



BE/BAT 485/585

Remote Sensing Data and Methods

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vegetation index & phenology Lab.
...Understanding a piece of the Earth system

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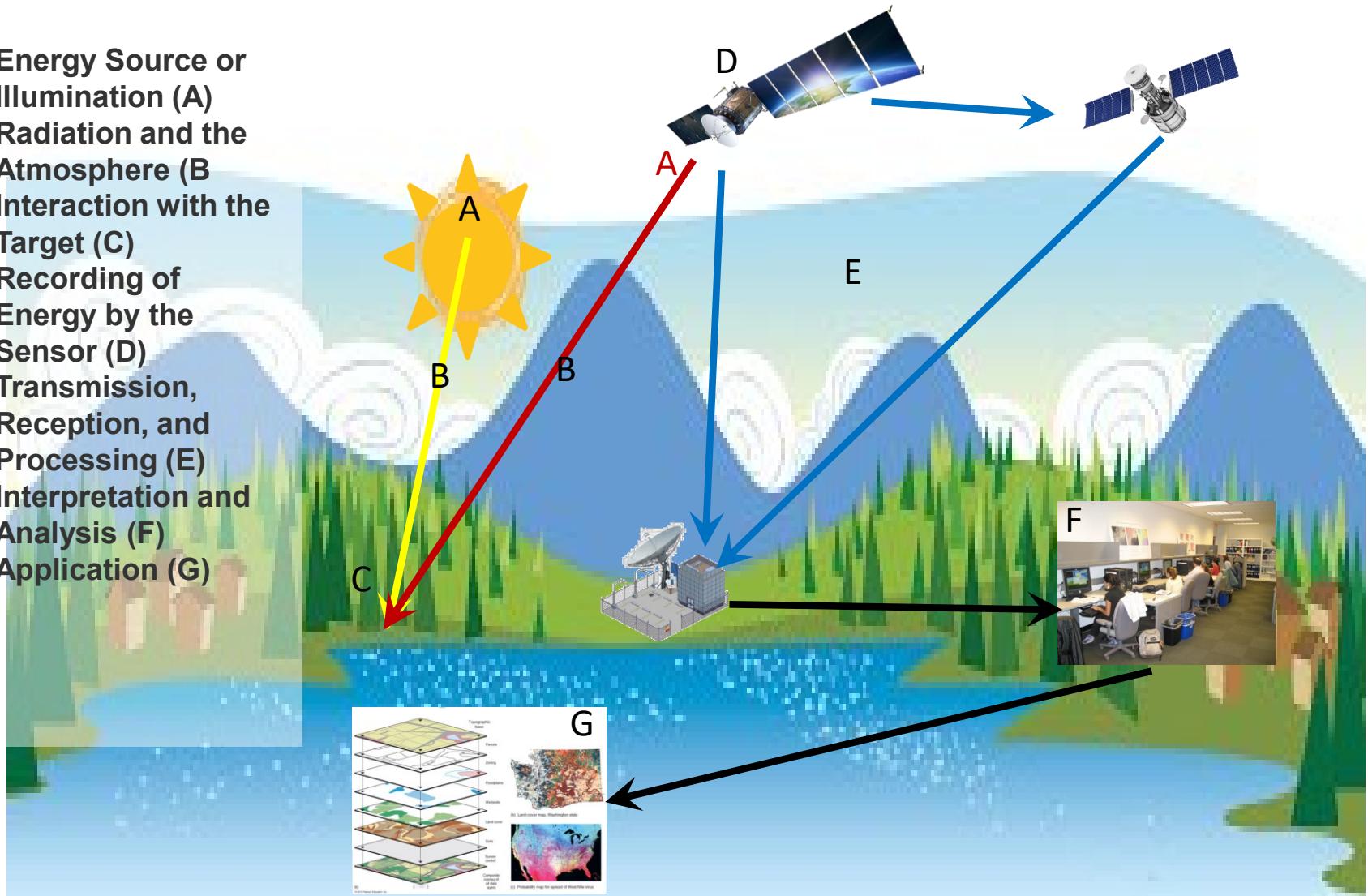
Earth Rise (from the Moon)

- Series of images taken during the Apollo 8/11 missions.
- Photographer Galen Rowell said "*the most influential environmental photograph ever taken*". How relative and how insignificant Earth suddenly becomes, yet how beautiful!

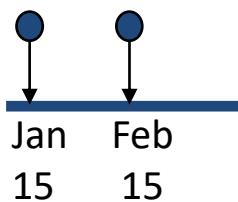
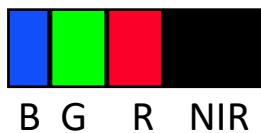
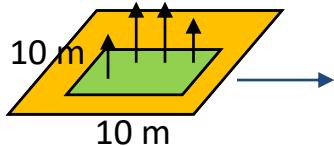


Remote Sensing System

1. Energy Source or Illumination (A)
2. Radiation and the Atmosphere (B)
3. Interaction with the Target (C)
4. Recording of Energy by the Sensor (D)
5. Transmission, Reception, and Processing (E)
6. Interpretation and Analysis (F)
7. Application (G)



Remote Sensing Resolutions



- **Spatial** - the size of the field-of-view, e.g. 10 x 10 m.
 - Currently from few km to few cm (commercial focused on urban or high value)
- **Spectral** - the number, locations, and size of spectral regions the sensor records data in, e.g. blue, green, red, near-infrared, thermal infrared, microwave (radar).
 - We are still stuck with Multispectral (few hyperspectral missions, AVIRSS-N, EO1-Hyperion, Fixed wing (NEON-AOP) etc. Very expensive and daunting.
- **Temporal** - how often the sensor acquires data, e.g. every 30 days, every day, every 15 minutes?
 - Few times a day to once a month, but mostly quasi-biweekly
- **Radiometric** - the sensitivity of detectors to small differences in electromagnetic energy.
 - 8 bits, 12 bits
 - Derived added value can become 16 bit and higher

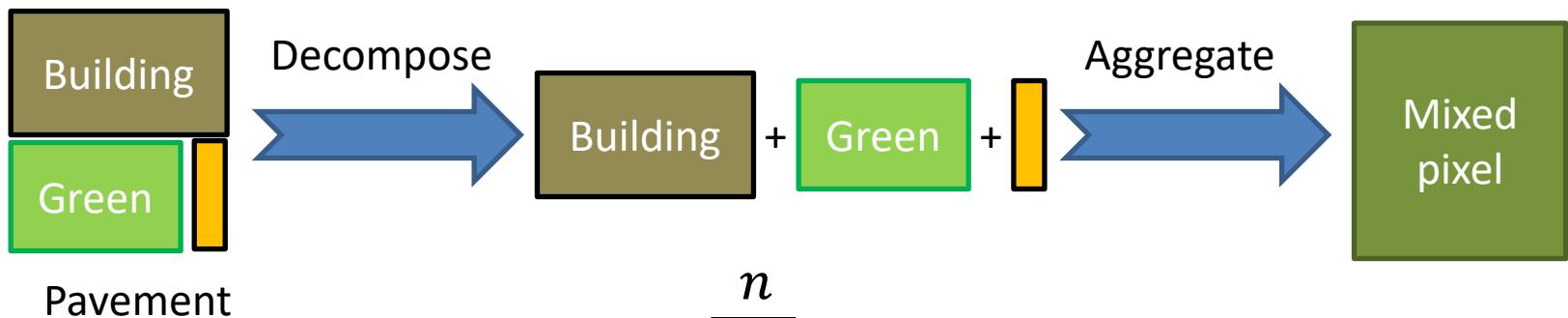
What is a BIT then?

- In remote sensing, a bit stands for the number of grey-scale values a spectral sensor can tell apart (separate).
- The greater the bit number, the greater the number of grey-scale values a spectral sensor can distinguish, and therefore, the higher the radiometric resolution of a spectral sensor.
- One bit stands for a sensor that knows only black and white.
- 2 bit equals 4 grey-scale values and
- 4 bit equals 16 values.
 - The equation is as follows:
 - 2^{bit} = Number of grey-scale values
 - **16 bit? How many values?**
 - 65536
 - **But what is the maximum value?**
 - 65536 or 65535? Why

Bits	Werteumfang	Grauwerte
1Bit	$2^1 = 2 \text{ (0-1)}$	0  1
4Bit	$2^4 = 16 \text{ (0-15)}$	0  15
8Bit	$2^8 = 256 \text{ (0-255)}$	0  255

Mixed Pixels – Spectral Mixing

- Depending on the sensor design and other factors the color (spectral) components of the pure objects are aggregated and result in mixed color/pixel (spectra), which is quite challenging to analyze.



$$\rho = \sum_{i=0}^n K_i \rho_i$$

ρ = Measured Total Radiance/Reflectance

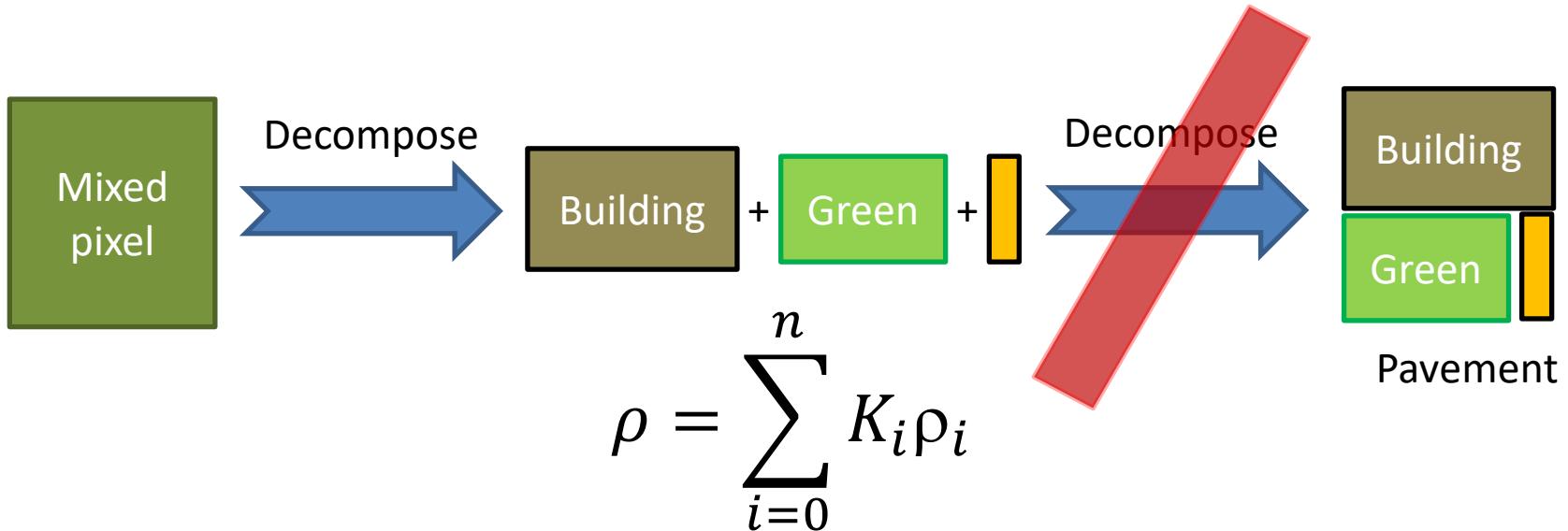
ρ_i = Radiance/Reflectance of pure pixels

K_i = % Contribution of pure pixels

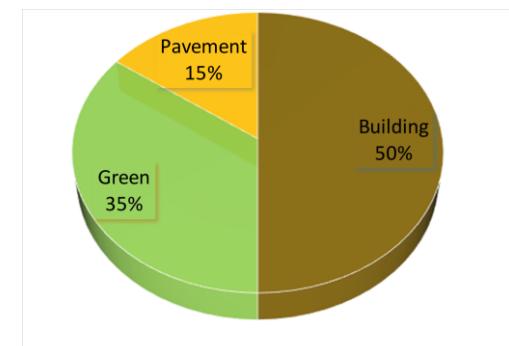
$$\sum_{i=0}^n K_i = 1$$

Decomposing a Pixel

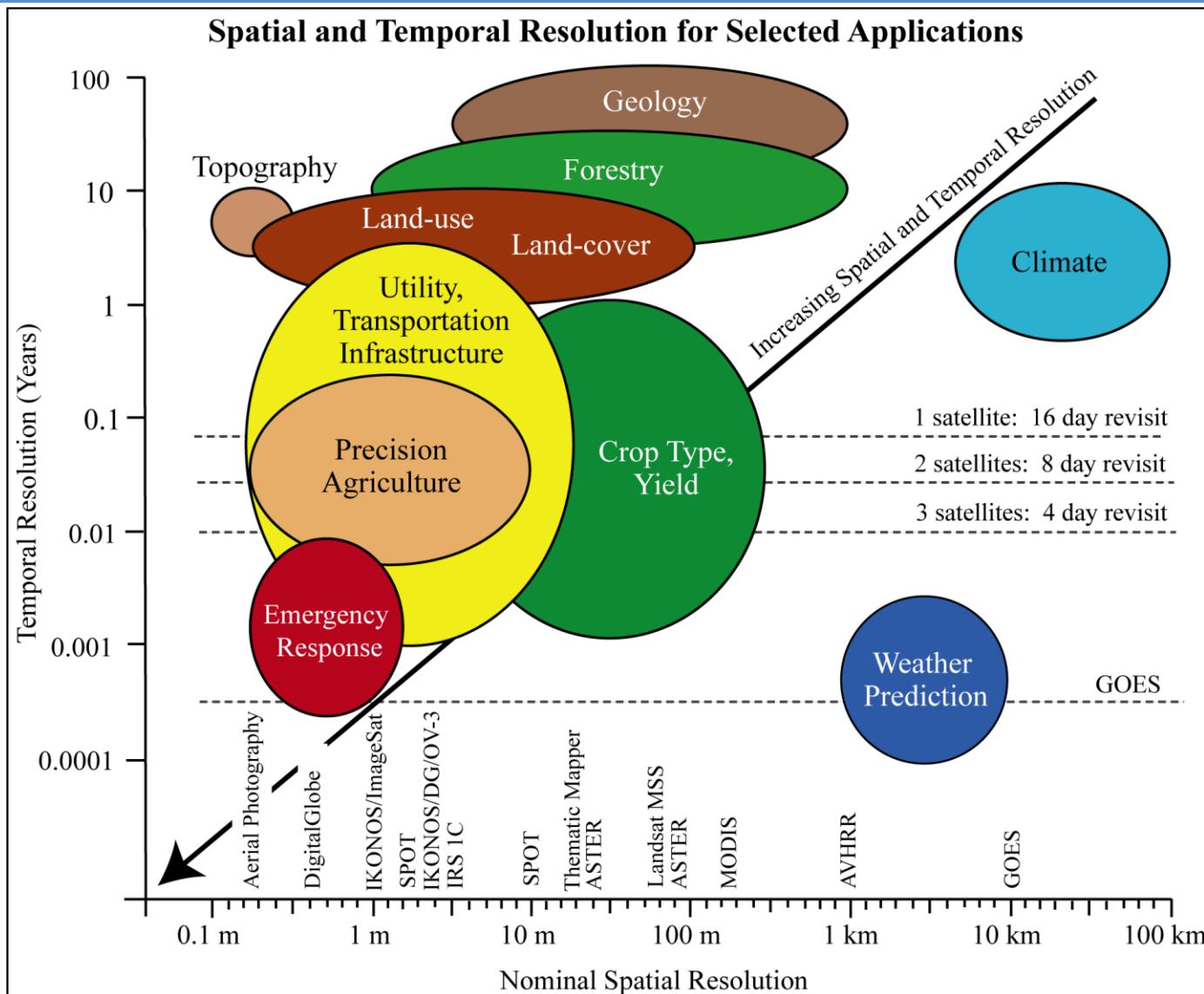
- Decomposing a pixel is to reproduce its constituents (spectral mixture analysis)



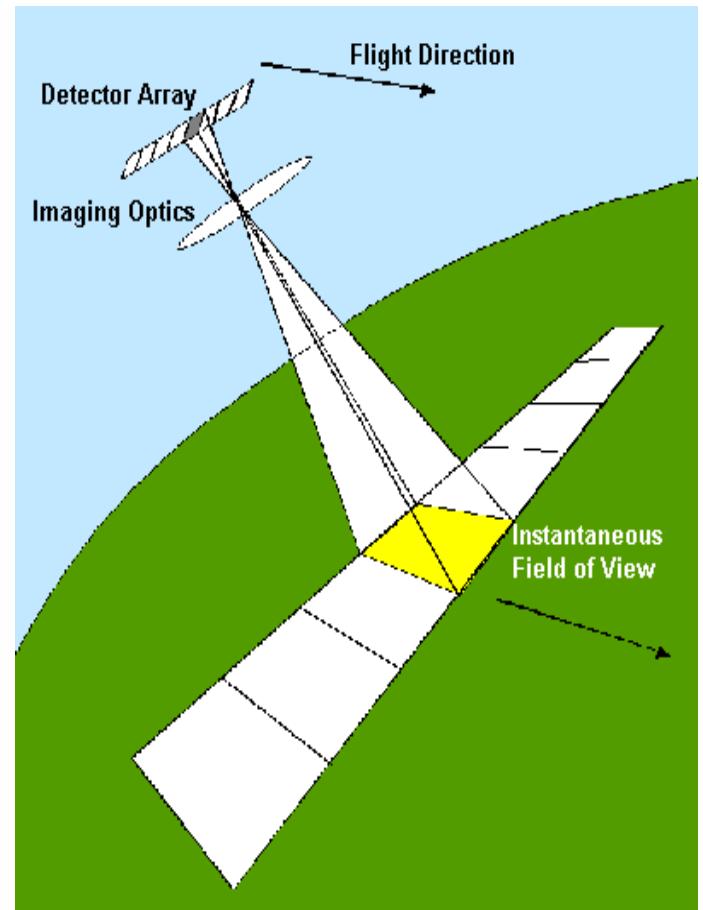
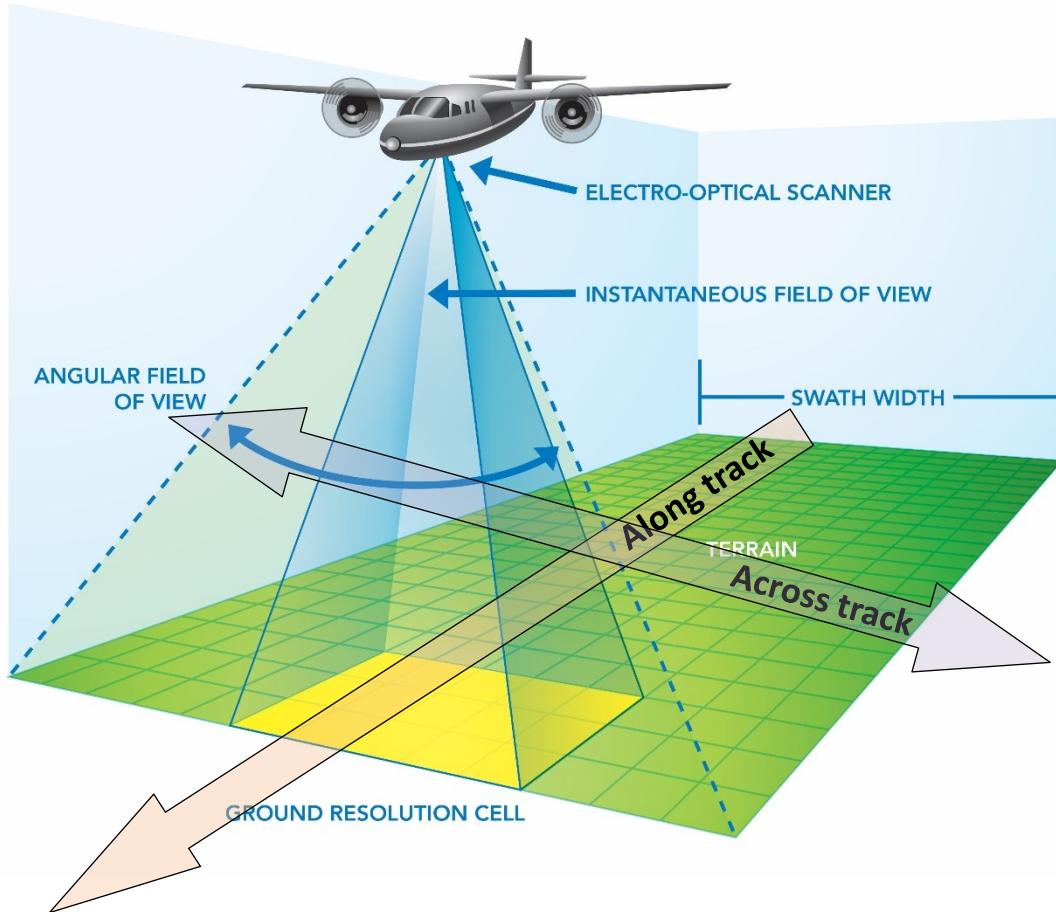
- Basically, find the K_i 's that when combined with the pure pixel signals (ρ_i) give us the total radiance measured (ρ)
- No idea on shape, location, or other pure signals within the mixed pixel
- You could think of a **research topic/project** that can actually predict the location of the 'pure signal' pixel?
 - Can you think of How?



