PC3020

THREE HOURS OR ONE HOUR

A list of constants is enclosed.

UNIVERSITY OF MANCHESTER

General Physics II

29th May 2001, 9.45 a.m. - 12.45 p.m.

THREE HOUR CANDIDATES
Answer **THREE** questions.
If four questions are attempted, the best three marks will be taken.
NOT more than four questions should be attempted.

ONE HOUR CANDIDATES
(Maths/Physics and Physics with Business and Management)
Answer **ONE** question from questions 1-6

Electronic calculators may be used, provided that they cannot store text.

The numbers are given as a guide to the relative weights of the different parts of each question.

1. What is the activity (in Bq) of a sample of 100μ g of ^{89} Sr, which decays (to a stable product) with a mean lifetime of 52 days? [4 marks]

Such a sample is spilled over a field of grass in a nuclear accident. 1.0 % of the sample is removed from the field every day by meteorological and biological processes. Calculate the time (in days) at which the activity will, due to the combined effect of these processes and radioactive decay, fall to 10% of its original value. [5 marks]

One part of the biological process is a sheep, which every day, through eating the grass, consumes 0.10 % of the sample present. All the strontium the sheep consumes is absorbed and retained. What does this tell you about the sheep's diet? [1 mark]

Write down the differential equation that describes the activity of the sheep, and solve it, using an integrating factor or otherwise. [8 marks]

At what time is this activity a maximum, and what is it? [7 marks]

2. Repeated measurements of the specific heat of a solid alloy were made at different temperatures. Results were tabulated in the lab notebook as follows:

T° C	c_p (J/kg)
110	226, 227, 225, 226, 229, 228, 225, 226, 225, 226
115	229, 228, 225, 227, 226, 225, 226, 227, 226, 228
120	231, 234, 232, 243, 233, 232, 233, 234, 231, 232
125	234, 232, 233, 235, 231, 233, 232, 234, 232, 233

Give the best estimate of the specific heat at each of the 4 temperatures. [4 marks]

In the lab notebook the experimental accuracy is quoted as being $\sigma = 1.3$ J/kg. Do you believe this? [6 marks]

Give the accuracy of the 4 specific heat values obtained above. [4 marks]

Discuss the variation of specific heat with temperature on the basis of the evidence. Discuss the physical process(es) that could cause such variation. [6 marks]

Draw a schematic diagram (with no numerical values) of the temperature of a specimen of this alloy as a function of time, if it is heated to 125°C and allowed to cool. [5 marks]

3. The speed of sound in an ideal gas is given by $c = \sqrt{K/\rho}$, where ρ is the density of the gas, $K = -V dp/dV$ is the bulk modulus and V is the volume. Assuming that changes in the pressure p are adiabatic, show that

$$c = \sqrt{\gamma p/\rho} = \sqrt{\gamma RT/M},$$

where γ is the ratio of specific heats, R is the gas constant, T is the temperature and M is the mass of one mole of the gas. [6 marks]

An empty wine bottle may be modelled as an oscillator, with the volume V of compressible air inside the bottle acting as the spring and the mass of air in the neck of the bottle acting as the mass. Show that the fundamental frequency of this oscillator (obtained by blowing across the top of the neck) is

$$\nu = \frac{c}{2\pi}\sqrt{\frac{\pi D^2}{4VL}}$$

where L is the length of the neck and D is the diameter of the neck. [12 marks]

A certain bottle has $V = 0.7$ l, $L = 6.8$ cm and $D = 1.8$ cm. Calculate the fundamental frequency (a) when the bottle is empty and (b) when the bottle is half full, assuming that the air temperature is 25° C and the sound speed is 340 m s^{-1}. [3 marks]

The half-full bottle is placed in a refrigerator to cool its contents to a temperature of 5° C. How much wine would need to be removed so that the bottle produces a perfect low E ($\nu = 165$ Hz) when taken straight from the refrigerator? [4 marks]

4. The following news items have appeared in the popular scientific press in the past year:

(a) CERN opens anti-matter factory

(b) Nothing goes faster than light

(c) Cosmologists think the universe is accelerating

(d) IT pioneers awarded Nobel Prize

(e) Levitating frog earns Ig Nobel Prize for Physics

(f) CERN extends search for Higgs Boson

Write short accounts of any *three* of the stories, at a level suitable for a first-year physics undergraduate.

