Evolved Parameterized Selection for Evolutionary Algorithms

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- EAs are highly parallelizable, and well-equipped to navigate complex fitness landscapes
- However, many EAs are highly sensitive to configurations/parameters

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- Parameter-tuning/Hyper-heuristics

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- Previous work has shown that many EA components are good candidates for targeted improvement: mutation/mating preference/genetic representation/crossover operators
- Our work targets selection for improvement

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- EAs and EC often use selection to control the path of evolution
- Performance depends heavily on good selection methods

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- Some selection functions are parameterized, adding more parameters to be set or tuned
- Using the right selection functions and selection parameters (where applicable) has a large impact on EA performance

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- The "No Free Lunch Theorem" implies that any possible selection function is optimal for some particular EA applied to some particular problem
- It is highly unlikely, for a given EA and problem, that the optimal selection exists within the conventional functions
- Therefore, we can expect a performance increase by developing a new selection function, tuned to the EA and problem

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- We develop a meta-EA and a new selection function representation to explore the space of possible selection functions
- We determine quality of evolved selection functions by running the EA with them, keeping all other parameters (if any) constant
- The EA's performance when using the evolved selection function (as opposed to a conventional selection function) is used as an indicator of the selection function's quality

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- Step 1: define a representation for selection functions to form a search space
- Step 2: discover which selection processes of the EA can be improved
- Step 3: explore the space of selection functions with a meta-EA
- Step 4: test the EA on new instances from the same problem class to test generalization of the performance increase

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  - Searching through an infinite space of functions
  - Ensuring that all outputs are valid valid selection functions

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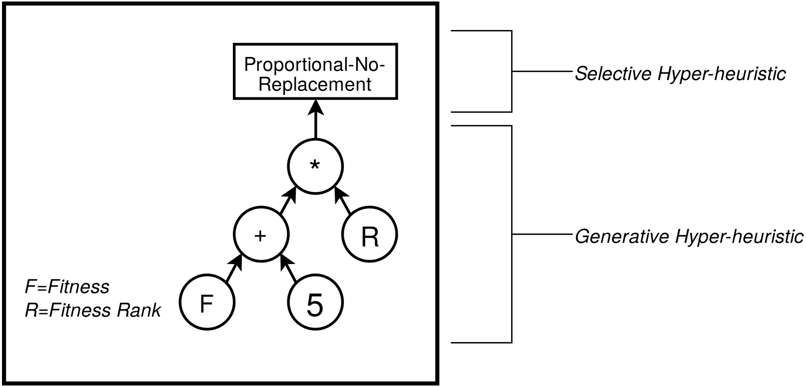
- The first component is a mathematical function, encoded in a Koza-style GP-Tree, that calculates a real-valued number corresponding to the individual's desirability
- The second component is a method of selecting individuals, based on their calculated desirability

Representation

A selection function is represented by a mathematical desirability function (encoded in a parse tree), and the final selection method.



Representation - Psuedocode

Algorithm 1 Probabilistic Selection Function

```
procedure EXAMPLESELECTION(P, W)
    W ← 0, ∀p ∈ P
    for all p ∈ P do
        W(i) ← (p.Fitness + 5) ∗ p.FitnessRank
    end for
    w_min ← minimum(W)
    s ← 0
    for all w ∈ W do
        if w_min < 0 then
            s ← s + (w − w_min)
        else
            s ← s + w
        end if
    end for
    selected ← ∅
    for j ← 1, m do
        r ← random(0, s)
        i ← 1
        while r > W(i) do
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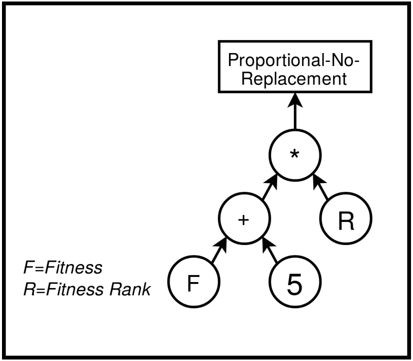

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Algorithm 1 Probabilistic Selection Function

procedure ExampleSelection(P, W)
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  for all $p \in P$ **do**
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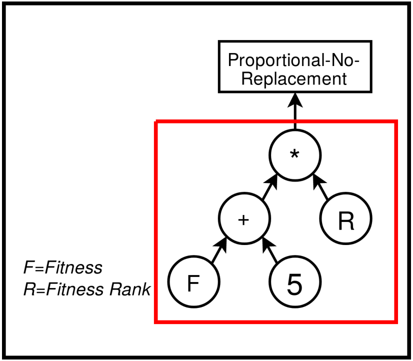

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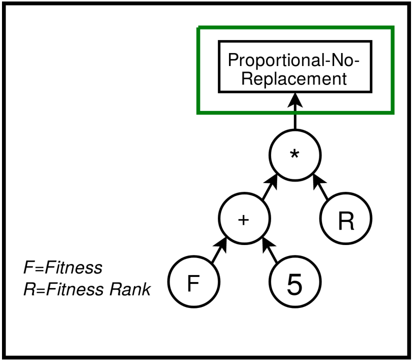

