THREE HOURS

A list of constants is enclosed.

UNIVERSITY OF MANCHESTER

General Physics

??th May 2015, 2 p.m. - 5 p.m.

You may answer as many questions as you can. Marks will be awarded for your best **THIRTEEN** answers

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Electronic calculators may be used, provided that they cannot store text.

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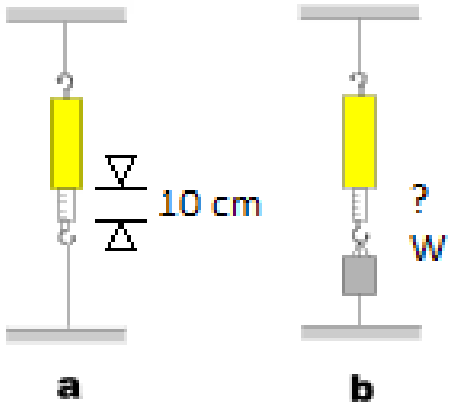
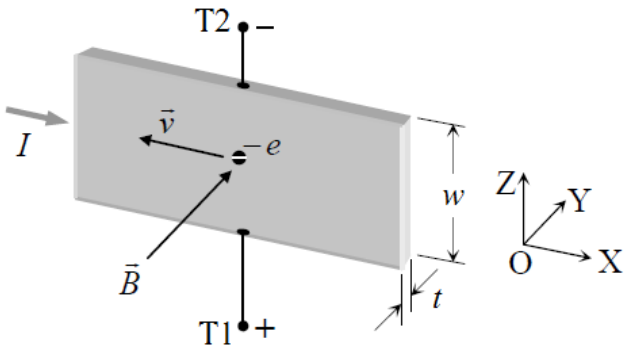
1. The total annual energy usage in the UK is about 10^{19} J. Assume that all of this energy is used to accelerate a small spacecraft with a mass of 250 kg. What would the final velocity of the spacecraft be?

2. The thickness d of an aluminium foil is to be determined by firing a beam of electrons at the foil and measuring how many pass through. The number of electrons that pass through the foil satisfies $n = n_0 \exp(-\lambda d)$, where $\lambda = (1.38 \pm 0.05) \times 10^3$ m^{-1}. An experimenter measures the number of counts, n_0, in the absence of the foil to be 572 and the number, n, of counts in the presence of the foil to be 417. Calculate the thickness of foil, d, and the standard error on this value.

3. A spring with spring constant $k = 100$ N m^{-1} and with hooks on its two ends is hung from the roof via a rope on the upper hook. It is then tightened to the floor via a second rope until the spring stretches by a distance $\Delta x = 10$ cm, as shown in figure (a) below. What will be the value of this distance if we hang a weight W from the lower hook as in figure (b), when: a) W = 6 N, and b) W = 14 N ? In the latter case, what is the value of the tension in the lower rope?

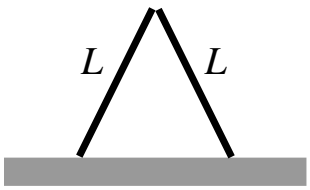


4. An electric current I in a conducting plate is composed of electrons flowing with an average speed v along the X axis, as in the diagram below. If we apply a magnetic field **B** parallel to the Y axis the electrons experience a Lorentz force that pushes them to the upper edge of the plate (terminal T2), where they will accumulate. The accumulation results in an electric field **E** parallel to the Z axis, that opposes the Lorentz force. At what value of **E** will these forces balance each other? If the plate has a width w, what will be the voltage difference between terminals T1 & T2? Express this voltage in terms of I, the plate thickness, t, and the number density of electrons, n.

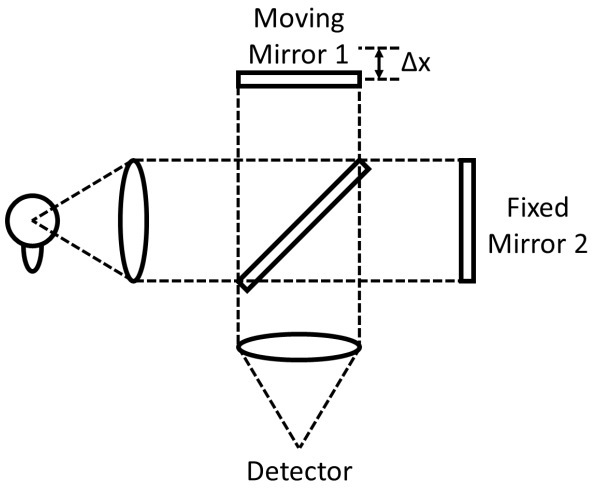


5. A pair of identical pendulums, each consisting of a mass m on a string of length l, are coupled by a spring with spring constant k, that is relaxed when the pendulums are vertical. Determine the two eigenfrequencies for small amplitude motion. A horizontal impulse I is applied to the mass attached to the left pendulum at $t = 0$. Find an expression for the subsequent horizontal displacement of this mass. You should assume that the pendulums are hanging at rest before the impulse is applied