5. Obtain an expression for the total energy of an Earth satellite in a circular orbit in terms of its mass m, orbital radius r, mass M of the Earth and Newton's gravitational constant G. [8 marks]

At a height of 200 km, the atmospheric density is $\rho = 10^{-10}$ kg m^{-3}. If a 1000-kg satellite at this height has a cross-sectional area of $a = 10$ m^2, estimate the loss of energy and hence the loss of height during each orbit. State clearly any assumptions you make in the calculation. [12 marks]

What is the change in speed during one orbit? [5 marks]

6. Write down the two equations which relate a function $f(x)$ to its Fourier transform $F(u)$ and vice versa. [4 marks]

Sketch the forms of the following functions and their Fourier transforms, indicating the relationships between the horizontal scales in each pair: a top hat, a regular infinite series of delta functions and a gaussian. [9 marks]

State the Convolution Theorem which relates the Fourier transform of the product of two functions to the Fourier transforms of the individual functions. Hence sketch the form of the Fourier transform of an isosceles triangle centred on the origin. [6 marks]

A finite diffraction grating consists of n lines having separation a and width b. Sketch the form of the aperture distribution and indicate how you would construct it from the functions above using appropriate convolutions and multiplications. Hence sketch the form of the fringe pattern seen when the grating is illuminated by a plane monocromatic wave. [6 marks]

7. In an interferometer a beam is divided into two distinct paths of equal length which are then recombined. Explain how interference in the transmitted beam demonstrates that a particle takes both paths at the same time. [6 marks]

Four orthonormal coherent states of a pair of photons, a and b, polarised individually either horizontally, $|H\rangle$, or vertically $|V\rangle$, may be written:

$$|\psi_+\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_a|V\rangle_b + |V\rangle_a|H\rangle_b\right)$$

$$|\psi_-\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_a|V\rangle_b - |V\rangle_a|H\rangle_b\right)$$

$$|\phi_+\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_a|H\rangle_b + |V\rangle_a|V\rangle_b\right)$$

$$|\phi_-\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_a|H\rangle_b - |V\rangle_a|V\rangle_b\right)$$

and these are known as entangled states.

In a crystal of beta barium borate an ultraviolet beam is partially converted into two waves each of twice the incident wavelength. A photon from the incident beam is converted into two photons, one of which is polarised horizontally and the other vertically. Where the two waves generated overlap there are pairs of photons for which the individual polarisations are not known, although it is known that the pair members have different polarisation states.

Which entangled states (above) could describe these pairs of photons? What is the probability that for such a pair the polarisation of photon a is observed to be vertical? Describe the wavefunction of the pair immediately after the polarisation of photon a is measured. [8 marks]

For each of the entangled states above, what is

(i) The probability that a photon pair is observed to have the same polarisation?

(ii) The exchange symmetry observed for the entangled state? [4 marks]

If a pair of photons is created in a state $|\psi_+\rangle$ and then separated, photon a may experience either a 90° change in polarisation or a phase change, reversing the exchange symmetry of the pair. If, when the photon pair is brought back together, it is desired to detect whether either of these effects has occured, what observables should be measured? Which must not be measured?

