1 (a) $(AB)_{ij} = A_{ik}B_{kj} \Rightarrow [(AB)^T]_{ij} = A_{jk}B_{ki} = A^T_{kj}B^T_{ik} = B^T_{ik}A^T_{kj} \Rightarrow (AB)^T = B^TA$.

(b) $\mathrm{Tr}(AB) = A_{ik}B_{ki}$

by renaming indices which are summed $i \leftrightarrow k$

$\mathrm{Tr}(BA) = B_{ik}A_{ki} = B_{ki}A_{ik} = \mathrm{Tr}(AB)$

2. $\ddot{r} - r\dot{\theta}^2 = 0$ & $\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta}) = 2\omega\dot{r} \Rightarrow \frac{d}{dt}[r^2(\dot{\theta}-\omega)] = 0$

$\Rightarrow r^2(\dot{\theta}-\omega) = \text{const} = 0$ since $\dot{\theta}(0) = \omega$ & $r(0) = R$

$\Rightarrow \omega = \dot{\theta}$ & hence $\ddot{r} - \omega^2 r = 0 \Rightarrow r = R\cosh\omega t$ by imposing BC's

3. (a) $\underline{v} = \dot{\underline{r}}_1 - \dot{\underline{r}}_2 = \begin{pmatrix} 6a_1t - 4a_2 \\ -9a_3t^2 \\ 0 \end{pmatrix} - \begin{pmatrix} 6a_1t + 3a_2 \\ -9a_3t^2 \\ 4a_6 \end{pmatrix} = \begin{pmatrix} -7a_2 \\ 0 \\ -4a_6 \end{pmatrix}$

(b) $\ddot{\underline{r}}_1 = \begin{pmatrix} 6a_1 \\ -18a_3t \\ 0 \end{pmatrix} = \ddot{\underline{r}}_2 \Rightarrow \underline{a} = 0$

(c) No relative acceleration $\Rightarrow$ if one is inertial then so is the other

4. 

Minimum frictional force is that required to balance the inertial forces

$\underline{F}_{inert} = -2m\,\underline{\omega}\times\underline{v} + m\omega^2\underline{r}$ since $\underline{r}\cdot\underline{\omega} = 0$

$\underline{\omega}\times\underline{v} = \omega\hat{\underline{z}} \times v\hat{\underline{r}} = v\omega \begin{vmatrix} \hat{\underline{x}} & \hat{\underline{y}} & \hat{\underline{z}} \\ 0 & 0 & 1 \\ \cos\theta & \sin\theta & 0 \end{vmatrix} = v\omega \begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{pmatrix} = v\omega\hat{\underline{\theta}}$

$\underline{r} = l\hat{\underline{r}}$

$\Rightarrow |F_{min}| = \left[(2m\omega v)^2 + (ml\omega^2)^2\right]^{1/2} = m\omega\left[4v^2 + (l\omega)^2\right]^{1/2}$

5. $\ddot{x} = -2\dot{z}\omega\cos\lambda$, $\ddot{y} = 0$, $\ddot{z} = -g$

$\Rightarrow y = 0$, $z = h - \frac{1}{2}gt^2$ & $\ddot{x} = 2gt\omega\cos\lambda \Rightarrow x = \frac{1}{3}g\omega t^3\cos\lambda$

$z = 0 \Rightarrow t = t_F = \left(\frac{2h}{g}\right)^{1/2} \Rightarrow x = \frac{1}{3}g\omega\left(\frac{2h}{g}\right)^{3/2}\cos\lambda$

Works because (i) $t_F \ll 1\,\text{day} \sim 1/\omega$

(ii) Motion starts in z direction and increases speed in that direction which causes an increase in x, but at next order so $\dot{x}, \dot{y} \ll 1$

