ONE HOUR THIRTY MINUTES

A list of constants is enclosed.

UNIVERSITY OF MANCHESTER

General Physics Skills

32nd January 2023, 2.00 p.m. - 3.30 p.m.

Answer as many questions as you can.
Marks will be awarded for the **SEVEN** best answers.

Each question is worth 10 marks.

Electronic calculators may be used, provided that they cannot store text.

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1. A horizontal spring, with spring constant $1500\,\mathrm{N\,m^{-1}}$ is mounted on top of a horizontal table and is fixed at one end. The table top is 1.8 m from the floor. A wooden block of mass 200 g is pushed against the free end of the spring and compresses it by 0.06 m. The block is released and slides off the end of the table and eventually hits the floor. Calculate the final speed of the wooden block on the table and also the speed when it hits the floor.

You can assume that the spring is massless and that the friction between the block and the table top is negligible.

2. An Earth-like planet orbits a star of mass 3.5×10^{30} kg and luminosity $1.5 \times 10^{27}\ \mathrm{J\,s^{-1}}$. The orbital velocity of the planet is determined to be $34\ \mathrm{km\,s^{-1}}$. Estimate the surface temperature of the planet. State reasons why your result is approximate, and comment on whether the surface could be covered with liquid water.

3. A long thin bar magnet has a uniform magnetisation $\mathbf{M}$. It is bent into a ring and the ends are joined. Assuming that $|\mathbf{M}|$ does not change, and ignoring any end effects from the join, find the $\mathbf{H}$ and $\mathbf{B}$ fields (a) inside the material of the magnet, and (b) just outside it. Justify your answers. What changes to the magnetic field would there be if a very small gap were to open up between the ends of the bar?

4. A particle of mass M decays at rest to two other particles with masses m_1 and m_2. The first travels at $\frac{4}{5}c$, the second at $\frac{3}{5}c$. Find the ratios of the three masses.

5. The state of a particle in a one-dimensional box of size L is a superposition of eigenfunctions. At time $t = 0$ the wavefunction $\psi(x,t)$ of the particle is given by:

$$\psi(x,0) = \sqrt{\frac{2}{L}}\left[\sqrt{\frac{2}{3}}\sin\left(\frac{\pi x}{L}\right) + \sqrt{\frac{1}{3}}\sin\left(\frac{2\pi x}{L}\right)\right].$$

a) Write down an expression for the wave function at time t.

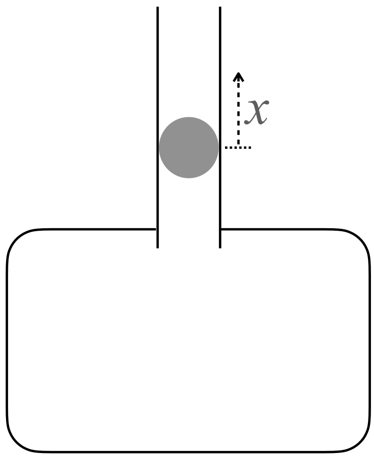
b) Find the probability (as a function of time) for the particle to be in the half-box $0 < x < L/2$. Can the particle be ever completely localised in that half box?

Hint: you may want to use the following integrals

$$\int_0^{L/2} \mathrm{d}x \sin^2\left(\frac{n\pi x}{L}\right) = \frac{L}{4}, \qquad \int_0^{L/2} \mathrm{d}x \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right) = \frac{2L}{3\pi}.$$

6. A cold, degenerate gas of N electrons is confined by a semiconductor quantum well to a two-dimensional square of lateral size $L = 10\,\mu$m. If the Fermi energy is 0.1 eV above the bottom of the band, estimate the value of N.
Hint: For a spinless particle, the two-dimensional density of states is given by $g(k) = L^2k/2\pi$.

7. Inside a container of volume V_0 is an unknown gas at pressure P_0. The container includes a narrow tube of cross-sectional area A, as shown in the diagram. In the tube is a metal ball of mass m that fits the tube closely but is free to move. If the ball is displaced by a small distance x from its initial position it oscillates with period τ.



Derive an equation to determine the specific heat ratio γ in terms of τ.

8. A Michelson interferometer is illuminated by light from a sodium lamp, which may be considered to consist of two sharp spectral lines at 588.995 nm and 589.592 nm. The movable mirror is initially positioned so that maximally high-contrast fringes are seen. The mirror is then moved so that lower-contrast fringes are seen at the detector, with the movement continuing until another maximally high-contrast fringe is observed. Calculate the distance through which the mirror has been moved, and the total number of fringes that have moved past the detector.

END OF EXAMINATION PAPER