PHYS30010X

THREE HOURS

A list of constants is enclosed.

UNIVERSITY OF MANCHESTER

General Physics

24th May 2007, 9.45 a.m. - 12.45 p.m.

THREE HOUR CANDIDATES

(Physics, Physics with Astrophysics, Physics with Theoretical Physics, Physics with Technological Physics)

Answer as many questions as you can. Marks will be awarded for your **THIRTEEN** best answers.

Each question is worth 10 marks.

Electronic calculators may be used, provided that they cannot store text.

The numbers are given as a guide to the relative weights of the different parts of each question.

P.T.O.

1. A particle accelerator produces protons at a speed of 0.9 c. What is the total energy of a proton in GeV and what is its kinetic energy?

2. The proposed Square Kilometer Array radio telescope will have antennas spread over a distance of 150 km. What is the resolution of the telescope in arc seconds at frequencies of a) 100 MHz and b) 10 GHz?

3. A container holds 100 litres of an ideal gas at 30°C and atmospheric pressure. The gas is compressed slowly, maintaining the same temperature, until it has reached half its original volume. How much work has been done on the gas?

4. Bose-Einstein condensation occurs when the de-Broglie wavelength is comparable to the inter-atomic spacing. A diffuse non-relativistic gas of $^{87}_{37}$Rb has 10^{18} atoms per m^3. Estimate the temperature at which condensation takes place.

5. The Maxwellian probability distribution for the speed of a gas molecule of mass m in a system in thermal equilibrium at temperature T is given by

$$P(v) = Av^2 e^{-\frac{mv^2}{2kT}}$$

where A is the normalisation factor. Derive an expression for the most probable speed of a gas molecule.

Use your result to estimate the most probable speed of hydrogen molecules at room temperature.

6. A cosmic ray particle travels straight across the Galaxy with a speed of v = 0.99c. If the Galaxy is 30 kpc across as measured in the Galaxy's rest frame, what is the distance as seen by the particle? How long does the journey take in the particle's rest frame?

P.T.O.

7. A 1 kg block of copper, with a specific heat capacity of 385 J kg^{-1} K^{-1}, at a temperature of 100°C is cooled reversibly to 10°C. Calculate the change of entropy of the block. What is the change of entropy of the universe as a result of the process?

8. Consider two bars (assumed massless) of length 1m, connected at their centres, each with a mass of 2 kg attached at each end. The bars are rigidly held at right angles to each other.

The system rotates about its axis through the centres of the bars, and perpendicular to the plane of the bars. If the total rotational energy is $E = 100$ J, what is the spin period?

9. The Coulomb self-energy due to a charged sphere of radius R and charge density $\rho(r)$ is given by

$$E = \frac{4\pi}{3\epsilon_0}\int_0^R [\rho(r)]^2 r^4 dr \quad .$$

Use this to show that the Coulomb energy of a nucleus is $\propto Z^2/A^{1/3}$ for large Z and estimate this energy for $^{238}_{92}$U. Z and A are the atomic and mass numbers respectively.

10. Glass used in optical fibres has a refractive index n of 1.5 at a wavelength of 1.5 micron and dispersion dn/dk $= -3.6 \times 10^{-9}$ m where k is the wavenumber. Estimate the group velocity of light in the glass as a fraction of the phase velocity.

11. Show that the period of a fictitious pendulum subject to the surface gravity g of the Earth and of length equal to the Earth's radius is equal to the minimum period of a satellite orbiting the Earth. What is its period?

12. A spherical satellite is in an orbit around the Earth such that it is in view of the Sun at all times. Assuming that the satellite behaves like a black body calculate its equilibrium temperature. You may neglect radiation coming from the Earth.

(The solar flux at the distance from the Earth is 1372 W m^{-2} .)

P.T.O.

13. A wire loop of resistance 0.5 Ω encloses an area of 0.1 m^2. It is located in a uniform magnetic field of 1 T, which is normal to the plane of the loop. How much charge flows round the loop when the field is switched off?