(iii) Centrifugal force negligible

6. $\ddot{\underline{r}}' = \underline{g} - 2\underline{w}\times\dot{\underline{r}}' + w^2\underline{r}' - (\underline{r}'\cdot\underline{w})\,\underline{w}$

$$\underline{W} = w\begin{pmatrix}0\\ \cos\lambda\\ \sin\lambda\end{pmatrix} \;;\; \underline{r}' = \begin{pmatrix}x\\ y\\ z\end{pmatrix} \;;\; \underline{g} = g\begin{pmatrix}0\\ 0\\ -1\end{pmatrix}$$

$$\underline{W}\times\dot{\underline{r}}' = w\begin{vmatrix}\hat{\underline{x}} & \hat{\underline{y}} & \hat{\underline{z}}\\ 0 & \cos\lambda & \sin\lambda\\ \dot{x} & \dot{y} & \dot{z}\end{vmatrix} = w\begin{pmatrix}\dot{z}\cos\lambda - \dot{y}\sin\lambda\\ \dot{x}\sin\lambda\\ -\dot{x}\cos\lambda\end{pmatrix}$$

$$w^2\underline{r}' - (\underline{r}'\cdot\underline{w})\,\underline{w} = w^2\begin{pmatrix}x\\ y\\ z\end{pmatrix} - w(y\cos\lambda + z\sin\lambda)\begin{pmatrix}0\\ \cos\lambda\\ \sin\lambda\end{pmatrix}$$

$$\Rightarrow \ddot{x} = -2w(\dot{z}\cos\lambda - \dot{y}\sin\lambda) + w^2 x$$
$$\ddot{y} = -2w\dot{x}\sin\lambda + w^2 y - w^2\cos\lambda(y\cos\lambda + z\sin\lambda)$$
$$\ddot{z} = -g + 2w\dot{x}\cos\lambda - w^2\sin\lambda(y\cos\lambda + z\sin\lambda) + w^2 z$$

$$\left.\begin{aligned}\alpha &= y\cos\lambda + z\sin\lambda\\ \beta &= -y\sin\lambda + z\cos\lambda\end{aligned}\right\} \Rightarrow \begin{aligned}y &= \alpha\cos\lambda - \beta\sin\lambda\\ z &= \alpha\sin\lambda + \beta\cos\lambda\end{aligned}$$

$$\Rightarrow \ddot{\alpha} = -g\sin\lambda$$
$$\ddot{\beta} = -g\cos\lambda + 2w\dot{x} + w^2\beta$$
$$\ddot{x} = -2w\dot{\beta} + w^2 x$$

$\alpha(0) = (R_E + h)\sin\lambda$

$\dot{\alpha}(0) = 0$

$\hookrightarrow \alpha = \sin\lambda\left[R_E + h - \frac{1}{2}gt^2\right]$

$Z = \beta + iX \Rightarrow \beta = \mathrm{Re}\,Z \;;\; x = \mathrm{Im}\,Z$

$$\ddot{Z} = w^2 Z + 2w(-i\dot{\beta} + \dot{x}) - g\cos\lambda = w^2 Z - 2iw\dot{Z} - g\cos\lambda$$

$$\Rightarrow Z = (A + Bt)e^{-iwt} + \frac{g}{w^2}\cos\lambda$$ NB A & B are complex.

$x(0) = y(0) = \dot{x}(0) = \dot{y}(0) = \dot{z}(0) = 0$ & $z(0) = h + R_E$ $\quad Z(0) = (R_E + h)\cos\lambda$, $\dot{Z}(0) = 0$

$$\Rightarrow A = \left(R_E + h - \frac{g}{w^2}\right)\cos\lambda \;;\; B = iwA = iw\left(R_E + h - \frac{g}{w^2}\right)\cos\lambda$$

$$\Rightarrow Z = \cos\lambda\left[\left(R_E + h - \frac{g}{w^2}\right)(1 + iwt)e^{-iwt} + \frac{g}{w^2}\right]$$

$$p = \mathrm{Re}[z] = \cos\lambda \left[\left(R_E + h - \frac{g}{\omega^2}\right)\left[\cos\omega t + \omega t \sin\omega t\right] + \frac{g}{\omega^2}\right]$$

$$q = \mathrm{Im}[z] = \cos\lambda \left(\frac{g}{\omega^2} - R_E - h\right)\cos\lambda\left[\sin\omega t - \omega t \cos\omega t\right]$$

$$\hookrightarrow \quad x = \left(\frac{g}{\omega^2} - R_E - h\right)\cos\lambda \, f_1(\omega t)$$

$$y = \cos\lambda \sin\lambda \left[\left(\frac{g}{\omega^2} - R_E - h\right) f_2(\omega t) - \frac{1}{2} g t^2\right]$$

$$z = h - \frac{1}{2} g t^2 - \cos^2\lambda \left[\left(\frac{g}{\omega^2} - R_E - h\right) f_2(\omega t) - \frac{1}{2} g t^2\right]$$

where $f_1(x) = \sin x - x\cos x$ & $f_2(x) = \cos x - 1 + x \sin x$.

