THREE HOURS

A list of constants is enclosed.

UNIVERSITY OF MANCHESTER

General Physics

30th May 2017, 2.00 p.m. – 5.00 p.m.

Answer as many questions as you can.
Marks will be awarded for your **THIRTEEN** best answers.

Each question is worth 10 marks.

Electronic calculators may be used, provided that they cannot store text.

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1. In an observer's inertial frame, event A occurs before event B. Both events occur on the x-axis, separated by a distance Δx and a time Δt. A second inertial frame moves along the x-axis with velocity v. Under what conditions would the two events

a) take place at the same point,

b) be simultaneous,

in the new frame?

2. A hydroelectric power station uses water to drive a turbine located at a height H below a reservoir. The water flow rate (volume per unit time) reaching the turbine is F, and the water density is ρ. What is the maximum electrical power that can be produced?

After the turbine, the water reaches a lake. If a fraction β of the maximum power is wasted heating the water that leaves the turbine, estimate the rate of entropy change in the lake. Assume that both the water in the reservoir and in the lake are at ambient temperature T.

3. A system of N atoms is in thermal equilibrium at temperature T, with each atom having a single unpaired electron at an energy level E. Under an external magnetic field B, the energy level is split into two levels with energies $E_{\mu_z} = E - \mu_z B$, where the magnetic moment per electron is equal to $\mu_z = -2m_s\mu_B$ and the spin quantum number $m_s = -1/2$ $(+1/2)$ for electrons aligned parallel (antiparallel) to B. Find an expression for the total magnetic moment M. What is the value of M for a large magnetic field ($\mu_B B \gg kT$)?

4. You are standing on an airport conveyor belt with your trolley. The conveyor belt propels you along with speed v. You give the trolley a push and jump onto it. As a result you and the trolley end up moving with speed V relative to the conveyor belt. The combined mass of you and the trolley is m.

a) How much work did you do?

b) From the point of view of an observer stationary on the ground how much kinetic energy did you gain?

c) If your answers to (a) and (b) differ, explain where the difference comes from; if they are the same, explain why.

5. Derive an expression for the energy levels of a particle in a 1D infinitely deep square well of width a.

At $t = 0$, a particle is in a superposition of the ground state and the second excited state,

$$\psi(x,0) = \sqrt{\frac{1}{3}}\,\phi_0(x) - \sqrt{\frac{2}{3}}\,\phi_2(x)\,.$$

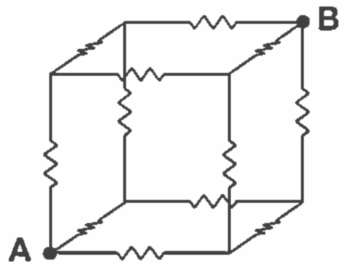
Derive an expression for the time-dependent probability density.

What is the time-dependent energy expectation value?

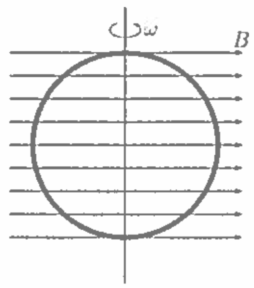
6. A perfect mirror is placed, normal to the incident sunlight, on the surface of Mercury. What is the ratio between the pressure due to solar radiation and atmospheric pressure?

The distance of Mercury to the Sun is $r_{\mathrm{M}} = 0.39\,\mathrm{AU}$, and its atmosphere pressure is $p_{\mathrm{M}} = 5 \times 10^{-15} \times p_{\mathrm{Earth}}$.

7. Twelve equal resistors, R, are joined together to form a cube (see figure below). A battery is connected to two opposite corners of the cube (A and B). What is the equivalent resistance?



8. A closed metallic ring is rotating about a diameter, in a constant magnetic field that is perpendicular to its axis of rotation. Explain why the magnetic field slows the rotation of the ring.
Such a ring, of area $3 \times 10^{-3}\,\mathrm{m}^2$, is rotating at a rate of 2000 revolutions per minute in a magnetic field of $2 \times 10^{-2}\,\mathrm{T}$. Calculate the root-mean-squared e.m.f. induced around the ring.



9. A scuba diver's pressure gauge fails. They can see a 20 m diameter transparent circle on the water surface directly above, surrounded by a reflective surface. What water pressure does the scuba diver experience?

The refractive index of water is $n_{\text{water}} = 1.33$.

10. A cuboid has sides of length a, b and c, which are measured to be:

$$\begin{aligned} a &= (29.7 \pm 0.3)\,\text{cm}, \\ b &= (21.0 \pm 0.3)\,\text{cm}, \\ c &= (\ 5.0 \pm 0.1)\,\text{cm}. \end{aligned}$$

Calculate the volume and the surface area of this cuboid, including their standard errors.

11. A galaxy cluster contains 1000 galaxies, which are gravitationally bound within a region of 3000 kpc. A typical galaxy is 15 kpc across, has a mass $5 \times 10^{11}\,M_{\odot}$, and contains 10^{11} stars, each of radius and mass similar to that of the Sun.

a) Assuming that this galaxy cluster and any galaxy are both virialised (i.e. stable under gravitational interactions), calculate the typical velocity of a galaxy within this cluster and of a star within a galaxy.

b) Using a simple dimensional argument, explain why galaxy collisions are frequent but stellar collisions very rare.

12. Consider a silicon crystal whose band gap energy is $E_g = 1.12\,\text{eV}$ and whose temperature is kept at $T = 300\,\text{K}$.

If the Fermi level, E_f, is located in the middle of the band gap, calculate the probability of a state that is an energy kT above the conduction band edge being occupied by an electron.

13. Photoelectrons with a maximum energy of 0.48 eV are observed when light of wavelength 500 nm is incident on a flat surface of a metal. Calculate the work function of the metal.

When the experiment is repeated using a spherical nanoparticle made of the same metal, we extract a slightly larger work function of 2.1 eV. Estimate the radius of the nanoparticle. Consider only the Coulomb potential due to the positive charge left as the photoelectron leaves the surface of the nanoparticle.

14. Hexane is found to behave as a non-ideal gas described by the van der Waals equation of state

$$\left(P + \frac{a}{V^2}\right)(V - V_{\min}) = RT,$$

where V is the molar volume and $V_{\min}$ and a are constants given by $V_{\min} = 0.1735\,\mathrm{l\,mol^{-1}}$ and $a = 24.71 \times 10^5\,\mathrm{Pa\,l^2\,mol^{-2}}$.

What are the physical meanings of $V_{\min}$ and a?

Calculate the values of the *critical temperature*, T_c, and *critical volume*, V_c, defined as the point at which the first and second derivatives of $P(V)$ with respect to V are both zero.

15. a) Two telescopes are pointed in the same direction. One uses a refracting lens and the other uses a reflecting mirror. Describe the image of a distant object formed by each: its orientation and whether it is real or virtual.

b) What is the wavelength-dependent effect that hinders the image quality of refracting lenses that have a very large aperture?

c) In order to minimise the previous effect one needs to use a thin lens with minimal curvature. Explain with the aid of the 'lens maker's formula' (below) the practical problem that arises for telescopes that require large collecting areas to see fainter objects.
The focal length of a biconvex lens with radii R_1 and R_2 is given by:

$$\frac{1}{f} = (n-1)\left(\frac{1}{|R_1|} + \frac{1}{|R_2|}\right),$$

where n is the refractive index of the glass.

END OF EXAMINATION PAPER