Don’t throw the associative baby out with the Bayesian bathwater: Children are more associative when reasoning retrospectively under information processing demands

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Abstract

Causal reasoning is a fundamental cognitive ability that enables children to learn about the complex interactions in the world. The mechanisms that underpin children’s causal reasoning, however, are not well understood. An open question is the extent to which children retrospectively reevaluate the causal effectiveness of an object given ambiguous information. Here, we report two experiments that test children’s capacity to engage in such inferences. We also fit those data to different computational frameworks – one more associative and one more Bayesian – to consider the strengths and weaknesses of each approach, and the possibility that these approaches together better explain children’s causal reasoning than either approach individually.

Keywords: causal reasoning; cognitive mechanisms; computational models; associative learning; Bayesian inference

Few capacities are more important than the ability to reason and make inferences about causal relations. Causal reasoning enables human learners to make predictions and inferences (e.g., Bullock, et al., 1982; Shultz, 1982), to intervene on those relations to generate new effects (e.g., Butler et al., 2020; Schulz et al., 2007), and to reason about counterfactual claims—both about what might have been and how events could have turned out differently (e.g., Harris et al, 1996; Walker & Nyhout, 2020). These, and many other studies (e.g., Bonawitz & Lombrozo, 2012; Gopnik et al., 2001; Legare et al., 2010; Meltzoff et al., 2012; Walker & Gopnik, 2014), posit that young children have sophisticated causal reasoning capacities.

A fundamental question that underlies this research is *how* children make such inferences. One answer to this question is that children’s causal inferences are best described by Bayesian inference. Although this view is often described as a computational level of analysis (cf. Marr, 1982), some advocates suggest that children use cognitive mechanisms that approximate or even represent Bayesian calculations (Bonawitz et al., 2014; Xu, 2019; see also Griffiths et al., 2015). Causal reasoning starts with statistical learning capacities that are present in infancy (e.g., Gomez, 2002; Kirkham et al., 2002; Marcus et al., 1999; Saffran et al., 1996) but that develop into a system that infers abstract patterns of coherent causal structure from probabilistic data (Gopnik & Wellman, 2012; Weisberg & Sobel, 2022).

An alternative perspective is that associative learning alone is sufficient to describe children’s causal inferences. On this view, children build up a representation of causal structure from connecting and processing multiple associative relations and statistical regularities. Connectionist models—which learn largely via associative learning—have provided a proof of concept that causal learning can emerge from such associative processes (e.g., Benton et al., 2021; McClelland & Thompson, 2007). Additionally, comparative investigation between non-human animals and adults (e.g., Heyes, 2012) and studies of instrumental action and conditioning on human infants (e.g., Greco et al., 1990; Rovee-Collier, 1999) provide behavioral support for associative learning as a candidate mechanism for how children reason in the world.

One way to illustrate the tension between these hypotheses in development is through investigations of retrospective reasoning, such as *backwards blocking* (Shanks, 1985). This is a form of reasoning that involves reevaluating the causal status of an ambiguous event based on learning more about the status of other unambiguous events (see also De Houwer et al, 2002; Larkin et al, 1998; Kruschke & Blair, 2000; Lovibond, 2003; Van Hamme & Wasserman, 1994, for other work on adults). One of the first studies to examine backwards blocking reasoning in children was carried out by Sobel et al. (2004). They introduced 3- and 4-year-olds to a machine called a “blicket detector” that lit up and played music when certain objects called “blickets” were placed on it (Gopnik & Sobel, 2000). Children were then shown that two novel objects, A and B, activated the machine when they were placed on it at the same time. Children were then shown that object A alone either did or did not activate the machine. On both types of trials, children were then asked whether each object was a blicket. Children judged that A was a blicket only when it activated the machine. Their judgments of object B also differed across these conditions. Children judged object B more likely to be a blicket when object A failed to activate the machine than when it did so. Using modified procedures, toddlers and even infants as young as 8 months showed a similar pattern of responses (Sobel & Kirkham, 2006).

These findings—and specifically the finding that children’s causal inferences are sensitive to base rates (e.g., Sobel et al., 2004, Exp. 3)—have been interpreted as support for a Bayesian description of causal reasoning rather than a description that appears more to associative learning. This is because some associative models (e.g., Rescorla & Wagner, 1972) predict that the strength between object B and the machine’s activation is equivalent between the backwards blocking (where A is effective) and another trial in which A is not effective (labeled indirect screening-off trials). Moreover, even a modified version of the Rescorla-Wagner model (e.g., Van Hamme & Wasserman, 1994) does not predict differences in such reasoning when the base rates of the causal effectiveness of an object is manipulated.

There are, however, two facets of these data that warrant further consideration. First, McCormack et al. (2009) questioned what exactly was being reevaluated in a backwards blocking inference. They showed 4- and 5-year-olds two objects (A and B) that activated the machine together, and then that object A activated the machine alone. They compared children’s causal status judgments for object B with a sequence in which a third object (C), unrelated to the compound set, activated the machine (i.e., AB+, C+). The 4-year-olds did not differ in their judgments (although 5-year-olds did, judging B less likely to be efficacious than C). This control measure—which we adopt here—is a superior measure of assessing whether children reevaluate their causal judgments, and specifically of examining whether children reevaluate the causal status of the object(s) shown independently or the object only shown as part of the initial ambiguous data.