7. $\underline{F} = q\left[\underline{E} + (\underline{v}' + \underline{\omega}\times\underline{r}')\times\underline{B}\right] - 2m\underline{\omega}\times\underline{v}' - m\underline{\omega}\times(\underline{\omega}\times\underline{r}')$

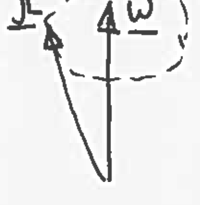
$= q\underline{E} + \underline{v}'\times(q\underline{B} + 2m\underline{\omega}) + q(\underline{\omega}\times\underline{r}')\times\underline{B} - m\underline{\omega}\times(\underline{\omega}\times\underline{r}')$

Choose $\underline{\omega} = -\frac{q\underline{B}}{2m}$ then $\underline{F} = q\underline{E} - 2m(\underline{\omega}\times\underline{r}')\times\underline{\omega} - m\underline{\omega}\times(\underline{\omega}\times\underline{r}')$

$= q\underline{E} + m\underline{\omega}\times(\underline{\omega}\times\underline{r}')$

$\Rightarrow \underline{E}_{eff} = \underline{E} + \frac{m}{q}\underline{\omega}\times(\underline{\omega}\times\underline{r}') = \underline{E} + O(B^2)$

If B is weak then $\underline{E}_{eff} \approx \underline{E} \Rightarrow$ elliptical orbits (it is a central force!) in the rotating frame with angular velocity $\underline{\Omega}$. In the non-rotating (Lab frame) $\underline{\Omega}$ precesses around $\underline{\omega}$



with $\omega = \frac{eB}{2m}$

(c) If $\omega > \Omega = \sqrt{\frac{g}{R}}$ then $\frac{g}{R\omega^2} < 1 \Rightarrow \cos\alpha_0$ exists

Write $\alpha = \alpha_0 + \delta\alpha$

$$\Rightarrow \sin(\alpha_0+\delta\alpha) = \sin\alpha_0\cos\delta\alpha + \cos\alpha_0\sin\delta\alpha$$
$$\simeq \sin\alpha_0 + \cos\alpha_0\,\delta\alpha$$
$$\cos(\alpha_0+\delta\alpha) = \cos\alpha_0\cos\delta\alpha - \sin\alpha_0\sin\delta\alpha$$
$$\simeq \cos\alpha_0 - \sin\alpha_0\,\delta\alpha$$

$$\Rightarrow \ddot{\delta\alpha} = -\frac{g}{R}\left(\sin\alpha_0 + \cos\alpha_0\,\delta\alpha\right) + \omega^2\left(\sin\alpha_0 + \cos\alpha_0\,\delta\alpha\right)\left(\cos\alpha_0 - \sin\alpha_0\,\delta\alpha\right)$$
$$\simeq -\frac{g}{R}\sin\alpha_0 + \omega^2\sin\alpha_0\cos\alpha_0 + \left[\omega^2\left[\cos^2\alpha_0 - \sin^2\alpha_0\right] - \frac{g}{R}\cos\alpha_0\right]\delta\alpha$$
$$= \omega^2\left[\cos^2\alpha_0 - \sin^2\alpha_0 - \cos^2\alpha_0\right]\delta\alpha \text{ using } \cos\alpha_0 = g/R\omega^2$$

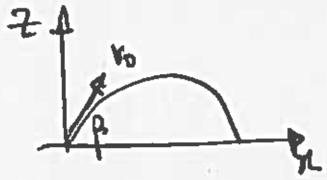
$$\Rightarrow \ddot{\delta\alpha} + \omega^2\sin^2\alpha_0\,\delta\alpha = 0$$

$\Rightarrow \alpha_0$ is stable when it exists ie when $\omega > \Omega$

9. If we assume that the Coriolis force is constant then we can estimate the change in distance due to it as

$\Delta y \approx \int\left(\int F_{corr}\,dt\right)dt$; $\frac{F_{corr}}{M} = 2\omega v_0\sin\alpha$ where α is the angle between $\underline{\omega}$ & $\underline{v}$