There are spatial and temporal resolution considerations that must be made for each remote sensing applications

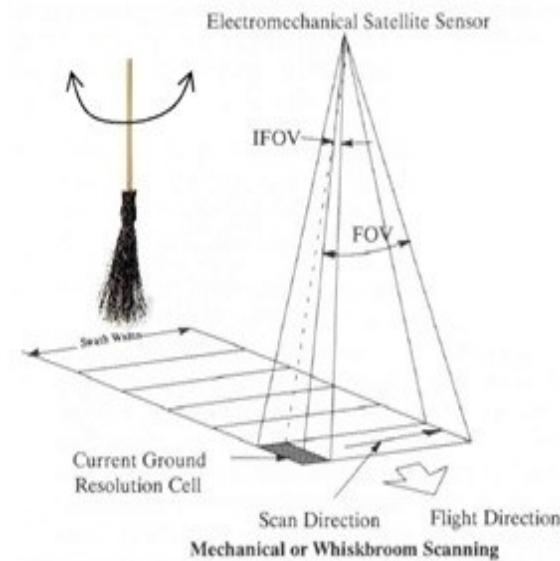
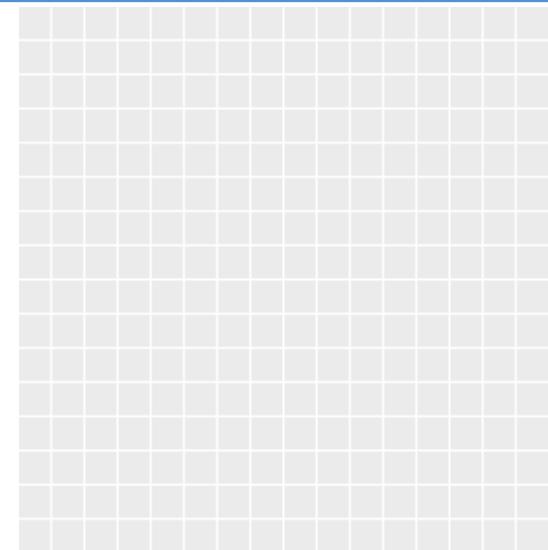


Observation Arrangement



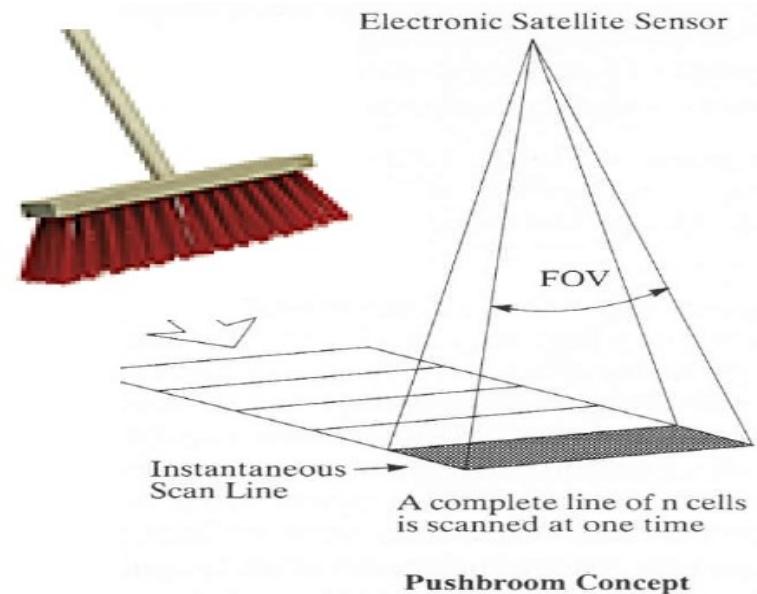
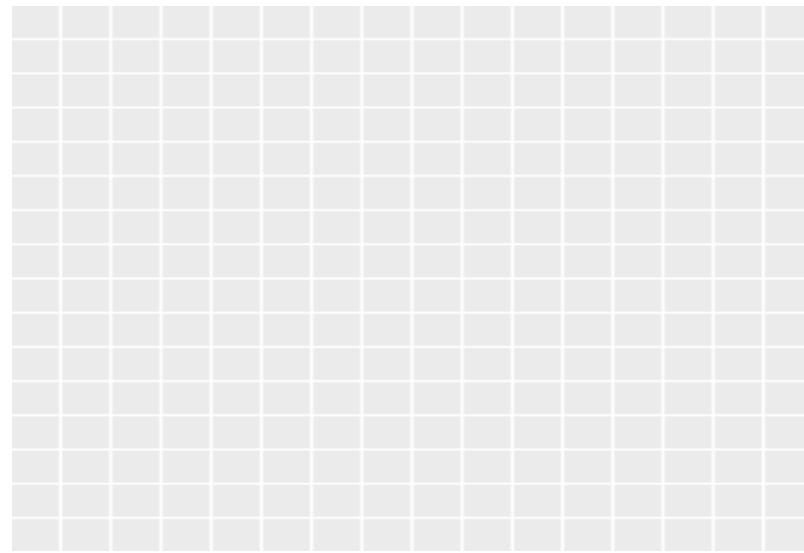
ACROSS-TRACK SCANNING

- These systems build 2-D images of the terrain beneath an aircraft.
- Uses a rotating or oscillating mirror, these scanning mirror systems scan back-and-forth along the flight line at right angles.
- Once incoming energy is reflected off the scanning mirror system, it is separated into several spectral components that are sensed independently.



ALONG-TRACK SCANNING

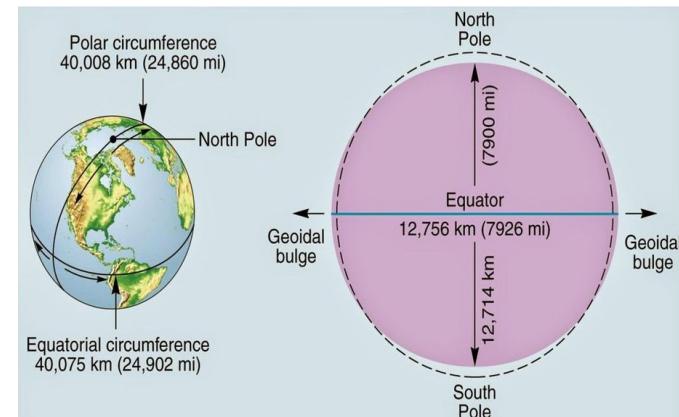
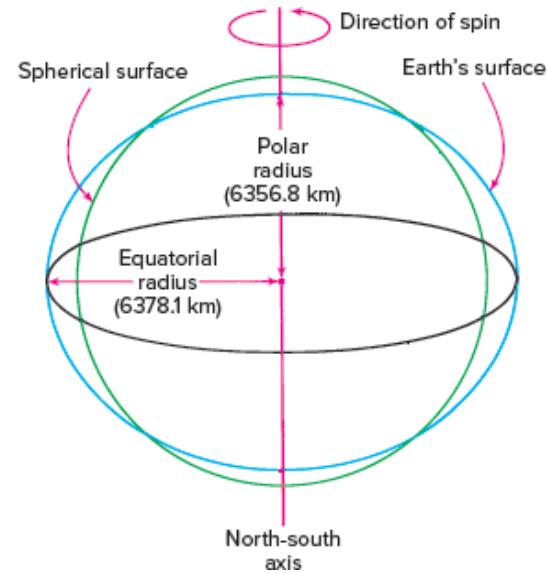
- Records multispectral image data beneath an aircraft, just like across-track scanners.
- Difference: linear array of detectors are used in replace of a rotating or oscillating mirror.
- Each spectral band of sensing requires its own linear array.



Pushbroom Concept

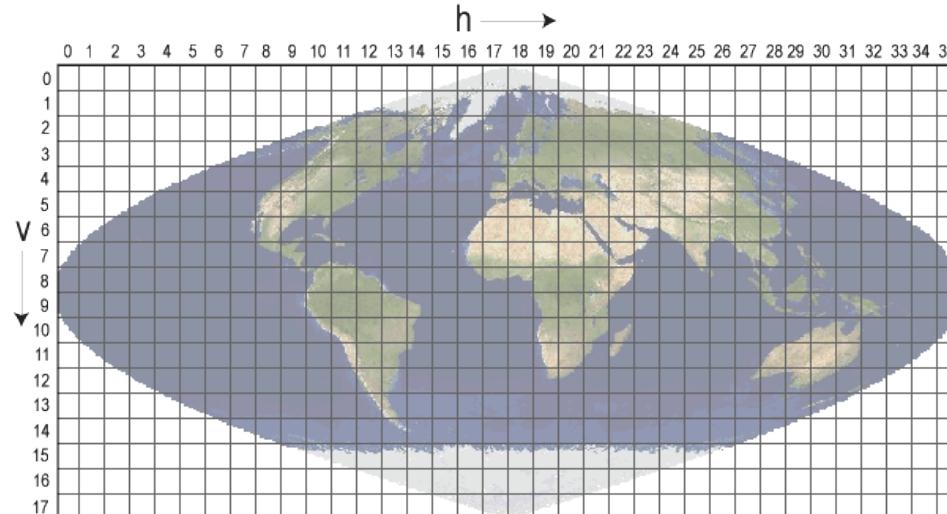
The Earth Dimensions

- **Radius, diameter and circumference**
 - The radius of Earth at the equator is 3,963 miles (6,378 kilometers), according to NASA's Goddard Space Flight Center.
 - However, Earth is not quite a sphere. Rotation causes it to bulge at the equator.
 - Earth's polar radius is 3,950 miles (6,356 km) — a difference of 13 miles (22 km).
- Using those measurements, the equatorial circumference of Earth is about 24,901 miles (40,075 km). However, from pole-to-pole — the meridional circumference — Earth is only 24,860 miles (40,008 km) around.
- This shape, caused by the flattening at the poles, is called an **oblate spheroid**.



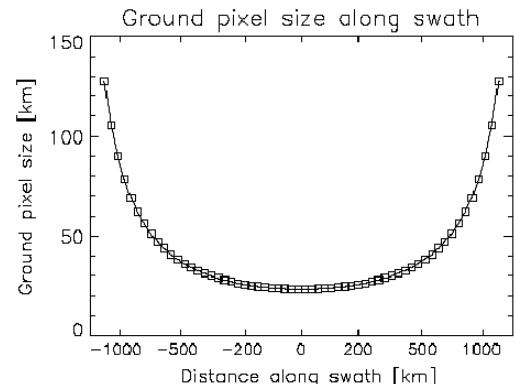
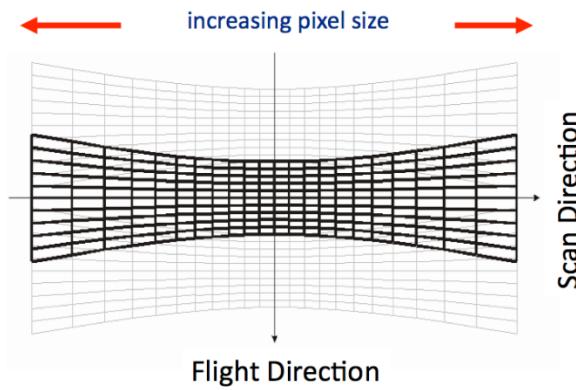
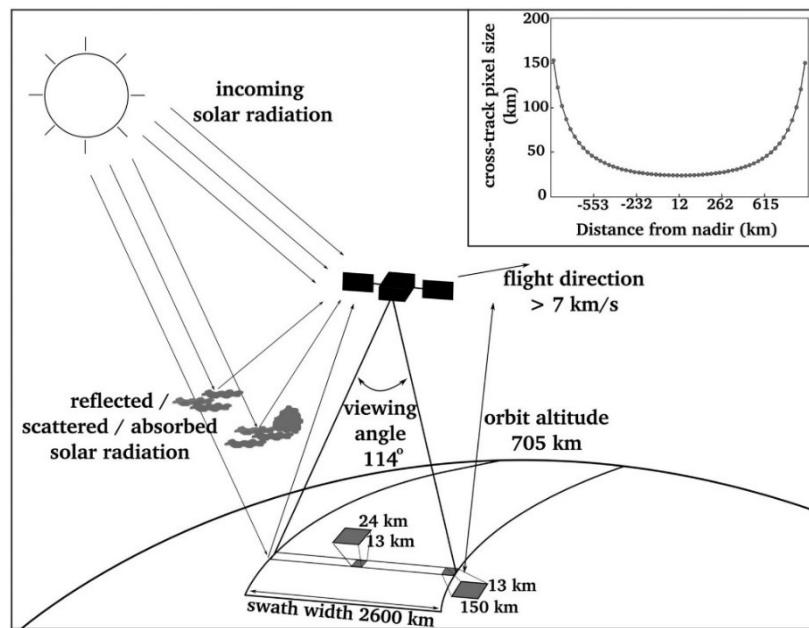
So, can you now convert between degrees and m/km?

- Well, we can use the equatorial circumference which is also equal to 360 Degrees?
 - 360 Degree = 40,075 km is about ~111.3194 km/degrees
 - Or 0.008983 degree/km
- This is only true at the Equator? So keep that in mind
- MODIS (the current work horse of atmosphere, land, and Ocean remote sensing) suggests a “**40,200**” ~km circumference at the equator? (1200 km x 36)
- Use 1200 x 1 km or 4800 x 250m tiles
 - Can you compute the actual MODIS Pixels size?
 - The pixel size is not exactly 1km (~926.62543305 m and ~232 m)
- Can you think or foresee another issue/challenge?
 - Elevation DEM?



Implications

- Due to this Earth's curvature and the large swath size (large view angles) the observation footprint changes from nadir to swath/scan edge.
- And it matters a lot?



Implications

- Here is an example of how that works out
- These are 2 Images from the same days 2 identical sensors (8 days apart) for Tucson (MODIS Terra vs. Aqua)



Terra



Aqua



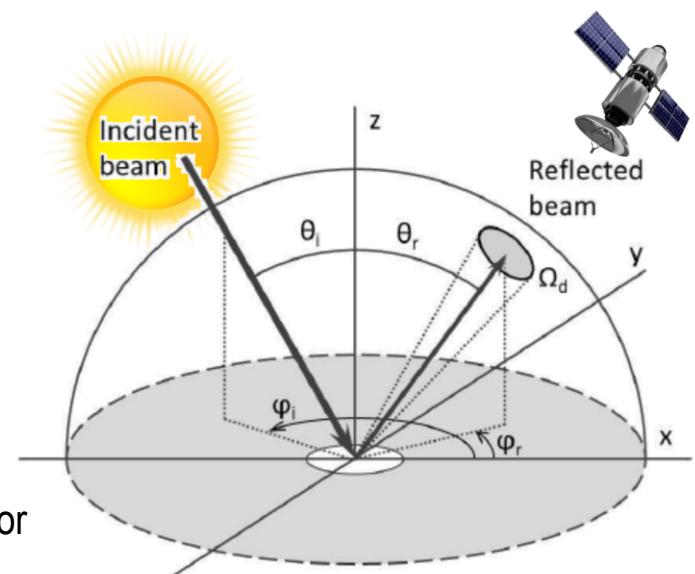
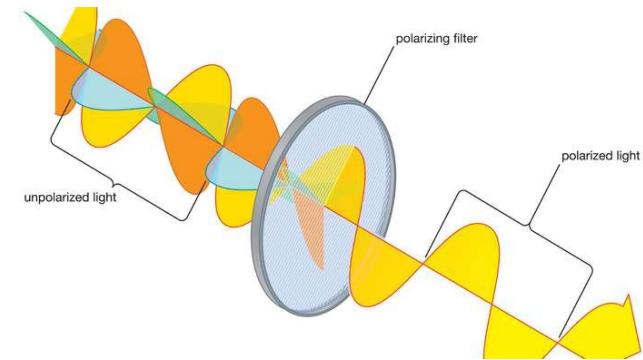
**LET'S ADD TO WHAT WE HAVE
LEARNED SO FAR**

Angular Information

- Remote sensing systems record very specific *angular characteristics* associated with each exposed pixel. The angular characteristics are a function of:
 - Location in a three-dimensional sphere of the illumination source (e.g., the Sun for a passive system or the sensor itself in the case of RADAR, LIDAR, SONAR, etc.) and its associated azimuth and zenith angles,
 - Orientation of the terrain (pixel) or terrain cover (e.g., vegetation), and
 - Location of the suborbital or orbital remote sensing system and its associated azimuth and zenith angles.

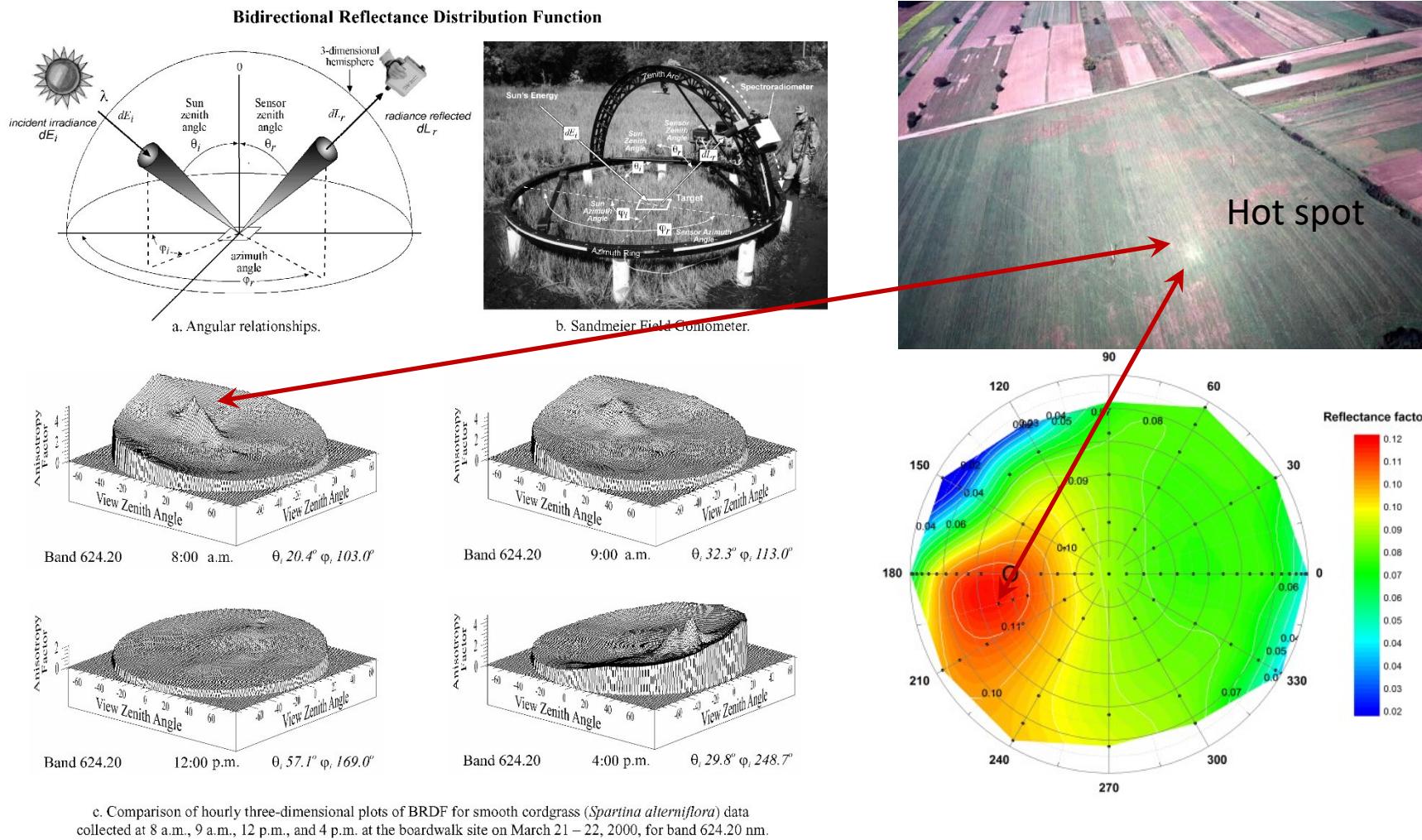
Angular Information

- There is always an angle of **incidence** associated with the incoming energy that illuminates the terrain (object of interest)
- And an angle of **exitance** from the terrain to the sensor system.
 - **Radiant exitance** is the **flux** emitted by a surface per unit area
 - Spectral **exitance** or spectral emittance is the radiant **exitance** of a surface per unit frequency or wavelength, depending on whether the spectrum is taken as a function of frequency or of wavelength.
- This ***bidirectional*** nature of remote sensing data collection is known to influence the spectral and polarization characteristics of the at-sensor radiance, L , recorded by the remote sensing system.
- There are **four angles** to remember (*will come back to this*):
 - **Incident beam**
 - Sun Zenith Angle (θ_i, θ_s)
 - Sun Azimuth Angle (ϕ_i, ϕ_s)
 - **Reflected beam**
 - Sensor/View Zenith Angle (θ_r, θ_v)
 - Sensor/View Azimuth Angle (ϕ_r, ϕ_v)
 - **Relative Azimuth** is a term that combines the Sun and Sensor Azimuth ($\phi_s - \phi_v$)



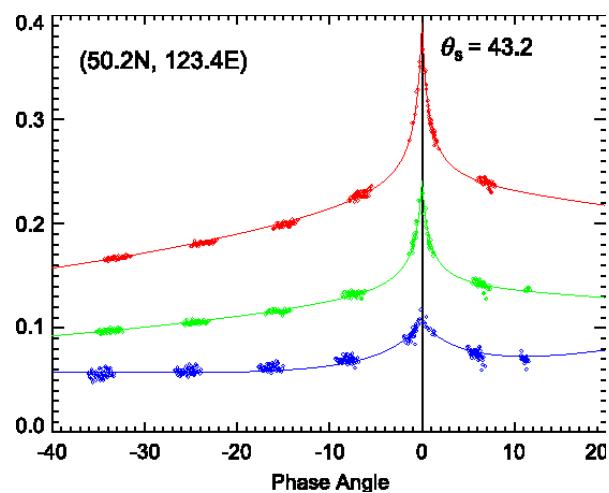
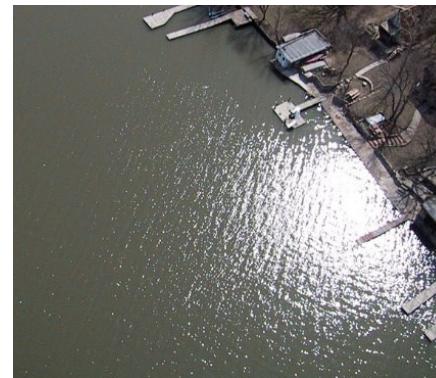
Angular Information - BRDF

- BRDF = Bidirectional Reflectance Distribution Function



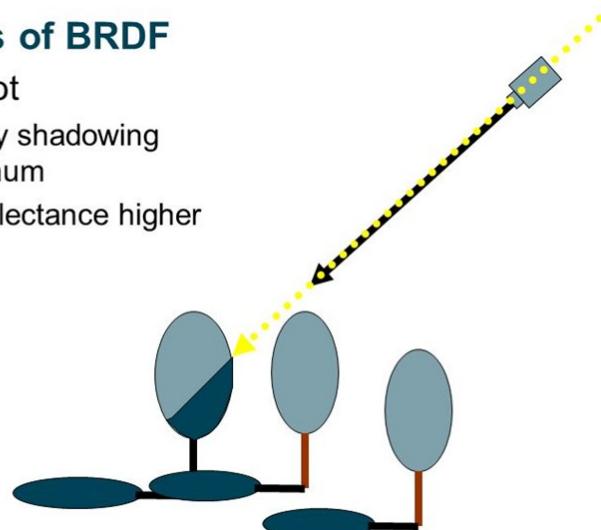
Angular Information - BRDF

- BRDF = Bidirectional Reflectance Distribution Function



Features of BRDF

- Hot Spot
 - mainly shadowing minimum
 - so reflectance higher



Angular Information

- BRDF Matters?