Second, backwards blocking and indirect screening-off are unique among retrospective causal inferences in that children only have to keep two potential causes in mind. When more than two objects are present, a Bayesian description demands that reasoners consider a larger hypothesis space, which may introduce additional information processing demands. The goal of the present study is to consider children’s retrospective causal inferences when more than two objects are present. Our prediction is that this will introduce a set of information processing demands that might affect the way in which children engage in causal inferences.

In particular, several studies suggest that the greater the complexity of the inference, the simpler the model of processing children use to reason (e.g., Doebel & Zelazo, 2015; Frye, Zelazo, & Palfai, 1995; Zelazo, Frye, & Rapus, 1996; Zelazo et al., 2003). Kenderla and Kibbe (2023) demonstrated that when faced with a challenging virtual memory game, 8- and 10-year-old children showed a decreased reliance on working memory and a greater dependence on manual exploration. The goal of this game was to find three cards with shared and differing features. Given that children were not required actively to maintain information in memory when manually exploring, manual exploration was an ostensibly simpler and less cognitively effortful strategy than one that required an already resource-limited system such as working memory. Similarly, Richland et al. (2006) found that 3- and 4-year-old children made more featural and relational errors when asked to reason about multiple relations or when the task included a salient distractor than when asked to reason about a single relation without a distractor.

Given the hypothesis that descriptions of causal reasoning that follow Bayesian inference start with more associative learning capacities, it is possible that when children’s causal reasoning systems are taxed, they look more associative in their reasoning. This idea is consistent with data from Sobel and Kirkham (2007): Although 8-month-olds exhibited backwards blocking inferences similar to preschoolers in an anticipatory eye-gaze measure, 5-month-olds’ inferences appeared more associative in nature (Sobel & Kirkham, 2007). Further, when infants make judgments about the reliability of others' information, their decision-making seems to be best explained by associative processing (Sobel et al., 2020; Tummeltshammer et al., 2014).

As children enter the preschool years, those judgments become more normative, although occasionally they will default to associative forms of processing, particularly under information processing demands (Hermes et al., 2018; Luchkina et al., 2020). In terms of children’s casual reasoning more generally, although there are cases in which children’s retrospective inferences look similar to adults and best described by Bayesian inference when asked about multiple objects, there are cases in which their performance on analogous control conditions is more associative in nature (Griffiths et al., 2011). Further, on other kinds of retrospective inferences, as the information demands of the procedure increase, only older children between the ages of 3-7 succeed (Fernbach et al., 2012; Erb & Sobel, 2014; Sobel et al., 2017). Beyond causal inference, preschoolers’ performance on theory-of-mind and social-problem-solving tasks was adversely affected when they first completed tasks that taxed their information-processing abilities compared to when such capacities were not taxed (Caporaso & Marcovitch, 2021; Powell & Carey, 2017; Steinbeis, 2018). Considered together, this research indicates that although children use different reasoning processes under different information-processing demands; the higher those demands, the simpler the process (e.g., Cohen, 1988; Cohen et al., 1998, 1999).

In the present study, we considered how children made retrospective inferences when first shown ambiguous data (i.e., three objects together produce an effect), followed by further evidence involving one of those objects (Experiment 1) or two of those objects (Experiment 2). In both cases, the logic of our design followed McCormack et al. (2009), in which we contrasted these retrospective inferences with control trials in which children saw the same initial ambiguous data, and then unrelated objects that had similar efficacy. The question across both experiments is the extent to which children show qualitative evidence for a Bayesian description of their causal inference, but an overall stronger fit of associative reasoning. After presenting these behavioral data across two experiments, we present a pair of computational models to illuminate possible cognitive mechanisms by which children arrived at their causal judgements.

**Experiment 1**

In Experiment 1, 5- and 6-year-olds observed three objects (A, B, and C) together cause a machine to activate. They then observed that object A either caused or failed to cause the machine to activate by itself. They were then asked whether each object caused the machine to activate. These trials were compared to control trials in which they observed three different objects (A’, B’ and C’) activate the machine, followed by an event in which a fourth object (D) either caused or failed to cause the machine to activate. Here, retrospective revaluation involves treating objects B and C differently from objects A’, B’ and C’; when A activates the machine alone, judgments of the causal effectiveness of B and C should be lower than the judgments of A’, B’ and C’. When A does not activate the machine alone, judgments of the causal effectiveness of B and C should be higher than B’ and C’. Because McCormack et al. (2009) found that 5 and 6-year-olds made such retrospective inferences about two candidate causes, we considered the same age range here to consider more than two causes.

