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The Automated Design of Probabilistic Selection Methods for Evolutionary Algorithms

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Introduction

- For Evolutionary Algorithms (EA's), the method of parent/survival selection has a significant impact on performance

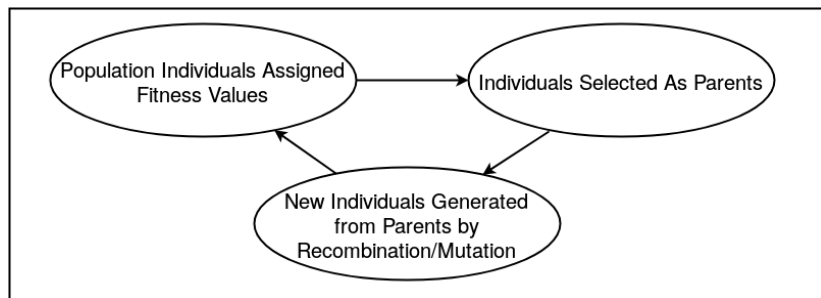
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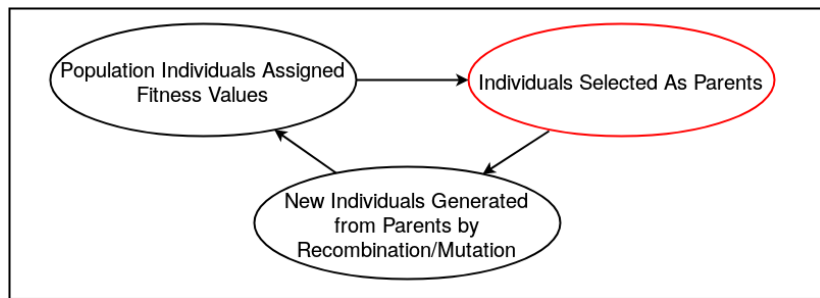
- For Evolutionary Algorithms (EA's), the method of parent/survival selection has a significant impact on performance
- Manual Tuning of selection strategy can lead to improvement but is still limited to existing/conventional methods
- Generation of new selection methods, specialized to particular problems, can improve performance further

Overview



Parent Selection and Survival Selection both strongly influence how genes survive in the population

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We wish to develop a customized selection function specialized to a particular problem class. For this study, we chose to specialize parent selection, while keeping survival selection static.

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- Step 2: explore this space and determine the quality of the selection functions to find the best one

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- However, the space of Turing-complete algorithms is large and complex, making it difficult to search through

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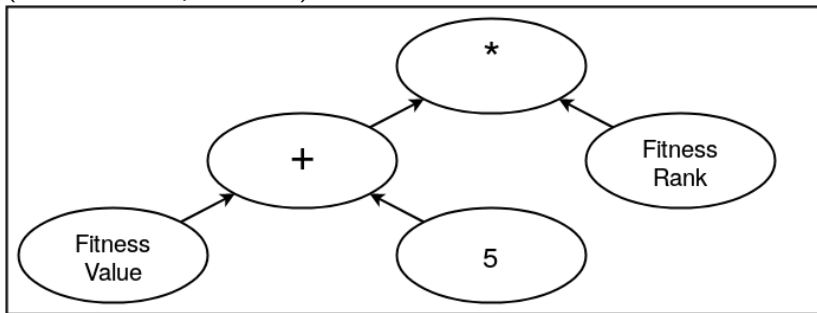
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- The function is evaluated for each individual, and a weighted random selection is performed, using the function output as the weight for each individual
- An additional boolean variable controls whether selection is performed with replacement

Representation

A selection function is represented by a mathematical function (encoded in a parse tree), and the boolean variable.