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- The GP-Tree is evolved with a generative Hyper-heuristic, exploring a large, yet restricted, space of mathematical functions
- The selection method is evolved with a perturbative Hyper-heuristic, picking from a list of methods inspired by conventional selection functions

Representation

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- In addition, we can guarantee that all functions in this space are valid selection functions
- This format cannot represent all possible selections, but this is an acceptable trade-off for the more easily-searchable space of selection functions

Representation - Terminals

Terminal	Description
Fitness	Individual's fitness value
Fitness Rank	Rank assigned to the individual when the population is sorted by fitness (higher fitness = higher rank)
Relative Fitness	Individual's fitness value, minus the minimum fitness value in the population, divided by the range of the population fitness values
Birth Generation	Generation number that the individual first appeared in the population

Representation - Terminals

Terminal	Description
Relative Uniqueness	Cartesian distance between the individual's genome and the centroid of all genomes in the population.
Population Size	Number of individuals in the population
Min Fitness	Smallest fitness value in the population
Max Fitness	Largest fitness value in the population
Sum Fitness	Sum of all fitness values in the population
Generation Number	Number of generations of individuals that have been evaluated since the beginning of evolution

Representation - Terminals

Terminal	Description
Constant	Returns a constant number, which is generated from a uniform selection within a configured range when the selection function is generated and held constant for the entire lifetime of the selection function.
Random	Returns a random number, which is generated from a uniform selection within a configured range every time selection is performed.

Representation - Operators

Operator	Operands	Description
+	2	Adds the left and right operands.
-	2	Subtracts the right operand from the left operand.
*	2	Multiplies the left and right operands.
/	2	Divides the left operand by the right operand. If the right operand is 0, the left operand is instead divided by a very small number, returning a large number while preserving the sign of the dividend.
Min	2	Returns the minimum of the left and right operands.

Representation - Operators

Operator	**Operands**	**Description**
Max	2	Returns the maximum of the left and right operands.
Step	2	Returns 1 if the left operand is greater than or equal to the right operand, and 0 otherwise.
Absolute Value	1	Returns the absolute value of the operand.

Representation - Selection Methods

Method	Description
Proportional-Replacement	A weighted random selection, with each individual's weight equal to its desirability score
Proportional-No-Replacement	As with Proportional-Replacement, but an individual is removed from the selection pool after being selected
k-Tournament-Replacement	A random subset of k individuals is considered, and the individual with the highest desirability score in the subset is selected
k-Tournament-No-Replacement	As with k-Tournament-Replacement, but an individual is removed from the selection pool after being selected

Representation - Selection Methods

Method	**Description**
Truncation	Individuals with the highest desirability score are selected, with no individual being selected more than once
Stochastic-Universal-Sampling	A variant of proportional selection that chooses individuals at evenly spaced intervals, reducing sampling bias

Representation - Example

Selection Function: $(Fitness + 5) * FitnessRank \rightarrow$ Proportional-No-Replacement

Population Member	1	2	3	4
Fitness	300	250	200	350
Fitness Rank	3	2	1	4

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$(Fitness + 5) * FitnessRank$	915	510	205	1420

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Desirability scores: 915, 510, 205, 1420. Individuals are selected with probability proportional to their desirability score.

