Determining Stock Trend Using Hidden Markov Model

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Abstract

In this paper, we model a stock dynamic using Hidden Markov Model (HMM) where weekly return is normally distributed with mean and variance depending on the hidden random variable representing the stock trend. Using Expectation-Maximization algorithm, we estimate all the parameters and design a simple trading strategy based on it. We then compare its performance with Buy and Hold and Resistance and Support strategy. The result shows that our proposed HMM strategy outperforms the other two strategies in most cases.

1 Introduction

Terms such as bull or bear are widely used in the investment world to describe the trend or the market "mood". Even though most investors have a good sense of the current state of the stock, the actual trend is not an observable value. Knowing the current state has an obvious benefit. Traders can formulate a trading strategy based on that information and be able to profit from the upward movement and avoid the downfall of a stock. As a result, the problem of determining the current stock trend has drawn lots of attention from investors. Many technical indicators, such as Short Interest/Total Market Float and the Put/Call ratio, has been developed to measure this hidden value.

In this paper, motivated by the hidden feature of the problem, we modeled the stock return using Hidden Markov Model (HMM). Since the underlying parameters of the model were not given, Expectation-Maximization algorithm was used to estimate the model parameters. Subsequently, hidden state, i.e. current stock trend, was estimated using Viterbi algorithm. Furthermore, to test the performance of the proposed model, we backtested a simple trading strategy based on the HMM model and compared the trading performance with other well known strategies to evaluate the model. For related applications of HMM in financial market, see [6], [4], [3].

The paper is organized as follows. In section 2, we describe a model for stock return based on HMM. In section 3, we explain the parameter estimation procedure and how to find the conditional probability distribution of the hidden state. We discuss how to calibrate the model to the data and all rising issue in section 4. Then in section 5, we propose a simple trading strategy based on the model as a proxy to evaluate the performance of the model. The result is also discussed in section 5. We then conclude our work and propose possible extensions in section 6.

2 Hidden Markov Model for Stock Return

In this section, we describe the stock dynamic based on Hidden Markov Model. Let Z_t be a Markov chain taking values from a state space $\mathcal{S} = \{s_1, s_2, \ldots, s_N\}$. We think of $\mathcal{S}$ as a set of all possible stock trend descriptions. For example, $\mathcal{S}$ could be $\{$bull, random walk, bear$\}$. Let π be the initial distribution and $\mathbf{P}$ denote the transition probability of Z_t, that is

$$\pi_i = \mathbb{P}[Z_0 = s_i], \qquad \mathbf{P}_{ij} = \mathbb{P}[Z_{t+1} = s_j | Z_t = s_i]$$

We assume a time-homogeneous Markov chain so $\mathbf{P}$ is independent of t. When Z_t is in state s_i, we assume that the stock return is given by a Gaussian random variable with mean μ_i and variance σ_i^2. Formally, let S_t denote the stock price at time t and $R_t = S_{t+1}/S_t - 1$ denote the stock return, then the conditional density function of R_t is given by

$$f_{R_t|Z_t=s_i}(r) = \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{-\frac{(r-\mu_i)^2}{2\sigma_i^2}}$$

The main assumption of the model is that Z_t is unobserved. Our goal is to find the most likely sequence Z_t by computing the conditional distribution of Z_t given the observed sequence of R_t (or equivalently S_t). In addition, the initial distribution π and the transition matrix $\mathbf{P}$ of the unobserved Z_t is also not known apriori. As a result, we need to estimate this set of parameters as well. This task can be done using Expectation-Maximization algorithm. We describe the method in the next section.

3 Parameter Estimation Method

In this section, we describe two main tools widely used to solve problems relating to Hidden Markov Models (HMM). The first tool is the Expectation Maximization (EM) algorithm and it is used to estimate parameters of the underlying Markov chain. The second tool is called the Viterbi algorithm and it is used to find the most likely sample path of unobservable Markov chain. The summary of how to implement these two methods follows below,

3.1 Expectation Maximization Algorithm

The EM algorithm is based on the maximum likelihood idea. It is an iterative method that alternates between performing an expectation (E) step, which computes the expectation of the log-likelihood using the current estimates, and a maximization (M) step, which computes parameters by maximizing the expected log-likelihood computed in the (E) step. Please refer to [2] for details on how to implement EM algorithm. The EM algorithm is an iterative method that leads to a *local* maximum of likelihood function. As a result, it is sensitive to the initial conditions and one must be careful in selecting a good initial condition. We discuss this issue in the next section.