$\approx t^2\omega v_0\sin\alpha$



Time taken is $t \approx \frac{2v_0\sin\beta}{g}$

$$\Rightarrow \Delta y \approx \frac{4v_0^3\omega\sin\alpha\sin^2\beta}{g^2}$$
$$\approx 95\text{m} \times 4\sin\alpha\sin^2\beta$$
$$\sim 100 \text{ yards}.$$

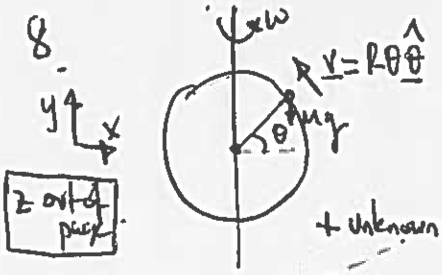
NB $\frac{v_0^3\omega}{g} \simeq \frac{(500\,\text{ms}^{-1})^3 \times 7.3\times10^{-5}\,\text{rads}^{-1}}{(9.8\,\text{ms}^{-1})^2} \simeq 95\text{m}.$

doing this we are assuming $t \ll 1$day!

To do better use the approximate solution derived in exercise 5.3

$$\underline{r} = \underline{r}_0 + \frac{\underline{v}_0}{\omega}T + \underline{A}T^2 + \dots \text{ where } T = \omega t;\ \underline{A} = \frac{\underline{g}}{2\omega^2} - \frac{\hat{\underline{\omega}}\times\underline{v}_0}{\omega} + \frac{1}{2}\left[\underline{r}_0 - (\underline{r}_0\cdot\hat{\underline{\omega}})\hat{\underline{\omega}}\right]$$

8.



$\underline{\omega} = \omega \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $\qquad$ $\underline{g} = g \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$

$\dot{\underline{v}} = R\ddot{\theta}\hat{\underline{\theta}} - R\dot{\theta}^2\hat{\underline{r}}$

with $\hat{\underline{\theta}} = \begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{pmatrix}$, $\hat{\underline{r}} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}$

+ unknown reaction force in $\hat{\underline{r}}$ & $\hat{\underline{z}}$ directions

(a) $\underline{F} = \underline{F}_{reaction} + m\underline{g} - 2m\underline{\omega}\times\underline{v} - m\underline{\omega}\times(\underline{\omega}\times\underline{r})$

$$\underline{F}_{cor} = -2m\underline{\omega}\times\underline{v} = -2mR\dot{\theta}\omega \begin{vmatrix} \hat{\underline{x}} & \hat{\underline{y}} & \hat{\underline{z}} \\ 0 & 1 & 0 \\ -\sin\theta & \cos\theta & 0 \end{vmatrix} = -2mR\dot{\theta}\omega \begin{pmatrix} 0 \\ 0 \\ \sin\theta \end{pmatrix}$$

$$\underline{F}_{cent} = -m\underline{\omega}\times(\underline{\omega}\times\underline{r}) = m\left(\omega^2\underline{r} - (\underline{r}\cdot\underline{\omega})\underline{\omega}\right) = m\left[\omega^2 R \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} - R\omega^2\sin\theta \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right]$$

$$= mR\omega^2 \begin{pmatrix} \cos\theta \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \underline{F} = \underline{F}_{react} + \begin{pmatrix} mR\omega^2\cos\theta \\ -mg \\ -2mR\dot{\theta}\omega\sin\theta \end{pmatrix} = \underline{F}_{react} - 2mR\dot{\theta} \text{[redacted]} \omega\sin\theta\,\hat{\underline{z}} + \left(-mg\sin\theta + mR\omega^2\cos^2\theta\right)\hat{\underline{r}}$$
$$+ \left(-mg\cos\theta - mR\omega^2\cos\theta\sin\theta\right)\hat{\underline{\theta}}$$

$\parallel$

$$m\dot{\underline{v}} = m\left[R\ddot{\theta}\hat{\underline{\theta}} - R\dot{\theta}^2\hat{\underline{r}}\right]$$

$$\cdot\hat{\underline{\theta}} \Rightarrow mR\ddot{\theta} = -mg\cos\theta - mR\omega^2\cos\theta\sin\theta$$

Define $\theta = \alpha - \frac{\pi}{2}$ $\Rightarrow$ $\cos\theta = \cos\alpha\cos\frac{\pi}{2} + \sin\alpha\sin\frac{\pi}{2} = \sin\alpha$

