Problem 1

The minimum I must win 1.

One toss

Let’s assume that I only have one toss. This is pretty trivial. Assume the dice is fair so I have 1/6 probability on each outcome {1,2,…,6}. Then the expected payoff I will get is:

Two tosses

Knowing that I can get $3.5 (at least in the long run…) in one toss, and suppose now I am given 2 tosses instead, so my strategy is S1: “I should only take the money on the first roll if my roll is larger than 3.5, i.e {4, 5, 6}”. There is 3/6 = 0.5 probability this can happen. So based on my strategy (accept first toss only if >3.5), the expected winning on a 2 toss game is:

Three tosses

By the same logic, I should only play strategy S3: “accept tosses > 4.25 in the first toss” if I know I have three tosses. So, my expected winnings if I have three tosses (assume I play out both strategies S1 and S2) will be:

E(Accept the first toss) + E(reject the first toss:=play the two tosses game)

Please refer to py file, “prob1.py” for solution to general case