14. The first excited state of the one dimensional simple harmonic oscillator has wavefunction

$$\psi = Ax \exp\left[-\frac{x^2}{2a^2}\right] \quad .$$

(i) Normalize the wavefunction;

(ii) compute the expectation value of x^2.

You may use without proof:

$$I_n = \int_{-\infty}^{\infty} dx\, x^{2n} \exp\left[-\frac{x^2}{2}\right] = \frac{(2n)!}{2^n n!}\sqrt{2\pi} \quad ,$$

when n is an integer.

15. The radial velocity of a 1 solar mass star is observed to vary sinusoidally by ± 30 m s^{-1} with a 10 day period. Calculate (i) the radius of the circular orbit of the planet causing the stellar motion; (ii) the mass of the planet in solar masses.

END OF EXAMINATION PAPER

PHYSICAL CONSTANTS AND CONVERSION FACTORS

SYMBOL	DESCRIPTION	NUMERICAL VALUE
c	Velocity of light in vacuum	299 792 458 m s^{-1}, exactly
μ_0	Permeability of vacuum	$4\pi \times 10^{-7}$ N A^{-2}, exactly
ϵ_0	Permittivity of vacuum where $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$	8.854×10^{-12} C^2 N^{-1} m^{-2}
h	Planck constant	6.626×10^{-34} J s
$\hbar$	$h/2\pi$	1.055×10^{-34} J s
G	Gravitational constant	6.674×10^{-11} m^3 kg^{-1} s^{-2}
e	Elementary charge	1.602×10^{-19} C
eV	Electronvolt	1.602×10^{-19} J
α	Fine-structure constant, $\frac{e^2}{4\pi\epsilon_0\hbar c}$	$\frac{1}{137.0}$
m_e	Electron mass	9.109×10^{-31} kg
$m_e c^2$	Electron rest-mass energy	0.511 MeV
μ_B	Bohr magneton, $\frac{e\hbar}{2m_e}$	9.274×10^{-24} J T^{-1}
R_∞	Rydberg energy $\frac{\alpha^2 m_e c^2}{2}$	13.61 eV
a_0	Bohr radius $\frac{1}{\alpha}\frac{\hbar}{m_e c}$	0.5292×10^{-10} m
Å	Angstrom	10^{-10} m
m_p	Proton mass	1.673×10^{-27} kg
$m_p c^2$	Proton rest-mass energy	938.272 MeV
$m_n c^2$	Neutron rest-mass energy	939.566 MeV
μ_N	Nuclear magneton, $\frac{e\hbar}{2m_p}$	5.051×10^{-27} J T^{-1}
fm	Femtometre or fermi	10^{-15} m
b	Barn	10^{-28} m^2
u	Atomic mass unit, $\frac{1}{12} m(^{12}C$ atom)	1.661×10^{-27} kg
N_A	Avogadro constant, atoms in gram mol	6.022×10^{23} mol^{-1}
T_t	Triple-point temperature	273.16 K
k	Boltzmann constant	1.381×10^{-23} J K^{-1}
R	Molar gas constant, $N_A k$	8.315 J mol^{-1} K^{-1}
σ	Stefan-Boltzmann constant, $\frac{\pi^2}{60}\frac{k^4}{\hbar^3 c^2}$	5.670×10^{-8} W m^{-2} K^{-4}
M_E	Mass of Earth	5.97×10^{24} kg
R_E	Mean radius of Earth	6.4×10^6 m
g	Standard acceleration of gravity	9.806 65 m s^{-2}, exactly
atm	Standard atmosphere	101 325 Pa, exactly
$M_\odot$	Solar mass	1.989×10^{30} kg
$R_\odot$	Solar radius	6.961×10^8 m
$L_\odot$	Solar luminosity	3.846×10^{26} W
$T_\odot$	Solar effective temperature	5800 K
AU	Astronomical unit, mean Earth-Sun distance	1.496×10^{11} m
pc	Parsec	3.086×10^{16} m
	Year	3.156×10^7 s