ONE HOUR AND THIRTY MINUTES

A list of constants is enclosed.

UNIVERSITY OF MANCHESTER

General Physics Skills

[redacted] 2024, [redacted] - [redacted]

Answer as many questions as you can.
Marks will be awarded for the **SEVEN** best answers.

Each question is worth 10 marks.

The use of calculators is permitted, as long as they cannot store text or perform algebra, and have no graphing capability.

The numbers are given as a guide to the relative weights of the different parts of each question.

1. A block of mass $m = 3\,\mathrm{kg}$ is attached to a spring and subject to a damping force given by $F = -bv$. The spring constant is $k = 15\,\mathrm{N\,m^{-1}}$. Initially it oscillates with an amplitude of 0.3 m but because of the damping the amplitude after four complete oscillations is 0.2 m.

i) Calculate the value of b.

ii) How much energy is lost in these four oscillations?

[10 marks]

2. A Dyson sphere is a thin spherical shell constructed by extraterrestrials which entirely surrounds a star, identical to the Sun, at its centre. The sphere has a radius of 1.5×10^{11} m and absorbs all the radiation from the star. Calculate the equilibrium temperature of the sphere assuming it behaves as a blackbody.

Hence, estimate in which part of the spectrum astronomers here on Earth ought to look in order to most easily detect electromagnetic radiation from the sphere.

[10 marks]

3. An air-source heat pump is used to heat water to 35°C. The electric motor driving it has an efficiency of 60%. Estimate the ratio of heat delivered to electrical power input for an external temperature of 0°C.

[10 marks]

4. A spherical vessel of diameter 10 cm is filled with air that enters at 300 K. Estimate the maximum pressure at which it is likely that a molecule can travel across the vessel without colliding with another molecule.

[10 marks]

5. The output of a light source has a distribution of wavelengths such that equal power is output at all wavelengths between 495 nm and 505 nm. Collimated light from the source is used to illuminate two narrow slits of separation 0.2 mm, forming an interference pattern on the screen 2 m away. Estimate how many fringes will be observed, and the width of the fringe pattern.

[10 marks]

6. Two photons, travelling in opposite directions, collide to produce a Higgs boson, mass 125 GeV/c^2. One photon has an energy of 500 GeV. What is the energy of the other photon?

What should the energies of the photons be to produce the Higgs boson with the minimum possible energy?

[10 marks]

7. Particles of mass M move along a circle of radius R. For each particle, the Hamiltonian is

$$\hat{H} = -\frac{\hbar^2}{2MR^2}\frac{d^2}{d\phi^2},$$

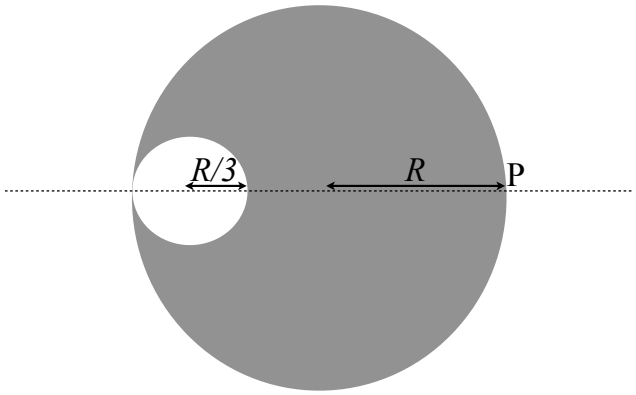
where (r, ϕ) are polar coordinates in the plane.

i. Find the single-particle energies and their degeneracies.

ii. For four identical particles, find the ground-state energy and its degeneracy if the particles are (a) bosons; (b) spin-1/2 fermions.

[10 marks]

8. The diagram shows a cross-section through a solid non-conducting sphere of radius R and uniform charge density ρ.



The sphere contains a spherical cavity of radius $R/3$ that is just beneath the surface of the sphere, as shown in the diagram. The dotted line passes through the centre of both the sphere and the cavity. Calculate the electric field at the point P where the dotted line meets the surface of the sphere.

[10 marks]

END OF EXAMINATION PAPER