THREE HOURS

A list of constants is enclosed.

UNIVERSITY OF MANCHESTER

General Physics

6th June 2018, 2.00 p.m. - 5.00 p.m.

Answer as many questions as you can.
Marks will be awarded for your **THIRTEEN** best answers.

Each question is worth 10 marks.

Electronic calculators may be used, provided that they cannot store text.

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1. A laser emits an ultrashort pulse of light centred at 532 nm with a duration of 5×10^{-16} s. What is the minimum spectral width of the pulse in nanometres (full-width at half-maximum)? What colour is this pulse?

2. A particle of mass 2 MeV/c^2 and kinetic energy 3 MeV collides with a stationary particle of mass 4 MeV/c^2. After the collision the two particles stick together. What is their final velocity?

3. Photoelectrons with a maximum energy of 0.5 eV are observed when light of wavelength 500 nm is incident on a heavily n-doped semiconductor. Calculate the work function of the n-doped semiconductor. When the experiment is repeated using a heavily p-doped semiconductor made of the same material, we now require a wavelength of 400 nm to observe the same maximum energy. Estimate the intrinsic bandgap of the semiconductor.

4. An experiment is set up to lift a small, spherical payload containing meteorological instruments using a helium balloon. The payload weighs 2 kg, has a radius 0.25 cm and a drag coefficient $C_d = 0.47$.

a) What volume of the balloon is required to lift the payload? Assume an ambient temperature 20° C and atmospheric pressure 101 kPa.

b) At an altitude of 5 km, the payload is released and is in free-fall. If the drag force is $F = \frac{1}{2}\rho v^2 A C_d$, with v its velocity, A its cross section and ρ the air density, what is the maximum velocity that the payload could reach?

5. The first two energy levels of a particle of mass m, which moves in a harmonic potential of classical frequency ω, correspond to normalised wave functions

$$\phi_0(x) = N\mathrm{e}^{-x^2/2a^2} \qquad \text{and} \qquad \phi_1(x) = \sqrt{2}N\left(\frac{x}{a}\right)\mathrm{e}^{-x^2/2a^2},$$

where N is a constant and $a = \sqrt{\hbar/m\omega}$. A particle is prepared at time $t = 0$ in the state

$$\psi(x) = \frac{N}{\sqrt{3}}\left(1 + \frac{2x}{a}\right)\mathrm{e}^{-x^2/2a^2}.$$

What is the wave function at time $t = \pi/\omega$? What is the expectation value of the energy?

6. The density of aluminium is $2700\,\mathrm{kg\,m^{-3}}$, its molecular weight is $27\,\mathrm{g\,mol^{-1}}$, and in an external magnetic field of $1\,\mathrm{T}$, the average magnetic moment per atom is $3 \times 10^{-5}\mu_B$. Find an expression for its magnetic susceptibility χ.

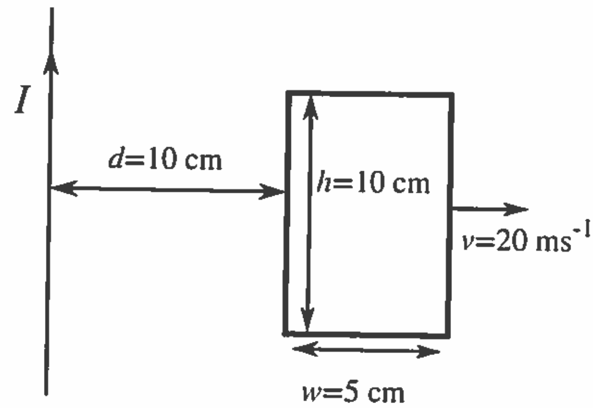
   Aluminium is a Pauli paramagnetic metal with its magnetic susceptibility given by $\chi = \frac{2}{3}\mu_0\mu_B^2 g(E_F)$, where $g(E_F)$ is the density of states. Evaluate χ and $g(E_F)$.

7. A curtain track is bent and laid so that the height of the track above the ground is

$$z(x) = h\left[1 - \cos\left(\frac{2\pi x}{L}\right)\right],$$

   where $h = 25$ cm and $L = 1$ m. A marble is placed on the track at $x = 1.1$ m. Describe its subsequent motion, and estimate the time at which it returns to its starting position. Justify any assumptions you use.

