THREE HOURS

A list of constants is enclosed.

UNIVERSITY OF MANCHESTER

General Physics

23rd May 2022, 2.00 p.m. - 5.00 p.m.

Answer as many questions as you can.
Marks will be awarded for the **THIRTEEN** best answers.

Each question is worth 10 marks.

Electronic calculators may be used, provided that they cannot store text.

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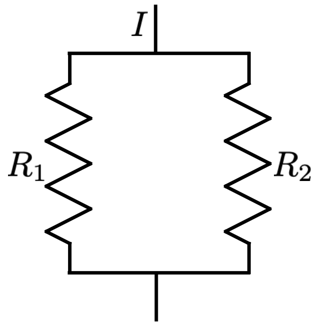
1. Make an estimate of the molar heat capacity of a three-dimensional crystal such as copper, at room temperature. What would be the molar heat capacity in a two-dimensional crystal, where the atoms can only move within the plane of the crystal? Justify your answers.

2. Give expressions for the total resistance of the circuit shown below and the potential drop between its ends, and find the current through the two resistors R_1 and R_2. Show that you can also obtain the same result by assuming that the currents distribute in such a way that the power dissipated in the circuit is minimum.



3. A two-slit system is set up, in which one of the slits is covered with a filter which absorbs 83% of the incident intensity of the incoming monochromatic light. Calculate the visibility of the fringe pattern produced by the slits.
(Visibility is defined as $(I_{\max} - I_{\min})/(I_{\max} + I_{\min})$, where $I_{\max}$ and $I_{\min}$ are the maximum and minimum intensities in the fringe pattern).

4. Due to its stellar wind, a star with a radius of 1 $R_\odot$ has an overall positive charge of 70 C. Assuming the charges reside at the surface and that $\varepsilon_r = 1$ in the space surrounding the star, calculate the potential of the stellar surface compared to infinity. Also calculate the capacitance of the star, and the stored electric energy.

5. When measured with a particular detector, a radioactive source gave 784 counts in one hour. The experiment was repeated exactly one year later, with the same source and detector, and 324 counts were recorded in one hour. Find the lifetime of the source, and the error on that lifetime.

6. An AC current source is used to apply a current of amplitude 10 mA. Consider the load to be a 10 Ω resistor with an exposed surface area of 1.0×10^{-7} m^2. Assuming that all the power generated is lost by radiation from the exposed surface and the surroundings are at room temperature, estimate the value of the steady-state temperature of the resistor. How does the temperature of the resistor change if we now connect an ideal inductor in series with the resistor?

7. Two spaceships each of length 300 m measured before launch, pass each other travelling in opposite directions along the x-axis, at velocities +0.8c and −0.8c as measured from the Earth's frame. How long do they take to pass each other completely a) in the Earth's frame and b) in one of the ships' frames? ("Pass completely" refers to the time between the front of the ships being at the same x coordinate and the back of the ships being at the same x coordinate).

8. Use dimensional analysis to find an expression for the central pressure of the Sun in terms of its basic properties. Estimate the central pressure of the Sun. Assuming the particle density is 10^{31} m^{-3} at its centre, also estimate the central temperature. Effects caused by degeneracy pressure can be ignored in this calculation.

9. At the LHC, a new particle has been observed which decays to a proton and a J/Ψ meson. In one event, the J/Ψ was produced at rest while the proton had kinetic energy 375 MeV. What is the mass of the new particle?

Why was the observation of this new particle considered significant?

[The mass of the proton is 0.938 GeV/c^2, and of the J/Ψ, 3.096 GeV/c^2.]

10. A quantum particle of mass m moves in a harmonic oscillator potential

$$V(x) = \frac{1}{2} m\omega^2 x^2.$$

At time $t = 0$, it is prepared in the state

$$\Psi(x, 0) = \frac{1}{\sqrt{2}}\Big(\phi_0(x) + \phi_2(x)\Big),$$

where $\phi_0(x)$ is the ground state and $\phi_2(x)$ is the second excited state. Write down an expression for $\Psi(x, t)$, the wave function of the particle at a later time t. Sketch ϕ_0 and ϕ_2, and hence, without doing any calculations, describe the time dependences of $\langle x \rangle$ and $\langle x^2 \rangle$ for this state.

11. A thin uniform circular hoop of radius 0.5 m hangs vertically from a horizontal nail, and swings from side to side (in its own plane) with a small amplitude. What is the period of the oscillation?

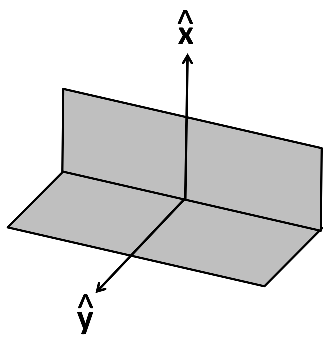
12. A converging lens produces an image, 15 cm behind the lens, of an object 5 cm in front of the lens. Derive an expression for small shifts in image position resulting from small shifts in object position parallel to the axis of the lens. Find the shift in the image position if the object is moved by 1 μm.

13. Consider the atomic masses of the following isotopes in atomic mass units:

$$ {}^{124}_{52}\mathrm{Te}: \quad 123.90282, \qquad {}^{124}_{53}\mathrm{I}: \quad 123.90621, \qquad {}^{124}_{54}\mathrm{Xe}: \quad 123.90589 $$

Discuss the possible $\beta^{\pm}$ decays linking these three isotopes.

14. Two large metal plates are attached at a right angle to each other as shown in the figure.



An electron is placed at $\mathbf{r} = a\hat{\mathbf{x}} + a\hat{\mathbf{y}}$, where $a = 1$ cm. Ignoring edge effects, calculate the force $\mathbf{F}$ acting on the electron.

15. A massive object bends the light from a more distant galaxy around it, so that more light reaches the Earth. Due to this gravitational lensing, the flux received from the galaxy increases by a factor of 2.

The galaxy has absolute magnitude -18 and a velocity of $2.8 \times 10^4\,\mathrm{km\,s^{-1}}$. What is the apparent magnitude of the galaxy as seen from Earth?

You may use a Hubble constant $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. The distance modulus equation is $m - M = -5 + 5\log r$, with r in parsecs.

END OF EXAMINATION PAPER