Representation - Psuedocode

Algorithm 1 Probabilistic Selection Function

```
1: procedure PROBABILISTICSELECTION( $P$ )
2:   for  $i \leftarrow 1, n$  do
3:      $W(i) \leftarrow (P(i).Fitness + 5) \cdot P(i).FitnessRank$ 
4:   end for
5:    $selected \leftarrow \text{WEIGHTEDSELECTION}(P, W)$ 
6:   remove selected from  $P$ 
7:   return  $selected$ 
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9: procedure WEIGHTEDSELECTION( $P, W$ )
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11:   $s \leftarrow 0$ 
12:  for all  $w \in W$  do
13:    if  $w_{min} < 0$  then
14:       $s \leftarrow s + (w - w_{min})$ 
15:    else
16:       $s \leftarrow s + w$ 
17:    end if
18:  end for
19:   $r \leftarrow \text{random}(0, s)$ 
20:   $i \leftarrow 1$ 
21:  while  $r > W(i)$  do
22:     $r \leftarrow r - W(i)$ 
23:     $i \leftarrow i + 1$ 
24:  end while
25:  return  $P(i)$ 
26: end procedure
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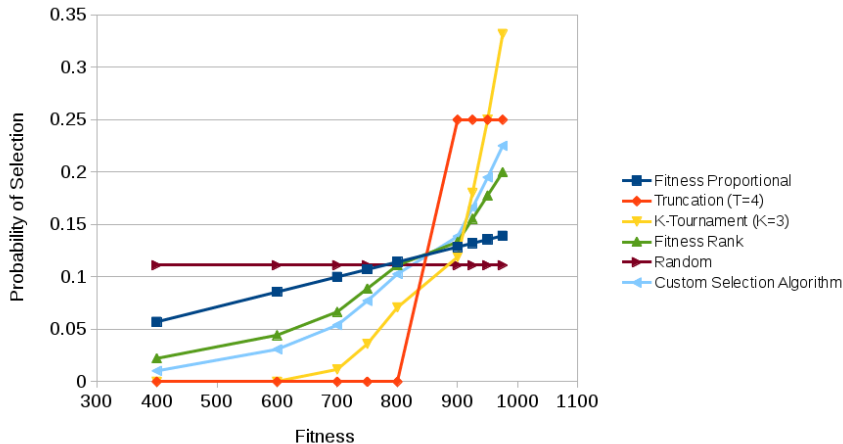
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$P(\text{fitness-proportional})$	0.273	0.227	0.182	0.318
$P(\text{fitness-ranking})$	0.3	0.2	0.1	0.4

Representation

Different selection functions, including conventional and our evolved selection functions, offer different probability distributions over the population



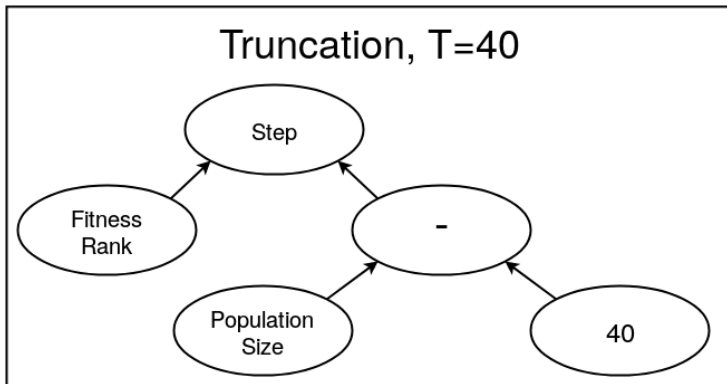
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Many conventional selection functions can be represented in this format



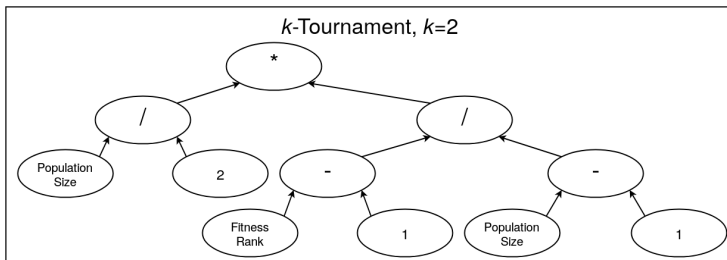
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- In addition, we can guarantee that all functions in this space are valid selection functions
- We cannot guarantee that all possible selection functions can be represented in this format, but this is an acceptable tradeoff for the more easily-searchable space of selection functions

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- Each strategy is evaluated by running a sub-level EA utilizing that strategy on a fixed set of benchmark instances from the NK-Landscape problem class, with $N=40$ and $K=8$
- After the meta-EA is run, the selection strategies are tested for generalization on a separate set of “testing” instances from the same problem class

Meta-EA Parameters

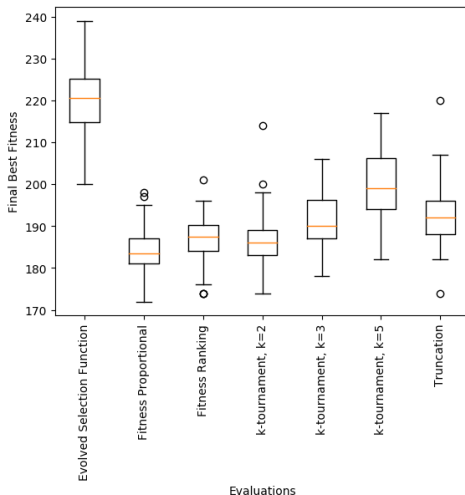
Parameter	Value
Population Size	50
Offspring Size	50
Evaluation Count	2500
Max GP-Tree Initialization Depth	3
Parent Selection	k -tournament, $k=10$
Survival Selection	Random
Mutation	Subtree Regeneration
Crossover	Subtree Crossover