**Method**

**Participants.** Participants were 32 5-year-olds (16 boys and 16 girls; *M* = 64.81 months, range = 60-71 months, SD = 3.48) and 31 6-year-olds (17 boys and 15 girls; *M* = 77.81 months, range = 72-83 months, SD = 3.78). Sample size was determined based on previous studies on backwards blocking reasoning in human children (e.g., Griffiths et al., 2011; Sobel et al., 2004). Two children were excluded from analysis for failing to participate (*N* = 1) or missing video (which made coding their responses impossible) (*N* = 1). We did not collect demographic information about the sample, but the demographic information about sample of children collected by the laboratory during this time was as follows: 82% White/Caucasian (compared with 83%), 3% Black/African American (9%), 4% Asian/Asian American (4%), 0.5% Native American (1%), and 11% of Mixed Descent (3%). Sixteen percent of the sample identified as Hispanic/Latinx (compared with 17% of the population). Similarly, the overall household income level of families tested in the lab during this time was as follows: Less than 30K: 7%, 30-50K: 7%, 50-70K: 14%, 70-90K: 9%, 90-120K: 25%, Over 120K: 38K. The median income for the population as measured by the 2020 Census was ~$74K.

**Materials.** The “device” used in the current study was a computer-animated version of the blicket detector (Gopnik & Sobel, 2000). The device was a white rectangle with a black border that measured 5.99 cm × 23.47 cm, presented on a computer screen. If the device was “on”, the white region of the rectangle turned blue when objects came into contact with it. If the device was “off”, the white region remained white. A maximum of 4 differently colored circles were shown on the screen. Each circle measured 2.67 cm × 2.67 cm (see Figure 1 below). The machine was designed such that it activated immediately when the bottommost edge of a circle—predetermined to be a blicket—contacted it. At the start of any given trial, three or four equally spaced circles appeared above the machine. Finally, the videos contained a built-in script, which experimenters read. All video events were created in Microsoft PowerPoint.

**Procedure.** Participants were tested in a quiet room in local children’s museum. At the beginning of the experiment, all participants were shown a pretraining video. The video consisted of a rectangular base (i.e., the previously mentioned “blicket detector”) and two shapes (i.e., a gray triangle and a gray pentagon). Crucially, these shapes were unrelated to the circles used during the experimental portion of the experiment. The pretraining phase began with the triangle (object A) and pentagon (object B) above the machine and next to one another. Object A then descended until it contacted and immediately activated the machine (i.e., the white region changed from white to blue). Object A then returned to its starting position above the machine. Object B then descended until it contacted and failed to activate the machine. Object B then returned to its starting position. Finally, both objects descended until they contacted and activated the machine. Participants were then asked whether each object was a blicket. This event ensured that participants understood the task and recognized that individual objects could activate the machine and that the it activated if at least one effective object was placed on it.

Following the pretraining phase, participants were given four trials. Half the participants received two backwards blocking trials and two backwards blocking control trials. The other half received two indirect screening off trials and two indirect screening off control trials. The order of these trials within each condition was counterbalanced using a Latin square design. Different colored objects were used across all trials to prevent carryover effects. A schematic of this procedure is shown in Table 1.

Diagram

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Figure 1. Schematic of a Backwards Blocking experimental trial. The upper-right portion of the figure shows the backwards blocking event as it unfolded across time. The lower-left portion of the figure shows the three objects and the text, “Is this one a blicket?” above each object across time.

**Backwards Blocking Experimental and Control Trials.** The two backwards blocking experimental trials began with three differently colored objects, which were located above the machine. The text, “Look, I have these three toys. Let’s find the blickets. Watch what happens” appeared above the objects. All three objects (i.e., objects A, B, and C) then descended until they contacted and activated the machine. At this point, the text, “Look, these also make the machine go!” appeared above the objects. The objects then returned to their starting positions.

The left- or right-most (counterbalanced) object (which we will refer to here as object A) then descended until it contacted and immediately activated the machine. The text, “Look, this one makes the machine go!” then appeared above the objects. This object then returned to its starting position. Children were then asked whether each object was a blicket. Specifically, the text, “Is this one a blicket?” with a downward-facing arrow then appeared above each object, and participants were asked to indicate whether each object was a blicket. Children received two of these trials, which were identical except for the color of the objects.

The two backwards blocking control trials began with four differently colored objects (i.e., objects A, B, C, and D), which were located above the machine. Objects A, B, and C then descended until they contacted and activated the machine; object D remained in place while objects A-C descended onto the machine. Object D then descended by itself until it contacted and activated the machine. The left-right position of object D was counterbalanced. Children were then asked whether each object was a blicket. Children once again received two trials, which were identical except for the color of the objects.

**Indirect Screening-Off Main and Control Trials.** The procedures for the indirect screening-off experimental and control conditions were identical to the backwards blocking trials except that object A (experimental trials) and D (control trials) failed to activate the machine.

**Results**

Figure 2 shows the number of times children responded “yes” to the question “Is this a blicket” for each object. Using this dependent measure, the data were entered into a five-way linear mixed-effects model with Age as a continuous fixed effect, Condition (Backwards blocking vs. Indirect screening-off) as the between-participants fixed effect, Trial Type (Experimental vs. Control), Objects (A vs. B vs. C vs. D), and Trial Number (Trial 1 vs. Trial 2) as the within-participants fixed effects, and participant as the random effect. This analysis yielded several experimental-effects and two-way interactions, which were qualified by 3 three-way interactions. These included a three-way interaction among Age, Condition, and Object, χ*2*(3) = 7.90, *p* = .05, a three-way interaction among Condition, Trial Number and Object, χ*2*(3) = 13.31, *p* = .006, and a three-way interaction among Condition, Trial Type, and Object, χ*2*(2) = 64.85, *p <* .001.