Representation - Example

Selection Function: $FitnessRanking * 100/(100 - Fitness) \rightarrow$ k-tournament, k=3

Population Member	1	2	3	4
Fitness	300	250	200	350
Fitness Rank	3	2	1	4
$FitnessRanking * 100/(100 - Fitness)$	-1.5	-1.33	-1.0	-1.6

Desirability scores: -1.5, -1.33, -1.0, -1.6. Selection picks a random set of three individuals, and selects the individual among them with the highest desirability.

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- After the meta-EA is run, the selection strategies are tested for generalization on a separate set of "testing" instances from the same problem class

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- We targeted improvement of the parent selection stage of the meta-EA, keeping all other factors constant
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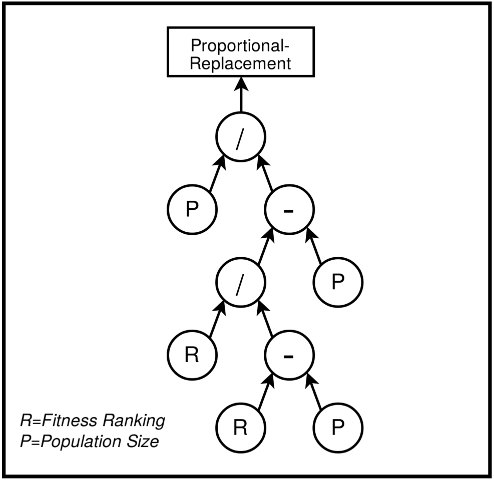
- For our benchmark problem class, we used the NK-Landscape problem class

Initial Experiment - Results

Our meta-EA evolved a parent selection function that significantly outperformed typical selection on 46 of the 50 benchmark functions we tested.



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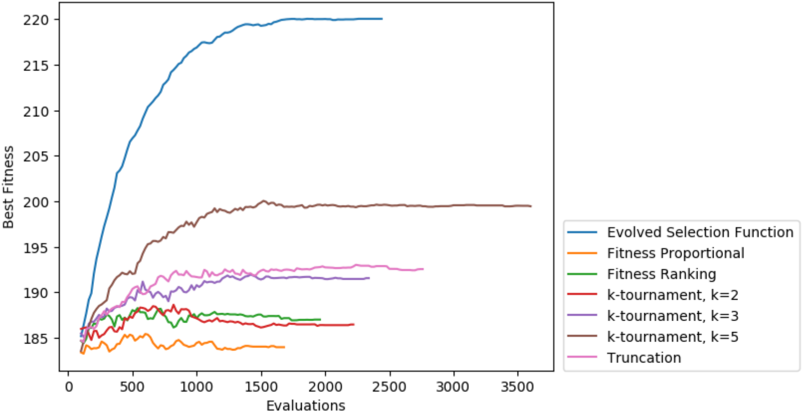
Best fitness vs. EA evals, averaged over all runs:



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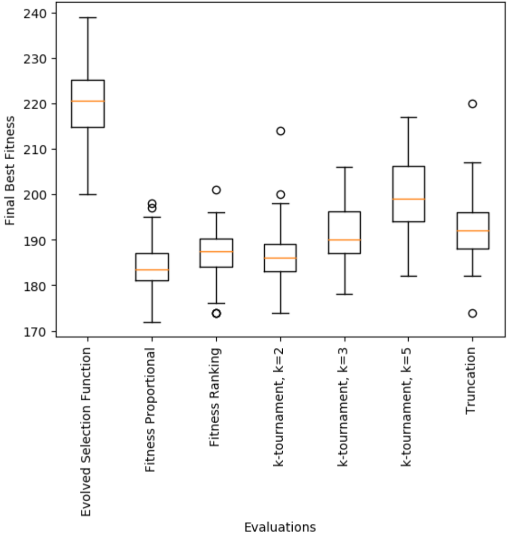
Final best fitnesses achieved by the EA:



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- We also needed to show that our methodology works with other problem classes, and other EAs
- We built these takeaways into our next three experiments

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- Evolving both parent and survival selection for a basic EA, solving MK-Landscapes
- Evolving both parent and survival selection for a basic EA, solving real-valued benchmark functions
- Evolving a new mean-update scheme for CMA-ES, solving real-valued benchmark functions

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- We use the iRace tuning package to find the best set of parameters to run the EA with, including the best choice of parent and survival selection from a list of conventional selection functions
- We use the MK-Landscape problem class, a generalization of the NK-Landscape class
- At the end of the meta-EA, we test the evolved selection function against the conventional selection functions selected by iRace

Experiment 1 - Results

Results: Our meta-EA evolved a parent selection function that significantly outperformed typical selection on 45 of the 50 benchmark functions we tested.

