<u>THREE HOURS</u>

A list of constants is enclosed.

UNIVERSITY OF MANCHESTER

General Physics (Trad)

27th May 2002, 9.45 a.m. - 12.45 p.m.

THREE HOUR CANDIDATES
Answer any **<u>FOUR</u>** questions

ONE HOUR CANDIDATES
(Maths/Physics and Physics with Business and Management)
Answer **<u>ONE</u>** question from questions 1-6

Electronic calculators may be used, provided that they cannot store text.

The numbers are given as a guide to the relative weights of the different parts of each question.

1. Explain why the maximum force that can be produced by braking a car without skidding is independent of its speed. [3 marks]

Show that the shortest stopping distance D (as measured from the point at which the driver starts braking) for a car travelling with speed v is proportional to v^2. [3 marks]

Under such circumstances a car travelling at a speed of 50 km/h stops in a distance of 15 m. Find the constant of proportionality and show that a car travelling at 125 km/h will stop in 94 m. [3 marks]

Consider a single stream of traffic travelling at uniform and constant speed. Suppose that each driver leaves a gap between the front of their car and the rear of the car in front equal to the stopping distance $D(v)$. Taking the length L of a car as 4 m, calculate the current J (i.e. the number of cars per second passing a given point) as a function of v. [5 marks]

Find the speed v (in km/h) for which J is a maximum. [6 marks]

How is this result affected if the formula for stopping distance is modified to include a term describing the finite reaction time of the driver? [5 marks]

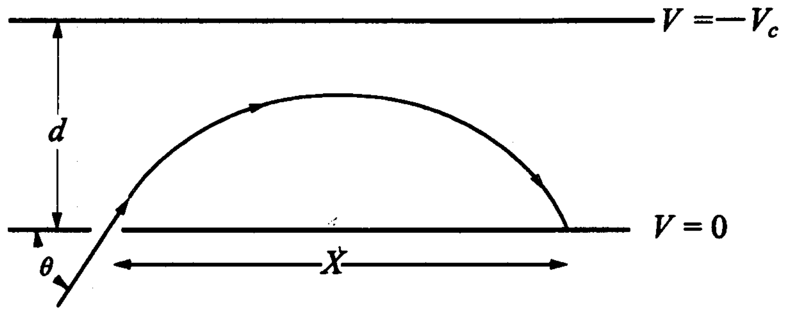
2. It is the year 2042, and you are the Minister for Science in the newly-formed republic of North-West England, which is surrounded on all sides by hostile states (Scotland, Wales, Greater Birmingham and the Yorkshire Democratic Republic).

In your first cabinet meeting, you are apprehensive that one or more of these states may wish to build nuclear weapons. Write a **memorandum** detailing

- How nuclear weapons work.
- What technical facilities are required to build them.
- What steps the intelligence services should take to gather evidence to determine whether they are being developed.

Some of the states have a nuclear power programme and some do not, but all are under suspicion.

3.



Non-relativistic electrons of kinetic energy E are injected at an angle θ through a small hole in one of two parallel plates. A cross section of the arrangement is shown schematically above. The separation of the plates is d, which is small compared to the length and width of the plates. The upper plate is held at potential $-V_c$ and the lower plate is earthed. The electrons are deflected so that they strike the lower plate at a distance X from the hole. [You may neglect the force of gravity.]

(a) Write down the magnitude and direction of the force on an electron between the plates. [2 marks]

(b) Find an expression for X in terms of the parameters given. [11 marks]

(c) Find θ_0, the value for which X has its maximum value, X_0 and show that $X_0 = \frac{2d}{eV_c} E$. [5 marks]

(d) Electrons are injected with angles that lie within a range $\Delta\theta$ around θ_0, where $\Delta\theta \ll \theta_0$. Explain why, to first order in $\Delta\theta$, they are all brought to a focus at X_0. [3 marks]