To explore the interaction among Age, Condition, and Object, we constructed separate two-way linear mixed-effects models between Age and Object for each condition. Age was included as a continuous fixed effect, Condition as a between-participants fixed effect, Object as a within-participants fixed effect, and participants as a random effect. Both linear models only yielded experimental effects of Objects, both χ2’s > 31.88, both *p*-values < .001, which indicated that participated treated the objects differently. Specifically, in the backwards blocking condition, participants considered object A (*M* = .89, *SD* = .31) to be more of a blicket than object B (M = .67, SD = .47), *t*(30) = 4.95, *p* < .001, and C (*M* = .71, *SD* = .46), *t*(30) = 3.89, *p* < .001*.* However, participants treated objects A and D (*M* = .85, *SD* = .36) equivalently, *t*(30) = .76, *p* = .45. In contrast, in the indirect screening-off condition, participants were less confident that object A (*M* = .54, *SD* = .50) was a blicket than object B (*M* = .79, *SD* = .41), and C (*M* = .84, *SD* = .37), both *t*’s > -5.03, both *p*-values < .001. However, participants were more confident that object A was a blicket than object D (*M* = .36, *SD* = .48), *t*(30) = 2.36, *p* = .02. This reflected the fact that the scores for object A were collapsed over Trial Type (in which A was seen by itself during the experimental trials but in combination with other objects during the control trials). Finally, participants were less confident that object D was a blicket than objects B and C, both *t*’s > 6.10, both *p*-values < .001.

To explore the second interaction among Trial Number and Object for each condition, Trial Number and Object were included as within-participants fixed effects and participants were included as a random effect. Although both linear models yielded experimental effects of Object, both χ2’s > 31.86, both *p*-values < .001, only the two-way linear mixed-effects model for the Indirect Screening Off condition yielded an interaction between Trial Number and Object, χ*2*(3) = 9.57. This interaction reflected the fact that participants treated the objects differently between the two trials. During trial 1 (when participants were asked to provide their first set of responses) participants treated object D (M = .55, SD = .50) as less of a blicket than object A (M = .72, SD = .45), *t*(114.97) = 2.31, *p* = .02, and object C (*M* = .78, *SD* = .42) as more of a blicket than either object B (*M* = .68, *SD* = .47), *t*(127) = -2.46, *p* = .02, or D, *t*(107.25) = 3.23, *p* < .01. In contrast, during trial 2, participants treated all the objects equivalently, all *t*’s < 1.78, all *p*-values > .08.

Chart, bar chart

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Figure 2. Participants’ mean responses to whether each object was a blicket across the conditions and trial types. Bars show standard error.

To examine the final interaction among Condition, Trial Type, and Object, we constructed a set of one-way linear mixed-effects models for the experimental and control trials within the backwards blocking and indirect screening-off conditions. The Objects factor was treated as the sole within-participants fixed effect in these follow-up analyses. Participants were once again treated as a random effect to control for the within-participant variance from multiple responses. The one-way linear model for the control trials within the backwards blocking condition did not reveal a significant effect of Objects,χ2(3) = 1.34, *p* = .72. This means that participants treated the objects similarly in the control trials of the backwards blocking condition. In contrast, the second one-way linear model for the experimental trials within the backwards blocking condition revealed a significant experimental effect of Objects, χ2(2) = 55.20, *p* < .001. This experimental effect reflected the fact that participants considered object A to be more of a blicket (*M* = .98, *SD* = 0.13) than object B (*M* = .55, *SD* = 0.50), *t*(30) = 6.45, *p<* .001, or object C (*M* = .61, *SD* = 0.49), *t*(30) = 5.62, *p* < .001. Participants treated objects B and C equivalently, *t*(30) = -1.07, *p* = .29.

The third and fourth one-way linear models for the experimental and control trials within the indirect screening-off condition both revealed a significant experimental effect of Objects, both χ*2*-values > 76.81, both *p*-values < .001. Participants considered object A (*M* = 0.26, *SD* = 0.44) in the ISO experimental trials and object D (*M* = 0.36, *SD* = 0.48) in the ISO control trials to be less likely to be blickets than any of the other objects, all *t*-values > -7.45, all *p*-values < .001. Participants treated object B and C equivalently in the experimental trials, *t*(30) = -0.77, *p* = .29, and objects A-C equivalently in the control trials, all *t*-values < -1.07, all *p*-values > .29.

**Evidence of retrospective reasoning.** To examine whether participants engaged in backwards blocking reasoning—operationalized as higher combined ratings of objects A-C in the control trials than of objects B and C in the experimental trials—data were entered into a two-way linear mixed-effects model with Trial Type and Object as the within-participants fixed effects and participants as the random effect. This analysis revealed only a main effect of Trial Type, *χ2*(1) = 21.97, *p* < .001. This result indicated that participants did engage in backwards blocking reasoning: they provided higher combined ratings of objects A, B, and C in the backwards blocking control trials (*M* = 0.80, *SD* = 0.40) than the combined ratings of objects B and C in the backwards blocking experimental trials (*M* = 0.58, *SD* = 0.49).

