THREE HOURS

A list of constants is enclosed.

UNIVERSITY OF MANCHESTER

General Physics - Traditional

4th June 1997, 9.45 a.m. - 12.45 a.m.

THREE HOUR CANDIDATES
Answer **FOUR** questions
ONE question out of each Section and any other

ONE HOUR CANDIDATES
(Maths/Physics and Chemistry/Physics)
Answer Any **ONE** question

Use a SEPARATE answer book for each question

Electronic calculators may be used, provided that they cannot store text.

The numbers indicate the relative weights of the different parts of each question and do **NOT** represent a marking scheme.

P.T.O.

SECTION A

A1. Derive an expression for the Moment of Inertia of a solid cylinder about its symmetry axis.

[7 marks]

A solid steel cylinder of mass 720 kg rolls down a ramp inclined at $30°$ to the horizontal. What is the acceleration of the centre of mass of the cylinder down the slope if it rolls without slipping?

[9 marks]

Calculate the rotational kinetic energy of the cylinder about its centre of mass when it has rolled 5 meters down the slope. How much work is done by the frictional force which prevents the cylinder from slipping?

[9 marks]

A2. Give a qualitative explanation of the behaviour of a dielectric material when it is placed in an external electric field. Hence explain why the electric field between the plates is greater when the space between the plates of a parallel plate capacitor is empty than when it is filled with a uniform dielectric material.

[8 marks]

Define the polarisation vector P and the displacement vector D. By applying Gauss' Law to a parallel plate capacitor, obtain the relation between D and the free charge density σ_f per unit area on the capacitor plates when it is filled with the dielectric material.

[8 marks]

A parallel plate capacitor has square plates of side a separated by a distance d. The plates carry free charge densities $\pm\sigma_f$ per unit area. Calculate the energy of the capacitor when empty and when completely filled with the dielectric material. How do you reconcile the energy difference with energy conservation?

[9 marks]

A3. An image of an object at infinity is formed by a thin plano-convex lens of focal length f with the convex surface facing the object. How far from the lens is the image formed for light that is parallel and close to the optic axis (i.e. within the paraxial approximation)? If the lens is turned round so that the flat surface faces the object where is the image now formed. Give reasons for your answer.

[5 marks]

Two thin lenses of focal lengths f_1 and f_2 respectively are separated by a distance t. Show that their equivalent focal length, f_e is given by

$$1/f_e = 1/f_1 + 1/f_2 - t/(f_1 f_2)$$

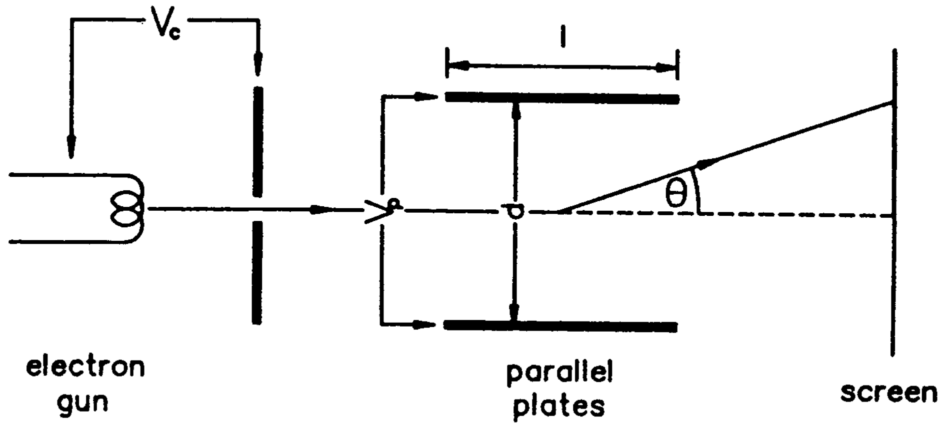
[10 marks]

again within the paraxial approximation.

The formula above applies to both mirrors and lenses. You are given i) a concave mirror of focal length 20 metres with a centrally placed hole and ii) a convex mirror of focal length −6 metres. How would you combine them to form a telescope with equivalent focal length approximately 120 metres. Draw a sketch of your answer.

[10 marks]

A4. One hundred years ago this year, J.J. Thomson discovered the electron using the apparatus sketched in the figure. The electrons are accelerated through a potential difference $+V_c$ (a few hundred) volts and then travel towards a screen, where they are detected.



(a) An electric field is produced by applying a potential difference V_p volts across a pair of parallel plates of length l and separated by a distance d. This causes the electrons to be deflected by an angle θ with respect to their original direction. Show that θ is given by the expression:

$$\tan\theta = \frac{eV_p l}{mdv^2},$$

where e, m and v are, respectively, the charge, rest mass and speed of the electrons.

[9 marks]

(b) The experiment is repeated using a beam of protons (hydrogen nuclei) accelerated through a potential difference of $-V_c$ volts. Is the magnitude of the deflection angle smaller than, greater than or equal to that obtained with electrons? Explain why.

[6 marks]

(c) A magnetic field of strength B perpendicular to the electron flight direction is applied to exactly the same portion, length l, of the electron path. The value of B is adjusted so that the resulting force on the electrons exactly balances that produced by the electric field from the parallel plates. Derive an expression for e/m in terms of B, V_p, d, l and θ.

[6 marks]

(d) Discuss briefly one source of systematic uncertainty you think might limit the accuracy with which the value of e/m could be determined using this method.