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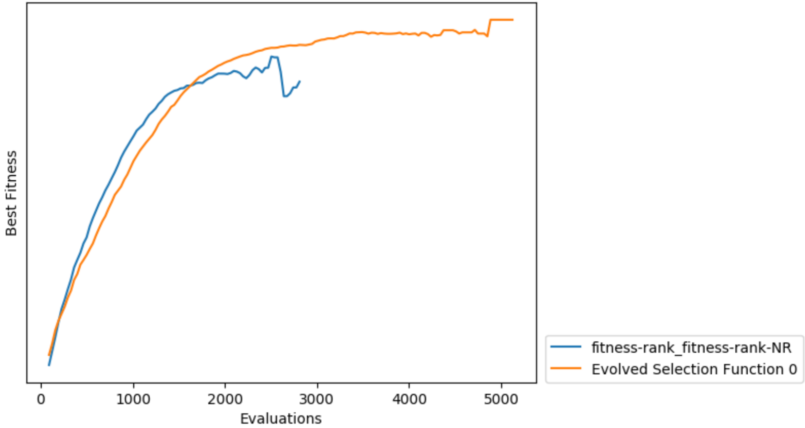
Best fitness vs. EA evals, averaged over all runs:



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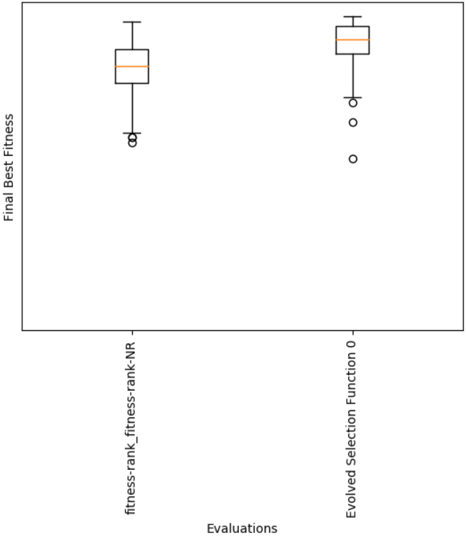

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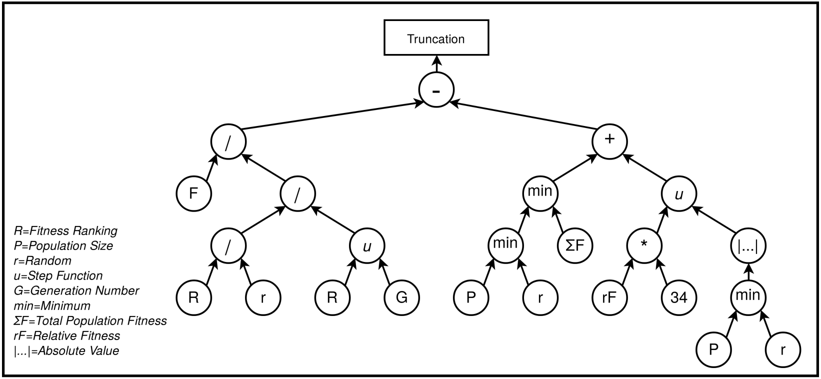
Parent Selection of the best evolved selection function:



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- Our Hyper-heuristic performs well, even against state-of-the-art parameter tuning
- Next, we need to test against more benchmark functions

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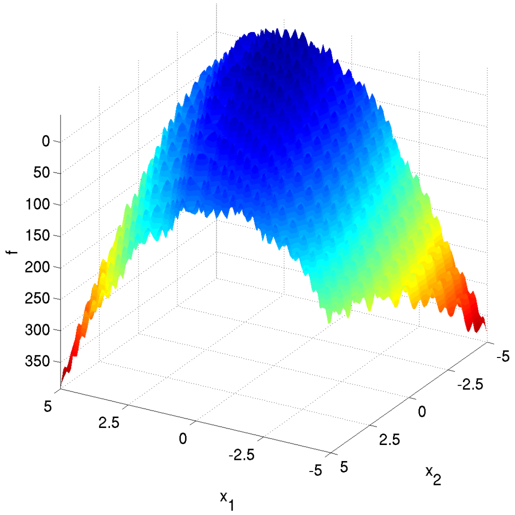
- As with Experiment 1, we test for generalization at the end of the meta-EA

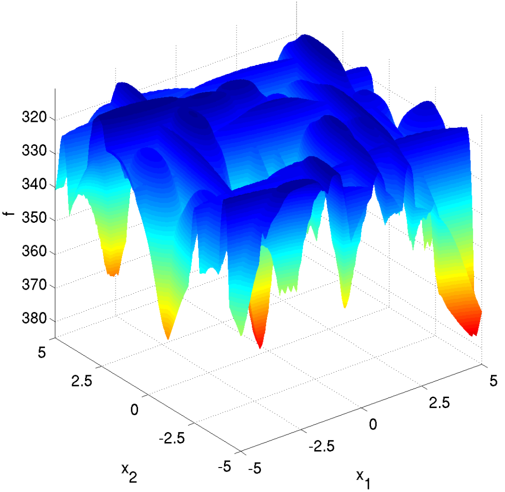
Experiment 2

COCO F #15: Rastrigin Function