For completeness, we ran the same analysis as above, but this time for the indirect screening-off condition. This analysis also only revealed a main effect of Trial Type, *χ2*(1) = 4.42, *p* = .04. The results mirrored the results for the backwards blocking condition. Participants provided higher combined ratings of objects A, B, and C in the indirect screening-off control trials (*M* = 0.84, *SD* = 0.36) than the combined ratings of objects B and C in the backwards blocking experimental trials (*M* = 0.77, *SD* = 0.42). Similar to the results above for the backwards blocking condition, this result indicated that when the object that is shown in isolation was also shown in combination with other objects participants show stronger retrospective reevaluations.

**Discussion**

In the experimental trials of Experiment 1, children were shown three objects that together activated a machine, and then that one of those objects was or was not efficacious on its own. When that object was efficacious, children reevaluated the efficacy of the other two objects: They stated that they were less likely to have efficacy than objects in a control condition in which a fourth, unrelated object was efficacious. When that object was not efficacious, children did not retrospectively reevaluate the efficacy of the other objects and treated their judgment of those objects no differently than in the control condition.

Before discussing the different mechanisms that might describe these data, we want to consider a second, related retrospective inference. In Experiment 1, the second piece of evidence children observed in the Experimental trials involved only one object being placed on the machine. In Experiment 2, we reproduce this procedure presenting children with evidence that the three objects together were efficacious, but then that two of those objects either were or were not together.

**Experiment 2**

Experiment 2 was similar to Experiment 1 except for the number of objects that were placed on the machine during the second part of the experimental trials. In the experimental trials here, children were shown that three objects activated the machine together, and then two of those three objects either did so or did not. These data were compared with a control condition in which three different objects activated the machine, and then two additional novel objects either did so or did not in tandem.

**Method**

**Participants.** Participants were 32 5-year-olds (18 boys and 14 girls; *M* = 65.31 months, range = 60-75 months, SD = 3.65) and 32 6-year-olds (10 boys and 22 girls; *M* = 76.56 months, range = 65-83 months, SD = 4.33). Participants were recruited in the same manner as Experiment 1. Participants were 12% Asian/Asian American, 9% Black/African American, 10% Hispanic, and 69% White/Caucasian.

**Materials & Procedure.** The materials and procedure for Experiment 2 was identical to that for Experiment 1 with the following exceptions: During the backwards blocking experimental events following an event in which objects A, B, and C together activated the machine, two objects A and B descended onto and subsequently caused the machine to activate (i.e., turn blue). Likewise, during the backwards blocking control events, two objects D and E descended onto and subsequently caused the machine to activate. D and E did not descend onto the machine during the initial event in which A, B, and C activated the machine and in this way were “unrelated” to objects A, B, and C. The indirect screening-off experimental and control trials were identical to the backwards blocking trials except that the machine neither activated when objects A and B descended onto the machine during the indirect screening-off experimental trials nor when objects D and E descended onto the machine during the indirect screening-off control trials. The left- and right-most positions of objects A and B during the experimental trials and objects D and E during the control trials were counterbalanced.

**Results**

Figure 3 shows the number of times children responded “yes” to the question “Is this a blicket” for each object. The data for this experiment were entered into a five-way linear mixed-effects model with Age as a continuous fixed effect, Condition (Backwards blocking vs. Indirect screening-off) as the between-participants fixed effect, Trial Type (Experimental vs. Control), Objects (A vs. B vs. C vs. D vs. E), and Trial Number (Trial 1 vs. Trial 2) as the within-participants fixed effects, and participant as the random effect. This analysis yielded several experimental-effects and two-way interactions, which were qualified by a single three-way interaction between Condition, Trial Type, and Object, χ*2*(2) = 185.38, *p <* .001.

A graph showing the different types of screening

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Figure 3. Participants’ mean responses to whether each object was a blicket across the conditions and trial types. Bars show standard error.

To examine the three-way interaction between Condition, Trial Type, and Object, we constructed a set of one-way linear mixed-effects models for the experimental and control trials within the backwards blocking and indirect screening-off conditions separately. Object was treated as the single within-participants fixed effect in these follow-up analyses, and participants were again treated as a random effect. The one-way linear model for the control trials within the backwards blocking condition did not reveal a significant effect of Objects,χ2(3) = 4.55, *p* = .34. Thus, as in Experiment 1, participants treated the objects similarly in the control trials of the backwards blocking condition. Also consistent with Experiment 1, the second one-way linear model for the experimental trials within the backwards blocking condition revealed a significant effect of Objects, χ2(2) = 14.26, *p* < .001. This result reflected the fact that participants considered object A to be more of a blicket (*M* = .84, *SD* = 0.37) than object C (*M* = .63, *SD* = 0.49), *t*(31) = 3.38, *p<* .01. Participants treated the remaining objects equivalently.