Radiometry vs. Photometry

Radiometry

is the measurement of optical radiation including visible light.

Photometry

is the measurement of visible light only.

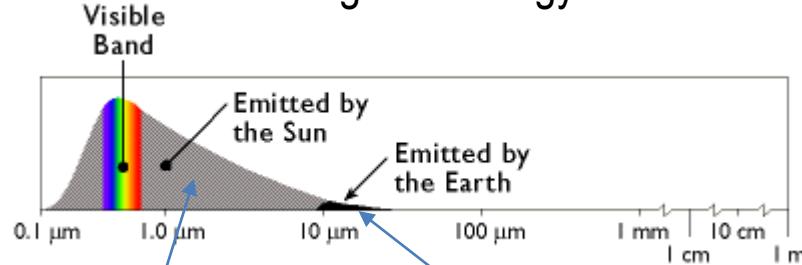
Both are forms of energy!

What is Energy?

- Energy is the ability to do work
- “Energy in motion is kinetic, energy that is waiting is potential”*
- Energy can be transferred from one place to another by **conduction**, **convection**, **radiation**
- **Radiation** is the only form of energy transfer that can take place in a vacuum (space, sun to Earth or other planets in our system)

Electromagnetic Spectrum

- Most remote sensing instruments measure “Electromagnetic radiation”
- Electromagnetic radiation is a form of energy emitted by all matter above absolute zero temperature (0 Kelvin or -273° Celsius).
 - You could think of absolute ZERO as hibernation
- X-rays, ultraviolet rays, visible light, infrared light, heat, microwaves, and radio and television waves are all forms of electromagnetic energy.

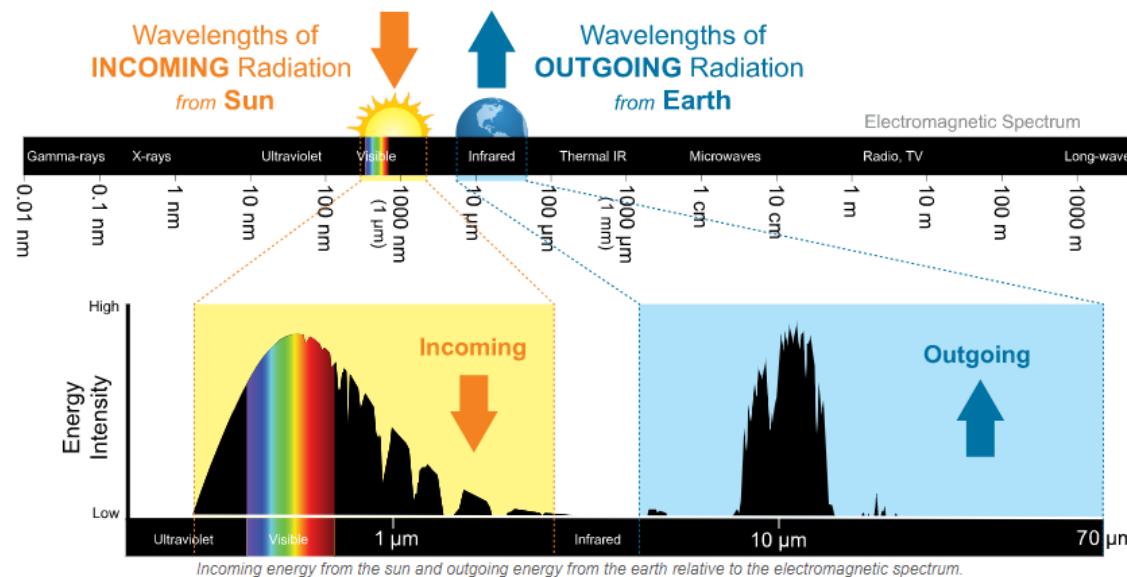


Electromagnetic energy emitted by the Sun and the Earth across the range of wavelengths called the electromagnetic spectrum.

- Hotter objects radiate **more electromagnetic energy** than cooler objects.
- Hotter objects also radiate energy at **shorter wavelengths** than cooler objects.
 - Example: The Sun emits more energy than the Earth, and the Sun's radiation peaks at shorter wavelengths.
- The portion of the electromagnetic spectrum at the peak of the Sun's radiation is called the **visible band** because the human visual perception system is sensitive to those wavelengths.
- Remote sensing technologies extend our ability to sense electromagnetic energy beyond the visible band, allowing us to see the Earth's surface in new ways, which, in turn, reveals patterns that are normally invisible.

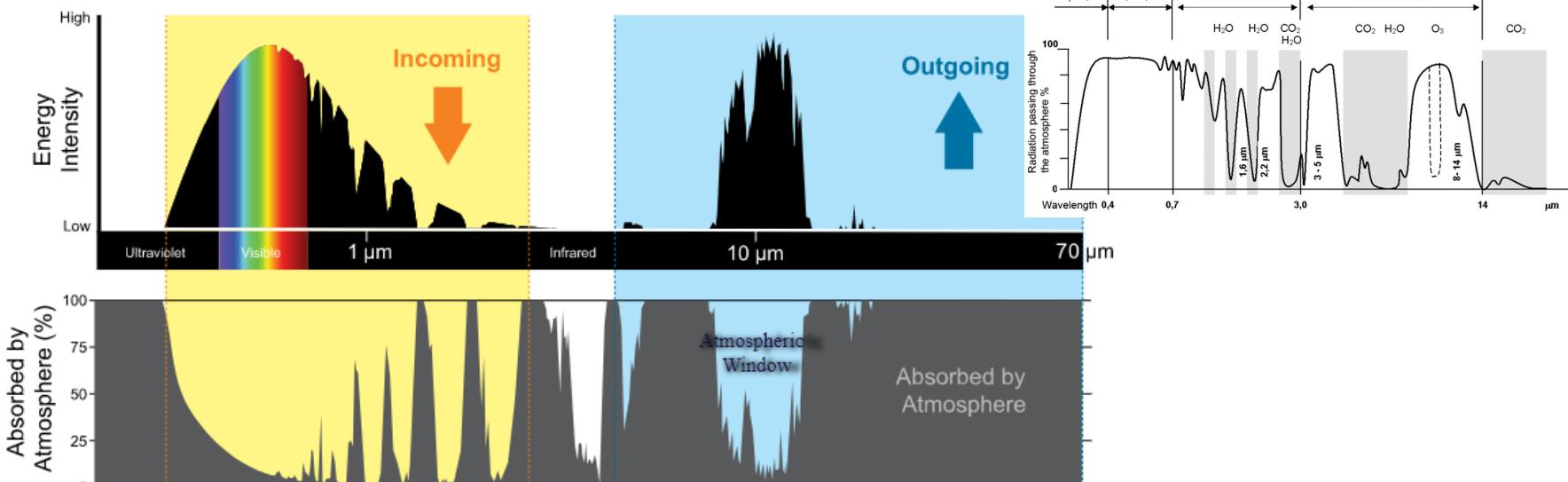
Transmissivity of the atmosphere

- **Yellow and Cyan areas indicate wavelengths at which the atmosphere is partially or wholly opaque (nothing passes).**
- At certain wavelengths, the atmosphere poses an obstacle to satellite remote sensing by absorbing electromagnetic energy.
- Sensing systems are therefore designed to measure wavelengths **within the windows** where the transmissivity of the atmosphere is greatest (some energy can be measured).



Atmospheric Windows

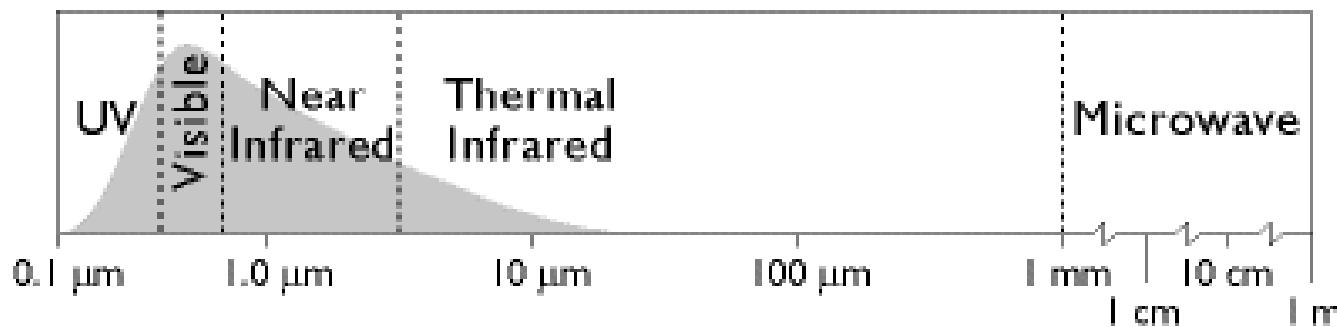
- Spectral sensors can only store the detected parts of the electromagnetic radiation **scattered back** from the Earth surface that **can pass the atmosphere**.
- The Earth's atmosphere contains gases :
 - Water, Ozone, Nitrogen, CO₂, etc. and
 - Small particles, like smoke, ice crystals, water droplets, dust, etc....
 - These so-called **aerosols absorb** and **scatter** all or some parts of the electromagnetic spectrum
- So, regions of the spectrum that are absorbed by the atmosphere are called **absorption bands**.
- Wavelengths getting through the atmosphere to the sensor are called **atmospheric window**.
- The figure below depicts how intense the solar radiation is per wavelength range.
- The intensity of the reflected **radiation energy** decreases the closer we get to the long-wave range of the electromagnetic spectrum.



The places with limited or almost no absorption by the atmosphere is known as the **atmospheric window** - allowing us to peer into the atmosphere at various wavelengths.

Electromagnetic Spectrum

- There are regions of the electromagnetic spectrum that are key to remote sensing.
- Remote sensing systems have been developed to measure reflected or emitted energy at various wavelengths for different purposes.
- Example for Land Use and Land Cover (LULC) mapping we usually rely on the visible, infrared, and microwave bands
- For Temperature we use the Infrared and Thermal



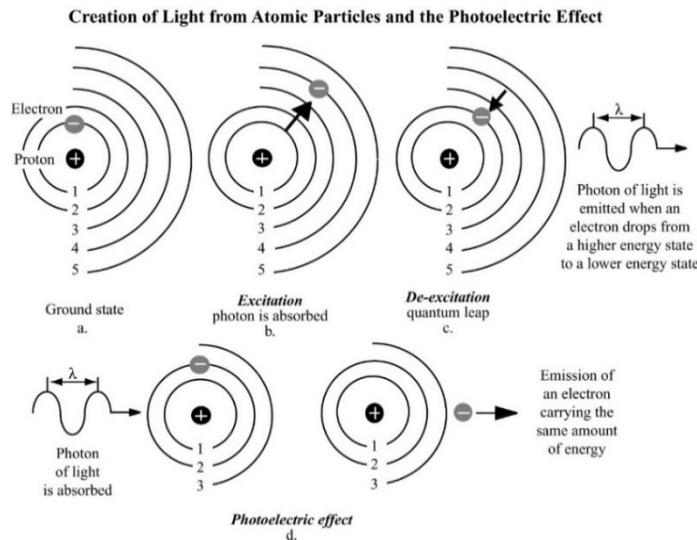
Electromagnetic radiation (EMR)

- Energy interacts with various objects before it is measured by a remote sensing system.
- To properly interpret remote sensing measurements, it is important to understand these interactions:
- E.g., if the source of the energy is the Sun then the energy
 - is radiated by atomic particles at the source (Sun)
 - travels through space at the speed of light,
 - interacts with Earth's atmosphere,
 - interacts with Earth's surface,
 - interacts with Earth's atmosphere again,
 - interacts with the remote sensor (captured)
- This needs to be considered during pre/post processing to generate the data we will use

**THE FOLLOWING YOU CAN READ ON
YOUR OWN – AND I WILL CLARIFY IT**

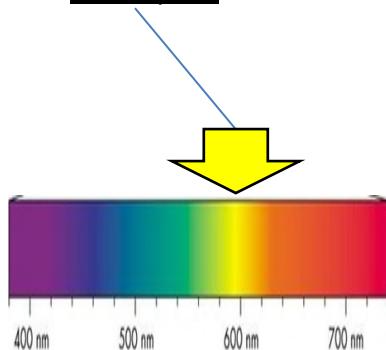
How Light is Created From Atomic Particles

- A **photon** of electromagnetic energy is emitted when an electron in an atom or molecule drops from a **higher-energy** state to a **lower-energy** state.
 - The light emitted (i.e., its wavelength) is a function of the changes in the energy levels of the outer, valence electron.
 - For example, **yellow** light may be produced from a sodium vapor lamp (electrons of sodium change/drop orbits, then regain their orbits via electric energy).
- Matter can also be subjected to high temperatures that electrons, which normally move in captured, non-radiating orbits, are broken free. When this happens, the atom remains with a positive charge equal to the negatively charged electron that escaped. The electron becomes a free electron, and the atom is called an ion. If another free electron fills the vacant energy level created by the free electron, then radiation from all wavelengths is produced, i.e., a continuous spectrum of energy.
- The intense heat at the surface of the Sun produces a **continuous spectrum** in this manner.



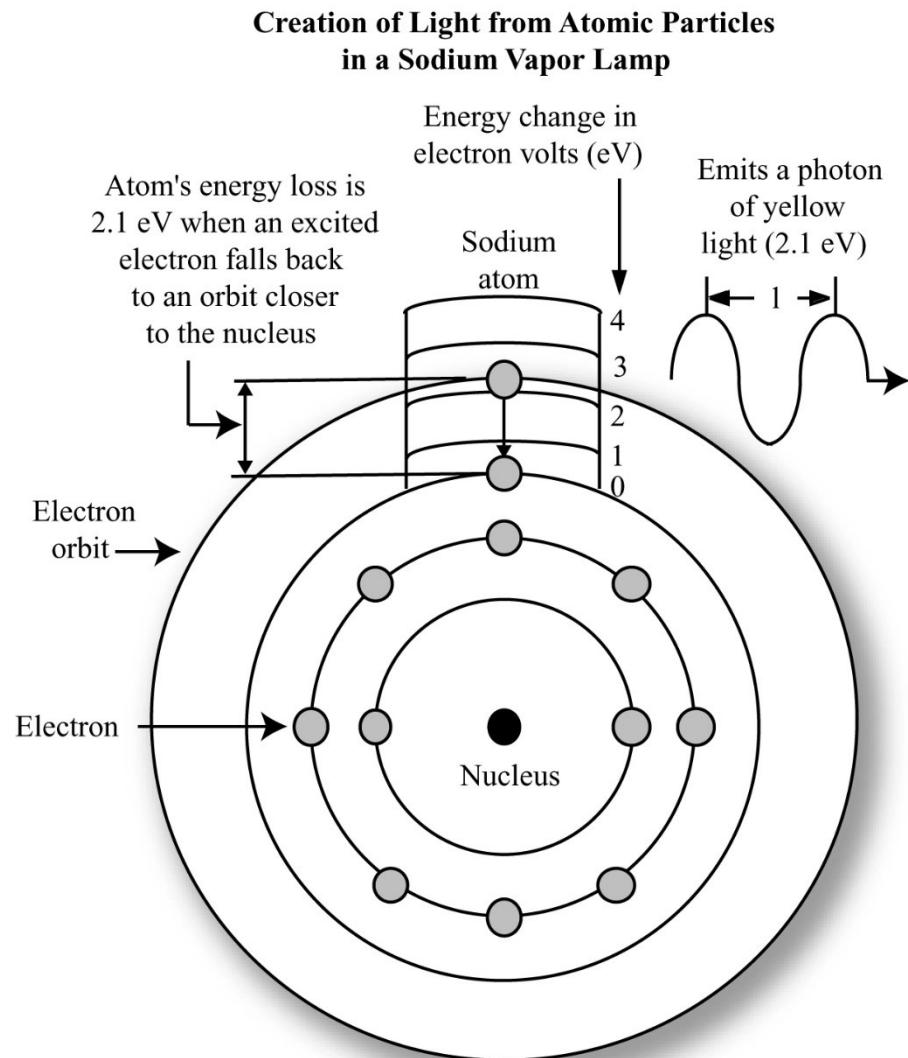
Particle Model of Electromagnetic Energy

- Substances have **color** because of **differences in their energy levels** and the selection rules.
- For example:
 - Consider energized **sodium vapor** that produces a bright yellow light that is used in some streetlamps. When a sodium-vapor lamp is turned on, several thousand volts of electricity energize the vapor. The outermost electron in each energized atom of sodium vapor climbs to a high rung on the energy ladder and then returns down the ladder in a certain sequence of rungs, the last two of which are 2.1 eV apart.
 - The energy released in this last leap appears as a photon of **yellow** light with a wavelength of $0.58 \mu\text{m}$ with 2.1 eV of energy.



Creation of Light

- After being energized by several thousand volts of electricity, the outermost electron in each energized atom of sodium vapor climbs to a high rung/orbit on the energy ladder and then returns down the ladder in a predictable fashion. The last two rungs in the descent are 2.1 eV apart. This produces a photon of **yellow** light, which has 2.1 eV of energy.

