THREE HOURS

A list of constants is enclosed.

UNIVERSITY OF MANCHESTER

General Physics

30th May 2008, 2.00 p.m. - 5.00 p.m.

THREE HOUR CANDIDATES
(Physics, Physics with Astrophysics, Physics with Theoretical Physics, Physics with Technological Physics)
Answer as many questions as you can. Marks will be awarded for your **THIRTEEN** best answers.

TWO HOUR CANDIDATES
(Maths/Physics, Physics with Business and Management, Physics with Finance, Physics with Philosophy)
Answer as many questions as you can from questions 1-10 inclusive. Marks will be awarded for your **NINE** best answers.

Each question is worth 10 marks.

Electronic calculators may be used, provided that they cannot store text.

The numbers are given as a guide to the relative weights of the different parts of each question.

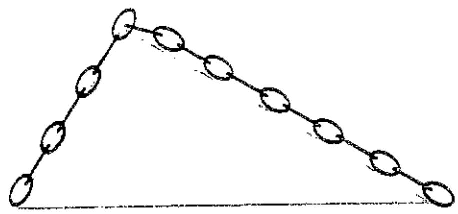
P.T.O.

1. What is the resolving power of the human eye at a wavelength of 500 nm. (Assume that the diameter of the pupil is 5 mm.)

What is the diameter of a radio telescope with the same resolution but operating at a frequency of 10 GHz?

2. An electron with kinetic energy of 5 eV moves in a circular orbit of radius 1 mm in a magnetic field. What is the magnitude and direction of the field?

3. A frictionless chain hangs on two inclined planes, as indicated in the drawing. Prove that the chain is in equilibrium, i.e. will not slip either to the left or to the right.



4. Compute the wavelength of a photon emitted when an electron transits from an energy level with $n = 167$ to that with $n = 166$ of the Hydrogen atom.

5. Write down an expression for the pressure in an isothermal atmosphere at temperature T as a function of the height Z, the surface pressure p_o and the average molecular weight m of the atmospheric constituents.

Calculate the scale height (at which the pressure has fallen to $1/e$ of its surface value) of the Earth's atmosphere, given that m = 29 a.m.u.

P.T.O.

6. A beam of 0.27 eV neutrons is directed onto the surface of a crystal with interatomic spacing 2×10^{-10} m. What is the angle between the surface and the incident beam at which strong diffraction will be observed?

7. A parallel beam of light of diameter 1 mm enters a concave lens of focal length 100 mm. The beam expands and is at a diameter of 5 mm where it enters a convex lens of focal length 100 mm. The beam is then brought to a focus.

Sketch a typical ray path through the lenses. What is the distance between the two lenses? What is the distance from the convex lens to the focus of the beam?

8. Write down an expresion for the entropy S of a system in terms of the number of quantum microstates Ω.

A system is initially in a state 1 and changes to a state 2 such that the number of quantum states available to the system is increased by a factor $e^{10^{23}}$. Calculate the increase in entropy of the system.

9. In 1987, a supernova exploded at a distance of 50 kpc ($1.545\ 10^{21}$ m). Twelve neutrinos were detected on Earth following this eruption. Assume they arrived 2 hours after the explosion was seen. Calculate (i) v/c of the neutrinos, and (ii) the travel time of the neutrinos in their own frame of rest: give your answer in years.

Hint: express v/c in the form $1 - \delta$.

10. For a particle of rest mass m, derive a formula for the total energy that would be required in order for it to resolve a structure of size λ. Estimate this for the case of resolving the internal structure of a nucleus using (i) an electron and (ii) a proton.

11. By treating them both as black bodies estimate the relative amounts of energy radiated by the Sun and the Earth.

At what wavelength should a distant observer choose to make measurements in order to maximise the visibility of the Earth against the Sun?

P.T.O.

12. The wavefunctions of two states of an atom are given by

$$\psi_1(r,\theta,\phi) = R(r)\cos\theta\cos\phi, \quad \psi_2(r,\theta,\phi) = R(r)\cos\theta\sin\phi.$$

Show that these are not eigenfunctions of the angular momentum operator $\hat{L}_z = -i\hbar\frac{\partial}{\partial\phi}$. By considering linear combinations of these two states construct two states with definite angular momentum in the z-direction.