Parsimony Pressure Coefficient	1
Mutation Rate	0.1
Training Instances	20
Runs per Training Instance	3
Testing Instances	50
Runs per Testing Instance	100
Range for Constant Terminals	$[-100, 100]$
Range for Random Terminals	$[-100, 100]$

Base EA Parameters

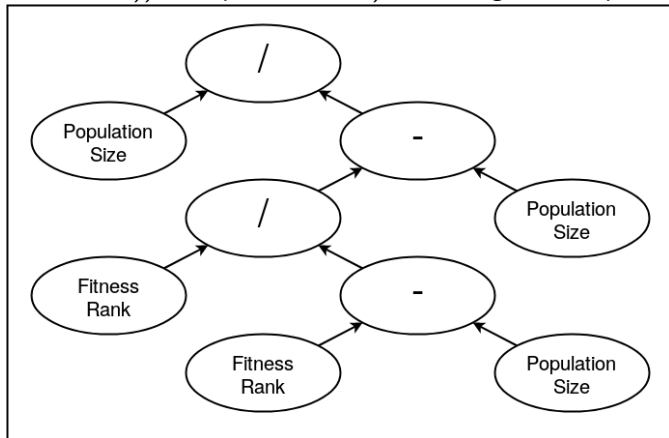
Parameter	Value
Population Size	100
Offspring Size	20
Genome Length (N)	40
Locus Length (K)	8
Locus Minimum Value	0
Locus Maximum Value	8
Parent Selection	(evolved in Meta-EA)
Survival Selection	Random
Termination Criteria	Convergence
Generations to Convergence	25
Mutation	Random Bit Flip
Mutation Rate	0.05
Crossover	Uniform Crossover

Results - Comparison with Conventional Selection Methods

The final best selection strategy evolved by the Meta-EA outperformed all conventional selection strategies tested on 94% of the 50 testing instances.

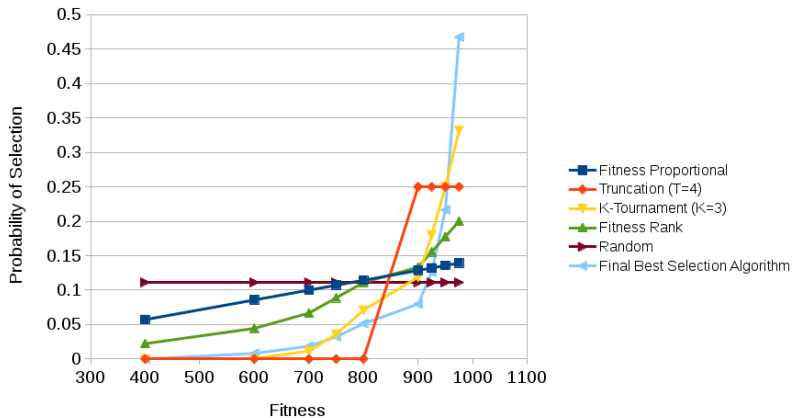


Results - Final Best Evolved Selection Strategy

$$P(\text{selection}) \propto \text{PopulationSize} / ((\text{FitnessRank} / (\text{FitnessRank} - \text{PopulationSize})) - \text{PopulationSize}), \text{ selecting with replacement}$$


The Final Best Evolved Selection Strategy

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- The Meta-EA can require large amounts of computing resources, especially if the sub-level EA is computationally expensive

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- The evolved selection methods are static, and do not adapt/self-adapt as the evolution progresses
- The sub-level EA uses static parameters for population size, number of offspring generated, mutation rate, and survival selection

Future Work

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- Investigate other weighted selection strategies apart from a weighted random selection.
- Investigate systems to encourage more diversity in parents selected, beyond a binary “with/without replacement” setting

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- An EA running on a problem class can gain a performance benefit with a selection function specialized to that problem class (with all other parameters unchanged)

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- An EA running on a problem class can gain a performance benefit with a selection function specialized to that problem class (with all other parameters unchanged)
- A Meta-EA Hyper-heuristic can be used to search through a space of selection functions to find one that significantly outperforms common selection functions for a particular problem class