As with Experiment 1, the third and fourth one-way linear models for the experimental and control trials within the indirect screening-off condition both revealed a significant experimental effect of Objects, both χ*2*-values > 1100.90, both *p*-values < .001. During the indirect screening-off experimental trials, participants considered objects A (*M* = 0.08, *SD* = 0.27) and B (*M* = 0.05, *SD* = 0.21) to be less likely to be blickets than object C (*M* = 0.98, *SD* = 0.13), both *t*-values > -21.10, both *p*-values < .001. Participants treated objects A and B equivalently, *t*(31) = 1.43, *p* = .16. During the indirect screening-off control trials, participants considered objects D (*M* = 0, *SD* = 0) and E (*M* = 0, *SD* = 0) to be less likely to be blickets than object A (*M* = 0.98, *SD* = 0.13), object B (*M* = 0.95, *SD* = 0.21), and object C (*M* = 0.97, *SD* = 0.18), all *t*-values > 35.79, all *p*-values < .001. Participants treated objects A-C equivalently.

**Evidence of retrospective reasoning.** We next examined whether participants engaged in retrospective reasoning using the operationalization of it from Experiment 1. Data were entered into a two-way linear mixed-effects model with Trial Type and Object as the within-participants fixed effects and participants as the random effect. This analysis revealed only a main effect of Trial Type, *χ2*(1) = 3.94, *p* = .05: Participants provided higher combined ratings of objects A, B, and C in the backwards blocking control trials (*M* = 0.79, *SD* = 0.41) than the combined ratings of object C in the backwards blocking experimental trials (*M* = 0.63, *SD* = 0.49). For the indirect screening-off condition, data were entered into a two-way linear mixed-effects model with Trial Type and Object as the within-participants fixed effects and participants as the random effect. This analysis revealed neither a main effect of Objects, *χ2*(2) = 1.40, *p* = .49, nor a main effect of Trial Type, *χ2*(1) = 0.35, *p* = .55.

**Discussion**

Similar to Experiment 1, Experiment 2 found that 5- and 6-year-olds engaged in retrospective reasoning about ambiguous data. In the experimental trials, children were shown three objects that together activated a machine and then that two of those objects was or was not efficacious on their own. When those objects were causally effective, children were less likely to state that the other two objects had efficacy than in a control condition in which a fourth, unrelated object was efficacious. However, when the pair of objects was not efficacious, children were not more likely to make this claim. Across these two experiments, children’s qualitative inferences were consistent with a Bayesian description, in that when objects were presented in compound, children did not appear to simply count the number of times any one individual object activated the machine. However, in neither experiment did children show clear quantitative inferences that suggests they understood and resolved the uncertainty they observed. In the next section, we present fits from two computational models that suggest other descriptions of causal inference might be a better quantitative fit of these data taken together.

**Computational Models**

We fit two different computational models to the behavioral data. The first was a model based on Bayesian inference. This model was described initially by Sobel et al. (2004) and in more detail in Griffiths et al. (2011). The second was a simple connectionist model, trained with the Delta Rule (Widrow & Hoff, 1960).

**Bayesian Model.** The Bayesian model starts with a set of hypotheses *H*. Each hypothesis *h* ∈ *H* is assigned a *prior probability*, *p*(*h*), which indicates the initial belief in that a learner has in a particular hypothesis prior to seeing data. After the learner observes data, *d*, the learner computes a posterior probability, *p*(*h* | *d*), given an updated belief about each hypothesis given the data. This is done using Bayes’ rule, shown in Equation 1:

(1)

In this formula, *p*(*d | h*) is the probability of the data *d* given each a particular hypothesis *h* (also known as the *likelihood*).

Forming the initial hypothesis space relies on assuming that there is a set of objects *O* and a set of detectors *D*, such that any object *o* ∈ *O* can potentially cause any detector *d* ∈ *D* to activate. Given that participants are shown that the machine activates when blicket objects are placed on its surface, a hypothesis *h* corresponds to a structure that posits whether individual objects have the causal efficacy to activate the detector (see Griffiths & Tenenbaum, 2005, for more computational details). Griffiths et al. (2011) describe the formal parameterization of this hypothesis space and model that results in the hypothesis space shown in Figure 3.

To instantiate the model, each hypothesis is given a prior probability *p*(*h*), which is a function of the child’s belief about the base rate of blickets *ρ*. This prior corresponds to the number of blickets posited by the hypothesis. For example, in the figure, Hypothesis 0 posits 3 blickets, so its *p*(*h*) = *ρ*3. Hypotheses 1, 2, and 4 posit exactly 2 blickets, so their *p*(*h*) = *ρ*2(1−*ρ*). Hypotheses 3, 5, and 6 each posit 1, making their *p*(*h*) = *ρ*(1−*ρ*)2. Finally, Hypothesis 7 posits no blickets, making its *p*(*h*) = (1-*ρ*)3.

**Timeline

Description automatically generated with medium confidence**

Figure 3. The eight different causal hypotheses indicating the possible causal relations for a causal event that involves three objects and one blicket detector. *A*, *B*, and *C* correspond to the three objects that were used on the machine and *E* indicates the activation of the machine.

