset, h is a discounted payoff function. Since γ is continuous, it is approximately a linear interpolation of discretely monitored asset price $S = S_{t_0} < S_{t_1} < ... < S_{t_m} = S_T$. Thus, we can write h as a function of $\mathbf{S} = (S_{t_k})_{k=1}^m$. That is,

$$C(S) = \mathbb{E}_Q[h(\mathbf{S})]$$

Assume that we can interchange the derivative, we then get

$$\Delta = \frac{\partial C}{\partial S} = \frac{\partial}{\partial S}\mathbb{E}_Q[h(\mathbf{S})] = \mathbb{E}_Q[\frac{\partial h}{\partial S}(\mathbf{S})] \tag{16}$$

The last quantity can then be estimated by usual Monte Carlo. This method is called *infinitesimal perturbation analysis* or *path differentiation*. As simple as it may seem, this method relies on a very crucial assumption that we can interchange differentiation and expectation. We will not discuss this issue here, but instead assume that we are allowed to do so, at least for our particular example of Asian option.

Consider an Asian option whose strike price is K, the discounted payoff function is then given by

$$h(\mathbf{S}) = e^{-rT}(\bar{S} - K)^+, \quad \bar{S} = \frac{1}{n}\sum_{i=1}^{n} S_{t_i}$$

Thus, if we let $Z_m = S_{t_m}/S_0$, then

$$\frac{\partial h}{\partial S}(\mathbf{S}) = \frac{\partial h}{\partial S_{t_0}}(\mathbf{S}) = e^{-rT}\bar{Z}\mathbf{1}(\bar{S} > K) = e^{-rT}\frac{\bar{S}}{S}\mathbf{1}(\bar{S} > K)$$

where $\mathbf{1}(A)$ is an indicator function. Then we can approximate (16) by usual Monte Carlo method. Note that the same method will not work for approximation of Gamma in the case of Asian option since the indicator function is not differentiable.

Next, we will discuss another method that is applicable for non-continuous function and even results in surprisingly simple equations.

D. The Likelihood Ratio Method

Note that an option price is merely an integral

$$C(S,\alpha) = \int h(\mathbf{S})\psi(\mathbf{S})d\mathbf{S} \tag{17}$$

where ψ is a risk-neutral density, $\mathbf{S} = (S_{t_i})_{i=1}^{m}$, and α is a parameter of our interest. In earlier method, we started by constructing a sample path using normal random variable so that

$$C(S,\alpha) = \int h(\mathbf{S}(z;\alpha)\phi(z)dz$$

Observe that we are interested in derivative with respect to α. Thus, the fact that α is contained in π, which could be discontinuous, gives us difficulty in getting an accurate approximation. The Likelihood Ratio method is based on the idea that we can shift the dependence on any of the parameters over into the density function $\psi = \psi(\mathbf{S},\alpha)$ in (17). Then for any parameter α, we have

$$\begin{aligned}\frac{\partial C}{\partial \alpha} &= \int h(\mathbf{S})\frac{\partial \psi(\mathbf{S},\alpha)}{\partial \alpha}d\mathbf{S} \\ &= \int h(\mathbf{S})\frac{\partial \psi(\mathbf{S},\alpha)/\partial \alpha}{\psi(\mathbf{S},\alpha)}\psi(\mathbf{S},\alpha)d\mathbf{S} \\ &= \mathbb{E}_{\psi}\left[h(\mathbf{S})\frac{\partial \psi(\mathbf{S},\alpha)/\partial \alpha}{\psi(\mathbf{S},\alpha)}\right]\end{aligned}$$

The last expression can then be approximated by usual Monte Carlo method. The name *likelihood ratio* comes from the term $\partial\psi(S,\alpha)/\partial\alpha$ that appears in the final estimate. Also note that the same method can also be applied to higher order derivative such as Gamma.

The difficulty of using this method is in a calculation of the density $\psi(\mathbf{S})$. This can be highly complicated for a complex derivative. Fortunately for our chosen examples - Delta and Gamma - the quantity $\frac{\partial\psi(\mathbf{S},\alpha)}{\partial\alpha}$ can be computed explicitly. First, we use the conditional distribution, to get

$$\psi(\mathbf{S}) = \psi(S_{t_1}|S_{t_0})\psi(S_{t_2}|S_{t_1})...\psi(S_{t_m}|S_{t_{m-1}})$$

Since all but the first term is independent of $S_{t_0} = S$, we get

$$\frac{\partial_S\psi(\mathbf{S})}{\psi(\mathbf{S})} = \frac{\partial_S\psi(S_{t_1}|S_{t_0})}{\psi(S_{t_1}|S_{t_0})} = \frac{z_{t_1}}{S_{t_0}\sigma\sqrt{\Delta t}}$$

Similarly, we get

$$\frac{\partial_S^2\psi(\mathbf{S})}{\psi(\mathbf{S})} = \frac{z_{t_1}^2 - z_{t_1}\sigma\sqrt{\Delta t} - 1}{S_{t_0}^2\sigma^2\Delta t}$$

Next, we present numerical result from applying all these methods to up-and-out Barrier and Asian call option.