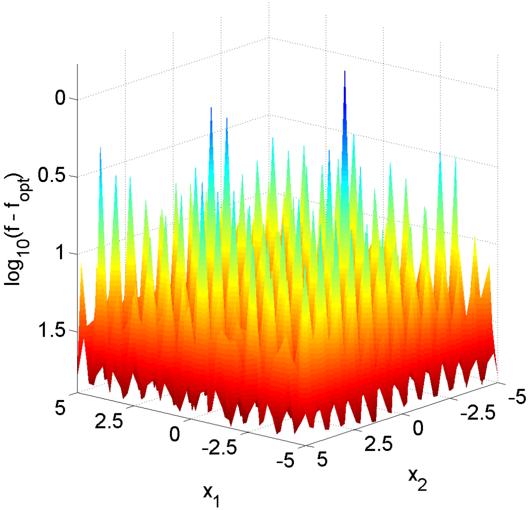
Experiment 2

COCO F #21: Gallagher's Gaussian 101-me Peaks Function



Experiment 2

COCO F #23: Katsuura Function



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- For 6 of the 24 function classes, the evolved selection function significantly outperformed the conventional selection on at least half of the problem instances
- For 2 of the 24 function classes, the evolved selection function significantly outperformed the conventional selection on all of the problem instances

Results:

Problem Index (D=10)	Number of Instances Improved
F=1	0 / 12
F=2	1 / 12
F=3	2 / 12
F=4	2 / 12
F=5	5 / 12
F=6	2 / 12
F=7	7 / 12
F=8	2 / 12
F=9	12 / 12
F=10	1 / 12
F=11	7 / 12
F=12	0 / 12

Results:

Problem Index (D=10)	Number of Instances Improved
F=13	2 / 12
F=14	0 / 12
F=15	0 / 12