3.2 Viterbi Algorithm

The Viterbi algorithm is a forward-backward programming algorithm for finding the most likely sequence of hidden states. First, we compute the maximal probability of being in state i at time n. This step is done forward in time. Then once we have computed these probabilities for every state and time, we find the optimal path by finding the argument that maximizes the values we found in the first step. This step is done backward in time. For details of how to implement the Viterbi Algorithm, please refer to [1].

4 Model Calibration

In this section, we explain how to fit the HMM to the data and address all issues involved.

4.1 Data Source

Our data consists of the daily stock prices obtained from CSRP during the period January 2000 to June 2011. We select a set of large-cap liquid stocks that encompasses different sectors.

4.2 Number of States

To fit the HMM, we need to specify number of states N, i.e. size of $\mathcal{S}$. One can try to optimize this parameter by fitting the model with several value of N and select the one that gives the best result. However, the optimal number of states might not give us any interpretation of what each states represent and we might also be overfiting the data. A popular alternative is to just specify it exogenously using the intuitive understanding of the real world property. In our problems, the number of states means the number of descriptions of stock trends. Thus, it is reasonable to assume that the appropriate number of state is three representing the upward, downward, and sideway (random walk) trend. One can also add strong upwards and strong downwards trends and have five states as done in [6].

We confirm this intuition mathematically as follows; we first take the set of weekly returns and apply the k-mean clustering. We then determine the number of clustering by the Elbow method [5]. Figure 1 shows the plot of the number of states against the variance explained. The plot suggests that the number of states is either three or five. However, to avoid the danger of overfitting, we decided to work with three states throughout the project.

4.3 EM algorithm

Having selected the number of states, we fit the weekly return data to the HMM using EM algorithm. We used the toolbox `mhsmm` [7] which provides an R function `hmmfit`. This function runs EM algorithm to Guassian Emission Hidden Markov Model and the outputs include the estimated parameters and the most likely hidden path.

5 Performance Analysis

In this section, we explain how to implement a simple trading strategy based on Hidden Markov Model as a tool to evaluate the model. We then compare the trading performance of the HMM strategy with simple Buy and Hold (BH) and Resistant Support Rule (RSR).

5.1 Trading Strategy based on HMM

The general idea is that we would like to buy a stock when it is on an upward trend and close the position when it exits that good condition. This can be done as follows; at each week t, we take a 300-week trading window and fit the

