

**THREE HOURS**

A list of constants is enclosed.

**UNIVERSITY OF MANCHESTER**

General Physics

23rd May 2005, 2.00 p.m. - 5.00 p.m.

**THREE HOUR CANDIDATES**

(Physics, Physics with Astrophysics, Physics with Theoretical Physics, Physics with Technological Physics)

Answer as many questions as you can. Marks will be awarded for your **THIRTEEN** best answers.

**TWO HOUR CANDIDATES**

(Maths/Physics, Physics with Business and Management)

Answer as many questions as you can from questions 1-10 inclusive. Marks will be awarded for your **NINE** best answers.

Each question is worth 10 marks.

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Electronic calculators may be used, provided that they cannot store text.

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The numbers are given as a guide to the relative weights of the different parts of each question.

PC3010 June 2005 continued...

1. What is the physical significance of group velocity? A wave has a group velocity equal to  $\frac{3}{2}$  of its phase velocity. How does  $\omega$  depend on  $k$  for this wave?

2. Estimate the kinetic energy of a constituent quark of mass  $300 \text{ MeV}/c^2$  confined inside a proton of radius  $1 \text{ fm}$ .

3. Write down the Maxwell distribution for the probability that a gas molecule of mass  $m$  has a speed  $v$ . There is no need to determine the overall normalisation factor. Determine the most probable speed of a gas molecule.

Use this result to estimate the most probable speed of oxygen molecules at room temperature.

4. A  $p$  state in a hydrogen-like atom is found to be split into two states by the spin-orbit coupling  $V_{ls} = \lambda \vec{l} \cdot \vec{s}$ .

Calculate the total angular momentum and energy shift of each of the two levels.

5. A  $2 \text{ kg}$  block of copper, with a specific heat capacity of  $385 \text{ J kg}^{-1} \text{ K}^{-1}$ , at a temperature of  $100^\circ \text{ C}$  is cooled reversibly to  $10^\circ \text{ C}$ . Calculate the change of entropy of the block. What is the change of entropy of the universe as a result of the process?

The same block is now cooled irreversibly from  $100^\circ \text{ C}$  by throwing it into a lake at  $10^\circ \text{ C}$ . What is the change of entropy of the block in this case?

6. All animals that have the capacity to jump (e.g. fleas and dogs) can jump approximately to the same height. By thinking about the chemical energy stored in muscles and by assuming that all these animals have the same proportion of their body mass as leg muscle, explain why this is not surprising.

7. Explain quantitatively how the wavelength for the  $n = 2$  to  $n = 1$  transition for deuterium differs from that for hydrogen, ignoring hyperfine structure.

The masses of  $\text{H}^1$ ,  $\text{H}^2$  and  $\text{e}$  are  $1.00728$ ,  $2.01355$  and  $5.58 \times 10^{-4} \text{ amu}$  respectively.

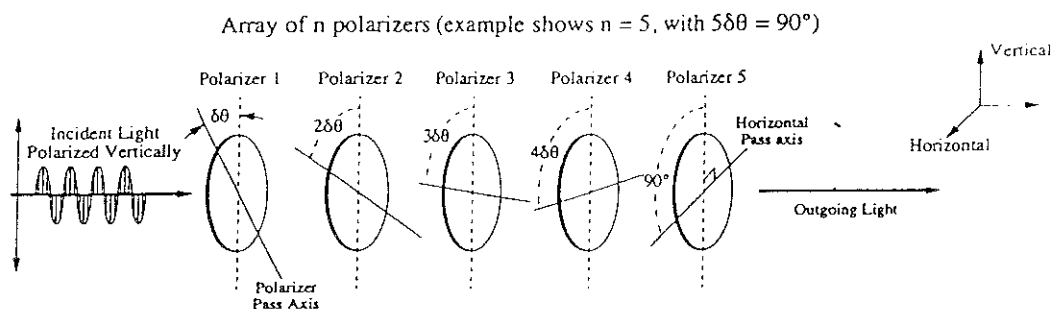
PC3010 January 2005 continued...

8. A colleague who is an enthusiastic scuba diver wishes to listen to BBC Radio 2 which transmits at a frequency of 98MHz, both on his boat and while diving in the sea off Devon. He purchases a special waterproof radio receiver and earphones to accomplish this. While meeting you at a party he keenly explains his ideas to you, intending to dive to a depth of 10 meters to explore the fish while listening to his favourite station.

At this stage you realise that he did not attend any of your favourite lectures in electromagnetic theory, and you have to gently explain to him why he will not be able to receive the music from the radio while diving. Share your reasoning with the examiner of this paper.

At the party you recall from your lectures that the conductivity of seawater in the sea off Devon is  $\sigma = 20$  siemens/m and that seawater is non-magnetic. You also recall from your 3rd year general paper that the skin-depth of a conductor may be written as  $\delta = (\mu\pi\sigma f)^{-1/2}$ , where all terms have their usual meanings.

9.



Vertically polarized light of intensity  $I_0$  is incident on an array of  $n$  perfect polarizers. The polarizers are oriented with respect to the incident polarization direction so that each polarizer has the same relative pass angle  $\delta\theta = \pi/2n$  radians, the final polarizer in the array being set to pass horizontal light. The example where  $n = 5$  is shown above.

If the number of polarizers in the array is increased to  $n = \infty$ , show that the array acts as a *light guide* which changes the light from vertical to horizontal polarization, with the output intensity being the same as that incident on the array.

You may find the approximation  $\cos^2 \beta \approx 1 - \beta^2$ , where  $\beta$  is small useful.

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**10.** Explain what is meant by the principle of equipartition of energy. Use it to determine the heat capacity of a monatomic ideal gas of  $N$  atoms.

The specific heat of hydrogen at a temperature of 55 K is measured to be approximately  $12.5 \text{ J mol}^{-1} \text{ K}^{-1}$ , but at a temperature of 320 K is found to equal approximately  $20.8 \text{ J mol}^{-1} \text{ K}^{-1}$ . Explain these results in the light of those predicted by equipartition.

**11.** A room has a volume of  $100 \text{ m}^3$ . Estimate the number of sound wave modes between 660 and 1320 Hz in the room if the velocity of sound is  $330 \text{ ms}^{-1}$ .

**12.** A galactanaut sets off for Proxima Centauri (4 light years away) at a speed of  $0.5 c$ . After 2 years, on Earth, mission control sends a radio message to abort the mission. How long does it take for the message to reach the galactanaut?

With fuel now in reserve from the aborted mission, the galactanaut can immediately accelerate to  $0.8c$  for the return trip. By how much has the galactanaut aged when she arrives back on Earth?

**13.** A vertical U-tube of uniform cross-section open at both ends contains a liquid which fills in total a length  $l$  of the tube. Calculate the period of the oscillations of the liquid if the liquid in one of the arms of the U-tube is pushed down a small distance and then released.

**14.** The function  $f$  is defined by

$$f(x) = \begin{cases} -1, & \text{if } -\pi < x < 0 \\ 1, & \text{if } 0 < x < \pi. \end{cases}$$

Is this function even or odd?

Find the Fourier series of  $f$  as an expansion in sines.

PC3010 January 2005 continued...

**15.** Show that the wave equation for the electric field in a dielectric, non-magnetic medium of relative permittivity  $\epsilon_r$  and refractive index  $n = \sqrt{\epsilon_r}$  (with no sources or sinks of radiation) may be written:

$$\nabla^2 \mathbf{E} \propto \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

What is the constant of proportionality in this equation in terms of the refractive index of the material? You may find the vector identity  $\nabla \times \nabla \times \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$  useful.

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