13. Show that if two springs with spring constants k_1 and k_2 are connected in series, the net spring constant of the combination is given by

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \; .$$

14. A reversible heat pump heats a building to 25°C using heat from a river at 5°C. Derive the efficiency (performance coefficient) for such a pump and calculate the number of Joules of heat produced in the building by the expenditure of 1J in the pump.

15. Sound waves in a solid obey the dispersion relation:

$$\omega = a\sin\left(\frac{k}{k_0}\right)$$

where ω is the angular frequency, k the wavenumber and a and k_o are constants. Show that

$$k^2\,V_p^2 + k_o^2\,V_g^2 = a^2$$

where V_p and V_g are the phase and group velocity of the waves.

END OF EXAMINATION PAPER

PHYSICAL CONSTANTS AND CONVERSION FACTORS

SYMBOL	DESCRIPTION	NUMERICAL VALUE
c	Velocity of light in vacuum	$299\,792\,458$ m s^{-1}, exactly
μ_0	Permeability of vacuum	$4\pi \times 10^{-7}$ N A^{-2}, exactly
ϵ_0	Permittivity of vacuum where $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$	8.854×10^{-12} C^2 N^{-1} m^{-2}
h	Planck constant	6.626×10^{-34} J s
$\hbar$	$h/2\pi$	1.055×10^{-34} J s
G	Gravitational constant	6.674×10^{-11} m^3 kg^{-1} s^{-2}
e	Elementary charge	1.602×10^{-19} C
eV	Electronvolt	1.602×10^{-19} J
α	Fine-structure constant, $\frac{e^2}{4\pi\epsilon_0 \hbar c}$	$\frac{1}{137.0}$
m_e	Electron mass	9.109×10^{-31} kg
$m_e c^2$	Electron rest-mass energy	0.511 MeV
μ_B	Bohr magneton, $\frac{e\hbar}{2m_e}$	9.274×10^{-24} J T^{-1}
R_∞	Rydberg energy $\frac{\alpha^2 m_e c^2}{2}$	13.61 eV
a_0	Bohr radius $\frac{1}{\alpha}\frac{\hbar}{m_e c}$	0.5292×10^{-10} m
Å	Angstrom	10^{-10} m
m_p	Proton mass	1.673×10^{-27} kg
$m_p c^2$	Proton rest-mass energy	938.272 MeV
$m_n c^2$	Neutron rest-mass energy	939.566 MeV
μ_N	Nuclear magneton, $\frac{e\hbar}{2m_p}$	5.051×10^{-27} J T^{-1}
fm	Femtometre or fermi	10^{-15} m
b	Barn	10^{-28} m^2
u	Atomic mass unit, $\frac{1}{12} m(^{12}\mathrm{C}$ atom)	1.661×10^{-27} kg
N_A	Avogadro constant, atoms in gram mol	6.022×10^{23} mol^{-1}
T_t	Triple-point temperature	273.16 K
k	Boltzmann constant	1.381×10^{-23} J K^{-1}
R	Molar gas constant, $N_A k$	8.315 J mol^{-1} K^{-1}
σ	Stefan-Boltzmann constant, $\frac{\pi^2}{60}\frac{k^4}{\hbar^3 c^2}$	5.670×10^{-8} W m^{-2} K^{-4}
M_E	Mass of Earth	5.97×10^{24} kg
R_E	Mean radius of Earth	6.4×10^6 m
g	Standard acceleration of gravity	$9.806\,65$ m s^{-2}, exactly
atm	Standard atmosphere	101 325 Pa, exactly
$M_\odot$	Solar mass	1.989×10^{30} kg
$R_\odot$	Solar radius	6.961×10^8 m
$L_\odot$	Solar luminosity	3.846×10^{26} W
$T_\odot$	Solar effective temperature	5800 K
AU	Astronomical unit, mean Earth-Sun distance	1.496×10^{11} m
pc	Parsec	3.086×10^{16} m
	Year	3.156×10^7 s