(e) A slit of width $w (w \ll X_0)$ is placed at X_0. Deduce an expression for the range of kinetic energies ΔE that would be transmitted by the device. Hence show that the energy resolution is

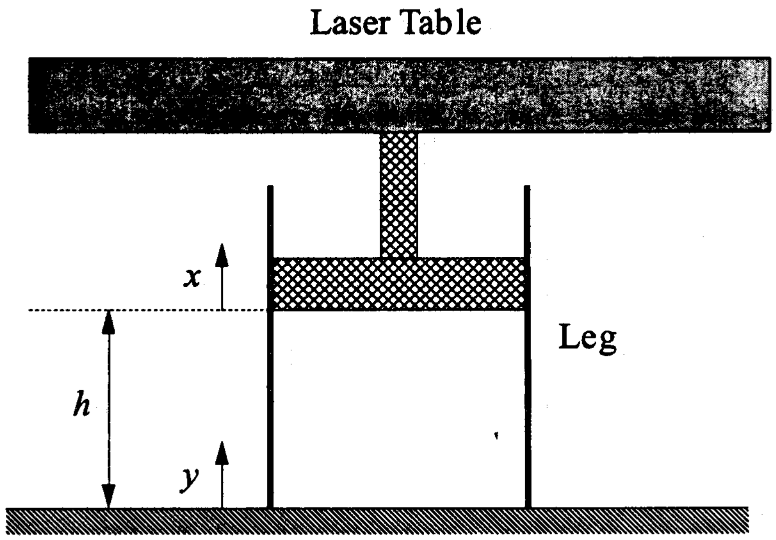
$$\Delta E/E = w/X_0 \ .$$

[4 marks]

4. The equation of motion of a damped harmonic oscillator has the form,

$$m\, d^2x/dt^2 + b\, dx/dt + kx = F_0 \cos \omega t \ ,$$

where the symbols have their usual meanings. Explain the physical significance of the terms in this equation. [4 marks]



The figure shows schematically the leg of a laser table that is used to isolate the table from vibrations of the floor. It consists of a piston in a cylinder of cross sectional area A containing air at pressure p. The piston supports a mass m. At equilibrium, the height of the column of air in the cylinder is h.

The upward displacements of the piston and floor from their equilibrium positions are denoted by x and y respectively. Assuming isothermal expansion and compression of the air, and neglecting the atmospheric pressure on the piston, find an expression for the net force on the piston. Hence show that, for small displacements of the floor and piston, the equation of motion of the piston is

$$d^2x/dt^2 + xg/h = yg/h$$

and find ω_0, the natural frequency of oscillation. [10 marks]

If the floor is oscillating according to

$$y = y_0 \cos \omega t$$

describe qualitatively the motion of the piston for the cases (i) $\omega \ll \omega_0$, (ii) $\omega \gg \omega_0$, (iii) $\omega \approx \omega_0$. What do these allow you to say about the optimum height of the cylinder when the floor oscillates with angular frequency ω? [8 marks]

Explain briefly what you would need to add to this setup to produce an effective way to reduce the oscillations of the table. [3 marks]

5. (a) Write down the energy E of a relativistic particle in terms of its mass m and its momentum p. Sketch E as a function of p and describe the form of $E(p)$ (i) for $p \ll mc$ and (ii) for $p \gg mc$. Rearrange $E(p)$ to obtain an expression for the momentum of the particle in terms of its energy and mass. [8 marks]

(b) Two highly relativistic particles (with $E \gg mc^2$) have identical energies but slightly different masses. Find an approximate expression for the difference between their momenta in terms of the difference between the squares of their masses, $\Delta(m^2)$. [5 marks]

Hence show that the difference between the de Broglie wavelengths of the two particles is approximately

$$\Delta\lambda \simeq \frac{hc^5}{2E^3}\Delta(m^2).$$

[4 marks]