6. A student starts building a house of cards. She puts down the first two cards as shown in the figure, on a horizontal surface with coefficient of friction μ. The length of the cards is L. Neglecting friction between the cards, what is the minimal angle with the horizontal for which the two cards are stable? Hint: balance the torques.



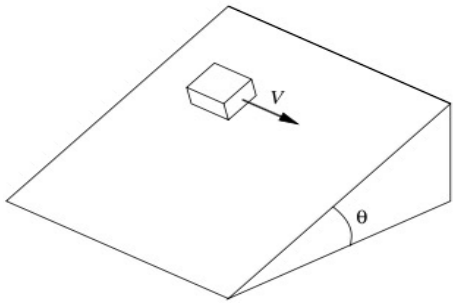
7. A lamp containing atomic Argon gas emits two closely spaced spectral lines at λ and $\lambda + \Delta\lambda$ where $\lambda = 810.4\,\text{nm}$. By using a Michelson Interferometer as shown in the diagram, compute how far mirror 1 must be moved from the equilibrium (zero offset) position to observe one full interference fringe for $\Delta\lambda$=1.1 nm.



8. A 5.0 kg block of copper, with a specific heat capacity of 385 J kg^{-1} K^{-1} and a temperature of 80°C is thrown into a lake at 10°C. Calculate the corresponding change of entropy of the block. What is the change of entropy of the universe?

9. A fridge is switched off and left with the door open to equilibrate with the air in a room at 20 °C and 1 atmosphere of pressure. The door, which is airtight and has a surface area of $1\,\text{m}^2$, is now closed. The power is switched on until the air inside is cooled to 2 °C. Assuming air is an ideal gas, estimate the force required to open the door. Comment on your result. You may assume that $1\,\text{atm} = 10^5\,\text{Nm}^{-2}$

10. A block is placed on a large inclined plane and given a kick so that it initially moves with speed V as shown in the diagram below. If the coefficient of friction between the block and plane is $\mu = \tan\theta$ determine the speed of the block after a long time has elapsed. Hint: Resolve the motion into a component down the plane and a component along the direction of motion.



11. An AC current source is used to apply a current $I_{\max}\sin(\omega t)$ which is independent of the load. If the load is a resistor R with an exposed surface area S. What is the average power generated in the resistor? Assuming that all of this power is lost via radiation through the exposed surface, estimate the steady state temperature of the resistor. What is the temperature of the resistor if we now connect an ideal capacitor C in series with the resistor?

12. A subsonic jet at an air show travelling parallel with the earth at 1 km altitude is observed to emit a 10 kHz hum by an observer directly below the plane, but a 14 kHz hum by an observer on the ground who is 2 km ahead of the jet in the direction of travel. How fast is the jet moving?

13. A particle of mass m moves in an infinitely deep one-dimensional square well of width a, extending from $x = -a/2$ to $x = a/2$. What is the normalised wave function for the ground state? The square well is then perturbed by adding a δ-function potential at the origin, $V_1(x) = W\delta(x)$. Find the ground state energy to first order in W. Explain why we can determine the energy of the first excited state exactly.

14. The Hubble Space Telescope (HST) has a mirror of diameter 2.4 m and observes at a wavelength of 100 nm. Calculate the resolution at this wavelength in radians. The closest approach of Mars to the Earth is 55 million km. How big must a spacecraft on Mars be to be resolved by the HST.

15. A refrigerator operates between cold and hot reservoirs at temperatures T_C and T_H. The refrigerator extracts heat at a rate Q from the cold reservoir and draws energy at a rate W from the power supply. What is the theoretical maximum value of the coefficient of performance, $\eta = Q/W$?

Estimate the cost per year of running a domestic refrigerator whose walls are 4 cm thick, have a total surface area of 8 m^2 and a mean thermal conductivity of 0.02 $\mathrm{W\,m^{-1}\,K^{-1}}$. The room temperature is 20 °C and the internal temperature is 2 °C. Assume that $\eta = 3$ and that the cost of electricity is 15p per kWh.

END OF EXAMINATION PAPER