F=16	0 / 12
F=17	4 / 12
F=18	4 / 12
F=19	12 / 12
F=20	0 / 12
F=21	8 / 12
F=22	6 / 12
F=23	0 / 12
F=24	1 / 12

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  - Little improvement on functions with weak global structure, which our EA setup depends on
- Using the same dimensionality for all 24 problems would inherently lead to "easy" and "hard" problems

Experiment 3

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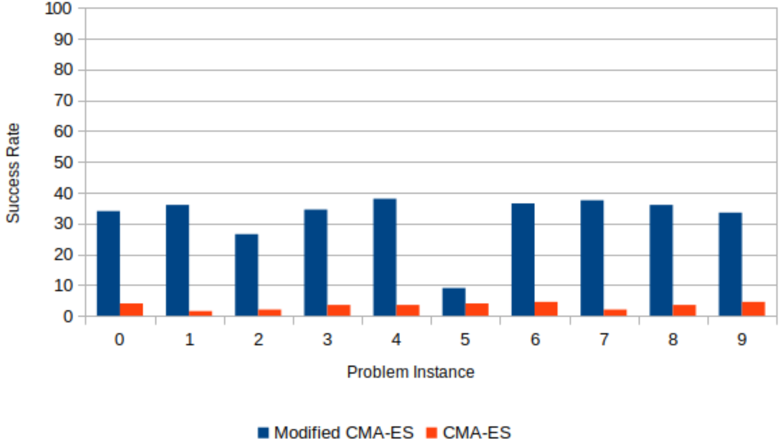
- We targeted the mean-update function of CMA-ES, to better select the points used for updating the mean (normally truncation)
- We select the COCO functions (class and dimensionality) which CMA-ES is unable to solve more than 50% of the time

Experiment 3 - Results

Results: For 6 of the 11 functions tested, the CMA-ES using the evolved selection function found the global best fitness more often than CMA-ES without it

