Evolving Mean-Update Selection Methods for CMA-ES

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ECADA @ GECCO, July 2019

Introduction

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- Search space mean is updated every generation using sampled points

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- We evolve a new method of selecting the points used to update the mean

Objective

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- Step 1: define a representation for selection functions to form a search space
- Step 2: explore this space and determine the quality of the selection functions to find the best one

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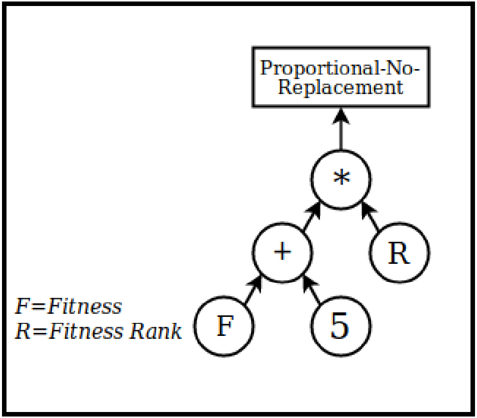
- The second part is a final selection step that selects points based on their desirability scores

Representation

A selection function is represented by a mathematical function (encoded in a parse tree) and a selection method.



GP-Tree Operators

Operator	Operands	Description
$+$	2	Adds the left and right operands.
$-$	2	Subtracts the right operand from the left operand.
$\times$	2	Multiplies the left and right operands.
$/$	2	Divides the left operand by the right operand. If the right operand is 0, the left operand is instead divided by a very small number, returning a large number while preserving the sign of the left operand.
Min	2	Returns the minimum of the left and right operands.
Max	2	Returns the maximum of the left and right operands.

GP-Tree Operators

Operator	Operands	Description
Step	2	Returns 1 if the left operand is greater than or equal to the right operand, and 0 otherwise.
Absolute Value	1	Returns the absolute value of the operand.

GP-Tree Terminals

Terminal	Description
Fitness	The individual's fitness value.
Fitness Rank	The individual's index in a list of the population members sorted by fitness, increasing.
Relative Fitness	The individual's fitness value divided by the sum of all fitness values in the population.
Birth Generation	The generation number that the individual first appeared in the population.
Relative Uniqueness	The Cartesian distance between the individual's genome and the centroid of all genomes in the population.

GP-Tree Terminals

Terminal	Description
Population Size	The number of individuals in the population.
Min Fitness	The smallest fitness value in the population.
Max Fitness	The largest fitness value in the population.
Sum Fitness	The sum of all fitness values in the population.
Generation Number	The number of generations of individuals that have been evaluated since the beginning of evolution.

GP-Tree Terminals

Terminal	Description
Constant	A constant number, which is generated from a uniform selection within a configured range when the selection function is generated and held constant for the entire lifetime of the selection function.
Random	A random number, which is generated from a uniform selection within a configured range every time selection is performed.

Selection Methods

Method	Description
Proportional-Replacement	A weighted random selection, with each individual's weight equal to its desirability score.
Proportional-No-Replacement	As with Proportional-Replacement, but an individual is removed from the selection pool after being selected.
k-Tournament-Replacement	A random subset of k individuals is considered, and the individual with the highest desirability score in the subset is selected.

Selection Methods

Method	Description
k-Tournament-No-Replacement	As with k-Tournament-Replacement, but an individual is removed from the selection pool after being selected.
Truncation	Individuals with the highest desirability score are selected, with no individual being selected more than once.
Stochastic-Universal-Sampling	Individuals are chosen at evenly spaced intervals of their desirability scores.

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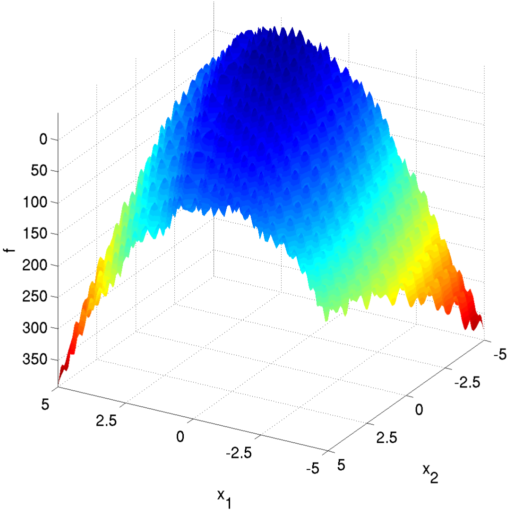
- Koza-style GP is used to evolve the trees, with an extra gene encoding the final selection step and any parameters to it
- Each run of the meta-EA targets one of the 24 noiseless test function classes in the Comparing Continuous Optimizers (COCO) function set
- Dimensions of 2, 3, 5, and 10 are used, each with their own meta-EA