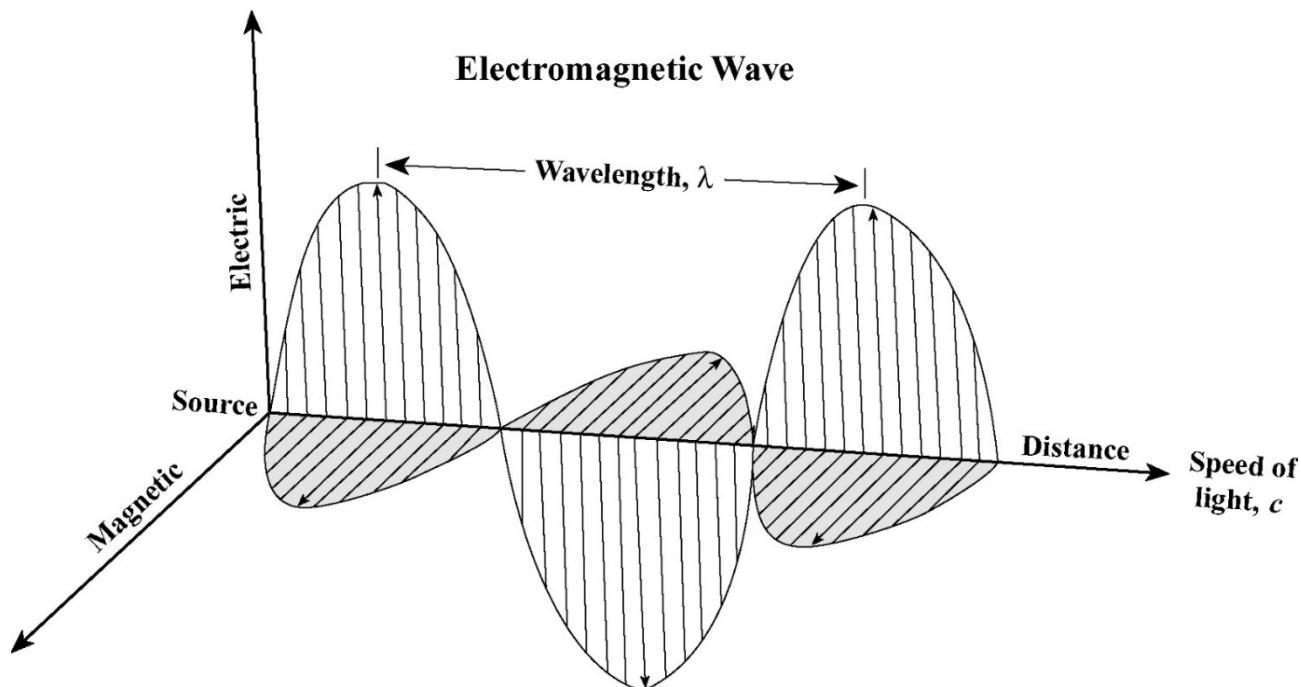


EMR models

- There are two different models used to help us understand and explain how EMR is created, how it travels through space and how it interacts with other matter
 - Wave model
 - Particle model

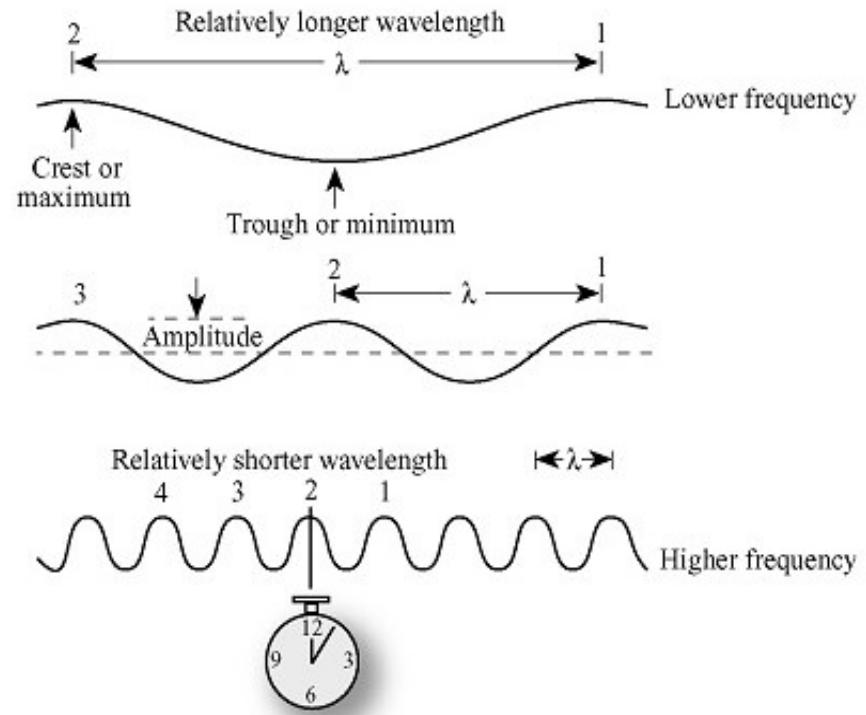
EMR : Wave model

- Wave Model > EMR conceptualized as an electromagnetic wave that travels through Space at the speed of light ($3 \times 10^8 \text{ m s}^{-1}$)
- Two fluctuating fields – one electric, one magnetic. They are at 90° angles to one another and both are perpendicular to the direction of travel



Wavelength & Frequency

- **Wavelength (λ ; nm - m)**
distance between crests (or troughs)
- **Frequency (ν ; Hz)**
The number of crests (or cycles) that pass a given point each second
- **Speed of light (c):** defined by the distance traveled by a crest in one second ($3 \times 10^8 \text{ m s}^{-1}$)
- These entities are **correlated by:**



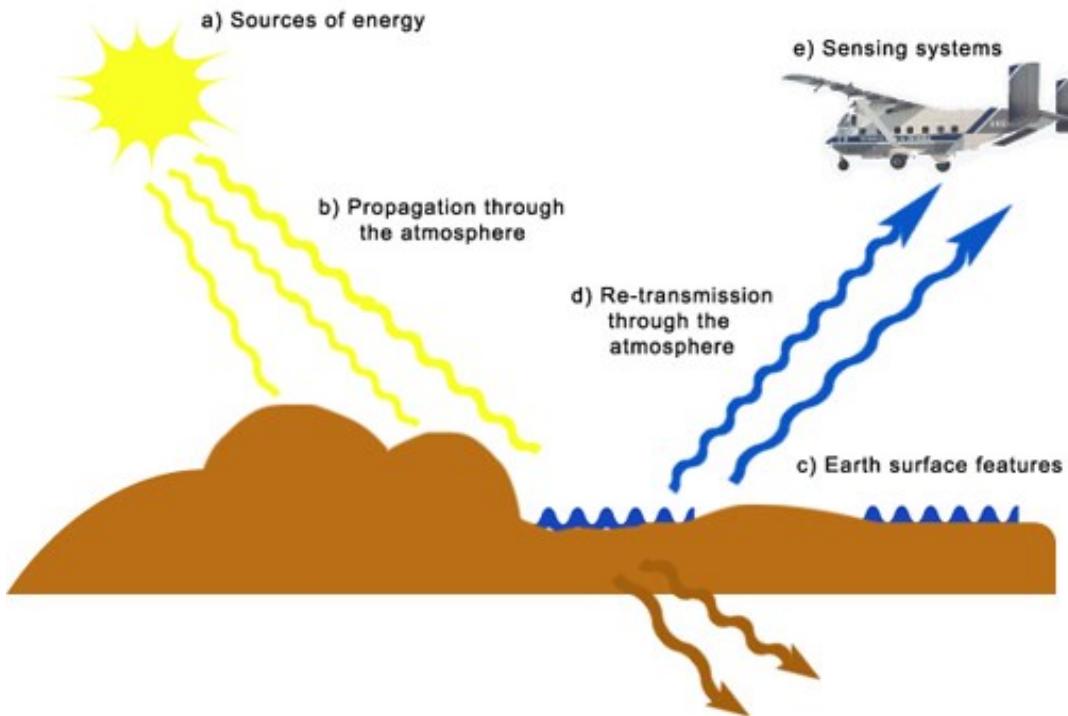
$$c = \lambda \cdot \nu$$

$$\lambda = \frac{c}{\nu}$$

$$\nu = \frac{c}{\lambda}$$

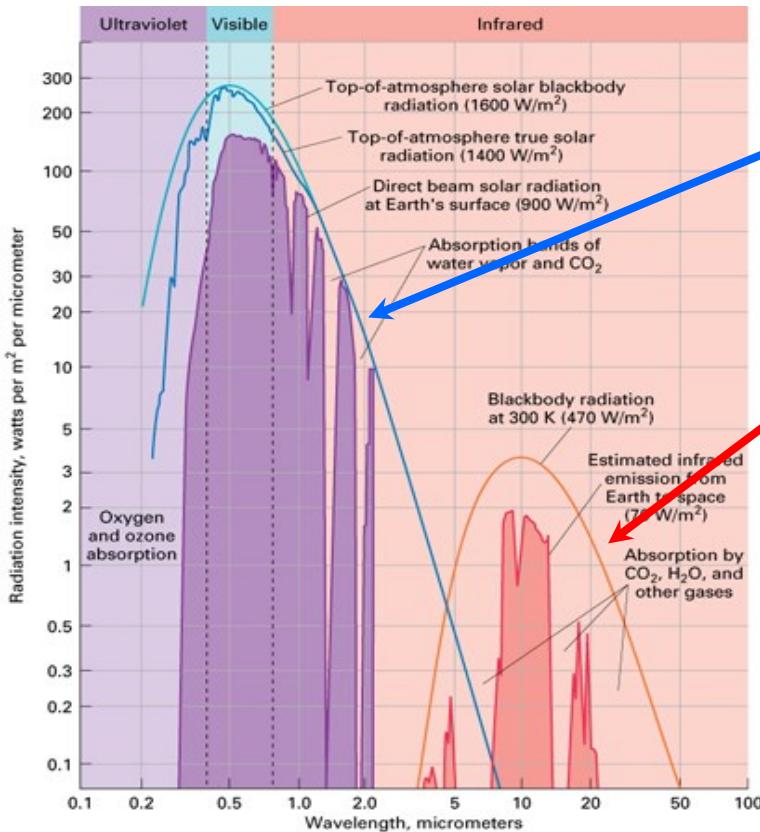
The relationship between temperature and radiated energy and dominant wavelength

- All objects with a temperature greater than 0 K (-273°C) emit EMR
- The Sun is the primary source of most EMR recorded by passive remote sensing systems



Temperature, Energy, and Waves

- The hotter the object....
 - (i) the more energy it radiates (**Stefan-Boltzmann Law**)
 - (ii) the shorter its dominant wavelength (**Wien's Displacement Law**)



Sun-to-Earth incoming shortwave radiation
Blackbody temperature = $6,000 \text{ K}$ ($\sim 6,000^\circ\text{C}$ or $11,000^\circ\text{F}$) Sun with a dominant wavelength of $0.48\mu\text{m}$

Earth-to-Space outgoing longwave radiation
Blackbody temperature = 300 K ($\sim 15^\circ\text{C}$ or 60°F); dominant wavelength of $9.66 \mu\text{m}$

A Blackbody is a theoretically perfect radiator and absorber of energy. (And it does not have to be black!)

- Radiates energy at the maximum possible rate per unit area at each wavelength for any given temperature
- Absorbs all the radiant energy incident on it (i.e., no reflected or transmitted energy)

Stefan-Boltzman Law

- The total radiation emitted from a blackbody

$$M_\lambda = \sigma \times T^4$$

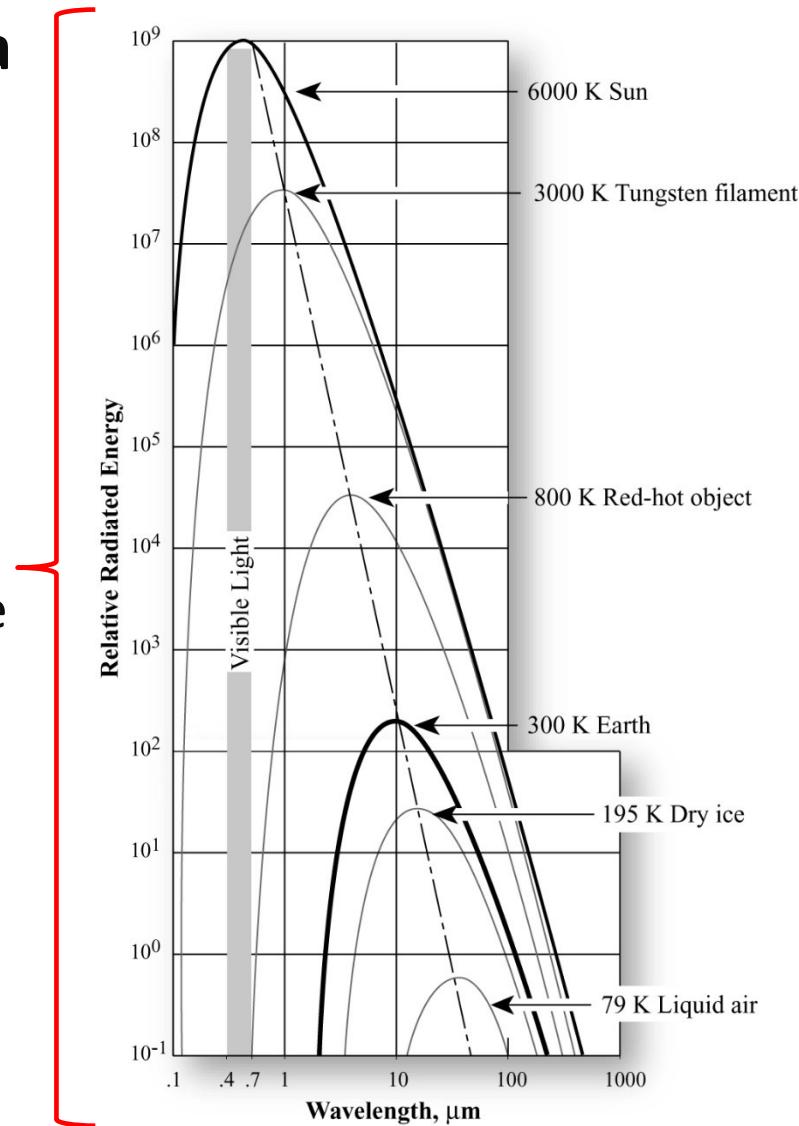
M_λ = total emitted radiation from a blackbody (W m^{-2})

σ = Stefan-Boltzmann constant ($5.6697 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$)

T = absolute temperature (degrees Kelvin)

- What does this mean?

- The hotter the object, the greater the amount of energy emitted by that object
- Actual amount of energy emitted corresponds to sum/integral of the area under its curve



Wien's Displacement Law

$$\lambda_{\max} = \frac{k}{T}$$

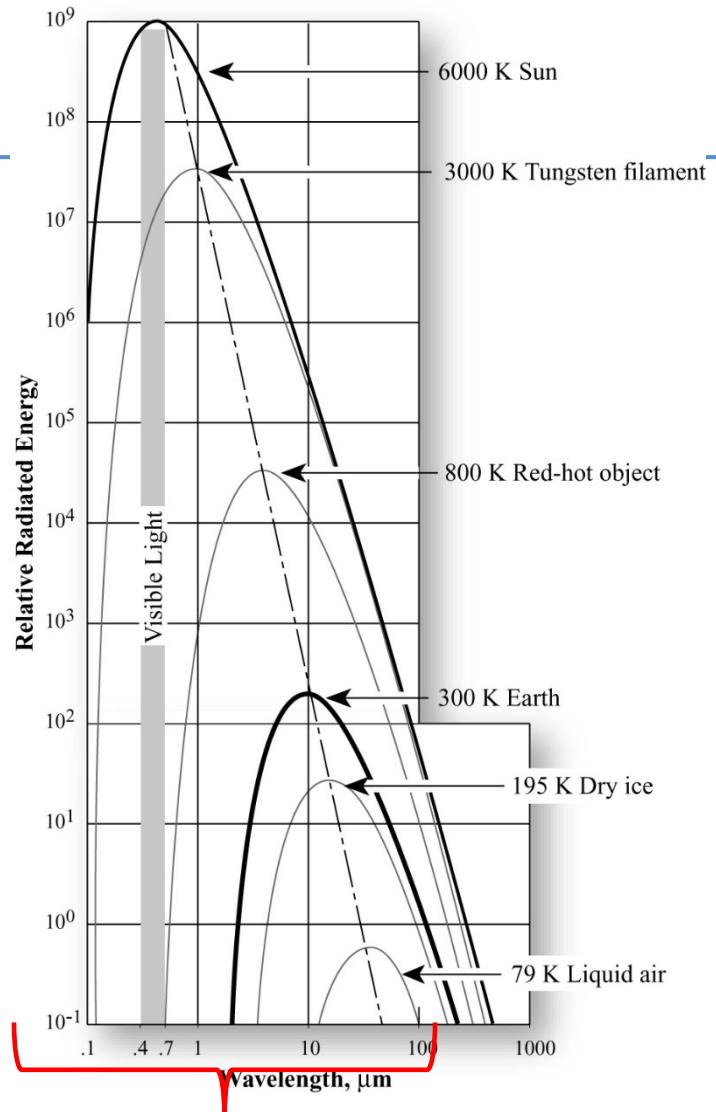
λ_{\max} = dominant wavelength

k = constant (2898 $\mu\text{m K}$)

T = absolute temperature (degrees Kelvin)

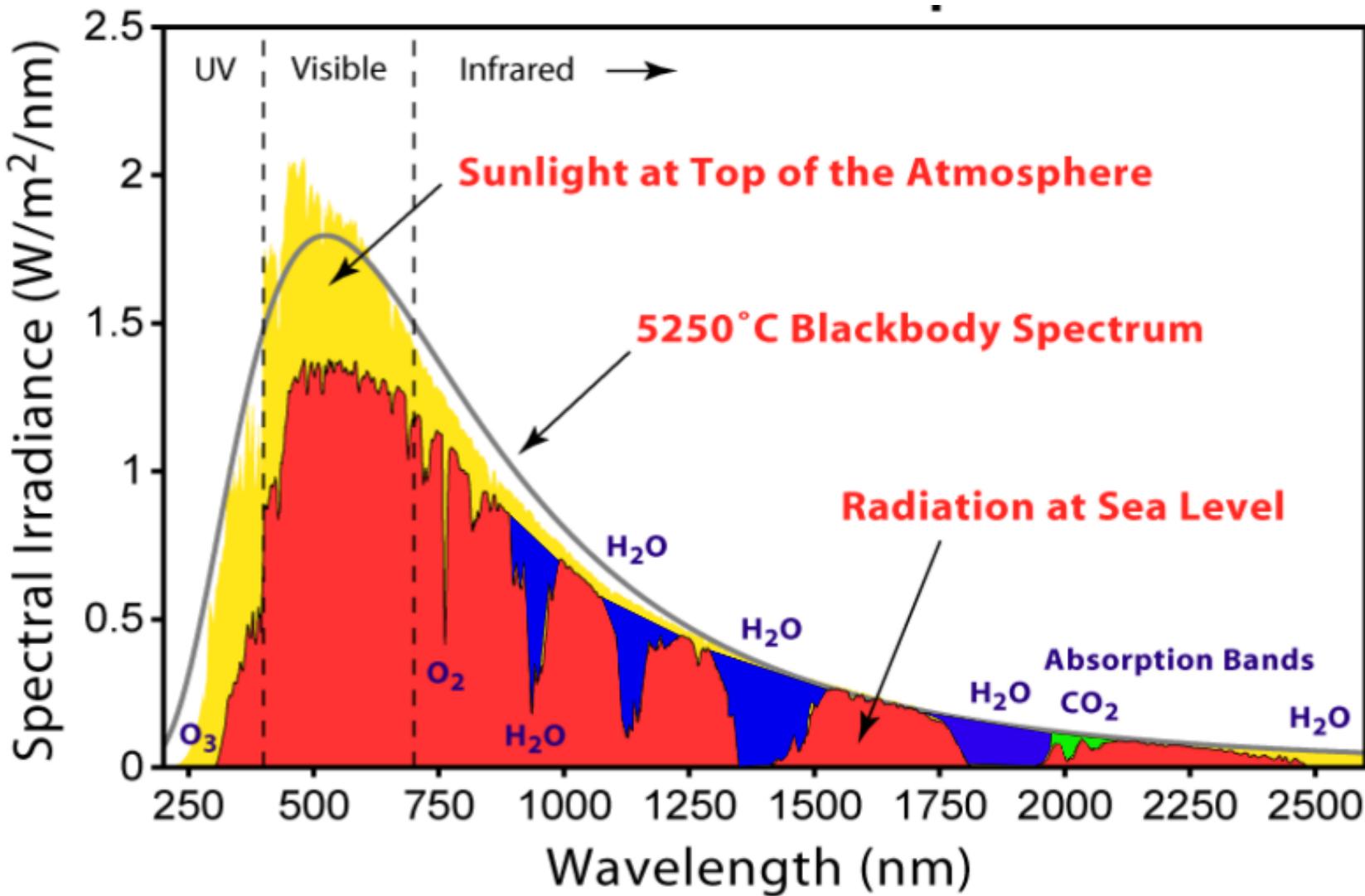
- **What does this mean?**

The hotter the object, the shorter its dominant wavelength or the higher its dominant frequency.



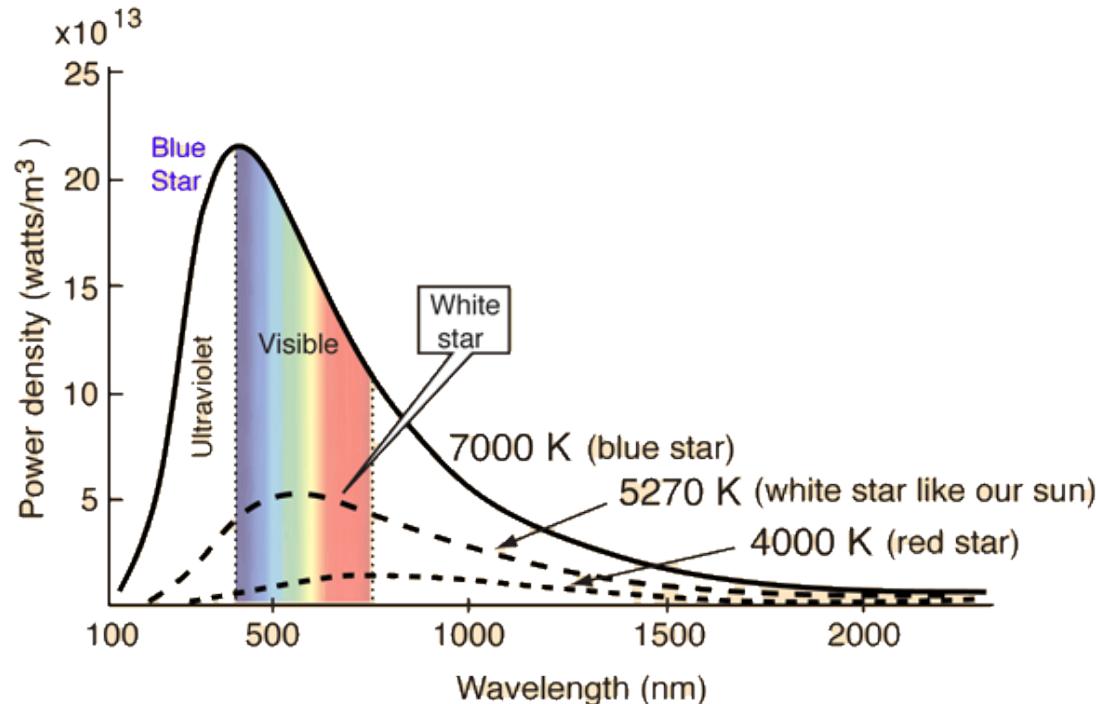
Note: Although the Sun's dominant wavelength is 0.48 μm , it produces a continuous spectrum from very short, very high frequency waves (e.g., gamma) to long, very low frequency waves (e.g., radio)

