THREE HOURS

A list of constants is enclosed

UNIVERSITY OF MANCHESTER

General Physics

24th May 2016, 2.00 p.m. – 5.00 p.m.

Answer as many questions as you can.
Marks will be awarded for your **THIRTEEN** best answers.

Each question is worth 10 marks.

Electronic calculators may be used, provided that they cannot store text.

P.T.O.

1. An electron with a kinetic energy of 1 eV moves in a circular orbit of radius 0.1 mm due to a magnetic field. What is the magnitude and direction of the magnetic field?

2. A sealed 200 g cup of tea is left to cool from boiling to room temperature (17 °C). What is the change in entropy of the universe due to this process?

   The specific heat of tea is $4.2\,\mathrm{kJ\,K^{-1}\,kg^{-1}}$.

3. A particle physics experiment is searching for a hypothesised rare event signal. To claim discovery, an excess of events (presumably due to the hypothesised rare signal) must be observed above background prediction, with a significance of at least 5 σ.

   The expected number of background events from known particle processes in this experiment, for one year of running, is 123 events. Assume that the background rate can be predicted with infinite accuracy, and that we expect a rare event signal rate of 100 events per year. How long would the experiment need to run, in order to have enough sensitivity to claim a discovery?

4. When studying a thick piece of glass in vacuum, you notice that no front-face reflections are observed for one polarisation of light when the glass is held at an angle of 60° to the incoming light. What is the fraction of power reflected for unpolarised light when the glass is held perpendicular to the incoming light?

5. A cylindrical bucket has a cross-sectional area of A and is filled with water to a depth d. A small hole of area S is made in its base. How long does it take for all of the water to leave the bucket?

6. Two rockets are drifting towards each other on a collision course. The rockets are each 100 m long. As measured by an observer on Earth, rocket 1 has a speed of $0.800c$, rocket 2 of $0.600c$ in the opposite direction, and initially they are 1.26×10^{12} m apart.

   According to the observer on Earth, how long does it take for the rockets to collide? If it takes 45 mins for each crew to evacuate their rocket, will they have enough time?

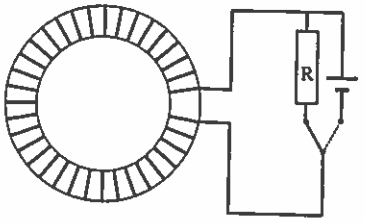
7. Consider the orbit of the Earth around the Sun, and approximate it as circular. Suppose that the Sun very slowly loses its mass, from an original mass of M_1 to a mass of M_2. Suppose also that the initial radius of the orbit is R_1 and the eventual one is R_2.

   Derive an expression for R_2 in terms of the other given parameters.

   Use the luminosity of the Sun to estimate the rate at which it loses mass, and hence the rate of change of the Earth's orbital radius. Express your answers in kilograms per year and in meters per year, respectively.

P.T.O

8. A cylindrical iron ring with $\mu_r = 1000$ of cross section 1 cm^2 and radius 10 cm is tightly wound with a conducting wire (100 windings per cm) as in the diagram.



The ring is magnetised by running a current of 1 A for a long time. The switch is then flipped, so that the battery is disconnected and replaced by the resistance $R = 100\ \Omega$. It is found that the magnetisation decays to zero exponentially, as $e^{-t/\tau}$. Find τ and hence the current that flows through the resistor at time t.

9. In one dimension, a particle moves freely inside an infinite square well of size a, i.e. its wavefunction is zero for $x < 0$ and $x > a$. What is the normalized ground state wavefunction?

If the particle has the normalized wavefunction:

$$\begin{aligned} \psi(x) &= 2\left(\frac{2}{3a}\right)^{1/2} \sin^2\left(\frac{\pi x}{a}\right) \qquad \text{for } 0 < x < a, \\ &= 0 \qquad\qquad\qquad\qquad\qquad\quad \text{otherwise,} \end{aligned}$$

what is the probability that the particle will be found in the ground state?

10. A cylinder of length L, radius R and mass density ρ rolls on a horizontal surface without slipping. A hole of radius $r \ll R$ has been drilled through the cylinder, with its centre a distance $R/2$ from the axis of the cylinder, parallel to the axis.

Consider the orientation of the cylinder specified by the angle θ between the vertical direction and a line connecting the centers of the cylinder and the hole.

If the cylinder is initially left at rest at a small angle $\delta\theta$ from its position of equilibrium ($\theta = 0$), show that the cylinder will exhibit simple harmonic motion. Find the corresponding frequency, ω.

11. The LHC collides pairs of protons each with a laboratory energy of 6.5 TeV. What is the heaviest particle that can be produced?

    What would your answer be if we were to collide one of these protons with another proton at rest?

12. An astronomer notices that a spectral line at 575.05 nm, resulting from a transition from a p state to an s state, is split into three lines, each separated by $\Delta\lambda = 0.0309$ nm. Estimate the strength of the external magnetic field causing the splitting. Ignore the spin quantum number.

13. A radio frequency signal at 100 MHz is split in half and sent along two lossless coaxial wires of equal length l, one with a mylar dielectric ($\epsilon = 3.1$) and one with a polystyrene dielectric ($\epsilon = 2.7$). When the wires are combined at their ends, no signal is observed. What is the minimum value of l that can explain this observation?

14. A galaxy can be approximated by a sphere of uniform density ρ and radius R. Find expressions for the velocity $v(r)$ of a star in a circular orbit of radius r about the galactic centre, considering the cases $0 < r < R$ and $r > R$. Make a sketch of $v(r)$ and suggest why its observed behaviour for $r > R$ might be different.

15. The pressure of water vapour over ice is 517 N/m^2 at -2°C and 611 N/m^2 at 0°C. What is the latent heat of vaporization of one mole of ice at a temperature of -1°C?

    You may use the fact that the Gibbs free energy, G, of a PVT system can be defined by the relation

    $$\mathrm{d}G = V\mathrm{d}p - S\mathrm{d}T\,,$$

    where S is the entropy, V is the volume, p is the pressure and T is the temperature and that, for two phases in equilibrium, the specific Gibbs free energy is the same in each phase.

END OF EXAMINATION PAPER