Introduction to Astrophysics & Cosmology Equations

This is a list of equations which you should know (or be able to derive) at the exam. It is not exclusive! You may need other equations which you can derive. The revision guide is much more complete. Note that for the definitions of the terms, you need the lecture notes. In the list, the same symbol can stand for different things in different equations (e.g. L, σ, a)! Beware of units! Basic equations such as $\rho = M/V$ or the volume of a sphere are not listed but assumed known. The symbol $\propto$ indicates that you do not need to know the value of the constant.

Newton's law

$$F = ma$$

Momentum and angular momentum

$$p = mv \quad L = r \times mv$$

Gravitational force

$$F = -\frac{GMm}{r^2}$$

Centripetal force

$$F = \frac{mv^2}{r}$$

Kinetic energy:

$$K = \frac{1}{2}mv^2$$

Potential energy

$$E_\mathrm{p} = -\frac{GMm}{r}; \quad \text{or} \quad E_\mathrm{p} = mgh$$

Escape velocity and orbital velocity

$$v_\mathrm{esc} = \sqrt{\frac{2GM}{r}}; \quad v_\mathrm{orb} = \sqrt{\frac{GM}{r}}$$

Distance and parallax

$$d = \frac{1}{p} \quad \text{d in parsec, p in arcsec}$$

Proper motion and velocity:

$$v = \mu \times 4.74 \times d$$

Flux and luminosity

$$F = \frac{L}{4\pi d^2}$$

Magnitude

$$m = -2.5 \times \log F + m_0$$

$$m_1 - m_2 = -2.5 \times \log \frac{F_1}{F_2}$$

Distance modulus:

$$m - M = -5 + 5 \times \log d \quad \text{d in parsec}$$

Photons: energy and momentum

$$E = hf; \quad p = \frac{E}{c}$$

Planck function: it will be given if you need it.
Wien's displacement law:

$$\lambda_\mathrm{max} \propto \frac{1}{T}$$

Stefan-Boltzman law:

$$F = \sigma T^4$$

Effective temperature

$$T_\mathrm{eff} = \left(\frac{L}{\sigma 4\pi R^2}\right)^{0.25}$$

Lens formula

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Lens magnification

$$M = -\frac{v}{u}$$

Telescopes:

$$M = \frac{f_o}{f_e} = \frac{\beta}{\alpha}$$

Resolution

$$\theta = 1.2\frac{\lambda}{D} \quad \text{or} \quad \frac{\lambda}{D}$$

Snell's law

$$\frac{n_2}{n_2} = \frac{\sin\theta_1}{\sin\theta_2}$$

Grating

$$d\sin\theta = m\lambda$$

Orbital period (Kepler):

$$P^2 = \frac{4\pi^2 a^3}{GM} \quad \text{or} \quad \left(\frac{P}{[\mathrm{yr}]}\right)^2 = \left(\frac{a}{[\mathrm{AU}]}\right)^3$$

$$\text{or} \quad P^2 = \frac{4\pi^2 a^3}{G(M_1 + M_2)}$$

$$v_s = \frac{M_p}{M_s}\sqrt{\frac{GM_s}{r_p}}$$

Temperature of a planet

$$T = \frac{1}{2r^{1/2}}\left(\frac{fL}{\pi\sigma}\right)^{1/4}$$

Virial theorem

$$E_{\mathrm{p}} + 2E_{\mathrm{k}} = 0 \quad \text{or} \quad Ep + 2E_{\mathrm{th}} = 0$$

Hydrogen line spectrum

$$E = hf = 13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\ \mathrm{eV}$$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Dynamical time scale

$$t_{\mathrm{dyn}} \approx \sqrt{\frac{1}{(G\bar{\rho}}}$$

Hydrostatic equilibrium and gravity

$$\frac{dP}{dr} = -\frac{Gm(r)\rho(r)}{r^2}$$

Ideal gas law

$$P = nkT$$

Thermal energy

$$E_{\mathrm{th}} = \frac{3}{2}NkT$$

Gravitational potential energy

$$E_{\mathrm{p}} = \int \frac{Gm(r)}{r}\rho(r)4\pi r^2\,dr \sim -\frac{GM^2}{R}$$

Kelvin-Helmholtz timescale

$$t = \frac{E_{\mathrm{p}}}{L} \approx \frac{GM^2}{RL}$$

Schwarzschild radius

$$R = \frac{2GM}{c^2}$$

Rotation curves

$$M(R) = \frac{v^2 R}{G}$$

Redshift

$$z = \frac{v}{c} = \frac{\Delta\lambda}{\lambda}$$

Hubble law

$$v = H_0 r; \qquad H_0 = 70 \pm 5\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

Hubble time

$$t = \frac{1}{H_0}$$

Critical density

$$\rho_{\mathrm{cr}} = \frac{3H_0^2}{8\pi G}$$

Scale factor

$$a = a_0\frac{r(t)}{r(t_0)}$$

$$a = \frac{1}{1+z}$$

Hubble 'constant'

$$H(t) = \frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}t}$$

Density parameter

$$\Omega_0 = \frac{\rho}{\rho_{\mathrm{cr}}} = \frac{8\pi G\rho_0}{3H_0^2}$$

Friedmann equation (know the first two terms)

$$\frac{1}{a^2}\left(\frac{\mathrm{d}a}{\mathrm{d}t}\right)^2 - \frac{8\pi G\rho}{3} - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{a^2}$$

CMB temperature

$$T = T_0(1+z) = \frac{T_0}{a}$$