Computational Fluid Dynamics With a Paper Airplane

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Abstract

In this paper, we investigate the relationship between an airplane's shape and its performance. Our results show that ...

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1 Introduction

This thesis covers the simulation of the aerodynamics of an airplane, using our own Computational Fluid Dynamics model (CFD).

The goal is to simulate the airflow around an airplane's body. CFD is a branch of fluid mechanics that uses numerical methods and algorithms to solve and analyze problems that involve fluid flows. It is used in many fields, including aerospace engineering, automotive engineering, and meteorology.

The application of CFD to an airplane is important because it allows testing of a model's aerodynamic performance without building a physical model, or have to set up a wind tunnel. The practical alternative is much more expensive and time-consuming.

The final aim is to determine how the shape of an airplane's body affects its performance. Part of the project is to dynamically generate 3D models of airplanes, and simulate the airflow around them. The intention is to use machine learning to optimize the shape of the airplane's body to maximize performance.

CFD is challenging in the sense that it requires a good understanding of fluid dynamics, as well as a good understanding of the math involved. CFD is also very expensive from a computational perspective, so code optimization is important.

The thesis questions are the following:

- How do the different aspects of fluid dynamics work and how do we implement it in a computer program?
- How do we dynamically generate 3D models?
- How does an airplane's wing shape influence its performance?

2 Execution

2.1 Preliminary: Lagrangian Fluid Simulation

One method of simulating fluid dynamics is the Lagrangian method. This method models the fluid as a particle collision system, where the air is represented by particles that interact with each other to emulate a fluid. Our implementation is based on Rigid Body Collision Resolution (Hakenberg, 2005). We used this paper as a guide for all the math involved.

The math relies on the momentum, inertia, and velocity of the particles to calculate the collision normal and point of contact. The collision normal is the direction in which the particles are moving away from each other, and the point of contact is the point at which the particles collide.

The following variables are necessary to perform the calculations:

Angular momentum L $(kg \cdot m^2/s)$:	$L = mvr$;
Inertia tensor I $(kg \cdot m^2)$:	$I = \frac{L}{\omega}$;
Angular velocity ω (rad/s) :	$\omega = \frac{\Delta\theta}{\Delta t}$;

Collision normal($n \in \mathbb{R}^3$) in world coordinates away from body;
Point of contact ($r_i \in \mathbb{R}^3$) in world coordinates with respect to p_i;
Orientation ($R_i \in SO(3)$) transforming from object to world coordinates;

Where i represents one of two particles in a given collision:

Velocity after collision	$\tilde{v}_i$,
Angular velocity after collision	$\tilde{\omega}_i$,
Constant	λ,

The following formulas represent the relation between particles:

$$\tilde{v}_1 = v_1 - \frac{\lambda}{m_1} n;$$
$$\tilde{v}_2 = v_2 + \frac{\lambda}{m_2} n;$$
$$\tilde{\omega}_1 = \omega_1 - \Delta q_1;$$
$$\tilde{\omega}_2 = \omega_2 + \Delta q_2;$$
$$\text{where } q_i := I_i^{-1} \cdot R_i^{-1} \cdot (r_i \times n),$$
$$\text{and } \lambda = 2\frac{nv_1 - nv_2 + \omega_1 I_1 q_1 - \omega_2 I_2 q_2}{(\frac{1}{m_1} + \frac{1}{m_2})n^2 + q_1 I_1 q_1 + q_2 I_2}$$