[5 marks]

How many bits of information are conveyed by the return of photon a? [2 marks]

8. Use the de Broglie wavelength and periodic boundary conditions to find the relationship between energy and wave-vector, k, for a free particle in a cube of side L, and the values of the wave vector that are possible. Use these to show that the Fermi energy at zero temperature is

$$E_F = \frac{\hbar^2}{2mL^2}\left(3\pi^2 N\right)^{\frac{2}{3}}$$

in a free electron gas of such particles where N is the number of electrons. [8 marks]

Sketch the relationship between energy and wavevector, $E(k)$, for an electron gas in a square well potential with a small periodic perturbation as produced by atoms in a crystal. Why are there values of electron momentum for which two energies are possible? How does the size of the band gap of forbidden electron energies depend on the periodic potential? [8 marks]

In analogy, an optically periodic medium can lead to an optical band gap due to the spatial modulation of refractive index, $n(\vec{r})$. By comparing the electromagnetic wave equation with the time-independent Schrödinger equation for a periodic potential, or otherwise, explain why there will be values of wavelength for which two magnitudes of photon momentum will be possible, and a band gap of photon momentum in between. What combination of materials could be used to maximise the size of the band gap? [9 marks]

 continued/7…

9. Consider a planet like Jupiter, of mass $10^{-3}M_{\odot}$, in orbit around a star of mass $1M_{\odot}$ at a distance 10 *pc* from Earth.

The existence of such planets can be inferred by two techniques:

(a) Wavelength shifts in stellar spectra

Derive an expression for the orbital velocity of the planet in terms of the stellar mass and the orbital radius. [3 marks]

Relate this to the orbital velocity of the star about the centre of mass of the system. [3 marks]

Spectroscopes can detect Doppler shifts in the spectral lines of a star for velocities down to 10 m s^{-1}. Calculate the limit of planetary orbital radius that can be detected using this method. [3 marks]

(b) Position wobbles in stars

Derive the distance the star wobbles about the centre of mass in terms of the planetary orbital radius and the masses. [3 marks]

Relate this to the angular wobble of the star, measured from Earth. [3 marks]

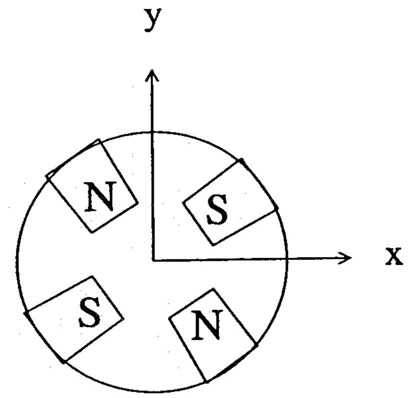
Astrometric measurements can detect wobbles in stars down to angles of 10 milliarcseconds ($4.9\,10^{-8}$ radians.) Calculate the limit of the planetary orbital radius that can be detected for this system. [3 marks]

Which technique would be better if the planetary orbital radius is the same as that from the Earth to the Sun? [2 marks]

How does the distance to the system affect each measurement? [2 marks]

Discuss the effect of orbital orientation for both techniques. [3 marks]

10.



The sketch shows the cross section of a symmetric quadrupole magnet. The magnet extends a distance L along the z axis, and charged particles moving approximately parallel to the z axis pass through the aperture of the magnet. Indicate how coils should be wound round the pole pieces to achieve the desired configuration, and sketch the magnetic field pattern. [5 marks]

For x and y small compared to the size of the aperture one can write

$$B_x = Ay$$

$$B_y = Ax$$

where A is a positive constant that depends on the magnet coils and geometry.