Solar Radiation Spectrum

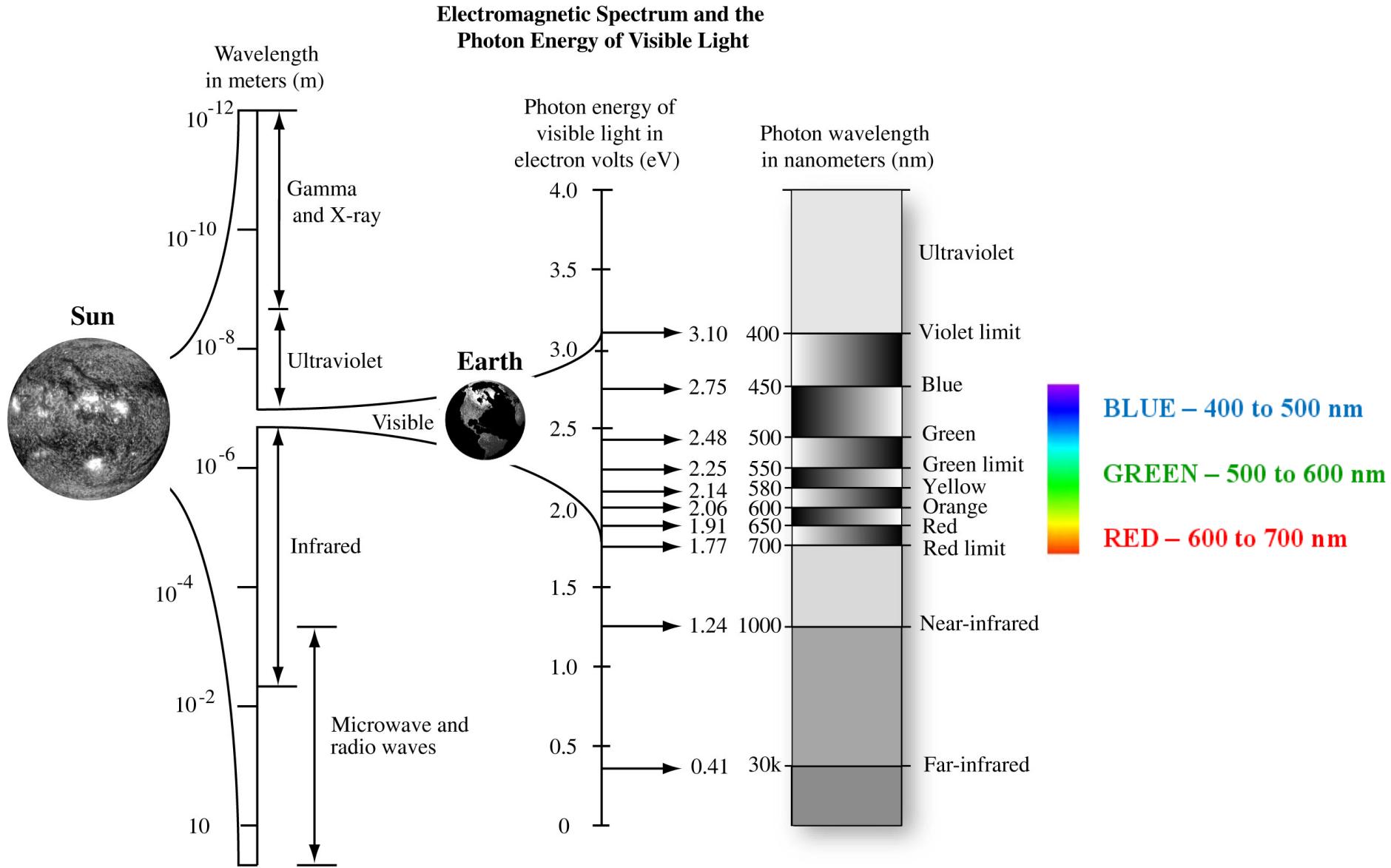


Star Temperature

- Stars approximate **blackbody radiators** and their visible color depends upon the temperature of the radiator.
- The curves show blue, white, and red stars.
 - The white star is adjusted to 5270K so that the peak of its blackbody curve is at the peak wavelength of the sun, 550 nm.
 - From the wavelength at the peak, the temperature can be deduced from the [Wien displacement law](#).



EMR Spectrum

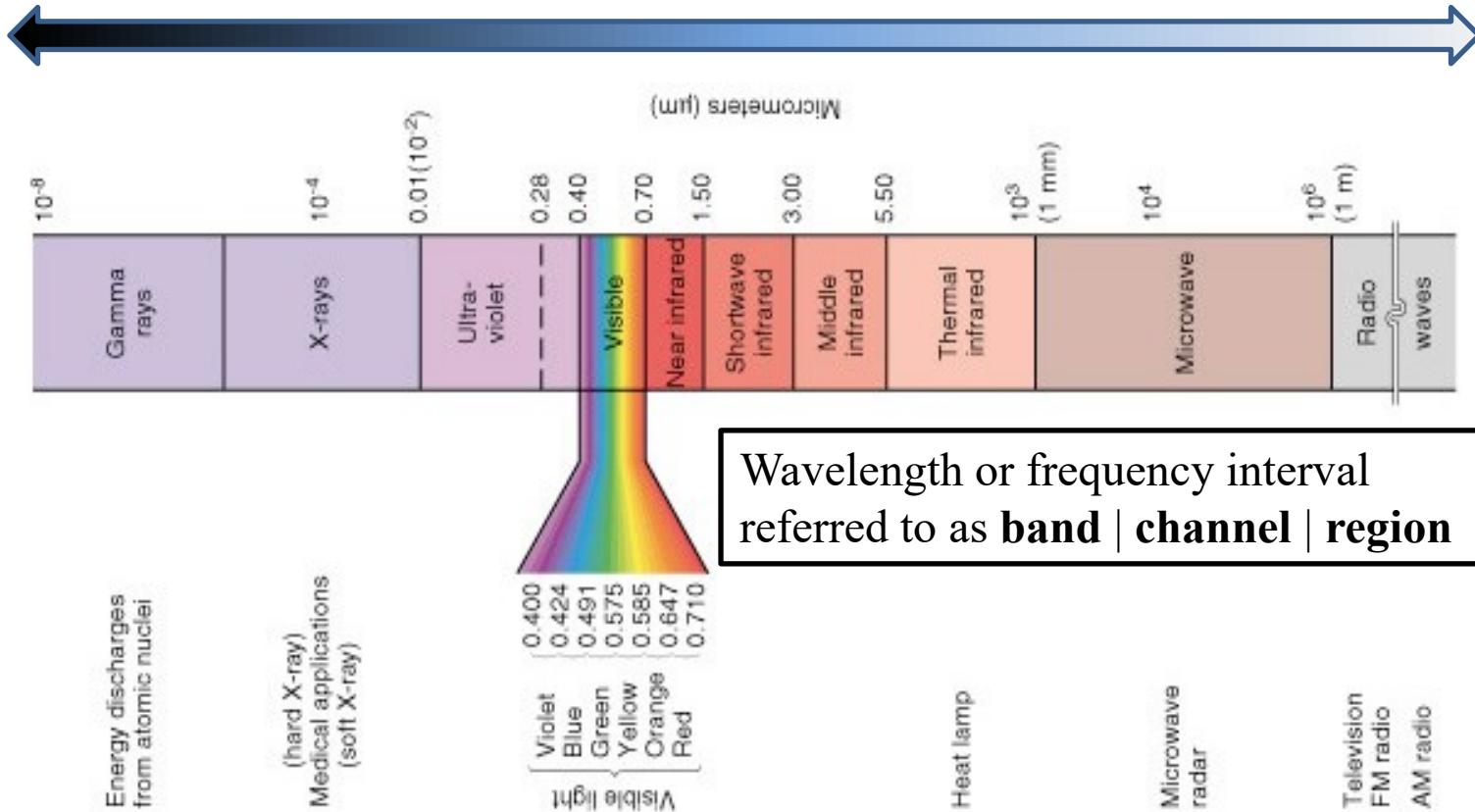


Electromagnetic Spectrum

Identify the different regions of the EMR spectrum

Shorter waves
Higher energy

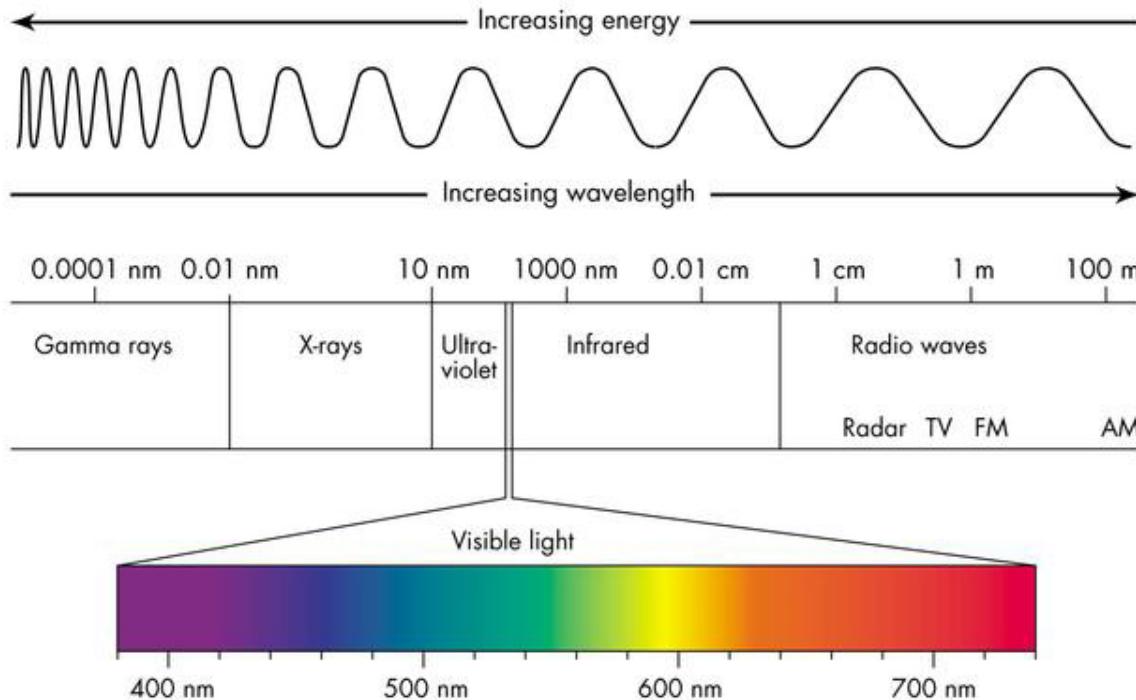
Longer waves
Lower energy



Sun's dominant wavelength: 0.48 μm
(blue portion of EMS)

Earth's dominant wavelength: 9.66 μm
(thermal infrared portion of EMS)

Units



Units to remember

$$\text{\AAngstrom} = 10^{-10} \text{ m}$$

$$\text{nm} = 10^{-9} \text{ m}$$

$$\mu\text{m} = 10^{-6} \text{ m}$$

$$\text{mm} = 10^{-3} \text{ m}$$

$$\text{cm} = 10^{-2} \text{ m}$$

$$\text{km} = 1000 \text{ m}$$

Color	Peak Wavelength (nm)
Blue	470
Cyan	525
Green	560
Yellow	585
Orange	600
Red	645
Deep Red	700

VISIBLE LIGHT

Violet: 400 - 446 nm

Blue: 446 - 500 nm

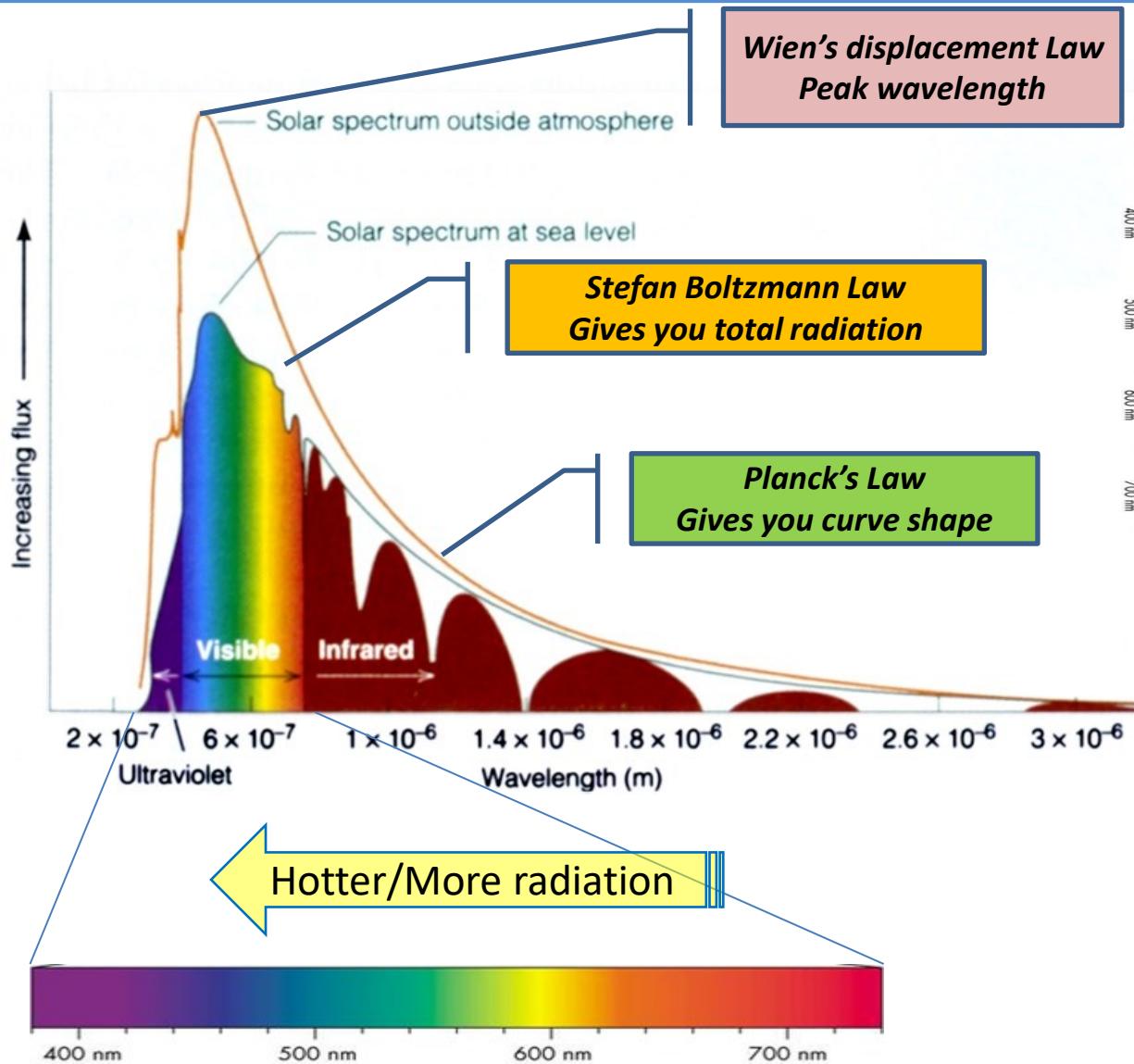
Green: 500 - 578 nm

Yellow: 578 - 592 nm

Orange: 592 - 620 nm

Red: 620 - 700 nm

Electromagnetic Spectrum



VISIBLE LIGHT

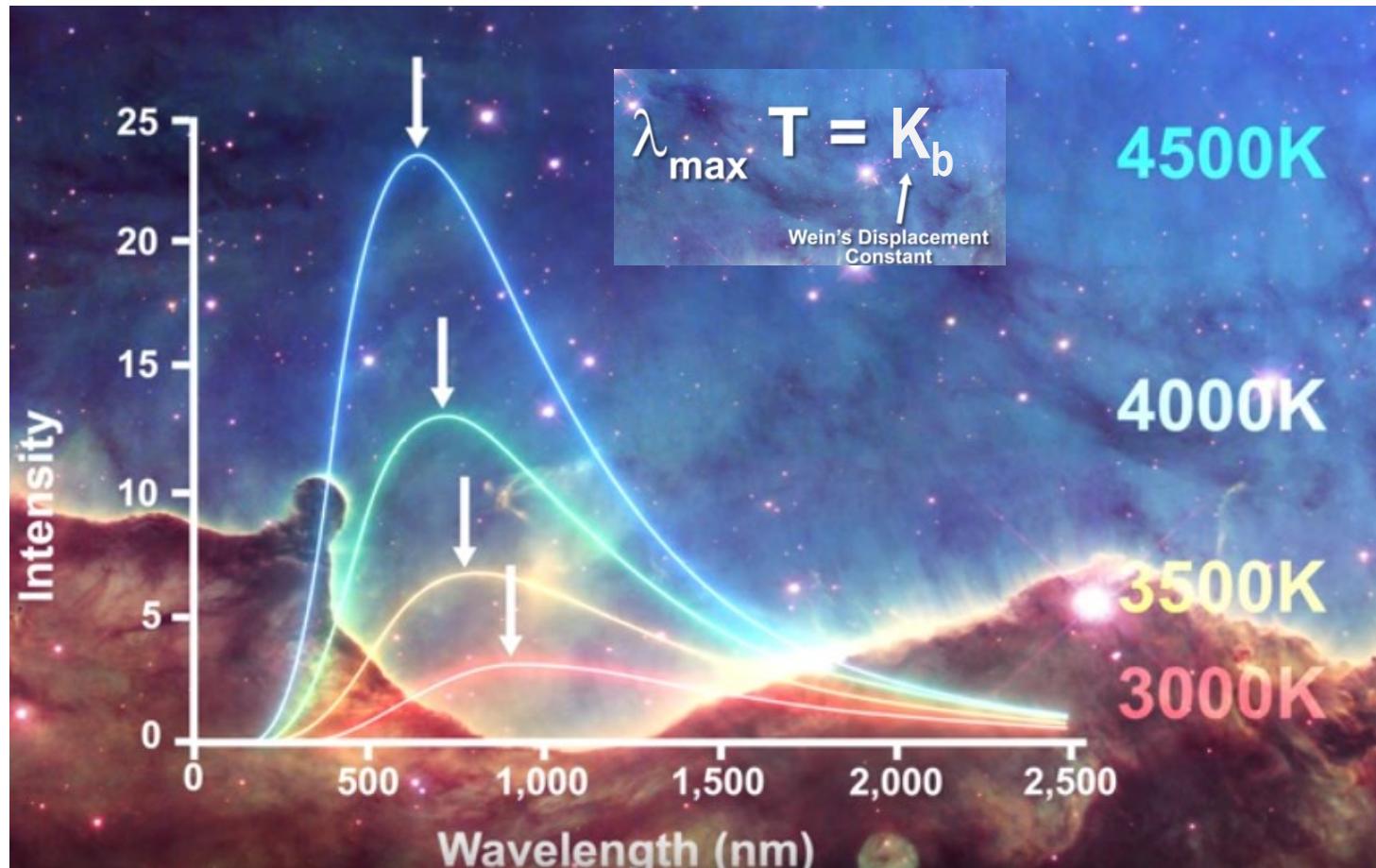


So what does this all mean and why is it important?

- Energy of a quantum is inversely proportional to wavelength
 - Longer wavelengths = lower energy content
- This is important in remote sensing because it is more difficult to measure longer-wavelength radiation (e.g., thermal - microwave)
 - They emit less energy
 - Example – passive microwave sensors have a very coarse spatial resolution (pixels cover a large area 3 km to 35 km) because the energy of radiation at microwave wavelengths is very low.
 - So, we must collect large amounts which is only possible if we image large areas
 - Example – thermal data from sensors such as Landsat Thematic Mapper are usually sampled at coarser spatial resolution (e.g., 90 m) than the visible, NIR and SWIR data (30 m), again so we can collect enough energy

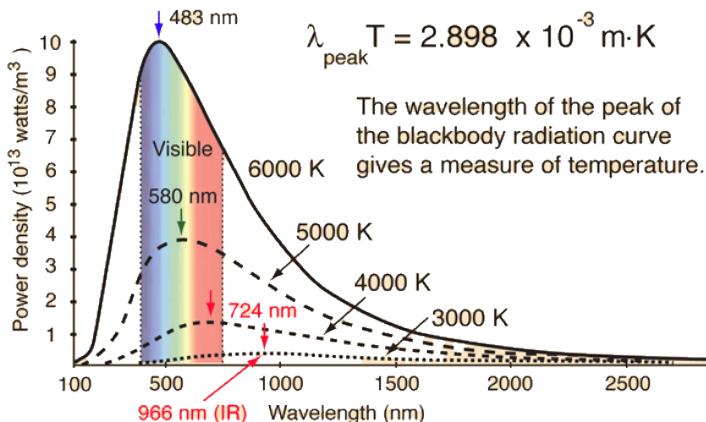
Wien's Displacement law (1896)

- Based on observations that hotter objects emit at shorter wavelengths
- Wien discovered that the product of T and Wavelength (λ) is always a constant (K_b)



Wien's Displacement Law

- When the temperature of a blackbody radiator increases, the overall radiated energy increases, and the peak of the radiation curve moves to shorter wavelengths.
- When the maximum is evaluated from the **Planck radiation formula** (we will get to this), the product of the peak wavelength and the temperature is found to be a constant.

