For practical reasons, this paper is not typical
of the usual form for PC302 Examinations.

The individual questions can be regarded as typical
but the structure of the paper is not.

PC3022

THREE HOURS OR TWO HOURS

A list of constants is enclosed.

UNIVERSITY OF MANCHESTER

General Physics

5 June 2000, 9.45 a.m. - 12.45 p.m.

THREE HOUR CANDIDATES

Answer as many questions as you can from Section A; marks for this Section will be awarded for your **SEVEN** best answers.
Answer **TWO** further questions from Section B.
The marks obtained from Sections A and B are combined with a relative weight of 1:1

TWO HOUR CANDIDATES

(Maths/Physics and Physics with Business and Management)

Answer as many questions as you can from questions 1 - 6 inclusive; marks for this Section will be awarded for your **FIVE** best answers.
Answer **ONE** further question from Section B.
The marks obtained from Sections A and B are combined with a relative weight of 7:5

Use a SEPARATE answer book for each Section.

Electronic calculators may be used, provided that they cannot store text.

The numbers are given as a guide to the relative weights of the different parts of each question.

SECTION A

A1. Define what is meant by the polarisation of an electromagnetic wave. State how two linearly polarised plane waves can be combined to form a circularly polarised wave.

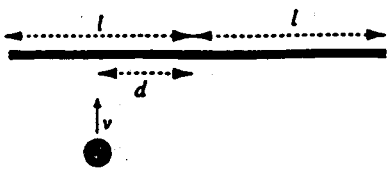
A2. Estimate the working temperature of a 50 W lamp whose filament has a radius of 10 μm and is 0.15 m long.

A3. A spin $\frac{1}{2}$ particle is observed (for example, in a Stern-Gerlach apparatus) to be in the state $s_z = \frac{\hbar}{2}$. The particle then passes through an apparatus that measures s_y and is found to be in the state with $s_y = -\frac{\hbar}{2}$. What was the probability of obtaining that result?

The particle then passes through a further apparatus that measures s_z. What are the possible results of this measurement, and with what probability will they be obtained?

A4. Estimate the pressure at which a gas of argon atoms, at a temperature of 300 K, will begin to show deviations from the ideal gas behaviour due to the finite size of the atoms.

A5. A hockey puck of mass m slides on ice with a speed v and hits a stationary stick of mass M and length $2l$ at a distance d from its centre, as in the diagram below. The puck comes to rest after the collision. Describe briefly the motion of the stick after the collision, assuming that both puck and stick slide without friction. Use the values $m = 0.5$ kg, $M = 0.875$ kg, $l = 1$ m, $d = 0.5l$ and $v = 0.7$ ms^{-1} to calculate the speed V and angular velocity ω of the stick after the collision.



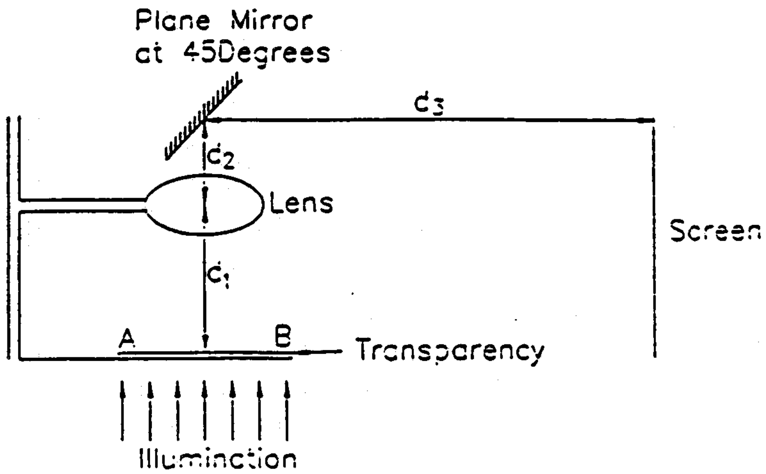
(Use $I = \frac{Ml^2}{3}$ for the moment of inertia of the stick about its centre of mass)

A6. The diagram shows a schematic representation of a simple, single-lens overhead projector system. The convex lens has a focal length of 25 cm, and the fixed distance between the lens and the centre of the plane mirror (d_2) is 5 cm.

