THREE HOURS

A list of constants is enclosed.

UNIVERSITY OF MANCHESTER

General Physics

22 May 2012, 14:00 - 17:00

You may answer as many questions as you can. Marks will be awarded for your **THIRTEEN** answers

---

Electronic calculators may be used, provided that they cannot store text.

The numbers are given as a guide to the relative weights of the different parts of each question.

1. An organ pipe is open at the top, and closed at the bottom, and has fundamental frequency 300 Hz. A brick falls from the church roof and closes the opening at the top. What is the new fundamental frequency of the pipe?

[10 marks]

2. A violin string of length l and mass per unit length μ is vibrating with amplitude A under a tension T. In the case where the frequency is independent of A, use the method of dimensions to show how the frequency depends on these parameters.

[10 marks]

3. A hollow ball and a solid ball of equal radius roll down an inclined plane. Determine the ratio of their linear accelerations down the plane.

You may use the fact that the moment of inertia of a solid sphere about an axis through its centre is $\frac{2}{5}MR^2$ and that for a hollow sphere is $\frac{2}{3}MR^2$.

[10 marks]

4. A spaceship moving away from Earth at a speed of $0.9c$ collides with a 1 g piece of space debris moving at a speed $0.3c$ towards Earth. Find the kinetic energy of the debris in the rest frame of the space ship.

[10 marks]

5. An ocean wave has a dispersion relation $\omega = \sqrt{gk}$, where ω is the angular frequency, k is the wavenumber and g is the acceleration due to gravity. Calculate the speed at which the crest of a wave of wavelength 15 m propagates.

[10 marks]

6. A golfer can hit a ball 250 m on level ground when the initial velocity of the ball is 30° to the horizontal. How far could the ball travel if it is hit with the same initial velocity and trajectory from a platform elevated 20 m above the ground? Ignore the effects of air resistance.

[10 marks]

7. A non-relativistic quantum particle of mass m in an infinitely deep 2D square well, $-L \leq x \leq L$, $-L \leq y \leq L$, has the wave function $\psi(x, y) = \frac{1}{L}\sin(\pi x/L)\cos(\pi y/2L)$. What are the expectation values of momentum (for both x and y components)? Show that ψ is an eigenstate of the Hamiltonian, and find the eigenvalue.

[10 marks]

8. Solid copper has density $\rho = 8.94\,\mathrm{g\,cm^{-3}}$, atomic weight $M = 63.5\,\mathrm{u}$. and valence (number of conduction electrons per atom) of 1. Estimate the thermal velocity of conduction electrons at room temperature. What is the de Broglie wavelength of these electrons? Hence, comment on whether quantum effects are important in copper at room temperature.

[10 marks]

9. A pond of water at 0°C is freezing from the top, when the air temperature is −5°C. Obtain an approximate expression for the depth of the ice as a function of time in terms of its thermal conductivity, K, latent heat of melting, L, and density, ρ. How long does it take for the pond to freeze to a depth of 1 cm?

The ice has a density $920\,\mathrm{kg\,m^{-3}}$, its latent heat of melting is $0.33\,\mathrm{MJ\,kg^{-1}}$ and its thermal conductivity is $2.3\,\mathrm{Wm^{-1}\,K^{-1}}$.

[10 marks]

10. An electric field is given by $\mathbf{E} = E(x, t)\hat{\mathbf{y}}$ where

$$\frac{\partial^2 E}{\partial x^2} = \mu_0\epsilon_0\frac{\partial^2 E}{\partial t^2}\,.$$

Write down a solution which is periodic with an angular frequency ω and wavenumber k and find the relationship between ω, k and $\mu_0\epsilon_0$. What is the physical significance of the relationship?

[10 marks]

11. Estimate the energy stored in a solenoid of radius $r = 20\,cm$ and length $l = 60\,cm$ that has a magnetic flux density $B = 8.0\,\mathrm{T}$ inside it. The solenoid is immersed in liquid helium, calculate the volume of liquid helium that will be evaporated after all the stored energy is isothermally released into the helium. For liquid helium, the latent heat of vapourization is $L = 90\,\mathrm{J\,mol^{-1}}$ and the molar volume is $V_\mathrm{m} = 28\,\mathrm{cm^3\,mol^{-1}}$.