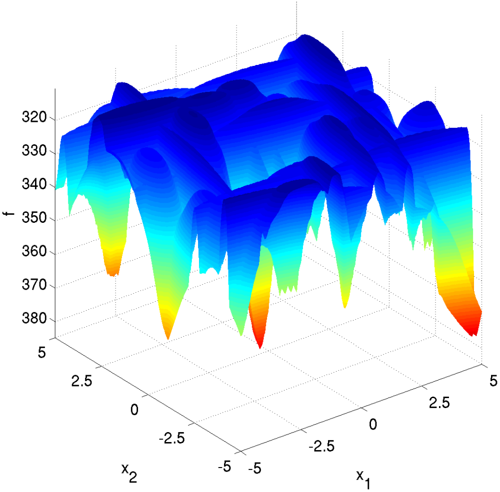
Meta-EA

COCO Function Class #15: Rastrigin Function



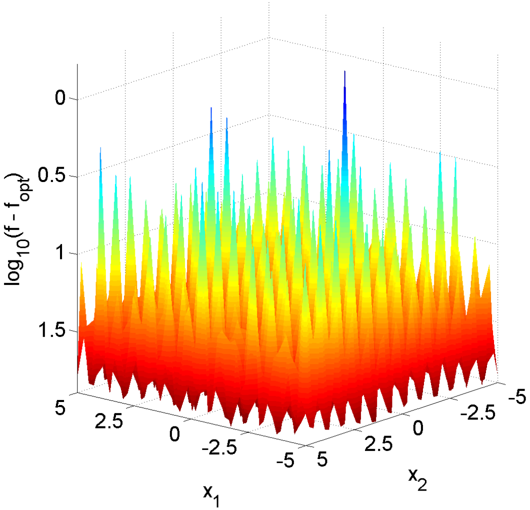
Meta-EA

COCO F. C. #21: Gallagher's Gaussian 101-me Peaks Function



Meta-EA

COCO F. C. #23: Katsuura Function



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- CMA-ES is run on a number of instances from the problem class. The solution rate is used as the fitness of the selection function
- After the meta-EA concludes, CMA-ES is run with the best selection function on new instances, to test for generalization

Meta-EA Parameters

Parameter	Value
Population Size	40
Offspring Size	40
Evaluation Count	4000
Max GP-Tree Initialization Depth	4
Parent Selection	k-tournament, k=4
Survival Selection	Truncation
Mutation	Subtree Regeneration
Crossover	Subtree Crossover
Parsimony Pressure Coefficient	0.0005
Mutation Rate	0.25
Range for Constant Terminals	[-100, 100]
Range for Random Terminals	[-100, 100]
Number of Runs (Training)	5
Number of Runs (Testing)	200

Results

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Results

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- For function class 6 and $D = 10$, success rate increased from 0% to 96%
- For function class 12, success rate increased from 44% to 100%
- Very few cases where the tuned CMA-ES performs worse, and only 7.4% worse at worst

Results: Selected Examples

Percentage of Runs Solved By Unmodified CMA-ES/Modified CMA-ES, averaged over all instances (selected examples)

F. C.	D=2	D=3
4	3.3% → 32.15%	0.25% → 0.15%
6	100.0% → 100.0%	100.0% → 100.0%
12	100.0% → 99.55%	100.0% → 99.5%
14	100.0% → 100.0%	100.0% → 100.0%
21	27.65% → 56.8%	28.7% → 55.4%
24	1.45% → 1.6%	0.3% → 0.1%

F. C.	D=5	D=10
4	0.0% → 0.0%	0.0% → 0.0%
6	100.0% → 99.1%	0.0% → 96.0%
12	100.0% → 99.85%	44.05% → 44.05%
14	100.0% → 100.0%	100.0% → 100.0%
21	3.3% → 36.05%	36.0% → 28.6%
24	0.0% → 0.0%	0.0% → 0.0%

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- Tune selection strategies for other problems, including real-world problems
- Reduce *a priori* computation required for tuning
- Tune more CMA-ES variants

“Take Home Message”

Performance of CMA-ES can be improved on a particular problem class by using a meta-EA to tune the method by which sampled points are selected and used to update the search space mean