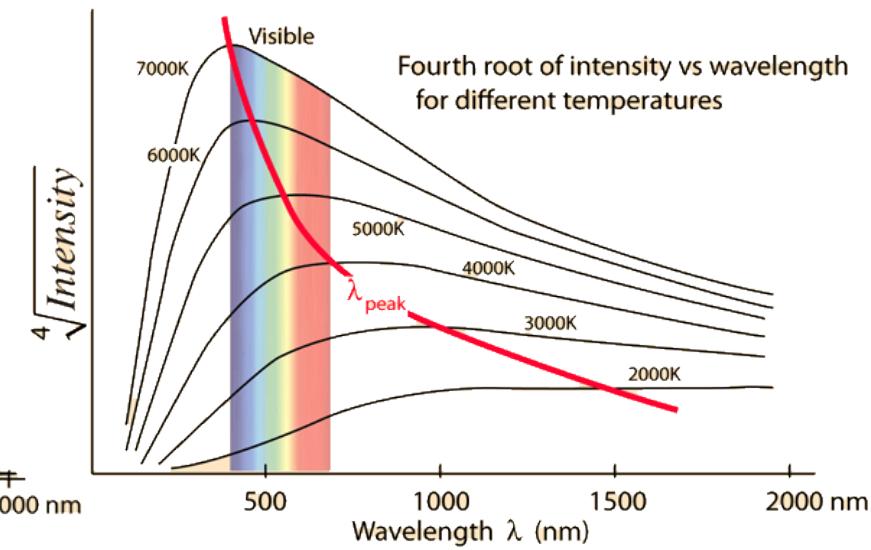
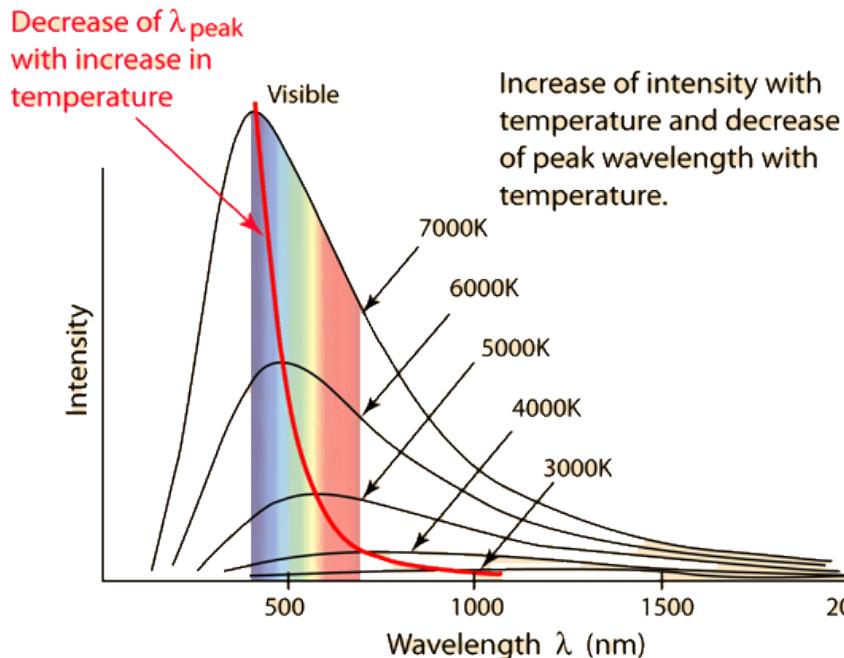


$$\lambda_{peak} T = 2.898 \times 10^{-3} m \cdot K$$

- This relationship is called **Wien's displacement law** and is useful for determining the **temperatures** of hot radiant objects such as stars, and for the determination of the temperature of any radiant object whose temperature is far above that of its surroundings.
- Use the online calculator = <http://hyperphysics.phy-astr.gsu.edu/hbase/wien.html#c2>
 - Can you estimate the peak wavelength of the Earth radiation?
 - What do you need?
 - Earth average temperature? $T = ?$
 - $\lambda = ?$

Radiation Curve

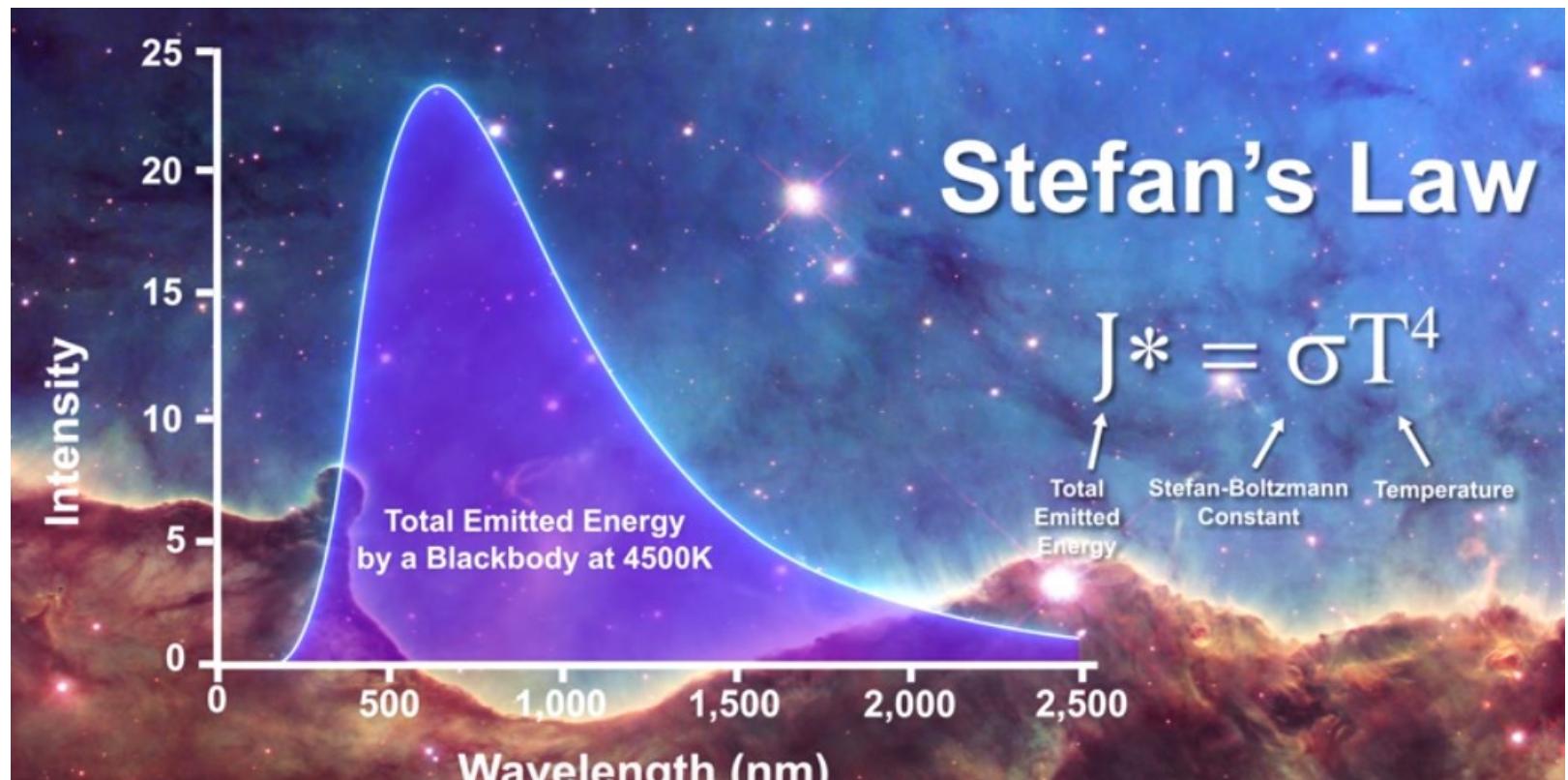
- The wavelength of the peak of the blackbody radiation curve decreases in a linear fashion as the temperature is increased (Wien's displacement law).
 - Why?
- This is not evident when intensity is plotted
- Using the fourth root of the intensity shows the variation of the wavelength more clearly than the plot of the full radiated intensity.



Stefan–Boltzmann law

- Stefan discovered another law : Total emitted energy (by a blackbody) is the product of T^4 and a constant (σ)

$$J^* = \sigma \cdot T^4$$



Stefan-Boltzmann Law

- The thermal energy radiated by a blackbody radiator per second per unit area is proportional to the fourth power of the absolute temperature and is given by

$$\frac{P}{A} = \sigma T^4 \text{ j/m}^2\text{s} \quad \text{Stefan-Boltzmann Law}$$

$$\sigma = 5.6703 \times 10^{-8} \text{ watt/m}^2\text{K}^4$$

- For hot objects other than ideal radiators, the law is expressed in the form:

$$\frac{P}{A} = e\sigma T^4$$

- Where e is the emissivity of the object ($e = 1$ for ideal radiator).
- If the hot object is radiating energy to its cooler surroundings at temperature T_c , the net radiation loss rate takes the form

$$P = e\sigma A(T^4 - T_c^4)$$

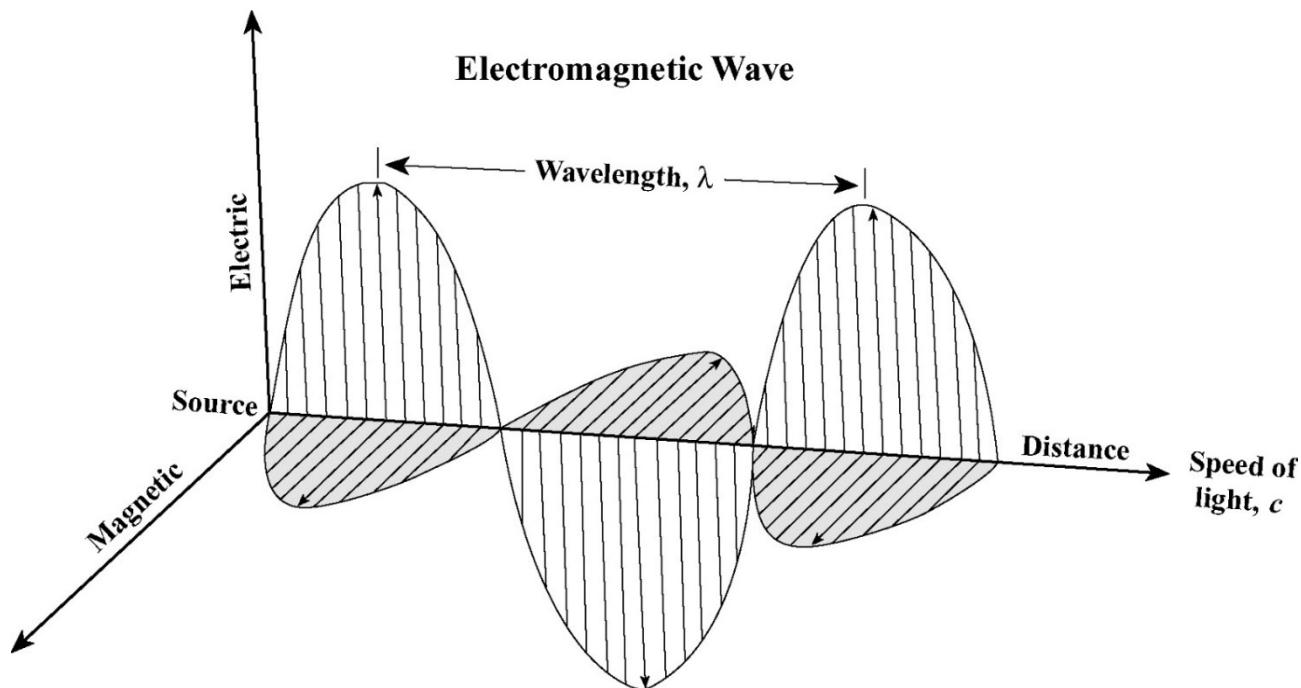
- The Stefan-Boltzmann relationship is also related to the energy density in the radiation in each volume of space.

EMR models

- There are two different models used to help us understand and explain how EMR is created, how it travels through space and how it interacts with other matter
 - Wave model
 - Particle model

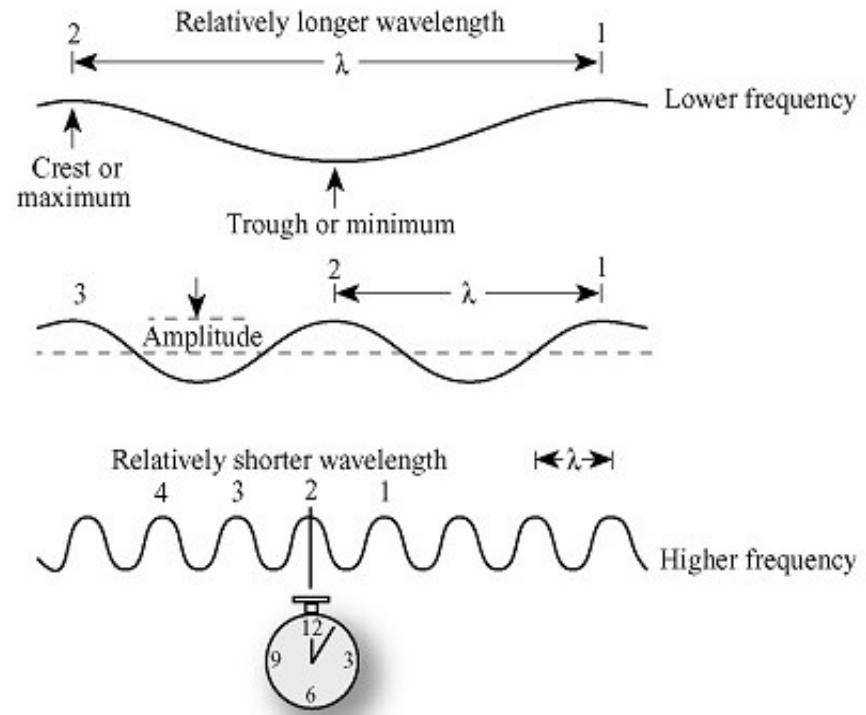
EMR : Wave model

- Wave Model > EMR conceptualized as an electromagnetic wave that travels through Space at the speed of light ($3 \times 10^8 \text{ m s}^{-1}$)
- Two fluctuating fields – one electric, one magnetic. They are at 90° angles to one another, and both are perpendicular to the direction of travel



Wavelength & Frequency

- **Wavelength (λ ; nm - m)**
distance between crests (or troughs)
- **Frequency (ν ; Hz)**
The number of crests (or cycles) that pass a given point each second
- **Speed of light (c):** defined by the distance traveled by a crest in one second ($3 \times 10^8 \text{ m s}^{-1}$)
- These entities are **correlated by:**



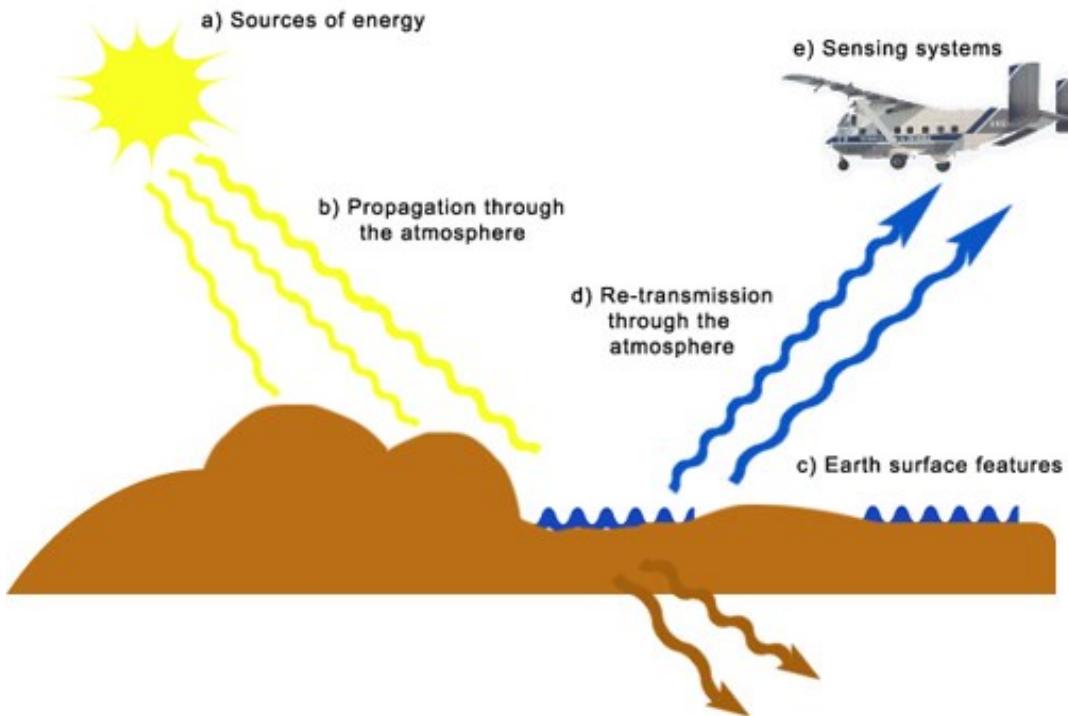
$$c = \lambda \cdot \nu$$

$$\lambda = \frac{c}{\nu}$$

$$\nu = \frac{c}{\lambda}$$

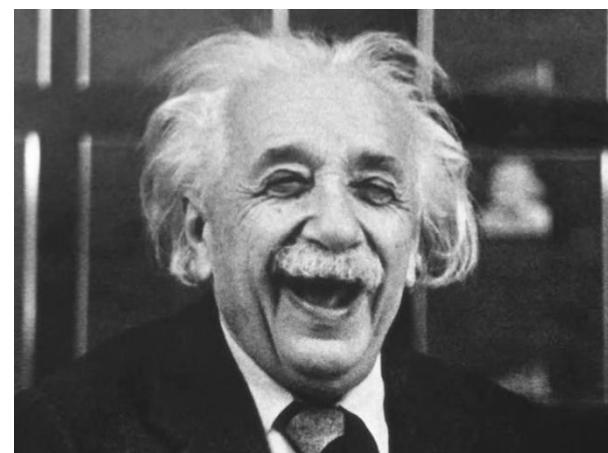
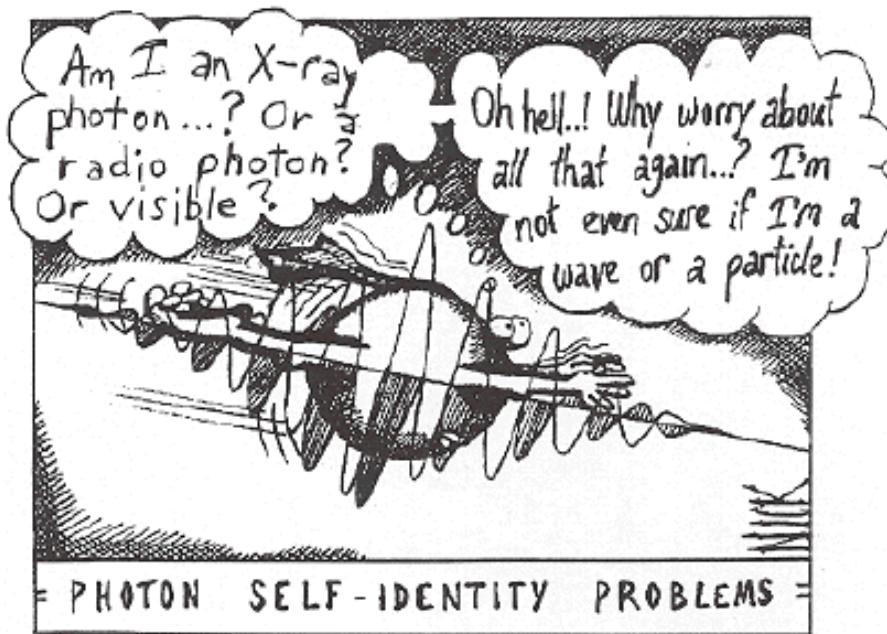
The relationship between temperature and radiated energy and dominant wavelength

- All objects with a temperature greater than 0 K (-273°C) emit EMR
- The Sun is the primary source of most EMR recorded by passive remote sensing systems



EMR : Particle model

- **Particle Model**
 - Conceptualizes EMR as a stream of discrete packets of energy, or quanta, called **photons** (i.e., transporters of energy that can be emitted or absorbed by charged particles)



Photons, Energy, and Waves

- Relationship between the frequency of radiation expressed by wave theory and the quantum:

$$Q = h \times v$$

- Q = energy of a quantum (J)
- h = Planck constant (6.626×10^{-34} J sec)
- c = speed of light
- v = frequency of radiation
- λ = dominant wavelength

- From wave theory we know that wavelength (λ) of radiation is equal to the speed of light divided by the frequency of radiation

$$\lambda = \frac{c}{v}$$

- We can multiply the equation with the Planck constant without changing its value

$$\lambda = \frac{hc}{hv}$$

- Then substitute Q for hv , which gives $\lambda = \frac{hc}{Q}$ or $Q = \frac{hc}{\lambda}$

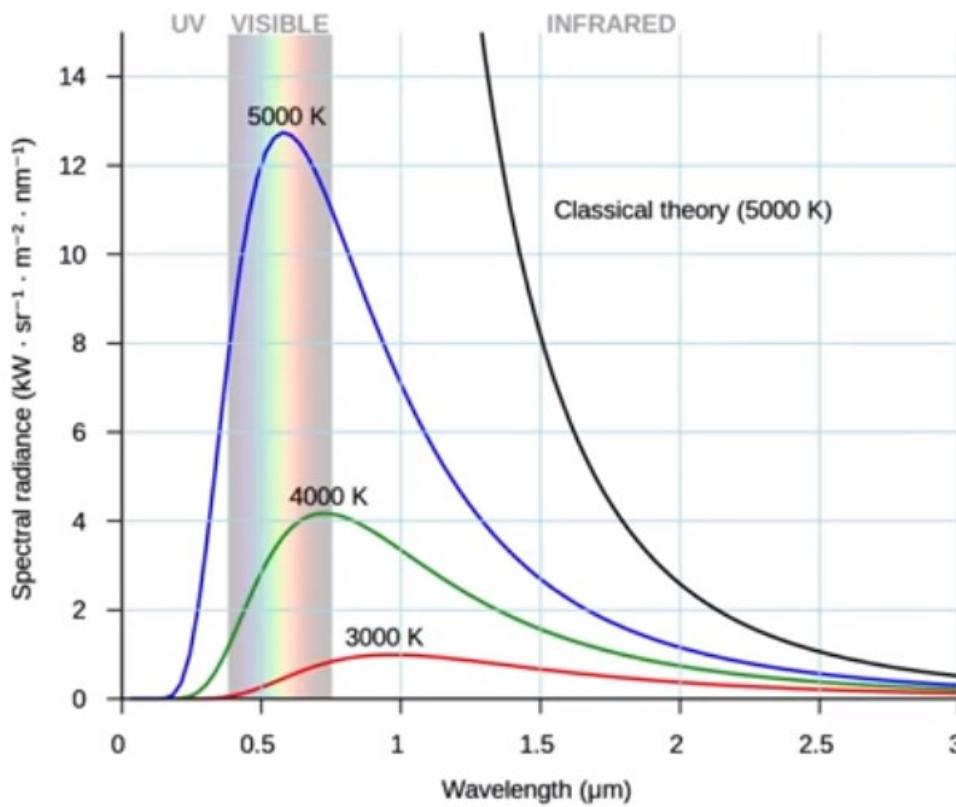
Classical Physics

- Classical physics was our ultimate understanding of the physical world around us for about 300+ years