Suppose a positively charged particle of momentum p proceeds parallel to the z axis (i.e. out of the page) and enters the magnet with $x = h, y = 0$. Assuming the length L of the magnet can be taken as small, calculate the angle through which the particle is deflected. Where does it cross the $y = 0$ plane? Describe the analogy with a converging lens. [7 marks]

Show that for particles with $x = 0, y = h$ the effect is similar to a diverging lens. If positively charged particles of momentum p are produced by a target a distance u from the magnet, describe the particle trajectories on exit from the magnet. [3 marks]

Two quadrupole magnets in a beam line are separated by an empty drift space of length d. The magnets are identical except that A is positive for the first and negative for the second. Particles of momentum p are again produced by a target a distance u from the first magnet. Find a condition that the beam is brought to a single focus, (i.e. it converges in both the x and y planes, and at the same distance) and show that it is satisfied for $u = 500m$, $d = 562.5m$, and $f = p/LAq = 300m$. [10 marks]

11. Write down expressions for the relativistic energy, E, and momentum, p, of a particle of mass, m, moving with speed v. Show that the quantity $E^2 - c^2p^2$ is an invariant, i.e. it is independent of the speed of the particle. [4 marks]

Write down the Lorentz transformation relating the space and time coordinates of an event in frames S and S' which move at speed V relative to each other along the x axis. Show that the Lorentz transformation leads to the phenomenon of time dilation. [4 marks]

At the Babar experiment electrons of energy E^- collide head on with positrons of energy E^+. The two particles can interact and produce a single new particle (the upsilon 4S) which has a mass of 10.56 GeV/c^2. Calculate the energy that the positrons must have if the electrons have an energy of 9.0 GeV. The electron mass may be ignored. What is the speed and direction of the produced upsilons? [7 marks]

The upsilon decays (in an unobservably short time) into two B mesons each having almost exactly half the mass of the upsilon. Thus the speed of the B mesons in the rest frame of the upsilon is essentially zero. The mean life of B mesons is 1.65×10^{-12} seconds. What is the mean distance, in the lab, that either B meson travels before it decays? In what fraction of these events does one B travel less than 0.15 mm and the other more than 0.30 mm? [10 marks]

12. Write down an expression for the electric field (a vector) at distance R from a point charge (at the origin and in free space). Show how this expression follows from Maxwell's equations. What is an electric dipole moment? Sketch the electric field due to a dipole. How does the field (at large distances) fall off with distance? [7 marks]

Discuss why stationary charges and charges moving at constant speed do not radiate whereas accelerated charges do radiate. What is meant by retarded time? [6 marks]

The general expression (the Heaviside-Feynman equation) for the electric field of a moving charge, q, is:-

$$\mathbf{E} = \frac{q}{4\pi\epsilon_\circ}\left(\frac{\hat{\mathbf{e}}}{r'^2} + \frac{r'}{c}\frac{d}{dt}\left(\frac{\hat{\mathbf{e}}}{r'^2}\right) + \frac{1}{c^2}\frac{d^2\hat{\mathbf{e}}}{dt^2}\right)$$

where $\mathbf{r}'$ is the displacement vector from the field point (point of observation) to the apparent position of the charge and $\hat{\mathbf{e}}$ is the unit vector along $\mathbf{r}'$. Thus, $\hat{\mathbf{e}}$ gives the direction the source would appear to be, if it were emitting light.

Use this expression to show that the electric field at a point $\boldsymbol{R}_{obs} = (R, 0, 0)$, due to a moving charge whose position is $\mathbf{r}_s(t_s) = \hat{\mathbf{k}} a \sin\omega t_s$, is transverse and is given by:-

$$\mathbf{E} = \hat{\mathbf{k}}\frac{qa\omega^2}{4\pi\epsilon_\circ c^2 R}\sin\omega t.$$

Assume that the observer is far away i.e. that $a << R$. $\hat{\mathbf{k}}$ is the unit vector $(0, 0, 1)$.

What is the field at the point $(R\sin\theta, 0, R\cos\theta)$? What would be the field due to two moving, equal and opposite charges (constituting an oscillating electric dipole), whose positions are given by $\pm\mathbf{r}_s(t_s)$? [7 marks]

Suppose that accelerated masses radiate gravitational waves in an analagous way. Comment on the angular distribution of the radiation from two equal masses with positions $\pm\mathbf{r}_s(t_s)$.

[5 marks]