For the purposes of this demonstration, we will assume that the model itself assumes that objects with causal efficacy will act deterministically on detectors.[[1]](#footnote-1) As a result, the likelihood of each hypothesis is equal to 1 if that hypothesis could produce the data and 0 if not. This allows each model to be updated based on Bayes’ rule given the data. The way the model determines the probability that an object is a blicket is based on the posterior probability of the models in the hypothesis space; that is, the probability that any object *o* is a blicket given the data *d* can be calculated by the equation in (2)

where *p*(*o*→*E* | *h*) is 1 if there is an edge between that object and the detector in h, and 0 otherwise.

Crucially, because the predictions of this (or any) Bayesian model will depend on the prior probability that any given object is a blicket, we fit a Bayesian model with the following prior probabilities: .5, .65, .8, .95, and 1. We considered a range of prior probabilities because it was unclear what participants’ baseline assumptions were about the prior probability of blickets in the absence of explicit manipulations to those probabilities. Thus, by deriving the model’s predictions for various prior probabilities, it was possible to compare the model’s predictions for the different probabilities to children’s actual treatment of the objects. The quantitative fit of this model to the data in Experiments 1 and 2 are shown below in Table 1 in Results.

**Connectionist model**. We also built a set of two-layer connectionist models. One set of these models corresponded to Experiment 1 and the other set corresponded to Experiment 2. The model architecture for the Experiment 1 simulations is shown in Figure 4. The rationale for building only a two-layer model was to explore whether a simple learning model trained with the Delta Rule (Kruschke, 1992; Widrow & Hoff, 1960)—which is formally equivalent to the traditional Rescorla-Wagner model (Danks, 2003; Gluck & Bower, 1988)—could be used to explain these data. Similar to children, we trained 16 models (i.e., ‘participants’) per condition for both experiments (i.e., 32 total model runs for Experiment 1 and 32 total model runs for Experiment 2), and like the children, each model received two trials. Each new participant began with a fresh set of small random weights (sampled uniformly between ±0.1). Finally, data were aggregated over the responses of each model, as was the case for the children.

The input layer for the model consisted of four units for Experiment 1 (corresponding to the four objects) and five units for Experiment 2 (corresponding to the five objects), and the output layer consisted of a single unit for the simulation of both experiments (corresponding to the activation of the machine). When object was placed on the machine, the activation value of its corresponding input unit was set to a value of 1 (and 0 otherwise). The input units could not take on any other values beside 0 or 1. If an object that was a blicket was placed on the machine, then the model was trained to turn on the single output unit (i.e., to produce an activation of 1).

All simulations used a learning rate of .05 but no momentum. Model weights were initialized to small random values (distribution range = ± 0.1), and the output units used sum-squared activation functions (which enabled the weights to be modified with training). The activation of the single output unit was interpreted as the model’s confidence (or prediction) that a given object was a blicket and could range between 0 and 1 due to the sigmoid activation function (unlike the input units, whose input values were “hard clamped” or fixed).

A diagram of a machine

Description automatically generated  
Figure 4. The connectionist model used to simulate Experiment 1.

Turning on the first three input units simulated placing objects A, B, and C on the machine, and training the model to turn on the single output unit corresponded to teaching the model that the machine activated when objects A-C were placed on it. During the subsequent A+ trials in Experiment 1 or the AB+ trials in Experiment 2, only the first input unit (for the simulation of Experiment 1) or the first and second input units (for the simulation of Experiment 2) were turned on, but again the model’s task was to activate the single output unit. The backwards blocking control trials were identical to the experimental trials except that the fourth input unit (corresponding to object D in Experiment 1) or the fourth and fifth input units (corresponding to objects D and E in Experiment 2) were turned on following the ABC+ trial. The indirect screening off experimental and control trials were identical to the backwards blocking experimental and control trials except that the model was trained to turn off the single output unit (i.e., to produce an output activation of 0) during the A- and D- phases of the indirect screening-off experimental and control trials. Each phase of the simulations—which were shown twice to be consistent with the behavioral study—lasted anywhere between 200 and 1,000 epochs. This meant that one complete simulation lasted anywhere between 800 (i.e., 200 × 4) and 4,000 (i.e., 1,000 × 4) epochs. Networks were trained for different numbers of epochs to ensure that the model-fit results were not idiosyncratic to the precise number of training epochs. Below we show the quantitative fit of this model to the data in Experiments 1 and 2.

**Results**

To assess the quantitative fit of the predictions of the connectionist and Bayesian models to the data, we computed the root mean square (RMSE) and mean absolute error (MAE) between each model’s predictions (for the connectionist model these were the average activation of the single output unit in response to each object; for the Bayesian model these were point estimates) and participants’ mean responses to the objects across Experiments 1 and 2. One or both metrics have been used in previous simulation studies to assess a model’s quantitative fit to behavioral data (e.g., Bhat et al., 2022; Buss & Spencer, 2014; Spencer et al., 2022; Steyvers et al., 2003; Stojnic et al., 2023). Lower values on each metric indicate better model fit. Table 1 below shows the model fits for the different connectionist and Bayesian model instantiations across both experiments and for different subsets of the data (e.g. model fit to the data overall, to the backwards blocking data only, etc.).