8. A 10 cm by 5 cm rectangular loop is moving at a constant speed $v = 20\,\mathrm{ms^{-1}}$ away from a long, straight wire which carries a current $I = 6\,\mathrm{A}$. The loop has a resistance of $100\,\Omega$. What is the magnitude of the current in the loop at the instant shown in the figure?



9. A Mach-Zender interferometer is constructed from glass of temperature-independent refractive index $n = 1.5$, such that the incoming light is split equally along two paths of length $L = 1\,\mathrm{mm}$ and recombined at the exit port. The optical transmission spectrum of the device is measured. The glass has a linear thermal expansion coefficient of $\alpha_L = \frac{1}{L}\frac{\mathrm{d}L}{\mathrm{d}T} = 3.2 \times 10^{-6}\,\mathrm{K^{-1}}$. One path is at a fixed temperature $T = 300\,\mathrm{K}$ while the temperature of the other path can vary.

   What range of temperatures can be measured by looking for the position of a single destructive interference in the transmission of the device in the visible range (400-800 nm)?

10. An engineer submits a proposal to increase the efficiency of a Carnot engine that operates between two heat reservoirs at temperature T_H and T_C, with $T_H > T_C$. The proposal consists of running one engine between T_H and an intermediate temperature T_I, and a second engine between T_I and T_C using as input the heat expelled by the first engine. Calculate the efficiency of the proposed engine and compare it to that of the original engine.

11. The semi-empirical mass formula gives the nuclear binding energy as

$$B(A,Z) = \alpha_v A - \alpha_s A^{2/3} - \alpha_c \frac{Z(Z-1)}{A^{1/3}} - \alpha_a \frac{(A-2Z)^2}{A} + \delta_p.$$

Give the physical meaning of each of the terms in this expression. Why is the term δ_p not a smooth function of A and Z?

12. A length of light rope is wound around a large cylindrical pulley of mass M and radius R. A block of mass m hangs from the end of the rope and is falling down with an acceleration $a = \frac{2}{3}g$. Find the ratio of the two masses, assuming there is no slip.

13. A binary system consists of two neutron stars that are being driven together by gravitational waves. Each neutron star has a mass $1.4\,\mathrm{M}_\odot$ and radius $10\,\mathrm{km}$.

a) Ignoring the effects of relativity and tides, at what orbital period (in seconds) will the neutron stars touch?

b) At that moment, what would be the orbital velocity (in ms^{-1}) of each neutron star?

c) If you include the effects of tides, would the neutron stars touch at a larger or smaller orbital period? Briefly justify your answer.

d) In this case would the velocity be larger or smaller?

14. A spin-half particle travels through a series of Stern-Gerlach apparatuses, oriented to make measurements of the spin component along various axes. The first measurement gives $\frac{1}{2}\hbar$ along the z-axis. The second, third and fourth measure the spin components along, in turn, the x, x and z axes.

a) What is the probability that the second measurement also gives $\frac{1}{2}\hbar$?

b) If instead the second measurement is found to give $-\frac{1}{2}\hbar$, what is the probability that the third measurement will give $\frac{1}{2}\hbar$?

c) If the first two measurements give $-\frac{1}{2}\hbar$ along the z-axis and then $\frac{1}{2}\hbar$ along the x-axis, what is the probability that the fourth measurement will give $\frac{1}{2}\hbar$?

In all cases give a brief justification for your answer.

15. In the Sahara desert, we can approximate surface temperatures by $T_s = \Delta T \sin(\omega t)$, where $\Delta T = 20°$, $\omega = 0.26\,\mathrm{hr}^{-1}$ and t is the time since sunrise. Treating the Earth as a solid with thermal diffusivity $\alpha = 4 \times 10^{-3}\,\mathrm{m}^2\,\mathrm{s}^{-1}$, we may write

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2},$$

where z is the depth. By considering a solution of the form $T(z,t) = U(z)e^{\pm i\omega t}$, calculate how deep one must dig to reach a maximum to minimum temperature variation of less than $1°$.

END OF EXAMINATION PAPER