(c) Neutrinos are emitted by the decay of ^{7}Be in the Sun with an energy of 0.86 MeV. They are described by a superposition of waves corresponding to two kinds of neutrino with slightly different masses. These waves are in phase at the point where the neutrinos are emitted. Suppose that the rate at which the neutrinos are detected on Earth can be explained if the waves are 180° out of phase when they arrive here. Assuming that this is the first point at which the phase difference is 180°, estimate the difference between the squares of the masses, $\Delta(m^2)$, for the two kinds of neutrino. Give your answer in $(\mathrm{eV}/c^2)^2$. [8 marks]

6. Write down the time-independent Schrödinger equation for a particle moving in the vertical (z) direction in a potential $V(z)$. [Ignore any horizontal motion of the particle.] [2 marks]

Recent experiments have shown that neutrons above a horizontal mirror exhibit quantum behaviour in the Earth's gravitional field. Assume that the mirror provides a potential $V(z) = \infty$ for $z \leq 0$. Write down the potential for $z > 0$ and hence the Schrödinger equation for the wave function $\psi(z)$ of a neutron in the region $z > 0$. Explain briefly why $\psi(z)$ satisfies the boundary conditions $\psi(0) = 0$ and $\psi(z) \to 0$ as $z \to \infty$. [5 marks]

Sketch $V(z)$ and the expected shapes of the first two energy eigenfunctions of the neutron. [5 marks]

Show that by changing variables to

$$u = \frac{z - z_0}{L},$$

where

$$z_0 = \frac{E}{mg} \quad \text{and} \quad L = \left(\frac{\hbar^2}{2m^2 g}\right)^{\frac{1}{3}},$$

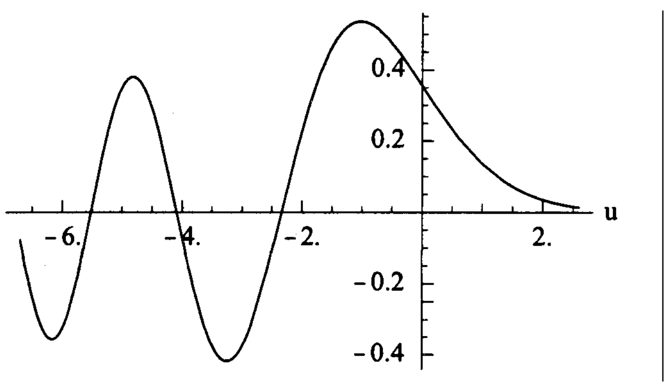
and E is the energy, the Schrödinger equation can be rewritten in the form

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}u^2} - u\,\psi = 0\ . \tag{1}$$

[4 marks]

Verify that L has the dimensions of length. Find the value of L in μm. [3 marks]

The figure below shows the solution to equation (1) that satisfies the boundary condition on $\psi(z)$ as $z \to \infty$. By considering the boundary condition on $\psi(z)$ at $z = 0$, use this figure to estimate the most likely height above the mirror of a neutron in its ground state. Estimate also the energy in peV of this state. [6 marks]



7. Consider a point at a distance r from the centre of the Earth, lying directly between the Earth and the Sun. What is the magnitude and direction of the combined gravitational force of the Sun (mass M) and the Earth (mass m), acting on a mass at this point? Assume that the distance from the Sun to the Earth R is a constant. [3 marks]

Assuming that $m \ll M$ and hence $r \ll R$, show that it is possible for a spacecraft to follow a circular orbit around the Sun with the same angular velocity as the Earth, if it lies directly between the Earth and the Sun at a distance

$$r = R\left(\frac{m}{3M}\right)^{1/3}$$

from the Earth. Calculate the value of r.

[11 marks]

A spacecraft at this point transmits data at a frequency of 10 GHz to groundstations distributed over the whole Earth. Estimate the optimum size of the spacecraft's antenna required to minimise the transmitter power. [7 marks]

One of the groundstation antennae has a diameter of 30 m. Estimate the power received when the spacecraft transmit 10 W from an antenna with the optimum size.