Ultraviolet Catastrophe

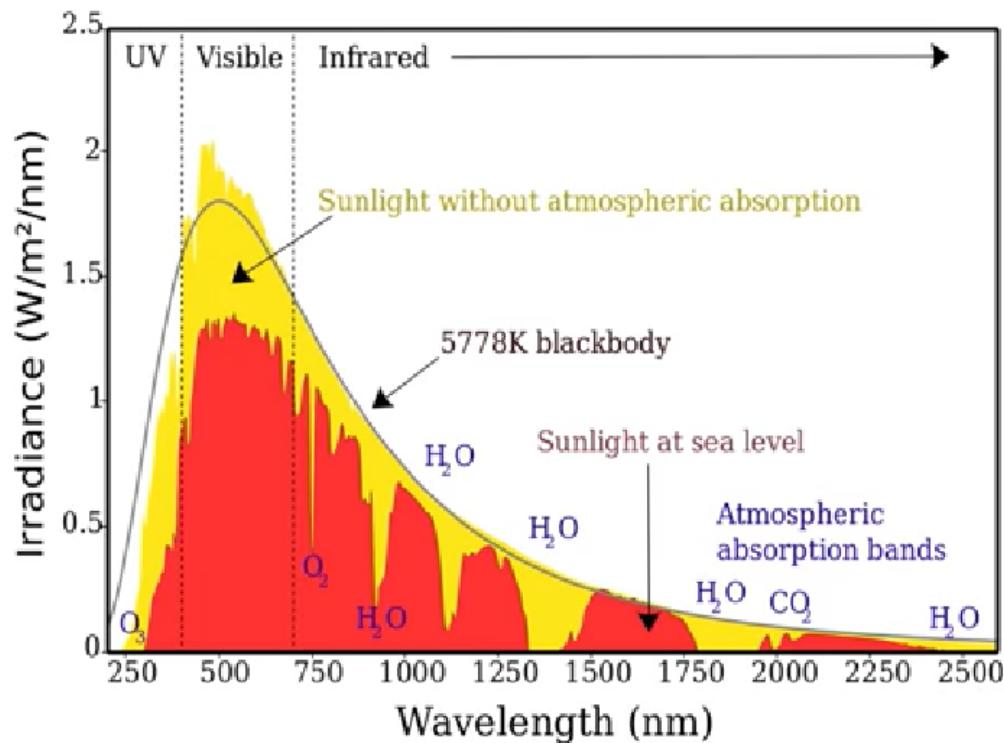
- We realized that the energy emitted depends not on the material but on the temperature of the material
- But?



Ultraviolet Catastrophe

- Ex: Sun is a blackbody

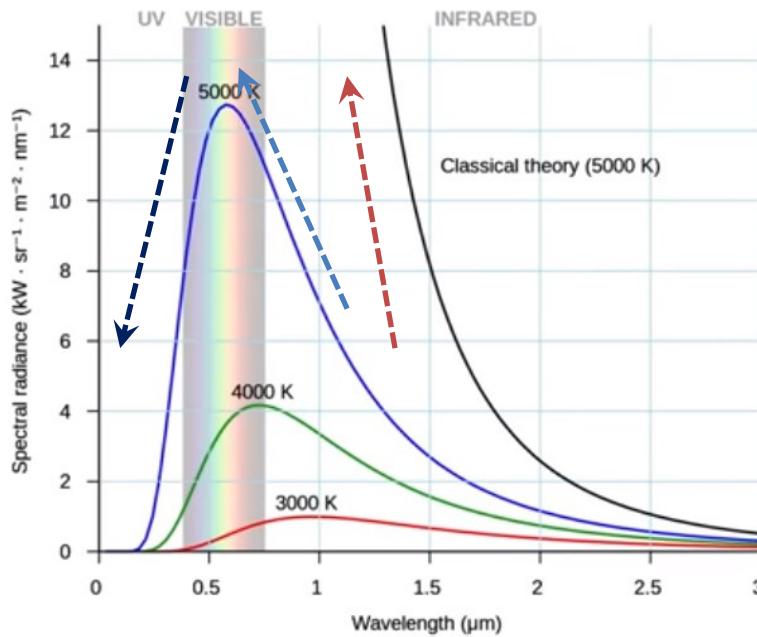
Spectrum of Solar Radiation (Earth)



**the light emitted by the sun (in yellow)
matches the blackbody spectrum for 5778 K**

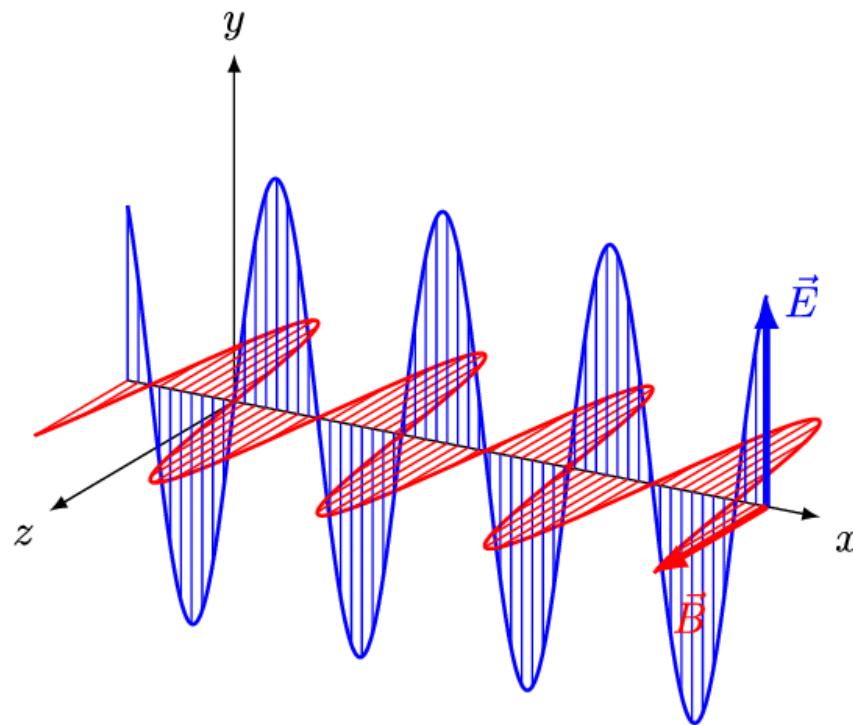
Ultraviolet Catastrophe

- Mathematical models were able to predict the intensity at long wavelengths but failed at shorter waves?
 - They could not predict that the intensity drops at shortwaves
 - Intensity keeps going up infinitely (UV)?
 - Observations were otherwise
 - This is called **Ultraviolet Catastrophe**



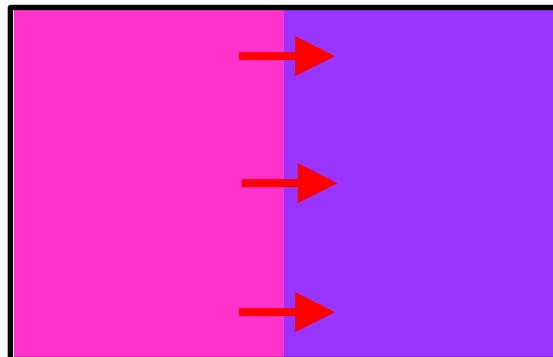
Classical Physics

- We realized that the theory of electromagnetism is incomplete?

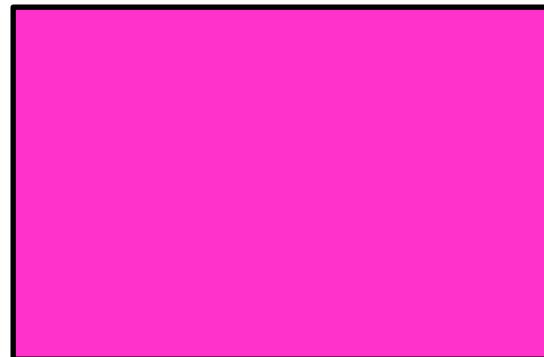


Max Plank Solved the Problem

- He introduced a concept called quantization (precursor to quantum physics. That is why he is celebrated)
- It went like this
 - From Classical physics heat is transfer via kinetic energy between objects



Hot Cold



Thermal Equilibrium

Max Plank Proposed

- In solid objects there is no **translational** motion but there is **vibrational** motion
- These vibrations are what generate the light we see



- Plank proposed that vibrational energy of these atoms must be quantized (discrete and not a continuum of values)
- They can only take specific discrete values



Max Plank Solved the Problem

- So he proposed a famous equation to express energy as:

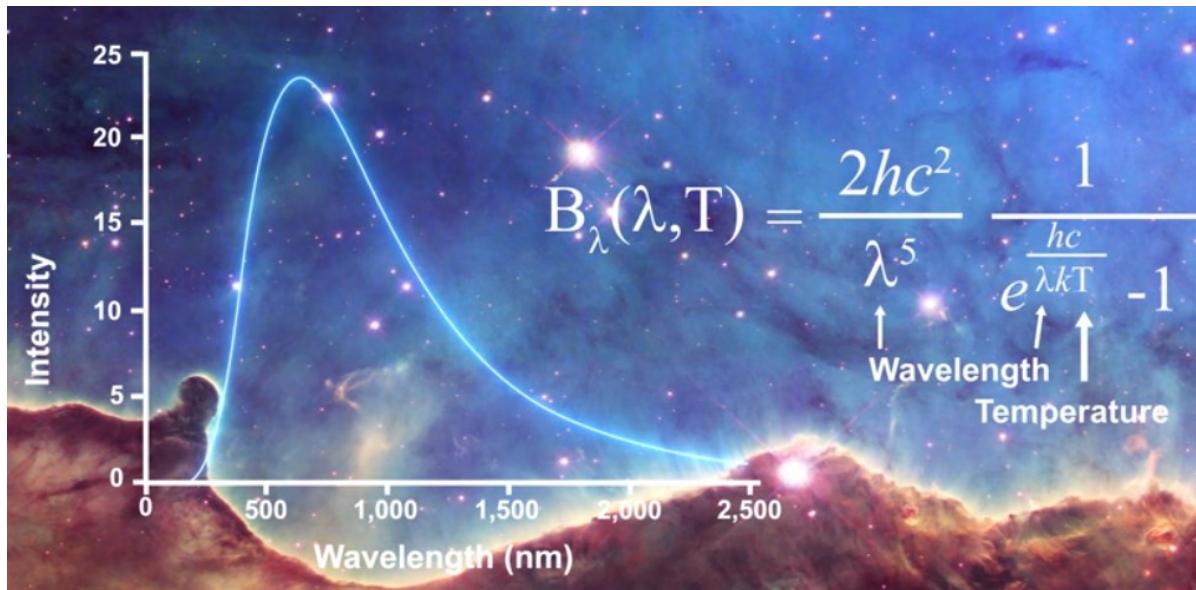
$$E = n \cdot h \cdot \nu$$



- Where:
 - n is any integer (0, 1, 2,)
 - h is Plank's constant = 6.626×10^{-34} J.s
 - It was empirically derived and quite confusing for its small value
 - Later it became a foundational constant for quantum physics with specific meaning (photon energy)
 - ν = Wave frequency
 - **NOTE :** $h \cdot \nu$ = Energy of a photon (not known back then)
- So energy is only from preset discrete values ($n = 0, 1, 2, \text{etc...}$) anything in between is forbidden

Max Plank Solved the Problem

- He proposed his famous equation that describes the continuous energy radiation curve without the ultraviolet catastrophe
- Expressed in terms of wavelength



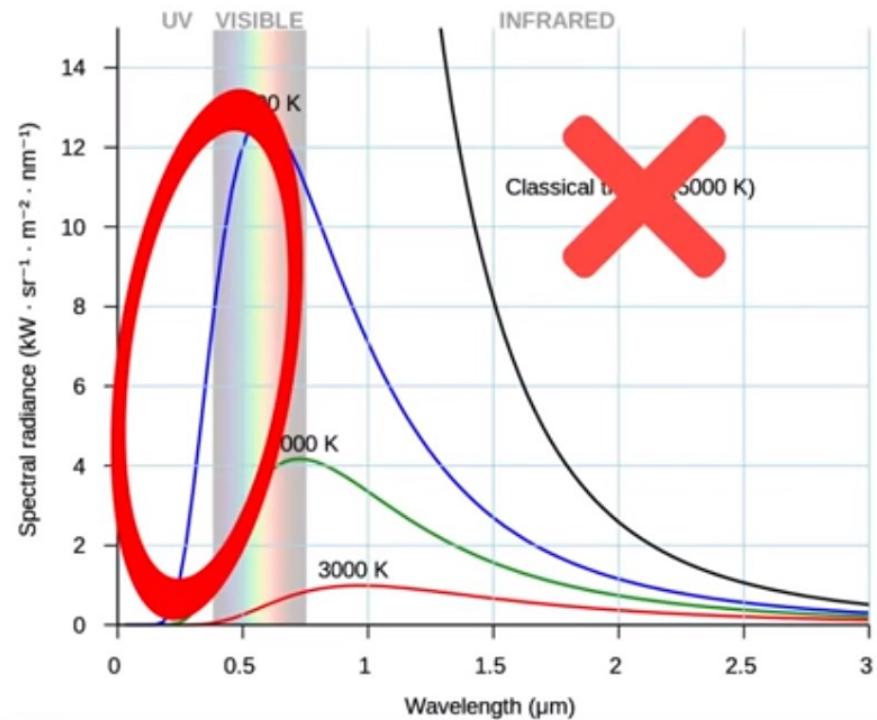
- We will work with this in the upcoming lab and you will see this in action
 - Look at <[BE_BAT_484-585-Lec_Exercie_Plancks_Law.ipynb](#)> and
 - Experiment and try to develop the Earth Radiation curve (you will need the Earth's Avg. Temp)?
 - Can you add the Wien's displacement law to this Python code (either points or curve that links the max)?
 - And turn in as optional HW3

This led to the solution of the EMR energy

- Now models line up with observations
- This led to **Plank's Law**
 - Expressed in terms of frequency

$$B_v(T) = \frac{2h\nu^3}{c^2(e^{\frac{h\nu}{KT}} - 1)}$$

- c is the speed of light, 3×10^8 m/s
- ν is the frequency
- T is temperature, in Kevin
- h is Plank constant
- K is Boltzmann constant



- Can you express this in terms of wavelength $B_\lambda(T)$?

Field of Quantum Physics/Mechanics

- Foundation for quantum physics/mechanics

$$B_v(v, T) = \frac{2hv^3}{c^2} \cdot \frac{1}{e^{\frac{hv}{kT}} - 1}$$

$c = \text{speed of light} \rightarrow c^2$

$h = \text{Planck's Constant}$

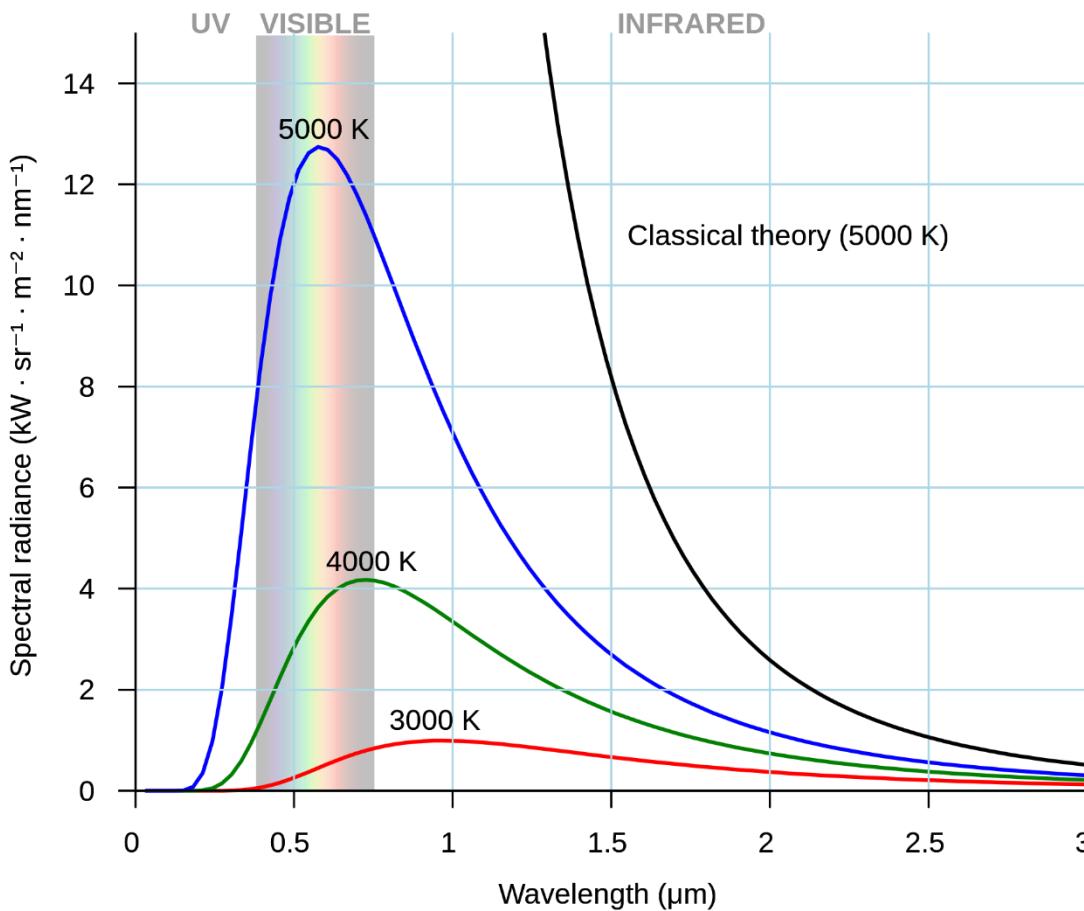
$hv = \text{Photon Energy}$

backup

Radiation Laws

Radiation Law - 1

1. Planck's Law:



$$B_v(T) = \frac{2hv^3}{c^2(e^{\frac{hv}{kT}} - 1)}$$

c is the speed of light, $3 \times 10^8 \text{ m/s}$
 v is the frequency
 T is temperature, in Kevin
 h is Planck constant
 K is Boltzmann constant

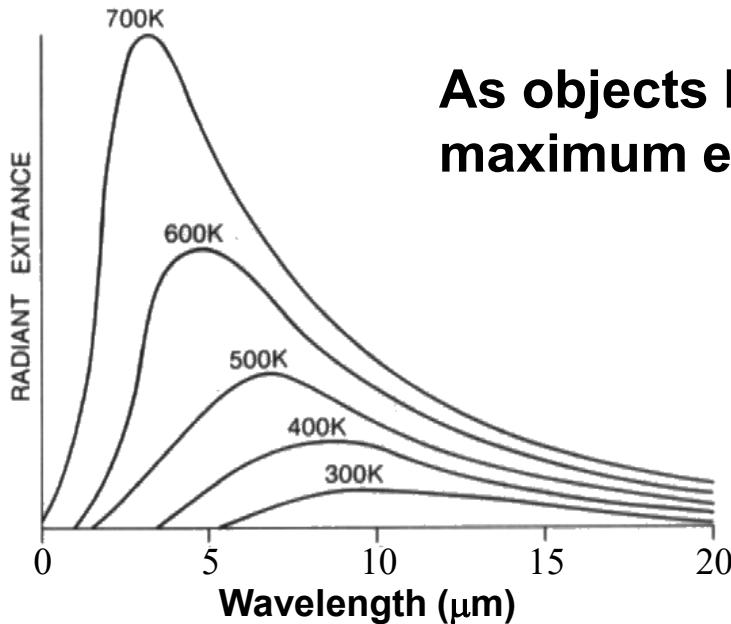
$$\lambda = c/v$$

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

Radiation Law - 2

3. Wien's Displacement Law: $\lambda = 2,897.8/T$

λ = wavelength (μm) at which radiance is at a maximum



As objects become hotter, the wavelength of maximum emittance shifts to shorter wavelengths.

