THREE HOURS

A list of constants is enclosed.

UNIVERSITY OF MANCHESTER

General Physics

24th May 2013, 2 p.m. - 5 p.m.

You may answer as many questions as you can. Marks will be awarded for your best **THIRTEEN** answers

---

Electronic calculators may be used, provided that they cannot store text.

---

1. An electron with kinetic energy of 1.0 eV moves in a circular orbit of radius 0.10 mm. What is the magnitude and direction of the magnetic field?

2. A capacitor is made of two metal disks of radius 1.0 cm separated by a slab of mica of thickness 0.10 mm and relative permittivity 7.2. The capacitor is charged so the potential difference between the disks is 100 V. The disks are then connected by a wire. How much heat will be released?

3. A flat circular coil of 2.0 cm diameter and 5000 turns is held between the poles of a large permanent magnet with a magnetic field of 0.030 T. The coil is then withdrawn to a point well outside the magnetic field at a constant speed of 0.50 ms^{-1}. Estimate the maximum e.m.f. that could be induced in the coil.

You may assume that the magnetic field lines are always perpendicular to the coil and that the field falls sharply to zero at the edge of the magnet.

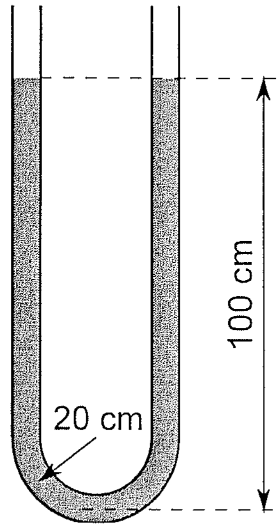
4. The television transmitter for Manchester is on Winter Hill near Bolton. Assuming that this is 30 km away and that the signal is transmitted in all directions with a power of 100 kW, calculate the r.m.s. amplitude of the corresponding electric and magnetic fields in Manchester.

5. A ball of mass M and radius R is a solid sphere made of metal. Show that its moment of inertia for rotation about any axis through the centre is $\frac{2}{5}MR^2$. The ball has a mass of 1.0 kg and radius 10 cm, and rolls without slipping down an incline of length 1.0 m, making a 30° angle with the horizontal. What is its speed at the bottom of the incline?

6. In 1947 G.I. Taylor estimated the energy of an atomic bomb explosion using the method of dimensions applied to a set of photographs that showed the propagation of the shockwave. Relate the energy released to the radius of the shockwave, the density of air and the time elapsed since the explosion. At $t = 0.006$ s the radius of the shock wave was approximately 80 m. Estimate the energy released in the explosion assuming the density of air 1.2 $\mathrm{kg\,m^{-3}}$.

7. An electron is injected (at negligible initial velocity) into a uniform electric field of 10 $\mathrm{V\,cm^{-1}}$. What distance should it travel for its de Broglie wavelength to become equal to 1.0 Å?

8. A thin U-shaped tube as shown in the diagram below is made of two vertical pipes connected by a semi-circular pipe. It is filled with water up to the height of 1.0 m. What is the period of small vertical oscillations of the water level?



9. Estimate the time taken to increase the temperature of a copper wire, with cross-sectional area 1.00 mm^2 carrying a current of 30 A, from room temperature (300 K) to its melting point of 1358 K. The conductivity of copper is $5.96 \times 10^7\,\Omega^{-1}\mathrm{m}^{-1}$, the density is $8960\,\mathrm{kg\,m^{-3}}$ and its specific heat capacity is $0.40\,\mathrm{kJ\,kg^{-1}\,K^{-1}}$. Assume that all these properties are independent of temperature and that the wire is perfectly insulated, that is, there is no heat loss.

10. Two rockets, A and B, depart from the Earth in opposite directions at speeds of $0.6c$. After one year as measured in the Earth's rest frame, rocket B emits a light signal. At what time after the departure from Earth does an observer on rocket A record the arrival of the signal?

11. A galaxy can be modelled as a sphere of uniform density ρ and radius R. Find expressions for the velocity $v(r)$ of a star in a circular orbit of radius r about the galactic centre, considering the cases $0 < r < R$ and $r > R$. Make a sketch of $v(r)$ and suggest why its observed behaviour for $r > R$ might be different.

12. Show that the wave function $\psi(r, \theta, \phi) = A\exp(-r/a_0)$ is an eigenstate of the time-independent Schrödinger equation for the hydrogen atom. Hence find the value of a_0 and the energy E. Without computing its value, explain how one would determine the value of the normalisation constant A?

You may use $\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\theta}\left(\sin^2\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$.

13. A 1.0 kg block of copper, with a specific heat capacity of $385\,\mathrm{J\,kg^{-1}\,K^{-1}}$ and a temperature of 100°C, is thrown into a lake at 10°C. Calculate the change of entropy of the block. What is the change of entropy of the universe as a result of this process?

14. Estimate the energy per day that the human body needs to maintain a body temperature of 37°C in a room with background temperature of 17°C if the dominant process is radiative loss from bare skin. Assume that the surface area of the body is $1\,\mathrm{m}^2$ and write your answer in kilo-calories where $1\,\mathrm{kcal} = 4.18 \times 10^3\,\mathrm{J}$. Compare this to the work done against gravity by a person of mass 100 kg in climbing a distance of 100 m.

15. A small boat floats in a rectangular water tank of cross-sectional area $0.040\ \mathrm{m}^2$. The boat is initially loaded with a piece of iron of volume $1.0 \times 10^{-5}\ \mathrm{m}^3$. The iron is then removed from the boat and placed into the water, whereupon it promptly sinks to the bottom of the tank. Calculate the change in the depth of the water in the tank. You may assume that the density of water is $1000\ \mathrm{kg\ m^{-3}}$ and that of iron is $7900\ \mathrm{kg\ m^{-3}}$.

END OF EXAMINATION PAPER