THREE HOURS

A list of Constants is enclosed

UNIVERSITY OF MANCHESTER

Year 3 General Paper

3 June 2011

14:00 – 17:00

(Physics, Physics with Astrophysics, Physics with Theoretical Physics, Physics with Technological Physics)

Answer as many questions as you can.
Marks will be awarded for your **THIRTEEN** best answers.

Each question is worth 10 marks

Electronic calculators may be used, provided that they cannot store text.

P.T.O.

1. Calculate the wavelength in nanometres of the photon emitted when an electron in a hydrogen atom makes a transition from an energy level with $n = 2$ to one with $n = 1$. In what part of the electromagnetic spectrum does this lie?

2. A parallel plate capacitor has an area of 10^{-3} m^2 and a separation of 0.05 mm. It is filled with a dielectric of relative permittivity 5. What is its capacitance?

3. A string whose linear density is $\mu = 50$ g m^{-1} is maintained at a tension $T = 5$ N. A transverse wave with frequency $f = 100$ Hz is continuously excited along the string. Calculate the speed of propagation of the wave and its wavelength.

4. An electric field E is found to obey the equation:
$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$
Show that a solution exists which is periodic with an angular frequency ω and wavenumber k. Find the relationship between ω, k and $\mu_0\varepsilon_0$. What is the physical significance of this relationship?

5. The total annual energy usage in the UK is about $E = 10^{19}$ J. Assume that this energy is all put into a small spacecraft with a mass of $m = 250$ kg. What would the velocity of the spacecraft be?

6. Consider an electron confined in a box of size 10^{-15} m. What is the uncertainty in its momentum? Hence, by finding the typical kinetic energy of the electron, show that it is not possible for the electron to be confined within a nucleus (assume the typical binding energy of the electron in this case would be 1 MeV).

7. Write down an expression for the entropy S of a system in terms of the number of quantum microstates Ω.
For a change of entropy ΔS from state 1 to state 2, what is the ratio of microstates Ω_2/Ω_1 ?
If ΔS is 1 J K^{-1}, what is Ω_2/Ω_1 ?

8. A plane wave of wavelength λ propagating in the z direction is diffracted from a rectangular aperture lying at the origin of the xy plane. The aperture dimensions are $d_y = 2\, d_x$. Describe the intensity angular distribution of the diffraction pattern along the x direction and the y direction at a great distance from the aperture.

P.T.O.

9. A 1-litre spherical flask contains molecular hydrogen at a temperature of 300 K. Estimate the pressure at which the mean free path of a molecule is larger than the diameter of the flask. You may take the diameter of the hydrogen molecule to be 2.2×10^{-10} m.

10. A spring with spring constant $k = 120$ N m^{-1} is cut into two pieces, one twice as long as the other. What are the spring constants of the two pieces?

11. An electromagnetic plane wave propagating in free space hits a flat dielectric surface (with refractive index n) at an angle i to the direction normal to the surface. If the incident light is unpolarised, the degree of polarisation of the reflected wave is maximised when $i + r = 90°$, where r is the refracted wave angle. Find the relation between i and n in this condition. If the dielectric constant of the medium is equal to 2.25, calculate i.

12. A particle of mass m is constrained to lie within a one-dimensional box extending from $x = -a/2$ to $x = +a/2$, within which it can move freely. It has a wave function for its lowest energy state given by $\Psi(x,t) = A\cos\left(\frac{\pi x}{a}\right)\exp(-i\,Et/\hbar)$.

    Sketch the space dependence of the wave function and hence state the expectation value of x.

    By substituting the wave function into the Schroedinger equation $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} = i\hbar\frac{\partial\Psi}{\partial t}$, find an expression for the energy E of the particle.

13. In a cloud chamber for photographing the trails of α-particles, the temperature of the air is 10° C. If its volume is increased by 50% by a rapid expansion, calculate the final temperature of the air (the ratio of specific heats for air is 1.41).