Radiation Law - 3

3. Stefan-Boltzmann's law: $W = \sigma T^4$

W = total emitted radiation (watt m⁻²)

σ = Stefan-Boltzmann constant (5.6697×10^{-8} W m⁻² K⁻⁴)

T = temperature in Kelvin (T in Kelvin = $273.15 + {}^\circ\text{C}$)



Meaning: Hot blackbodies emit more energy per unit area than do cold blackbodies

Radiation Law - 4

4. Kirchhoff's law: $\epsilon = M/M_b$

ϵ = emissivity: a measure of the effectiveness of an object as a radiator of electromagnetic energy (0-1)

M = emittance of a given object

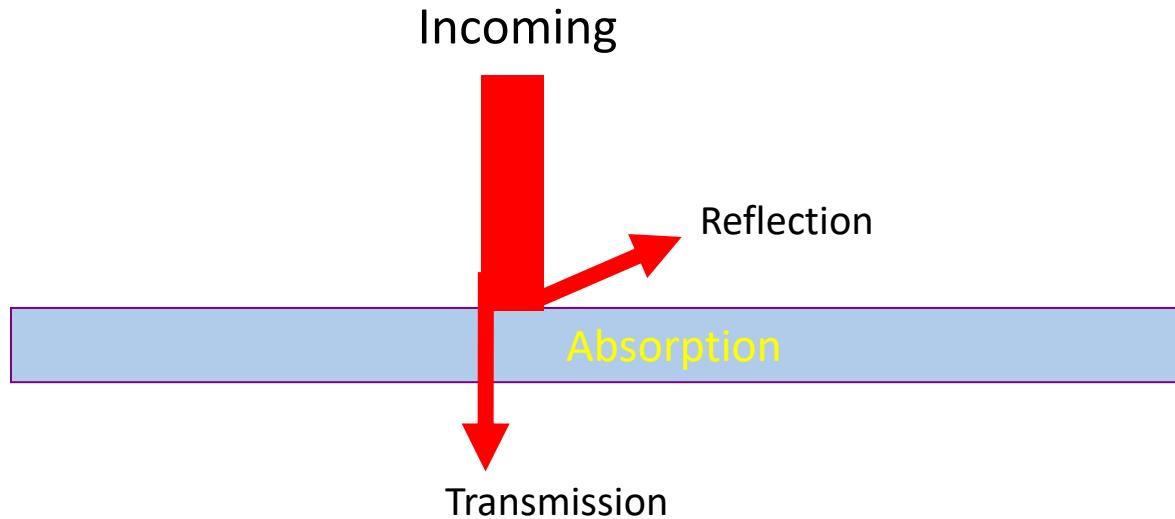
M_b = emittance of a blackbody

Blackbody: object that absorbs all incident radiation; none is reflected. Ideal emitter

Radiation Law - 5

5. Radiant Energy Conservation Law

Incoming Radiation=Absorption + Reflection + Transmission

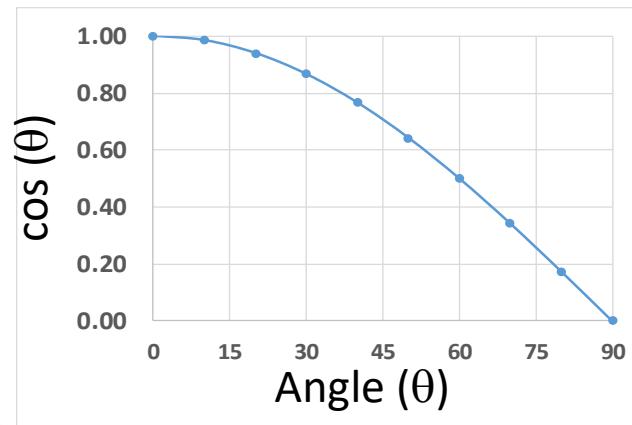


$$\text{Absorptivity} + \text{Reflectivity} + \text{Transmissivity} = 1$$

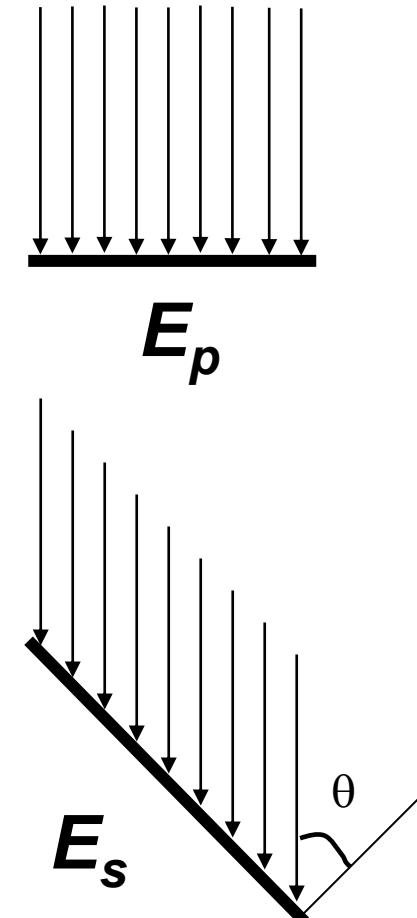
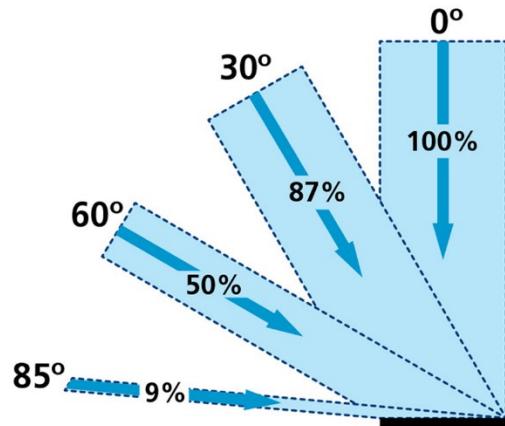
Radiation Law - 6

6. Cosine's Law

$$E_s = E_p \cos \theta$$

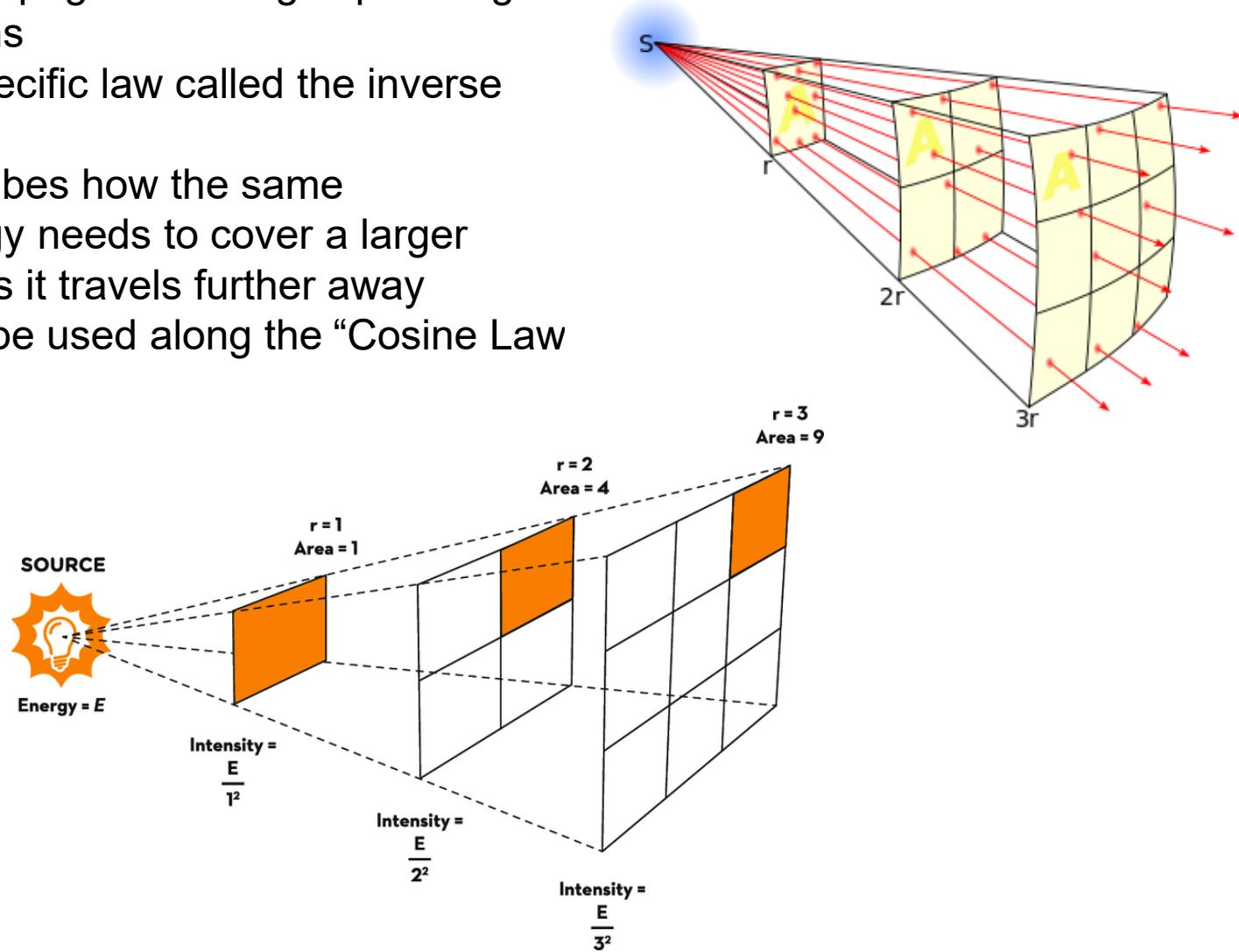


Cosine Law: $E_\theta = E * \cos (\theta)$



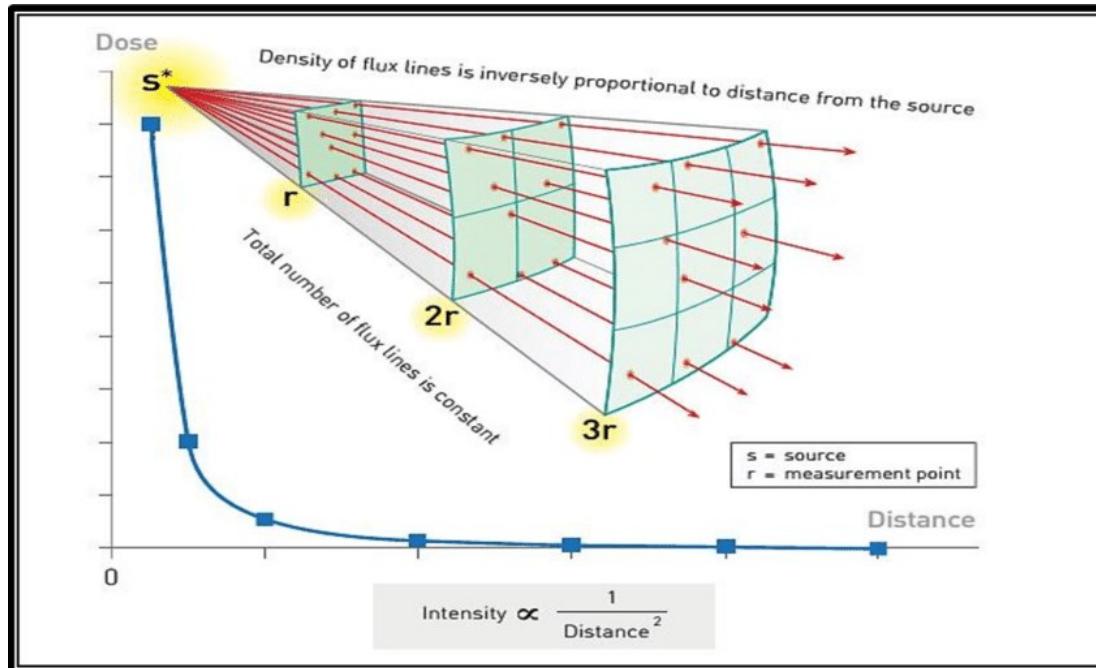
+ Inverse Square Law

- As radiation propagates through space it gets diluted/weakens
- Following a specific law called the inverse square law.
- This law describes how the same radiation/energy needs to cover a larger surface area as it travels further away
- This needs to be used along the “Cosine Law”



Inverse Square Law

- This law governs the propagation of waves, and applies to many fields
 - It states that the intensity of an effect such as illumination (radiation) or gravitational force changes in inverse proportion to the square of the distance from the source (it basically describes a dilution/weakening process).

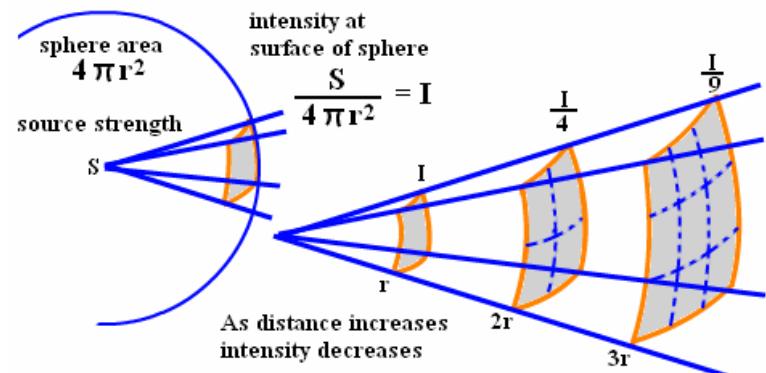
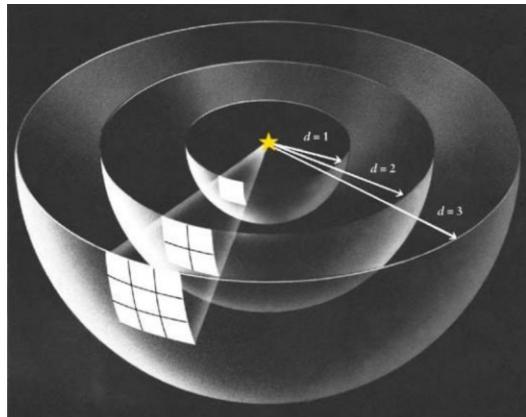
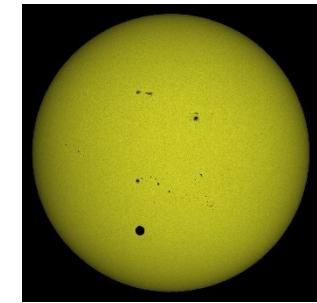


Derive the Inverse Square Law

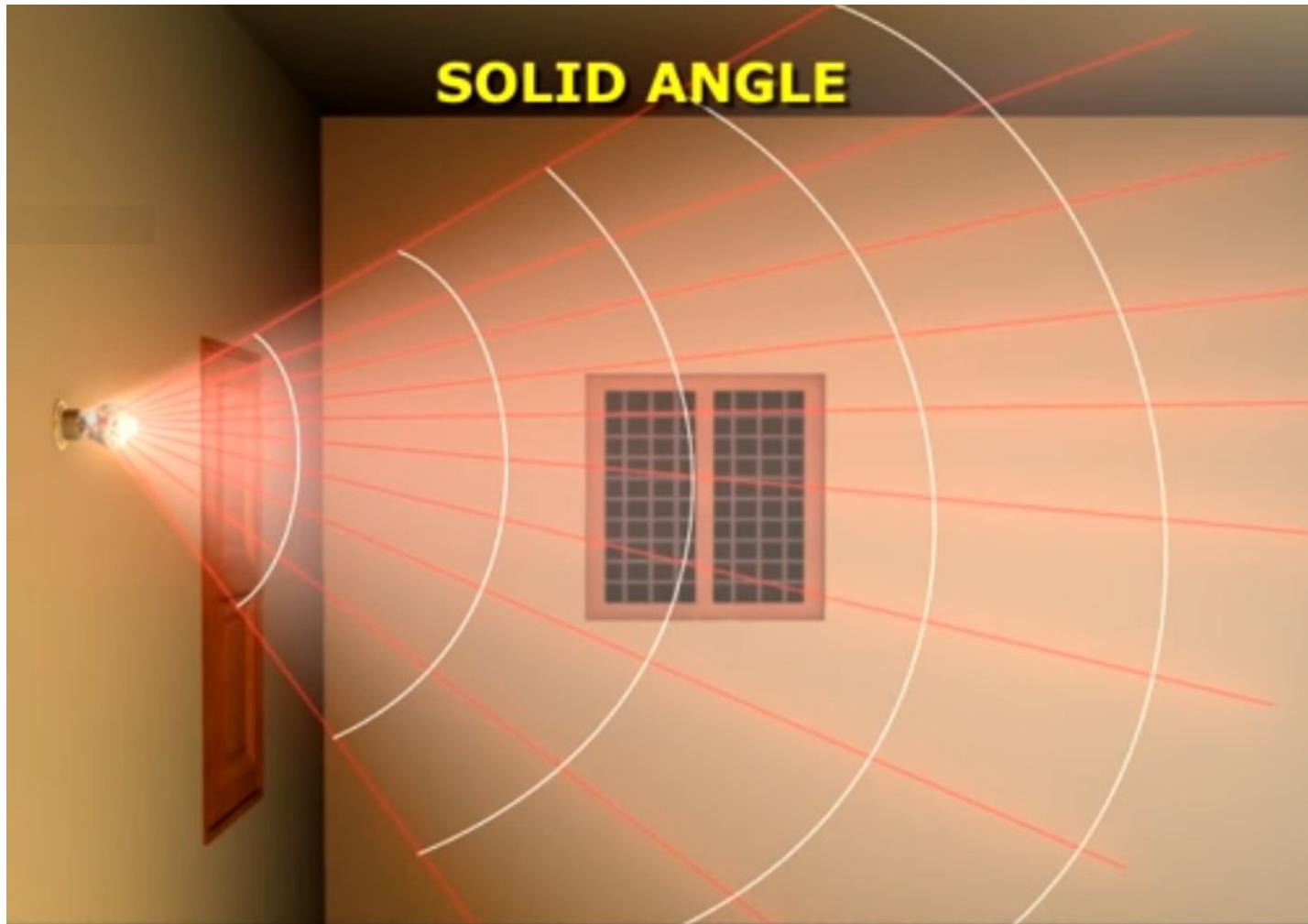
- What is the area of a sphere?

$$A = 4\pi r^2$$

- The energy (light/radiation) propagates in all directions
- So the Intensity at the surface is $I = \text{Energy}/A = S/4\pi r^2$
- That same amount of energy (S) needs to cover an area $= 4\pi d^2$ (d = distance from source)
- So at distance $2r$, $I = S/4\pi(2r)^2 = (S/4\pi r^2)/2^2$
- So at distance $3r$, $I = S/4\pi(3r)^2 = (S/4\pi r^2)/3^2$
- etc..

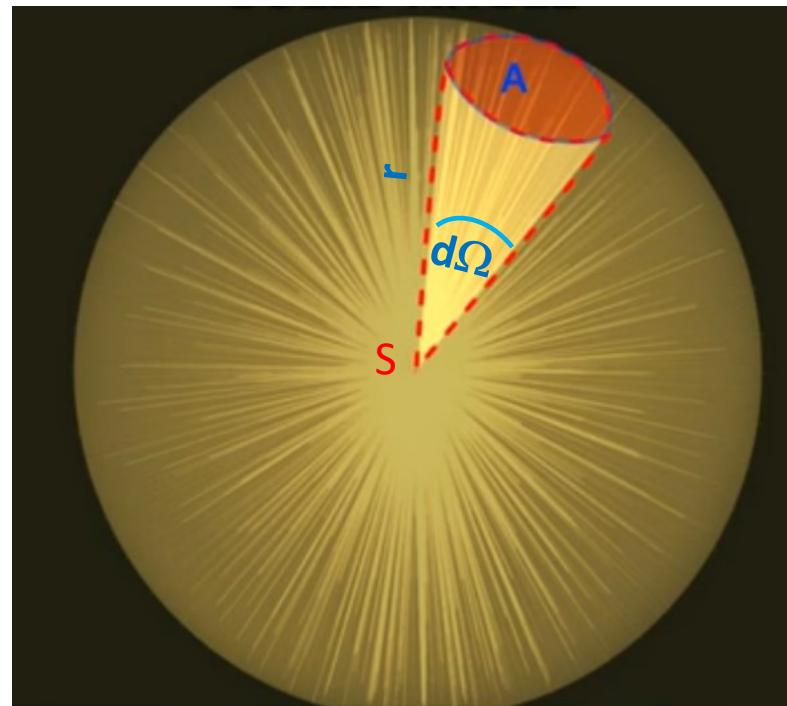


What is a Solid Angle?



What is a Solid Angle?

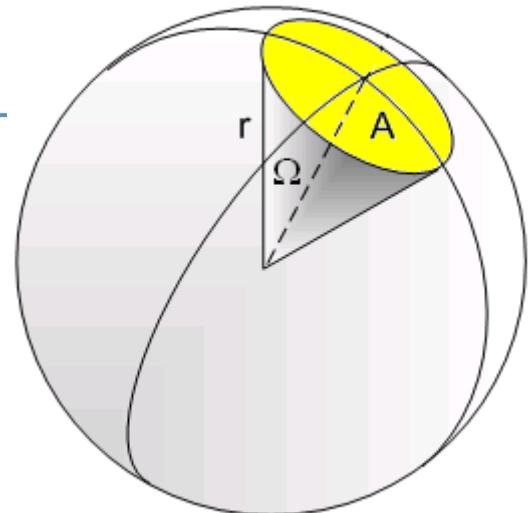
- Source : **S** and Region: **A**
- The light rays leaving source S and falling on Region A form a cone this also gives rise to a solid angle ($d\Omega$)
- $d\Omega = A/r^2$ (in steradians)
- If $A = 1$ unit and $r = 1$ Unit
 - Then $d\Omega = 1$ (sr)



Solid Angle

- The solid angle, Ω , is the 2D angle in 3D space that an object subtends at a point.
- Definition
$$\Omega = \frac{A}{r^2} \quad (\text{remember in 2D } \theta = k \frac{s}{r})$$
- It is a measure of **how large** that object appears to **an observer looking from that point**.
- SI unit is *steradian* (sr)
- The solid angle of a sphere measured from a point in its interior is 4π sr.

$$\Omega = \frac{A}{r^2} = \frac{4\pi r^2}{r^2} = 4\pi \text{ sr}$$



A : Surface area subtended from the center

r : Radius of the sphere

BACKUP SLIDES

HW #3

- The Moon has Surface area of $\sim 3.8 \times 10^7 \text{ km}^2$ and has a Mean orbital radius about the Earth of $\sim 3.8 \times 10^5 \text{ km}$.
 1. What is the solid angle subtended by the Moon in the sky?
 2. What fraction of the total sky does this solid angle represent?