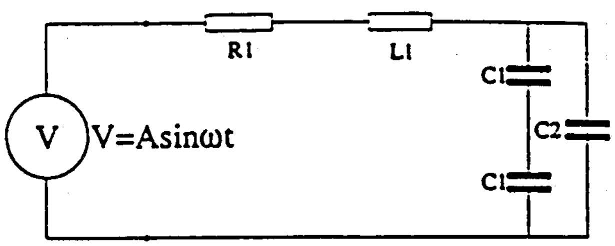
(i) If the distance between the lens and the transparency (d_1) is 27.5 cm what must the distance to the screen (d_3) be to produce a sharp image?

(ii) What is the magnification from transparency to screen?

(iii) If the transparency is to be projected the correct way up, should the top of the transparency be located at A or B?



A7. Find the resonant frequency of the RLC circuit shown below, given that C2 = 0.5 C1.



A8. Explain why the molar specific heat of a crystal of diamond is equal to 25 J $mol^{-1} K^{-1}$ at high temperature. How does the specific heat depend on temperature at very low temperatures?

SECTION B

B9. What is the heat capacity per molecule for air at room temperature and pressure? [3 marks]

Random motion or diffusion of molecules leads to the following expression for the coefficient of thermal conductivity in a gas

$$\kappa = \frac{5}{6} nk\lambda\bar{c}$$

where n is the number density of molecules, k is Boltzmann's constant, λ is the mean free path and $\bar{c}$ is the mean molecular speed. Estimate the thermal conductivity coefficient of air at room temperature and pressure.

[7 marks]

Calculate the heat loss per square metre due to thermal diffusion through 1 cm of air-filled insulating clothing, from skin at 35°C to air on the other side at 15°C.

[7 marks]

Suppose the wind drives air molecules (at 15°C) with a bulk velocity v onto the skin, where they are heated to 35°C, but protective clothing ensures that only 0.1% of the air molecules do this. Estimate the wind speed v necessary to produce a rate of heat loss equal to that due to diffusion.

[8 marks]

Assume the collision cross section of an air molecule to be 1.2×10^{-19} m^2.

B10. A simple model of an earthquake consists of a point source, some distance below the Earth's surface, radiating a wave train uniformly in all directions. Suppose that a seismic wave reaching a monitoring station directly above the source is given in terms of the ground displacement x at time t by

$$x(t) = A\cos\left(2\pi t/T\right) \ ,$$

where A and T are the amplitude and period of the wave respectively. Show that the average kinetic energy of the ground motion per unit volume is given by

$$\epsilon = \frac{\rho\pi^2 A^2}{T^2} \ ,$$

where ρ is the density of the medium.

[8 marks]

A wave train, of duration τ, will typically have many wave periods in it (i.e. $\tau = nT$ for some integer n). If it is propagated with velocity c what will be the energy flow per unit area at the station? Hence show, by considering a spherical shell, that the total kinetic energy from the origin of the earthquake at a distance h below the Earth's surface is

$$E = 4\pi^3 h^2 c\tau\rho(A/T)^2 \, .$$

[6 marks]

Give three assumptions that have been made in deriving the above results.

[5 marks]

The following empirical relations have been obtained for earthquakes in Southern California: $\log\tau = 0.25M - 0.92$ and $\log(A/T) = 0.63M - 5.80$, where M is the magnitude of the quake on the Richter scale and where τ and A/T are measured in SI units. Taking $c = 3.4\,\mathrm{km\,s^{-1}}$, $\rho = 2.7\times10^3\,\mathrm{kg\,m^{-3}}$ and $h = 16\,\mathrm{km}$, find a formula connecting $\log E$ to the magnitude of Californian earthquakes. How much more seismic energy is released by an earthquake measuring 8 on the Richter scale than by the 5×10^{13} J of energy released by the atomic bomb which was dropped on Hiroshima in 1945?

[6 marks]

B11. What is the relation between the energy E and the momentum p of a particle of mass m in Relativity? Draw a graph which shows how the momentum of a particle depends on its energy. Draw a graph which shows how the velocity of a particle depends on its energy.

[8 marks]

Suppose a galaxy at a distance of 10^{16} km is being observed optically when a star in it is observed to become a supernova. 20 seconds later a burst of neutrinos, of energy 10 MeV arrives. Assume that the photons and neutrinos are produced at the same time and calculate the mass (in eV/c^2) of the neutrino.

[12 marks]

If the energy of a neutrino is not exactly known, but measured as 10 ± 1 MeV, what is the error on the neutrino mass?

[5 marks]

B12. (a) Briefly describe the phenomenon of photoelectric effect and the operation of a photomultiplier.

[6 marks]

For sections (b) - (d) consider two plane electrodes, made of the same metal and separated by distance 1 cm, in vacuum. A short pulse of light of wavelength 600 nm strikes one electrode.