$\qquad\qquad\qquad\qquad \sin\theta = \sin\alpha\cos\frac{\pi}{2} - \cos\alpha\sin\frac{\pi}{2} = -\cos\alpha$

$$\Rightarrow \ddot{\alpha} = -\frac{g}{R}\sin\alpha + \omega^2\sin\alpha\cos\alpha$$

$\ddot{\alpha} = 0$ when $\sin\alpha\left(-\frac{g}{R} + \omega^2\cos\alpha\right) = 0 \Rightarrow \exists$ 3 equilibrium points $\alpha = 0, \pi$ & $\cos\alpha_0 = \frac{g}{R\omega^2}$

(b) $\alpha = 0$ is stable when ?

Assume $\alpha = \delta\alpha \ll 1$ then $\ddot{\delta\alpha} = \left(\omega^2 - g/R\right)\delta\alpha \Rightarrow \ddot{\delta\alpha} + \left(\frac{g}{R} - \omega^2\right)\delta\alpha$

$\Rightarrow$ stable if $\frac{g}{R} - \omega^2 > 0 \Rightarrow \omega^2 < g/R = \Omega^2 \Rightarrow \Omega = \sqrt{g/R}$

NB not asked for but consider $\alpha = \pi + \delta\alpha$ with $\delta\alpha \ll 1$

$$\Rightarrow \ddot{\delta\alpha} = -\frac{g}{R}\sin(\pi + \delta\alpha) + \omega^2\sin(\pi+\delta\alpha)\cos(\pi+\delta\alpha) \approx \left(\frac{g}{R} + \omega^2\right)\delta\alpha$$

$\Rightarrow$ Always unstable.

Set $\underline{r}_0 = \underline{0}$ & $\underline{v}_0 = v_0(\cos\beta\,\hat{\underline{x}} + \sin\beta\,\hat{\underline{z}})$

Assume the latitude is constant $\Rightarrow$ $\underline{\omega} = \omega(\cos\lambda\,\hat{\underline{y}} + \sin\lambda\,\hat{\underline{z}})$

Can calculate the time taken from $\underline{r}\cdot\hat{\underline{z}} = 0$

$$\Rightarrow T = -\frac{\underline{v}_0\cdot\hat{\underline{z}}}{\omega\,\underline{A}\cdot\hat{\underline{z}}}$$

$\underline{v}_0\cdot\hat{\underline{z}} = v_0\sin\beta$ & $\underline{A}\cdot\hat{\underline{z}} = \frac{\underline{g}\cdot\hat{\underline{z}}}{2\omega^2} - \frac{\hat{\underline{z}}\cdot(\hat{\underline{\omega}}\times\underline{v}_0)}{\omega} = \frac{-g}{2\omega^2} - \frac{\underline{v}_0\cdot(\hat{\underline{z}}\times\hat{\underline{\omega}})}{\omega}$ NB $\hat{\underline{z}}\times\hat{\underline{y}} = -\hat{\underline{x}}$

$$= \frac{-g}{2\omega^2} + \frac{v_0\cos\beta\cos\lambda}{\omega} = \frac{-g}{2\omega^2}\left(1 + \frac{2v_0\omega\cos\beta\cos\lambda}{g}\right)$$

$\Rightarrow T = \frac{2\omega v_0\sin\beta}{g}\left(1 + \frac{2v_0\omega}{g}\cos\beta\cos\lambda\right)$ NB $\frac{v_0\omega}{g} \simeq \frac{500\,ms^{-1}\times 7.3\times10^{-5}\,s^{-1}}{9.8\,ms^{-1}} \ll 1.$

Ignore

$\Delta y = \underline{r}\cdot\hat{\underline{y}} = \underline{A}\cdot\hat{\underline{y}}\,T^2$

$\underline{A}\cdot\hat{\underline{y}} = -\frac{\hat{\underline{y}}\cdot(\hat{\underline{\omega}}\times\underline{v}_0)}{\omega} = -\frac{\underline{v}_0\cdot(\hat{\underline{y}}\times\hat{\underline{\omega}})}{\omega} = -\underline{v}_0\cdot\omega\sin\lambda\,\hat{\underline{y}}\times\hat{\underline{z}} = -v_0\sin\lambda\cos\beta$

$$\Rightarrow \Delta y \approx -v_0\sin\lambda\cos\beta\cdot\left(\frac{2\omega v_0\sin\beta}{g}\right)^2 = \underline{\underline{-\frac{4v_0^3\omega\sin\lambda\cos\beta\sin^2\beta}{g^2}}}$$

Note British guns were calibrated for $\sim 50°$N but battle at $\sim 50°$S $\Rightarrow$ Error may have been $2\Delta y$!