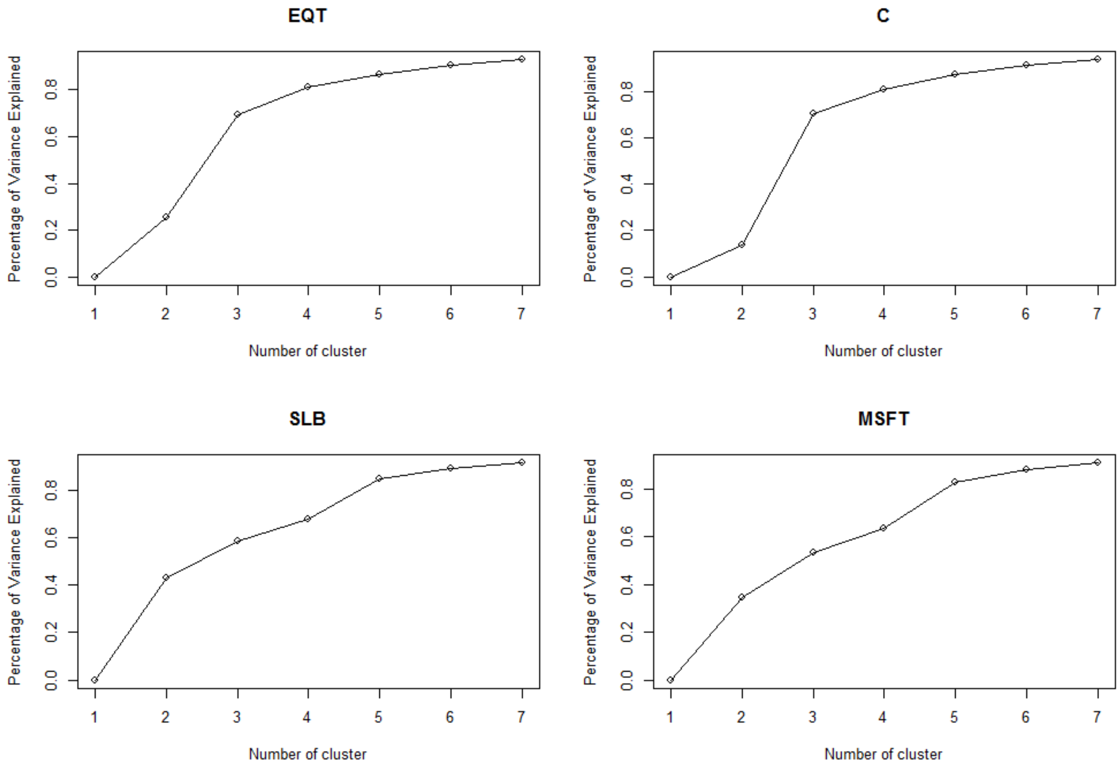

Figure 1: The typical plot of number of clusters against the variance explained. The plot suggests that the number of states is either three or five

HMM to that set of data. We then obtain the most likely current trend,

$$s_i = \text{argmax}_{s_k \in \mathcal{S}} \mathbb{P}[Z_t = s_k | R_{t-300}, R_{t-299}, \ldots, R_t]$$

If $\mu_i - \sigma_i$ is positive, we call s_i a "good" state and we initiate a long position, if not already. If $\mu_i - \sigma_i$ is negative, we close the existing position and hold cash. We did not allow a short position and we assume zero interest rate since it was extremely low during that period.

5.2 Result and Discussion

Table 1 summarizes the trading result during our test period. We record the annualized mean return and standard deviation. It can be seen from the table that in most cases the HMM strategy outperforms both BH and RS strategy. To see what happened in more detail, we plot the portfolio values of all three strategies. From figure 2, it is observed that the HMM method can detect the downward trend rather quickly. As a result, we are able to avoid the crisis at the early stage. Since we do not allow the short position, we have a long period of constant portfolio value as seen in the figures.

Table 1: Trading Performance of Hidden Markov Model (HMM), Buy and Hold (BH), and Resistant and Support (RS).

	HMM		BH		RS	
Stock	Mean	SD	Mean	SD	Mean	SD
HAL	-20.96%	46.18%	-5.21%	59.24%	8.65%	29.43%
SLB	-6.6%	31.93%	-2.99%	52.07%	-3.53%	39.37%
EQT	5.35%	18.66%	5.83%	41.49%	1.65%	21.14%
OXY	-5.73%	17.36%	3.86%	50.39%	-10.9%	39.26%
AAPL	29.63%	33.88%	27.75%	40.46%	26.62%	27.9%
CSCO	4.64%	28.15%	-2.45%	33.89%	-2.89%	24%
MSFT	6.96%	17.38%	-1.06%	29.3%	5.55%	17.59%
BAC	-4.01%	11.21%	-26.54%	81.23%	-7.95%	41.92%
JPM	10.34%	57.99%	23.97%	67.13%	-9.36%	25.7%
WFC	2.38%	7.99%	-5.24%	67.75%	-13.55%	41.53%
C	44.73%	108.85%	-5.48%	144.38%	-28.11%	32.39%
STI	-5.33%	22.93%	-26.66%	80.37%	-6.26%	39.22%
ANF	3.57%	19.98%	0.15%	53.79%	2.65%	30.41%
GES	4.24%	46.42%	-9.29%	64.94%	2.28%	49.2%
SKS	1.2%	22.05%	-10.06%	72.96%	4.42%	41.76%
URBN	4.43%	33.6%	4.29%	47.68%	8.45%	29.99%

Comparing HMM strategy with BH, one can see from figure 2 that during the normal period, there is no evidence of a superior performance of the HMM over BH. This is probably due to the fact that in our test period, except for the crisis, stocks are mostly in the upward trend resulting in the same return for both strategies. Note that the HMM strategy attempts to detect the trend and not short-term fluctuation, thus, it will tell us to buy and hold when the market is generally going upwards resulting in the same performance as BH. However, the result is opposite when the market is crashing. One can see that the HMM can detect the downfall or even medium-term downward movement at the early stage. Consequently, we can avoid the loss and stay invested only during the profitable period.