(b) A retarding voltage of 1.5 V is required to reduce the photocurrent to zero. What is the work function of the metal?

[6 marks]

(c) With zero voltage between the electrodes, how much time will be needed for the fastest electrons to reach the second electrode after the light strike?

[6 marks]

(d) An accelerating voltage of 100 V is applied between the electrodes. Estimate the number of electrons produced at the second electrode after the strike if the total energy of the pulse is 10^{-15} J. (You may assume 100% efficiency at both electrodes.)

[7 marks]

B13. Distinguish between a solid, a liquid and a gas according to simple kinetic theory.

[5 marks]

In a hail storm, water vapour condenses into ice at 0°C which then falls to the ground to melt. A 1 cm depth of ice covers a square kilometre of ground beneath a storm before the ice begins to melt. Estimate the heat taken from the ground when the ice melts.

[7 marks]

Estimate also the latent heat released into the cloud when the ice was formed.

[3 marks]

Assume that half of the total mass of air above this square kilometre is within the storm cloud. If 10% of the latent heat released into the cloud creates a wind of uniform speed, calculate this speed.

[10 marks]

Assume that the latent heat of conversion of ice to water is 340 kJ kg^{-1} and of conversion of water to water vapour is 2500 kJ kg^{-1}.

B14. (a) The energy of a charged battery, 25 kJ, is enough for 3 hours of talking over a mobile phone. Assuming isotropic radiation of electromagnetic waves by its antenna, estimate the power transmitted in a solid angle 1 steradian.

[5 marks]

(b) Aerials of a cellular network, receiving at a radio frequency 900 MHz, have height $H = 20$ m and are just capable of detecting signals from a mobile phone at a distance of 15 km. Assuming that their effective cross-section for absorption of the radio waves is $\sim \lambda H$, where λ is the wavelength of the waves, estimate the minimum power of the radio signals which can be detected.

[6 marks]

(c) A parabolic antenna of a diameter 100 m is sending a beam of electromagnetic waves at a frequency 900 MHz. By considering diffraction, estimate the solid angle into which most of the radiation is transmitted.

[6 marks]

(d) Imagine that on a distant planet, 4 light years from earth, there is a parabolic antenna (of diameter 100 m) which is sending a microwave signal directly to earth. On earth there is a similar receiving antenna pointed at the planet. How much power should be emitted from the planet to have a signal strong enough to be detected on earth? You may assume that the minimum detectable power of the signal is the same as for the receivers of the cellular network.

[8 marks]

B15. Consider a crystal of metal with a simple cubic lattice and interatomic distance 0.3 nm.

(a) Explain the meaning of terms in the Bragg formula

$$2d \sin\theta = n\lambda$$

in connection with diffraction of waves from such a crystal.

[4 marks]

(b) Calculate the largest momentum of X-ray photons, for which there will be no diffraction from adjacent planes of this crystal.

[6 marks]

(c) Explain how the Fermi statistics of electrons leads to the concept of the the Fermi sphere.

[4 marks]

(d) Consider a gas of conduction electrons in this metal, with density of one electron per atom. Show that there will be no diffraction of electrons from the crystal lattice. You may assume the formula for the Fermi momentum

$$p_F = \hbar(3\pi^2 n_e)^{1/3},$$

where $n_e = N/V$ is the density of free electrons.

[6 marks]

(e) Will the conduction electrons diffract if the metal has two electrons per atom?

[5 marks]

B16. Give expressions defining the one-dimensional Fourier transform $g(k)$ of the function $f(x)$ and the inverse Fourier transform giving $f(x)$ in terms of $g(k)$.

[5 marks]

Write down or evaluate the Fourier transform of each of the following functions (factors of order unity may be ignored), and in each case illustrate the relation between the function and its Fourier transform with labelled sketches.

$$
\begin{aligned}
\text{(a)}\quad f(x) &= A\cos\left(\frac{3\pi x}{\lambda}\right)\\
\text{(b)}\quad g(k) &= B\delta(k-k_0)\\
\text{(c)}\quad f(x) &= \begin{cases} A, & -a/2 < x < a/2\\ 0, & \text{otherwise}\end{cases}\\
\text{(d)}\quad f(x) &= Ae^{-x^2/2a^2}\,.
\end{aligned}
$$

In the above, A, B, λ, k_0 and a are real constants.

[14 marks]

In the last two cases ((c) and (d)), give an example of a physical phenomenon to which the transform is applicable.

[6 marks]