THREE HOURS

A list of constants is enclosed.

UNIVERSITY OF MANCHESTER

General Physics II

3rd June 1998, 9.45 a.m. - 12.45 a.m.

THREE HOUR CANDIDATES

Answer FOUR questions,
ONE question out of each section and any other.

ONE HOUR CANDIDATES
(Maths/Physics and Chemistry/Physics)
Answer any ONE questions.

Use a SEPARATE answer book for each question.

Electronic calculators may be used, provided that they cannot store text.

The numbers indicate the relative weights of the different parts of each question and do NOT represent a marking scheme.

SECTION A

A1. How is the momentum operator defined in quantum mechanics? Derive the un-normalised wavefunction for a particle moving with constant momentum P_x in the x direction.

[4 marks]

A particle with mass m is confined by a one-dimensional infinite potential well

$$V(x) = 0 \qquad 0 < x < a$$

$$V(x) = \infty \quad x \leq 0 \text{ and } a \leq x.$$

Write down the time-independent Schrödinger equation for the particle and find the normalised wavefunctions $\phi_n(x)$.

[10 marks]

What results could be found on making a measurement of P_x when the particle is in the ground state $\phi_1(x)$?

[4 marks]

The same particle is placed in a larger well of width L with state wavefunctions $\Phi_n(x)$:

$$V(x) = 0 \qquad 0 < x < L$$

$$V(x) = \infty \qquad x \leq 0 \text{ and } L \leq x.$$

The particle is initially in a state corresponding to the ground state of the original well

$$\psi(x) = \phi_1(x) \qquad 0 < x < a$$

$$\psi(x) = 0 \qquad a \leq x \leq L \text{ and elsewhere.}$$

Describe how you could calculate the probability of the particle being found in the state $\Phi_n(x)$.

[4 marks]

What results could now be found on making a measurement of P_x?

[3 marks]

A2. Define the mean free path of an atom or molecule in a gas and show how it is related to the collision cross section.

[8 marks]

In kinetic theory the thermal conductivity κ of a noble gas such as helium is given by the expression

$$\kappa = \frac{1}{2} n k_B \lambda \bar{c},$$

where n is the number density of the atoms, k_B Boltzmann's constant, λ the mean free path and $\bar{c}$ the mean speed of the helium atoms. Show that this expression leads to the prediction that the thermal conductivity at constant temperature is independent of pressure. How does the thermal conductivity depend on temperature for a perfect gas?

[8 marks]

A cylinder 10 cm long, with external diameter 5 cm, is thermally isolated and is placed inside a concentric tube with internal diameter 6 cm. The tube is held at a constant temperature of 4 K. The gap between the tube and the cylinder is filled with helium gas. Estimate the rate of heat transfer from the cylinder to the tube when the cylinder is at 5 K. (The diameter of the helium atom is 0.14 nm.)

[9 marks]

A3. A thin plano-convex lens forms a 'sharp' image of an object in a narrow range of wavelengths. Explain why the quality of the image declines as the wavelength range is increased.

[8 marks]

A thin lens of diameter 2 cm, and paraxial focal length 100 cm, forms a real image in monochromatic light on to a photographic film which is placed perpendicularly to the optic axis. The object is 200 cm from the lens. If the photographic film has grain sizes of 20 μm across how accurately must it be placed along the optic axis before the blurring effect of defocussing becomes apparent in the image? Ignore diffraction.

[10 marks]

Give two good reasons that could arise for using a concave mirror rather than a lens to form an image.

[7 marks]

A4. A slab of dielectric is placed in an external electric field $\mathbf{E}$ such that $\mathbf{E}$ is normal to the faces of the slab. By considering the surface polarisation charges, show that the polarisation vector may be expressed as:

$$\mathbf{P} = (\epsilon - 1)\epsilon_0 \, \mathbf{E}_{int},$$

where ϵ is the relative permittivity of the dielectric, and $\mathbf{E}_{int}$ is the electric field inside the dielectric.

[12 marks]

Carbon tetrachloride (CCl_4) is a liquid that has a relative permittivity of 2.24 and a density of 1600 kg m^{-3}. It can sustain an electric field of 10^7 V m^{-1} before electrical breakdown occurs. Use this value of the electric field to estimate the magnitude of the charge displacement caused by the electric field. Comment on the value obtained.

[13 marks]

C and Cl have atomic number 6 and 17 respectively; and the molecular weight of carbon tetrachloride can be taken to be 156.