Comparing the HMM strategy with RS, it is observed that HMM perform better in most cases. because of the same reason as explained above. However, it is interesting to note that during the period when the market does not show obvious trend but behave like a random walk, the RS performs slightly better

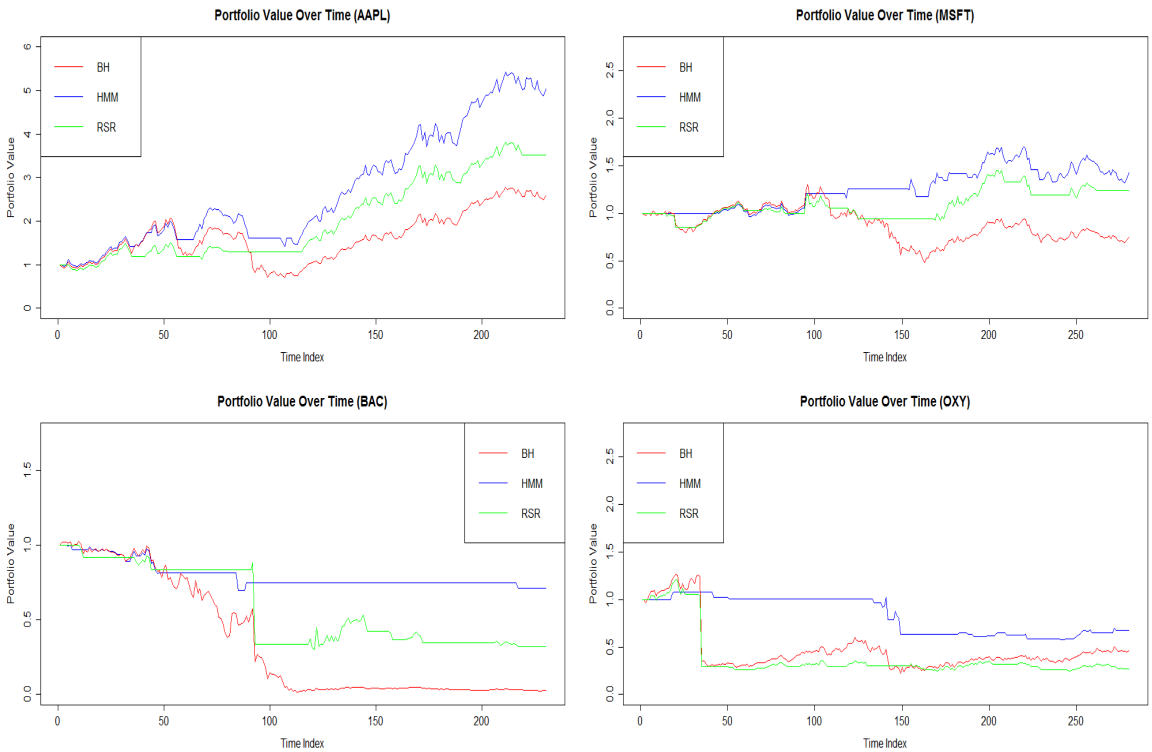

Figure 2: Portfolio Value when investing $1 in AAPL (top left), MSFT (top right), BAC (bottom left), and OXY (bottom right) using Hidden Markov Model (HMM), Buy and Hold (BH) and Resistant Support(RS) strategies.

since it could gain from short term fluctuation of the stock.

6 Conclusion and Future Work

In this paper, we have developed a simple HMM model to capture the stock trend. The stock weekly returns were modeled as independent processes and normally distributed with mean and variance depending the hidden state. Our goal was to find the most likely current state and invest during a positive return period. Since all parameters involved in the model were not known a priori, the EM algorithm was used to estimate the parameter. We then evaluated the proposed HMM model by proposing a simple strategy by holding a long position when the most likely current state yields a positive return. We found that, in most cases, the HMM strategy outperforms both Buy and Hold and Resistant and Support Rule during the test period. This result was mainly due to the fact that HMM method was able to detect the coming crash, both long and medium term, and suggested selling at the early stage of the crash.

Our work can be extended in various ways. For instance, throughout the work, we assumed that the return is Gaussian, but we can easily modify it to other distributions. Instead of replacing Gaussian, we can also add extra hidden states that will emit return with different distributions. As many literatures have shown that stock returns exhibit a heavy tail behavior, we can modify the model straightforwardly to incorporate this fact. Another direction is to include other factors such as volume into the model. It will require a more complicate model and we will leave this issue for future work.

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