FOUR HOURS

UNIVERSITY OF MANCHESTER

General Paper

21st May 2021, 9.00 a.m. - 1.00 p.m.

Answer as many questions as you can.
Marks will be awarded for the **THIRTEEN** best answers.

Each question is worth 10 marks.

You MUST NOT confer with anyone in answering the questions on this assessment.

The numbers are given as a guide to the relative weights of the different parts of each question.

Solutions must be handwritten and scanned, or handwritten on a tablet, and uploaded to Blackboard **as a single pdf file**.

Order the pages so that the answers to different questions are sequential and make clear on every page which question part is being addressed. Number the pages and write your student ID on the first page.

Ensure the scan is clear. Do not use green or red pens.

One hour of the exam duration has been allowed for accessing the exam and uploading the answers. Multiple submissions are allowed and you should upload your first attempt 30 minutes before the deadline. Only the final submission will be marked.

Late penalties will apply to work submitted after the deadline.

If you are a DASS-registered student with extra time, write your own submission time, which will have been communicated to you in advance, on the first page of your solutions, and submit before that deadline.

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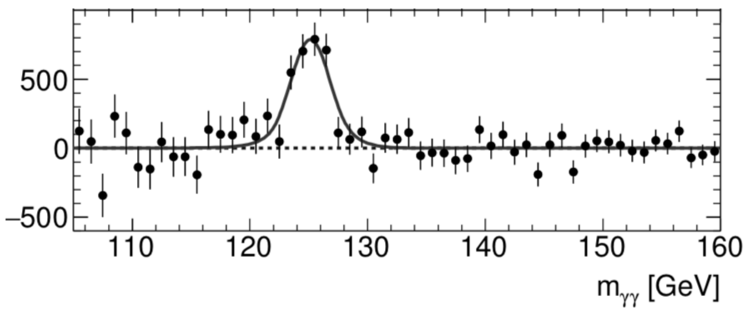
1. A circular loop of thin wire rotates about a diameter that is perpendicular to a uniform magnetic field. The loop has radius 0.1 m, and the strength of the magnetic field is a constant 2 T. If the loop rotates 5 times per second, calculate the maximum electro-motive force (EMF) created along the wire.

2. A compressor takes in air at 300 K and 1.0×10^5 Pa and delivers air at 2.0×10^5 Pa. Assuming that the compression is reversible and adiabatic, calculate the temperature of the air delivered.

3. An accelerator directs a beam of protons onto a hydrogen target. Find the minimum energy of the beam needed to create Z^0 bosons (with a mass of 91.2 GeV/c^2) in the resulting proton-proton collisions.

4. A solid cylinder of radius R and mass M is placed on a sloping surface which makes an angle of θ with the horizontal. Find an expression for the minimum coefficient of friction necessary for the cylinder to roll without slipping down the slope.

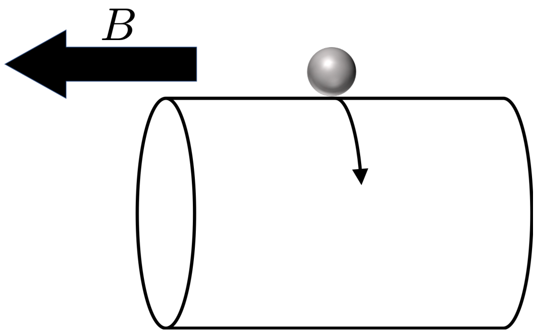
5. The figure below, from the ATLAS experiment at the LHC, shows the event rate (after background subtraction) as a function of final-state energy, for the decay of the Higgs boson to two photons. The solid line shows the best-fit resonance curve.



From the plot, estimate the lifetime of the Higgs boson. Give one reason why this might not be a good estimate of the true lifetime.

6. A block of ice, of mass 10 kg and initial temperature $-5\,°C$, melts after being dropped into a lake whose temperature is just above $0\,°C$. Calculate the entropy change of the universe. [The specific enthalpy of fusion of ice is $330\,\mathrm{kJ\,kg^{-1}}$ and its specific heat capacity at constant pressure is $2.0\,\mathrm{kJ\,kg^{-1}\,K^{-1}}$.]

7. A particle, of mass 10 g and charge 10 mC, is released from the top of a horizontal cylinder of radius 1 m, and slides frictionlessly along the surface under the influence of gravity. A uniform magnetic field B lies along the axis of the cylinder, as shown in the figure below.



Calculate the minimum magnetic field for the particle to remain in contact with the cylinder.

8. X-rays of wavelength 0.154 nm undergo Bragg diffraction from crystalline iron, which has the body-centred cubic structure. With the help of a diagram, show that one set of atomic planes has spacing $a/\sqrt{2}$, where a is the length of one edge of the cubic unit cell. First-order Bragg diffraction from these planes is observed at a scattering angle of 44.9°. Calculate the length a.

9. A physics experiment produces particles with velocity $(0.80 \pm 0.03)\,c$. From other experiments it is known that the half-life of such particles in their rest frame is $(2.0 \pm 0.1)\ \mu$s. Calculate the half-life of these particles in the lab frame, including the error.

10. A uniform spherical insulator, with $\epsilon_r = 1$ and radius 10 cm, contains a spherically symmetric charge distribution. The curl of the resulting electric field is zero and the energy density decreases as $1/r^2$ from the centre of the distribution. The electric field is $1\ \hat{\mathbf{r}}\ \mathrm{V\,m^{-1}}$ at 1 cm from the centre. Derive an expression for the charge distribution.

11. A slit system is composed of three slits, each separated by a distance d from the previous one. It is illuminated at normal incidence with light of wavelength λ to produce an interference pattern at a distant screen. If the intensity observed at the centre of the fringe pattern is I_0, calculate the intensity at a small angle $\lambda/(2d)$ from the centre of the pattern.

12. Consider an intrinsic semiconductor crystal at room temperature, where $k_B T$ is 0.025 eV. The probability of a state close to the valence-band edge being occupied by a hole is 1.0×10^{-5}. Calculate the band gap.

13. A rotating spherical space probe is designed to study the Sun. By considering the effects of radiation, determine how close it is able to orbit the Sun in a circular orbit, before the solder in the electronics will melt. Comment on whether an elliptical orbit could allow the probe to get closer to the Sun.
[The melting point of the solder may be taken to be 200° C.]

14. A particle of mass m moves in one dimension in a potential $V(x)$. The ground-state energy is zero and the ground-state wave function is $\psi(x) = A\mathrm{e}^{-bx^4/4}$, where A and b are constants. Determine the potential $V(x)$.

15. At 300 K, a metal has a total thermal conductivity $\kappa_{\mathrm{T}} = 250\ \mathrm{W\,m^{-1}\,K^{-1}}$, dominated by a 90% electron contribution, κ_{e}, with the rest given by phonons, κ_{ph}. Due to phonon-phonon scattering, $\kappa_{\mathrm{ph}} \propto T^{-1}$. Calculate the temperature at which the contributions of phonons and electrons become equal. Hence calculate the value of κ_{T} at this temperature.
[You may assume that the electrical conductivity of the metal is constant over the relevant temperature range.]

END OF EXAMINATION PAPER