SECTION B

B5. The sodium atom ($Z = 11$) has electron configurations $(1s)^2(2s)^2(2p)^6(n\ell)$ corresponding to closed shells plus one further electron. Sketch an energy level diagram showing the gross structure of the $(n\ell)$ states with $n = 3$ and 4.

[8 marks]

Indicate on your sketch how the level energies compare with the hydrogenic $(n\ell)$ values. Make an estimate of the ionization potential (in eV) of the highest level in your diagram.

[5 marks]

Indicate on your sketch the transitions between these states that are allowed by the electric dipole selection rules.

[5 marks]

Classically a photon of momentum $\mathbf{p}$ can carry away orbital angular momentum $\ell\hbar$ ($= \mathbf{r} \times \mathbf{p}$) if it were emitted a finite distance from the centre of the atom. Estimate this distance for a photon of energy 2eV for $\ell = 1$. What is the implication of this result for atomic transitions?

[7 marks]

B6. Calculate the work required to charge a capacitor, of capacity C, to a potential difference V.

[8 marks]

A sphere of radius R has a constant charge density ρ, uniformly distributed throughout the volume of the sphere. Calculate the electric potential at any point outside the sphere.

[7 marks]

Calculate the energy required to build up this charge distribution.

[10 marks]

B7. An object of mass m is dropped from rest in a medium which offers resistance, $-kv(t)$ where $v(t)$ is the magnitude of the instantaneous velocity of the object. Assuming the gravitational force to be constant, derive an expression for the velocity and position of the object at any time t.

[10 marks]

What is the limiting value of the velocity at $t = \infty$?

[5 marks]

The combined mass of a man and a motor boat is 200 kg. The thrust of the boat is equivalent to a constant force, F, of 100 Newtons in the direction of the motion and the resistance to the motion can be expressed as -kv where v is the velocity and k is 50 kg s^{-1}. If the boat is initially at rest find an expression for the velocity of the boat at time t and estimate the limiting velocity.

[10 marks]

B8. Show that the Earth's orbital velocity at a distance R from the Sun is given by $\sqrt{GM/R}$ where M is the solar mass.

[8 marks]

The diameter of the Sun subtends an angle of around half a degree to an observer on Earth. The Earth's orbital rotation period is one year. Use these two facts to estimate the mean solar density.

[17 marks]

SECTION C

C9. Explain what is meant by *thermal equilibrium* and *reversible, isothermal* and *adiabatic* changes in thermal physics.

[10 marks]

A rubber band has the following properties: (a) when it is extended by a length ℓ its tension τ is $AT\ell$, where A is a constant and T the temperature in kelvins, (b) the internal energy of the band at extension ℓ is $2\tau\ell$ and (c) the volume of the band does not change when it is stretched. Does the band get cooler or hotter when it is stretched adiabatically? Obtain expressions for the work done when an initially unstretched band is extended reversibly and isothermally by a length ℓ and the heat absorbed when this change is carried out.

[8 marks]

The fundamental theorem of thermodynamics which applies to the band is

$$\mathrm{d}E = T\mathrm{d}S + \mathrm{d}W.$$

Use this equation to find the relation between the tension and the extension when the band is stretched adiabatically.

[7 marks]

C10. State the special relativistic expressions for energy and momentum of a particle of rest mass m_0 and velocity v.

[4 marks]

How are they related?

[3 marks]

Demonstrate that the Newtonian relationship for kinetic energy of a particle is consistent with the predictions of Special Relativity in the appropriate limit.

[10 marks]

A proton is travelling at 0.9 the speed of light with respect to the observer. What is its kinetic energy?

[8 marks]

C11. Explain the following observations:

(a) bulk gold is opaque, and highly reflective to visible light;

[7 marks]

(b) white light appears blue-green after passing through a thin film of gold;

[6 marks]

(c) bulk gold has a distinctive colour;

[6 marks]

(d) bulk gold is transparent in part of the x-ray spectrum.

[6 marks]

(The free electron plasma frequency w_p for gold is 1.34×10^{16} rad sec^{-1}.)

C12. Define and explain the terms *input impedance* and *virtual earth* when applied to the inverting configuration of an operational amplifier. Sketch a circuit showing how an operational amplifier may be used to give an output voltage corresponding to the sum of several separate current inputs.

[10 marks]

The circuit shown below represents a simple two bit digital to analogue converter (DAC), set to give an output corresponding to the binary number 01. Explain its working, and draw up a table giving the voltage output for the four possible combinations of switch settings.

[15 marks]