[4 marks]

SECTION B

B5. A charged pion at rest decays into a muon and a neutrino ($\pi^+ \rightarrow \mu^+ + v_\mu$).

Calculate the energy and the momentum of the muon (in the pion rest frame).

[6 marks]

Now consider the decay, $\pi^+ - \mu^+ + v_\mu$, where the pion is moving in the lab with speed 0.6 c.

Calculate the energy and the momentum of the pion in the lab frame.

[6 marks]

In what directions would the muon and neutrino travel if the energy of the muon is to be: (a) maximized; (b) minimized?

[4 marks]

Hence calculate the maximum energy and the minimum energy of the muon (in the lab frame).

[9 marks]

(You should take the rest mass of the π^+ to be 140 MeV/c^2, that of the μ^+ to be 106 MeV/c^2 and the v_μ to be massless.)

B6. What is the physical interpretation of a quantum mechanical expectation value?

[5 marks]

The normalized wavefunction describing an electron in the ground state of a hydrogen atom is

$$\psi = \frac{1}{\sqrt{\pi a_0^3}} \exp(-r/a_0)$$

where r is the distance from the nucleus and a_0 is the Bohr radius.

Calculate: (a) the expectation value of r; (b) the most probable value of r.

[10 marks]

The wavefunction of an electron when it is in one of the excited states of a hydrogen atom may be expressed as

$$\psi = A\, z \exp(-r/(2a_0))$$

where A is a constant.

Show that this wavefunction corresponds to a state with a definite z-component of angular momentum, L_z.

[10 marks]

$$\left[\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}, \quad \text{for interger } n \text{ and } a > 0\right]$$

B7. Write notes on three of the following:

(a) What is a metal?

(b) The value of band models in describing material properties;

(c) Thermal and electrical conductivity in insulators;

(d) Electrical and electro-optical properties of semi conductor junctions.

[25 marks]

B8. What is the physical reason for the terms a and b in van der Waal's equation for an imperfect gas

$$\left(p + a/V^2\right)\ (V - b) = nRT\ ?$$

[9 marks]

From this equation obtain an expression for the isobaric coefficient of thermal expansion

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p .$$

[8 marks]

Using van der Waal's equation together with the thermodynamic relations $dE = TdS + PdV$ and

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

show that

$$\left(\frac{\partial E}{\partial V}\right)_T = \frac{a}{V^2} .$$

[8 marks]

SECTION C

C9. Write down a mathematical representation of a Fourier transform and its inverse.

[6 marks]

Give without derivation the Fourier transforms for the following functions. Factors of order unity may be ignored.

(a) $f(x) = A \exp\left(-x^2/2\, a^2\right)$
(b) $f(x) = A$ for $-a/2 < x < a/2$
$= 0$ otherwise.

[9 marks]

In the above A is a real constant.

Illustrate two physical applications of (a) and one of (b). Also give another example of the application of Fourier transforms in Physics.

[10 marks]

C10. Sketch the intensity of radiation emitted by a black body as a function of frequency at two different temperatures. State how the area under the curve varies with the temperature of the black body.

[7 marks]

Assume the Sun to be a black body radiator at 5800 K and calculate the total energy flux incident on the Earth in Watts. Assume that all the sunlight incident on the Earth is absorbed and hence estimate the total radiation pressure exerted on the Earth. By what factor does the force due to radiation pressure differ from the gravitation force that the Sun exerts on the Earth?

[9 marks]

A dust grain in the solar system has a similar density to the Earth. Hence estimate its size if the forces on it due to solar radiation pressure and solar gravity are to be comparable.

[9 marks]

C11. Write a short account of energy production in stars.

[6 marks]

Assume that the Sun is powered by the $p-p$ chain, which can be summarised as

$$4p \rightarrow {}^4He + 2\nu_e \, .$$

The neutrinos (ν_e) each have an average energy of 13 MeV. These reactions should result in a flux of 1370 W m^{-2} of energy carried by neutrinos at the Earth. Calculate the expected number of neutrinos leaving the Sun's core per second. You may assume that the Sun is transparent to neutrinos.

[7 marks]

On Earth a detector consisting of 1000 kg of gallium (^{71}Ga) can detect neutrinos through the reaction

$$\nu_e + {}^{71}Ga \rightarrow {}^{71}Ge + e^- \, .$$

The cross-section for this reaction is 10^{-47} m^2. Estimate the number of gallium nuclei in the detector, and hence the expected number of ^{71}Ge nuclei produced every day.

[8 marks]

The actual number of ^{71}Ge nuclei produced is 2.8 per day; comment on this value. [Radius of Earth's orbit = 1.5×10^{11} m.]

[4 marks]

C12. State Ampere's law and Faraday's law of induction.

[8 marks]

An infinitely long straight wire and a ten turn rectangular coil lie in a place. The sides of the coil parallel to the wire are of length a while the other two sides are of length b. The side closest to the wire is at a distance d from it. Show that the mutual induction between the wire and the coil is given by:

$$M = \frac{5\mu_o a}{\pi} \, ln \left[1 + \left(\frac{b}{d}\right)\right] .$$

[9 marks]

A form of high voltage generator is shown in the diagram below. The inner coil is of length 0.5 m, of cross sectional area 10 cm^2 and is closely wound with 1000 turns. The outer coil of 3 turns is closely wound around the centre of the inner coil. Calculate the e.m.f. induced in the outer coil if the current in the inner coil changes uniformly from -5 A to $+5$ A in 1 μs.

[8 marks]