[4 marks]

8.
Note:- This question requires some numerical manipulations with a calculator.

The voltage across a resistor was measured for several different currents in order to determine its resistance. The following appeared in the students laboratory logbook:-

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We think we can read the voltages to 0.8 V.
i.e. the S.D of the measurements is 0.8 V.

The ammeter has negligible error compared with the voltmeter.

Current            Voltage                                  added later
 amps (x)           volts (y)                               15.64 x   delta
 0.20                2.4                                      3.13     0.73
 0.40                4.4                                      6.26     1.86
 0.60                8.8                                      9.38     0.58
 0.80                9.6                                     12.51     2.91
 1.00               19.2 (Ouch! it's getting hot!)           15.64    -3.56
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The *Method of Least Squares* gives

$$a = \frac{\langle xy \rangle}{\langle x^2 \rangle} \quad \text{with} \quad \sigma_a^2 = \frac{\sigma^2}{N\langle x^2 \rangle}$$

for fitting a line of the form $y = ax$. Here, N is the number of points and σ is the S.D. of each measurement and it is assumed that all the points have the same weight.

What is the *Method of Least Squares* ? [4 marks]

Evaluate the slope of the best fitting line of the form $y = ax$. (Use a calculator as needed, but show your working in sufficient detail that it can be followed.) [4 marks]

Explain what is meant by χ^2 ? [4 marks]

Evaluate the χ^2 of the fit. (The column headed 'added later' is useful.) Comment on the value of χ^2. [4 marks]

Discuss whether or not the assumption that the S.D. of the voltage measurements is 0.8 V is justified. [4 marks]

Having considered all the data and the quality of the fit, give a value and accuracy (error) for the resistance. [5 marks]

Sketch any graphs using the lined paper of the answer books.

9. A ballistic galvanometer is very similar to an ordinary galvanometer except that the damping is minimised. It consists of a rectangular coil, with sides of length a (vertical) and b (horizontal), with n turns of fine wire. The coil is initially in the xz plane and it is suspended so that it can rotate about a vertical axis (the z direction) through its centre. A restoring torque is provided by the suspension (either a spiral spring or a torsion wire). There is a magnetic field, B, in the x direction provided by a permanent magnet. See the diagram.

Show that the torque on the coil when a current i is flowing through it is $\Gamma = abniB$. [5 marks]

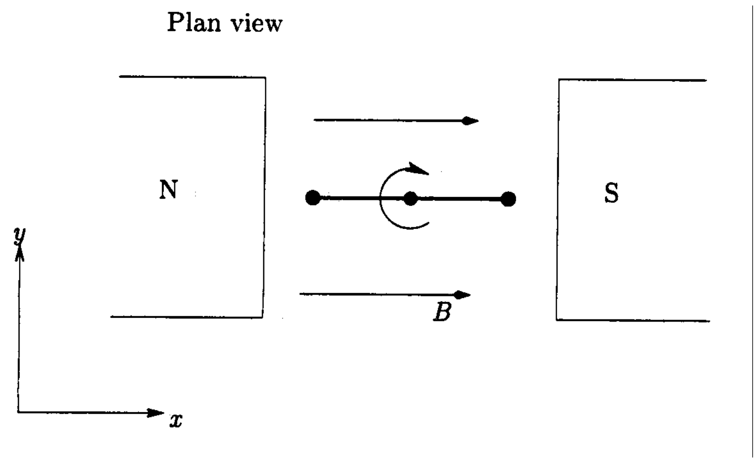
One way to measure the capacitance of a capacitor is to charge it to a known voltage and then discharge it through a ballistic galvanometer.