[10 marks]

12. State the efficiency of the Carnot heat engine in terms of the higher temperature T_h and the lower temperature T_c. T_h is either raised or T_c is lowered by a small shift of ΔT. Which gives the greater increase in efficiency? Explain your reasoning.

[10 marks]

13. A passenger in an aeroplane can just resolve a person of height 2 m lying on the ground. Assuming that the diameter of the pupil of the passenger's eye is 5 mm and the light has a wavelength of 500 nm, estimate the altitude of the aircraft.

[10 marks]

14. A cow faces the point where a flash of lightning hits the ground with its front feet 20 m from that point. Its back legs are a further 1.5 m away. Estimate the maximum potential difference that occurs between the front and back legs.

You may assume that the maximum current in the flash is $10^5\,\mathrm{A}$ and that it disperses equally in all directions through the ground. The conductivity of the earth is $0.03\,\Omega^{-1}\,\mathrm{m}^{-1}$.

[10 marks]

15. The acceleration due to gravity a distance r from the centre of the Earth is

$$g = \frac{GM(r)}{r^2},$$

where $M(r)$ is the mass enclosed within a radius r. Assuming that the density of the Earth is constant, calculate the pressure at the centre of the earth.

[10 marks]

END OF EXAMINATION PAPER

PHYSICAL CONSTANTS AND CONVERSION FACTORS

SYMBOL	DESCRIPTION	NUMERICAL VALUE
c	Velocity of light in vacuum	299 792 458 m s^{-1}, exactly
μ_0	Permeability of vacuum	$4\pi \times 10^{-7}$ N A^{-2}, exactly
ϵ_0	Permittivity of vacuum where $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$	8.854×10^{-12} C^2 N^{-1} m^{-2}
h	Planck constant	6.626×10^{-34} J s
$\hbar$	$h/2\pi$	1.055×10^{-34} J s
G	Gravitational constant	6.674×10^{-11} m^3 kg^{-1} s^{-2}
e	Elementary charge	1.602×10^{-19} C
eV	Electronvolt	1.602×10^{-19} J
α	Fine-structure constant, $\frac{e^2}{4\pi\epsilon_0\hbar c}$	$\frac{1}{137.0}$
m_e	Electron mass	9.109×10^{-31} kg
$m_e c^2$	Electron rest-mass energy	0 511 MeV
μ_B	Bohr magneton, $\frac{e\hbar}{2m_e}$	9.274×10^{-24} J T^{-1}
R_∞	Rydberg energy $\frac{\alpha^2 m_e c^2}{2}$	13.61 eV
a_0	Bohr radius $\frac{1}{\alpha}\frac{\hbar}{m_e c}$	0.5292×10^{-10} m
Å	Angstrom	10^{-10} m
m_p	Proton mass	1.673×10^{-27} kg
$m_p c^2$	Proton rest-mass energy	938 272 MeV
$m_n c^2$	Neutron rest-mass energy	939.565 MeV
μ_N	Nuclear magneton, $\frac{e\hbar}{2m_p}$	5.051×10^{-27} J T^{-1}
fm	Femtometre or fermi	10^{-15} m
b	Barn	10^{-28} m^2
u	Atomic mass unit, $\frac{1}{12} m(^{12}\mathrm{C}$ atom)	1.661×10^{-27} kg
N_A	Avogadro constant, atoms in gram mol	6.022×10^{23} mol^{-1}
T_t	Triple-point temperature	273.16 K, exactly
k	Boltzmann constant	1.381×10^{-23} J K^{-1}
R	Molar gas constant, $N_A k$	8.314 J mol^{-1} K^{-1}
σ	Stefan-Boltzmann constant, $\frac{\pi^2}{60}\frac{k^4}{\hbar^3 c^2}$	5.670×10^{-8} W m^{-2} K^{-4}
M_E	Mass of Earth	5.97×10^{24} kg
R_E	Mean radius of Earth	6.4×10^6 m
g	Standard acceleration of gravity	9.806 65 m s^{-2}, exactly
atm	Standard atmosphere	101 325 Pa, exactly
$M_\odot$	Solar mass	1.989×10^{30} kg
$R_\odot$	Solar radius	6.96×10^8 m
$L_\odot$	Solar luminosity	3.84×10^{26} W
$T_\odot$	Solar effective temperature	5.8×10^3 K
AU	Astronomical unit, mean Earth-Sun distance	1.496×10^{11} m
pc	Parsec	3.086×10^{16} m
	Year	3.156×10^7 s