14. You are holding a book by pushing it with your hand against a vertical wall. The force you apply is at an angle of θ with the upward direction (so $\theta = 0$ is up). The mass of the book is m and the coefficient of static friction between the book and the wall is μ.
    a. Show that the minimum force to keep the book stationary against the wall requires you to push at an angle $\theta = \arctan\mu$.
    b. Show that the force required to hold the book is given by $F = mg\cos\theta$.

15. A galaxy can be approximated by a sphere of uniform density ρ and radius R. Find expressions for the velocity $v(r)$ of a star in a circular orbit of radius r about the galactic centre, considering the cases $0 < r < R$ and $r > R$. Make a sketch of $v(r)$ and suggest why its observed behaviour for $r > R$ might be different.

END OF EXAMINATION PAPER

PHYSICAL CONSTANTS AND CONVERSION FACTORS

SYMBOL	DESCRIPTION	NUMERICAL VALUE
c	Velocity of light in vacuum	$299\,792\,458$ m s^{-1}, exactly
μ_0	Permeability of vacuum	$4\pi \times 10^{-7}$ N A^{-2}, exactly
ϵ_0	Permittivity of vacuum where $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$	8.854×10^{-12} C^2 N^{-1} m^{-2}
h	Planck constant	6.626×10^{-34} J s
$\hbar$	$h/2\pi$	1.055×10^{-34} J s
G	Gravitational constant	6.674×10^{-11} m^3 kg^{-1} s^{-2}
e	Elementary charge	1.602×10^{-19} C
eV	Electronvolt	1.602×10^{-19} J
α	Fine-structure constant, $\frac{e^2}{4\pi\epsilon_0 \hbar c}$	$\frac{1}{137.0}$
m_e	Electron mass	9.109×10^{-31} kg
$m_e c^2$	Electron rest-mass energy	0.511 MeV
μ_B	Bohr magneton, $\frac{e\hbar}{2m_e}$	9.274×10^{-24} J T^{-1}
R_∞	Rydberg energy $\frac{\alpha^2 m_e c^2}{2}$	13.61 eV
a_0	Bohr radius $\frac{1}{\alpha}\frac{\hbar}{m_e c}$	0.5292×10^{-10} m
Å	Angstrom	10^{-10} m
m_p	Proton mass	1.673×10^{-27} kg
$m_p c^2$	Proton rest-mass energy	938.272 MeV
$m_n c^2$	Neutron rest-mass energy	939.565 MeV
μ_N	Nuclear magneton, $\frac{e\hbar}{2m_p}$	5.051×10^{-27} J T^{-1}
fm	Femtometre or fermi	10^{-15} m
b	Barn	10^{-28} m^2
u	Atomic mass unit, $\frac{1}{12} m(^{12}\text{C atom})$	1.661×10^{-27} kg
N_A	Avogadro constant, atoms in gram mol	6.022×10^{23} mol^{-1}
T_t	Triple-point temperature	273.16 K, exactly
k	Boltzmann constant	1.381×10^{-23} J K^{-1}
R	Molar gas constant, $N_A k$	8.314 J mol^{-1} K^{-1}
σ	Stefan-Boltzmann constant, $\frac{\pi^2}{60}\frac{k^4}{\hbar^3 c^2}$	5.670×10^{-8} W m^{-2} K^{-4}
M_E	Mass of Earth	5.97×10^{24} kg
R_E	Mean radius of Earth	6.4×10^6 m
g	Standard acceleration of gravity	$9.806\,65$ m s^{-2}, exactly
atm	Standard atmosphere	101 325 Pa, exactly
$M_\odot$	Solar mass	1.989×10^{30} kg
$R_\odot$	Solar radius	6.96×10^8 m
$L_\odot$	Solar luminosity	3.84×10^{26} W
$T_\odot$	Solar effective temperature	5.8×10^3 K
AU	Astronomical unit, mean Earth-Sun distance	1.496×10^{11} m
pc	Parsec	3.086×10^{16} m
	Year	3.156×10^7 s