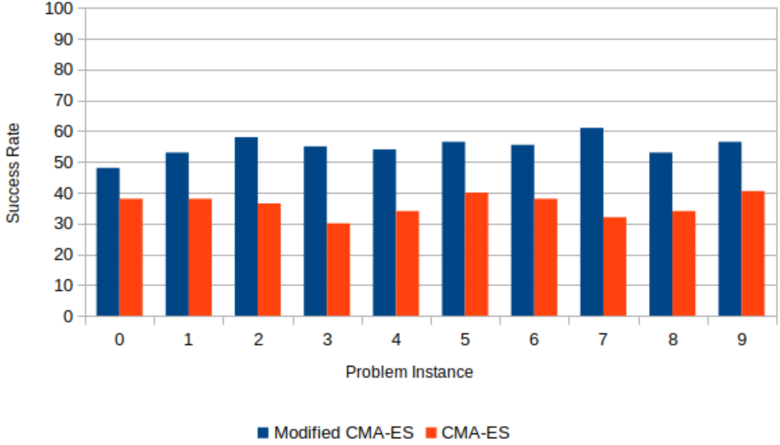
Experiment 3 - Results

F=4,D=2:



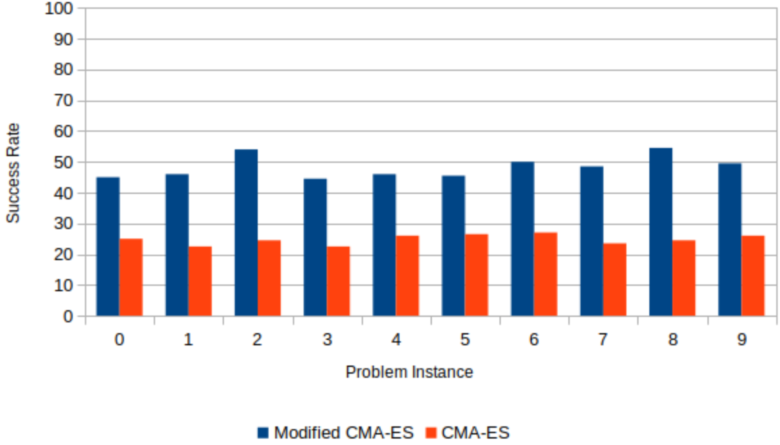
Experiment 3 - Results

F=19,D=2:



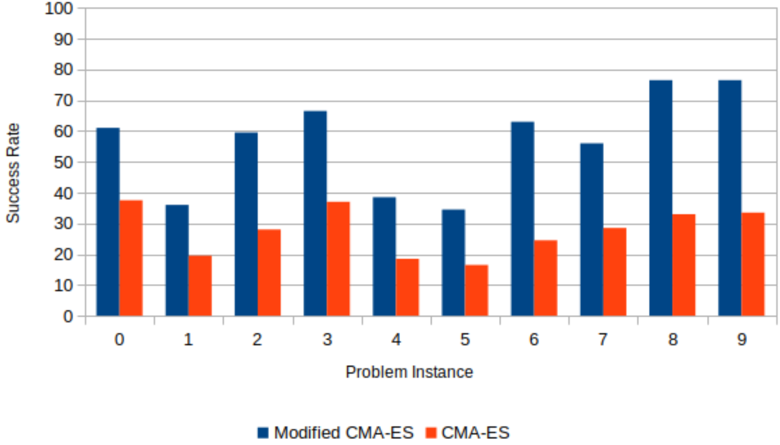
Experiment 3 - Results

F=20,D=2:



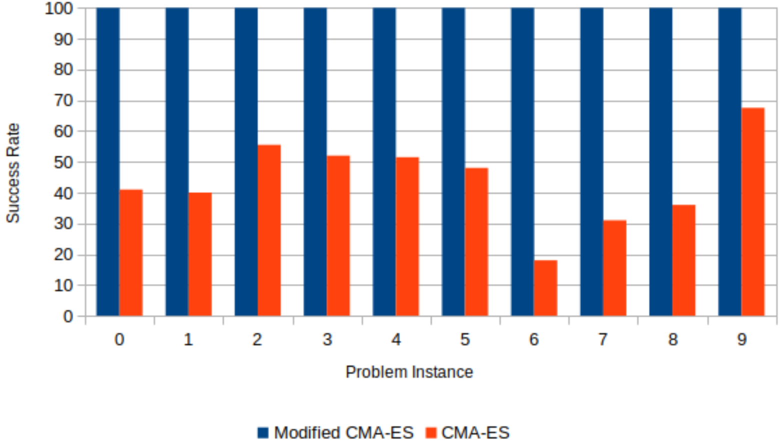
Experiment 3 - Results

F=21,D=2:



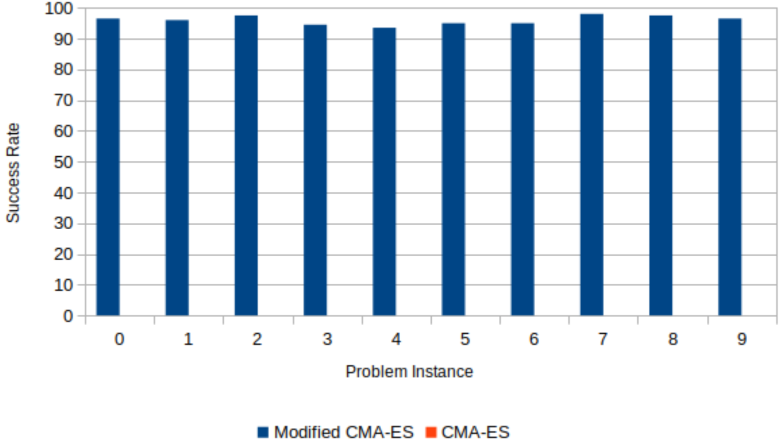
Experiment 3 - Results

F=12,D=10:



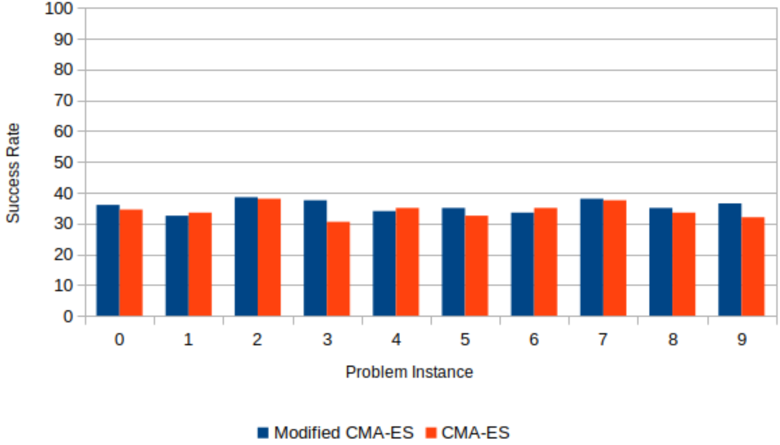
Experiment 3 - Results

F=6,D=10:



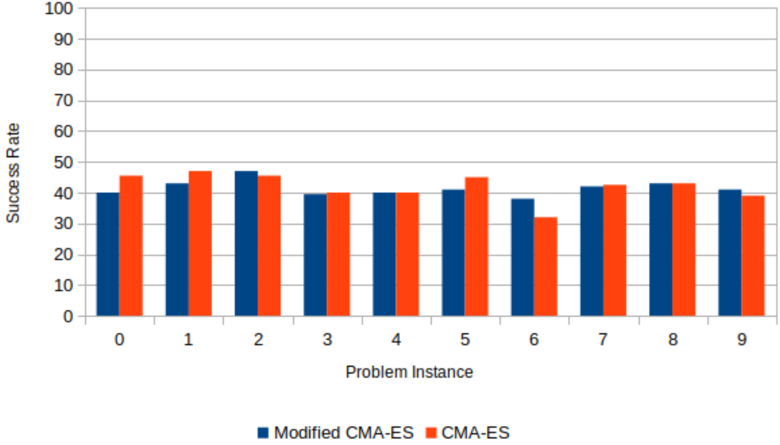
Experiment 3 - Results

F=3,D=2:



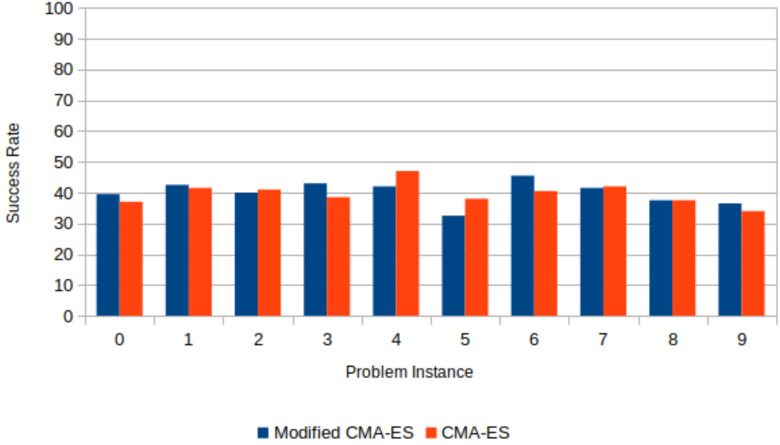
Experiment 3 - Results

F=15,D=2:



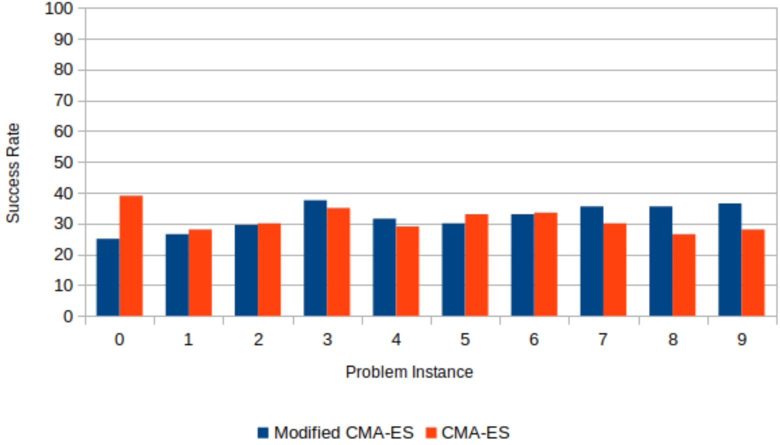
Experiment 3 - Results

F=16,D=10:



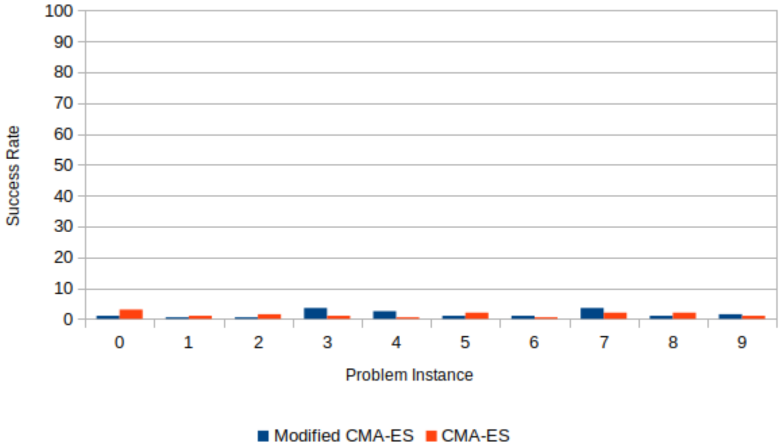
Experiment 3 - Results

F=23,D=5:



Experiment 3 - Results

F=24,D=2:



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- Performance on other benchmarks may depend more heavily on other CMA-ES update functions
- Functions with poor performance have poor local structure, require a more intelligence search of global scale (i.e., Rastrigin)

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- Potential performance gain likely depends strongly on the EA, the component being replaced, and the problem being solved

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- Some information encodable in GP-Trees is not typically considered for selection, not present in the conventional selection functions tested against.

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- Test with more terminals available to GP-Tree in meta-EA, including mate preference information