A capacitor that has been charged to a voltage V is connected to the coil. Assume that the coil was initially stationary, in the xz plane, with no current passing through it and with zero restoring torque from the suspension. By assuming that the capacitor discharges in a *short* time derive an expression for the angular velocity acquired by the coil. Hence show that the capacitance, C, is proportional to the amplitude, θ_{max}, of the subsequent swing and is given by

$$C = \alpha \frac{\theta_{max}}{V}, \qquad \text{where} \qquad \alpha = \frac{\sqrt{k\mathcal{I}}}{abBn}.$$

Here, $\mathcal{I}$ is the moment of inertia of the coil and k is the torsional constant of the suspension. [12 marks]

One way of calibrating the galvanometer is to measure the period, T, of free torsional oscillations and the steady deflection, θ_0, produced by a steady known current, i_0. Show how these measurements determine the constant α. [8 marks]



10. Describe briefly what is meant by a Fourier series. Write down a general mathematical expression for a Fourier series. How does this simplify for an odd function?

[6 marks]

A square wave of period τ is described by the expression:

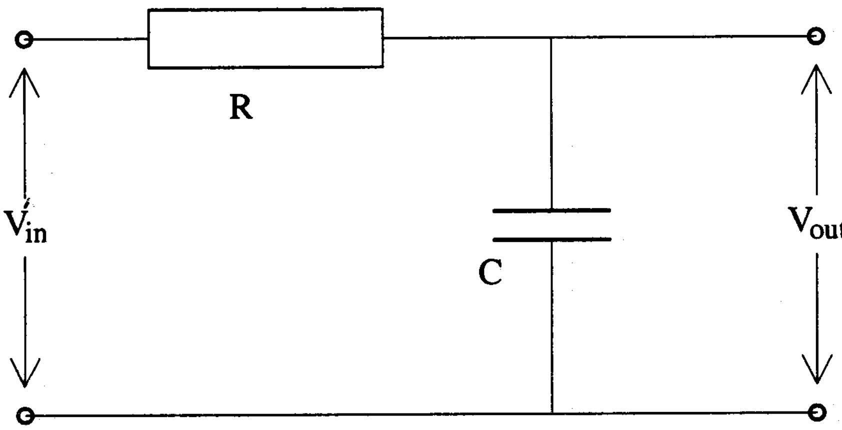
$$f(t) = \begin{cases} +1 & (0 < t \leq \tau/2); \\ -1 & (-\tau/2 < t \leq 0); \end{cases}$$

$$\text{and} \quad f(t) = f(t + n\tau). \qquad \text{for any integer } n.$$

Sketch $f(t)$, labelling the axes. Obtain the Fourier series for $f(t)$.

[10 marks]

A square wave of period 0.001 s and amplitude 1V is applied at the input to the R-C circuit shown below.



By using complex impedances or otherwise, find an expression for the output voltage V_{out}, produced by a general complex input voltage $V_{in} = V \exp(i\omega t)$.

[3 marks]

Hence calculate the amplitudes of the first three non-zero Fourier components of V_{out} with the lowest frequencies, when the above square wave voltage is applied at the input. The components have the values R = 1000 Ω and C = 0.1 μF.

[6 marks]

11. Explain the physical origin of the parameters a and b in van der Waal's equation of state for a non-ideal gas:

$$(P + a/V^2)(V - b) = nRT.$$

[6 marks]

What are the dimensions of the parameters a and b?

[3 marks]

Show that the quantity a/bRn has the dimensions of temperature (this is called the Boyle temperature).

[5 marks]

The internal energy E of a monatomic gas obeying van der Waal's equation of state is $3nRT/2 - a/V$. In a Joule-Kelvin expansion the gas is forced through a narrow tube without supplying any heat, and the enthalpy H does not change. Show that the enthalpy is

$$H = \frac{5}{2}nRT - \frac{2aP}{nRT} + bP$$

to first order. (You may assume that the parameters a and b are small and neglect terms of second order or higher, such as a^2, ab, etc.)

[6 marks]

Hence show that the Joule-Kelvin expansion produces cooling provided that the initial temperature of the gas $T < 2a/bRn$.

[*Hint*: Take differentials of the expression for H.]

[5 marks]