10. $\ddot{\underline{r}}' = \underline{g} - 2\underline{\omega}\times\dot{\underline{r}}' + \omega^2\underline{r}' - (\underline{r}'\cdot\underline{\omega})\underline{\omega}$

(a) $r_{\parallel} = \underline{r}'\cdot\hat{\underline{\omega}}$

$\Rightarrow \ddot{r}_{\parallel} = \underline{g}\cdot\hat{\underline{\omega}} - \cancel{2\underline{\omega}\times\dot{\underline{r}}'\cdot\hat{\underline{\omega}}} + \cancel{\omega^2(\underline{r}'\cdot\hat{\underline{\omega}})} - \cancel{\omega(\underline{r}'\cdot\underline{\omega})}$

$= \underline{g}\cdot\hat{\underline{\omega}}$

$= -g\sin\lambda.$

$\Rightarrow \underline{r_{\parallel}(t) = r_{\parallel}(0) + v_{\parallel}(0)t - \frac{1}{2}gt^2\sin\lambda}$

(b) $\dot{\underline{r}}'\cdot\ddot{\underline{r}}' = \underline{g}\cdot\dot{\underline{r}}' - \cancel{2\underline{\omega}\times\dot{\underline{r}}'\cdot\dot{\underline{r}}'} + \omega^2\left[\underline{r}'\cdot\dot{\underline{r}}' - (\underline{r}'\cdot\hat{\underline{\omega}})(\dot{\underline{r}}'\cdot\hat{\underline{\omega}})\right]$

$\frac{1}{2}\frac{d}{dt}(\dot{\underline{r}}'^2) \qquad \frac{d}{dt}(\underline{g}\cdot\underline{r}')$

$\underline{r}\cdot\dot{\underline{r}}' - (\underline{r}'\cdot\hat{\underline{\omega}})(\dot{\underline{r}}\cdot\hat{\underline{\omega}}) = r_i'\dot{r}_i' - r_i'\hat{\omega}_i\dot{r}_j'\hat{\omega}_j = (\delta_{ij} - \hat{\omega}_i\hat{\omega}_j)r_i'\dot{r}_j'$

$= P_{ij}r_i'\dot{r}_j' = \frac{1}{2}P_{ij}r_i'\dot{r}_j' + \frac{1}{2}P_{ji}r_j'\dot{r}_i'$ by renaming indices that are summed

But $P_{ij} = \delta_{ij} - \hat{\omega}_i\hat{\omega}_j = P_{ji}$

$\Rightarrow \underline{r}\cdot\dot{\underline{r}}' - (\underline{r}'\cdot\hat{\underline{\omega}})(\dot{\underline{r}}\cdot\hat{\underline{\omega}}) = \frac{1}{2}P_{ij}(r_i'\dot{r}_j' + r_j'\dot{r}_i') = \frac{1}{2}P_{ij}\frac{d}{dt}(r_i'r_j')$

$\Rightarrow \frac{1}{2}\frac{d}{dt}(\dot{\underline{r}}'^2) = \frac{d}{dt}(\underline{g}\cdot\underline{r}') + \frac{1}{2}P_{ij}\frac{d}{dt}(r_i'r_j')$

$\Rightarrow \frac{1}{2}\dot{\underline{r}}'^2 - \underline{g}\cdot\underline{r}' - \frac{1}{2}P_{ij}r_i'r_j' \equiv \text{const}$

NB $P_r^2 = (I - \hat{\underline{\omega}}\hat{\underline{\omega}}^T)(I - \hat{\underline{\omega}}\hat{\underline{\omega}}^T) = I - 2\hat{\underline{\omega}}\hat{\underline{\omega}}^T + \hat{\underline{\omega}}\underbrace{(\hat{\underline{\omega}}^T\hat{\underline{\omega}})}_{1}\hat{\underline{\omega}}^T$

$= I - \hat{\underline{\omega}}\hat{\underline{\omega}}^T = P$ ✓