TABLE I

DELTA : BARRIER OPTIONS

Method	Estimated Delta	Standard Error
Actual	0.5397	
Finite Differencing	0.5389	0.0017056
Important Sampling	0.5380	0.007156
Likelihood Ratio	0.5324	0.024758

IV. NUMERICAL RESULTS

We implemented the above described methods for up-and-out Barrier options and Asian options. In our implementation we used the following values: $S_0 = \$100, K = \$100, r = 5\%, \sigma = 0.3, T = 0.1, n = 100$, where S_0 is the initial price of the asset, K is the strike price, r is the interest rate, σ is the volatility, T is the time to maturity, n is the number of interval segments assumed between initial date and the date of maturity. We assume that this is a non-dividend paying stock. In the case of Barrier options, the Barrier was kept at \$180. The number of iterations in the Monte-Carlo simulations was varied upto 50,000. These simulations were effected in a Notebook PC with 2.27Ghz Intel(R) Core(TM) i5 processor, 4GB RAM and 64-bit Operating System. The programs were written in MATLAB version 7.6.0 (R2008a).

From the results of the Delta estimation for Barrier Options I, it can be seen that all the three methods converge close to the actual solution obtained using the closed form solution. Within them, the Finite Differencing and the Important Sampling methods are the most accurate. However, with important sampling the running time complexity is high as the simulation of the variates depend on the stock price in the interval preceding it.

The results of the Delta Estimation for Asian Options II again indicate that all the three methods converge more or less to the same value. However, Finite Differencing and Infinitesimal Perturbation methods are the most accurate and most consistent. We would not be able to use the importance sampling method easily for the Asian Options, because in importance sampling we change our measure to reflect the rare events. Unlike the Barrier Option, where in importance sampling it was easy to find the region where the call would be in-the-money by just looking at the stock price at maturity; in the Asian Option, however, we need to look at the average price of the whole stock path.

In the simulations for the Gamma estimates, in Barrier Options III, the estimates are however not much closer to the actual value from the closed form solution. Within the different methods, Finite Differencing is the worst performer

TABLE II

DELTA : ASIAN OPTIONS

Method	Estimated Delta	Standard Error
Finite Differencing	0.60461	0.001632
Infinitesimal Perturbation	0.60417	0.00163
Likelihood Ratio	0.63281	0.016159

TABLE III

GAMMA : BARRIER OPTIONS

Method	Estimated Gamma	Standard Error
Actual	0.0425	
Finite Differencing	3.979×10^{-9}	0.0039410
Important Sampling	0.0336	0.0038441
Likelihood Ratio	0.1042	0.038038

with a very negligible value for gamma. In the case of Asian Options IV, again the Finite Differencing method indicates a very negligible value while the Likelihood Ratio method gives a relatively better estimate.

The time taken by the different Monte Carlo methods to converge for their Delta estimates are tabulated in V. The corresponding graphs indicating the time to converge for Barrier and Asian Options are given respectively in 1 and 2. As can be seen the Monte Carlo methods converge by around 10000 iterations.

TABLE IV

GAMMA : ASIAN OPTIONS

Method	Estimated Gamma	Standard Error
Finite Differencing	7.3667×10^{-9}	0.004997
Likelihood Ratio	0.0773	0.024751

TABLE V

CPU TIMES (IN SEC)

Method	Barrier Option	Asian Option
Finite Differencing	75.72	73.72
Important Sampling	2432.242	-
Infinitesimal Perturbation	-	50.76
Likelihood Ratio	112.289	116.62

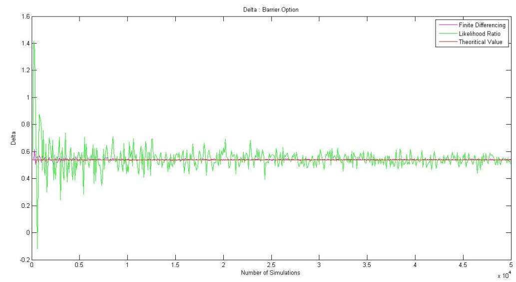

Fig. 1. Delta for Barrier Option

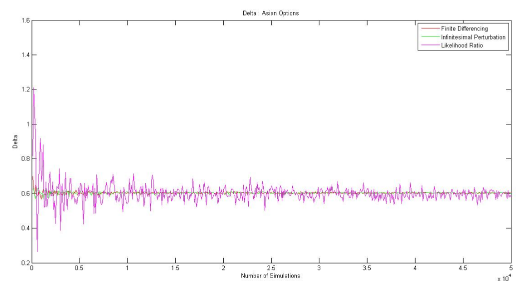

Fig. 2. Delta for Asian Options

V. CONCLUSION

Through the numerical results obtained from implementing the different Monte Carlo methods, we conclude that Finite Differencing and Important Sampling methods give the most accurate and consistent Monte Carlo estimates for Delta(Δ) for the Barrier Options. In the case of Asian Options, Finite Differencing and Infinitesimal Perturbation methods give almost same estimates for Delta(Δ). For Gamma, methods other than Finite Differencing give better estimates. We also find that Monte Carlo methods start converging after $10K$ Simulations.

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