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| (A) Model fit to the human data overall | | | | | | | |
| Experiment 1 | | | | Experiment 2 | | | |
| Connectionist‡ | | Bayesian Model | | Connectionist‡ | | Bayesian Model | |
| RMSE | MAE | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| .15 | .11 | .17 | .17 | .13 | .11 | .16 | .13 |
|  |  |  |  |  |  |  |  |
| (B) Model fit to the backwards blocking data only | | | | | | | |
| Experiment 1 | | | | Experiment 2 | | | |
| Connectionist‡ | | Bayesian Model | | Connectionist‡ | | Bayesian Model | |
| RMSE | MAE | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| .19 | .16 | .20 | .18 | .13 | .11 | .15 | .14 |
|  |  |  |  |  |  |  |  |
| (C) Model fit to the indirect screening-off data only | | | | | | | |
| Experiment 1 | | | | Experiment 2 | | | |
| Connectionist‡ | | Bayesian Model | | Connectionist | | Bayesian Model‡ | |
| RMSE | MAE | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| .08 | .07 | .18 | .16 | .11 | .11 | .12 | .03 |
|  |  |  |  |  |  |  |  |
| (D) Model fit to the experimental trials only | | | | | | | |
| Experiment 1 | | | | Experiment 2 | | | |
| Connectionist | | Bayesian Model | | Connectionist | | Bayesian Model‡ | |
| RMSE | MAE | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| .19 | .16 | .19 | .16 | .16 | .14 | .14 | .12 |
|  |  |  |  |  |  |  |  |
| (E) Model fit to the control trials only | | | | | | | |
| Experiment 1 | | | | Experiment 2 | | | |
| Connectionist‡ | | Bayesian Model | | Connectionist‡ | | Bayesian Model | |
| RMSE | MAE | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| .10 | .08 | .20 | .17 | .11 | .09 | .17 | .17 |

Table 1. Model fit indices for the various models and instantiations for the data overall and the data for the backwards blocking, indirect screening-off, experimental, and control trials in Experiments 1 and 2 data. ‡ Corresponds to the best fitting overall model based on average RMSE and MAE.

The main finding from Table 1 is that, although the connectionist model generally performed better than the Bayesian model (achieving higher performance in 7 of 10 total situations), the Bayesian model either outperformed the connectionist model in 2 situations or exhibited comparable performance in 1 situation. These model findings suggest that participants may simultaneously be relying on associative processing and Bayesian inference, even when there is a greater tendency to rely on associative learning to reason about multiple potential causes. Stated somewhat differently, these data neither clearly support the conclusion that children rely exclusively on Bayesian inference to reason causally nor do they permit the conclusion that children rely exclusively on associative learning to reason about causes. Instead, these data support the conclusion that children weigh these two cognitive mechanisms differently depending on the number of potential causes about which they are asked to reason. Bayesian inference may be given more weight than associative learning when there are a small number of potential causes (such as in Sobel et al., 2004), but as the number of causes and the information processing demands of the task increase participants give more weight to associative learning (such as in the current study).

**General Discussion**

The purpose of this study was to examine whether and how children engage in retrospective reasoning under more strenuous information processing demands, in which children must track the efficacy of more than two objects. In both experiments, when shown first that three objects activated a machine, and then that a subset of those objects did so on their own, the objects not in that subset were judged as less likely to do so than analogous objects in a control condition. When the subset of those objects did not activate the machine on their own, judgments of the efficacy of the other objects were not different from the control condition.

We subsequently fit a Bayesian model and a connectionist model to the data in both experiments. The Bayesian model did make some qualitative predictions about retrospective reevaluation that were seen in children’s data. However, overall, the connectionist model tended to provide better fits across the trials. In contrast to findings where children only have to reason about two objects, increasing the demand characteristics of the experiment might have made children default to a more associative strategy.

To illustrate why this might be the case, consider the underlying inferential process the two models use to account for the control trials in the blocking conditions. For example, in Experiment 1 where children see three objects activate the machine together and then a fourth do so independently, the associative model uses a relatively simple “counting” strategy. During the simulation of this trial, when all four objects were first presented to the model, the resulting difference at the output layer between the activation of the single output unit and the predicted activation of that unit was equivalent for all four objects. Thus, because the difference between the observed and predicted activation of the output unit was equivalent for all four objects, the model made equivalent weight adjustments in sign and magnitude to the connections between each object and the output unit. Crucially, these connections instantiated each object’s association with the machine’s activation. As such, because objects A-D were shown with the “machine’s activation” (i.e., the output unit = 1) an equal number of times, the strength of the association between each object and the machine’s activation was equivalent.

In contrast, the Bayesian model predicts a clear difference between the efficacy of the first three objects and the fourth. By virtue of the fourth object independently activating the machine, it has unambiguous efficacy. In contrast, one of the other three objects have efficacy, so the probability that each is efficacious is greater than the base rate, but not at ceiling. Whereas the Bayesian model makes qualitative predictions about retrospective reevaluation in the experimental trials that were mostly upheld (except in the indirect screening off trials in Experiment 1), this difference between ceiling and non-ceiling level responses was not present in the data.

What, however, is the nature of this associative processing? More broadly, these experiments suggest that the more objects children have to keep in mind, the more their inferences might indicate multiple reasoning processes. Despite a general tendency for learners to process information at the most sophisticated level possible, when tasks exceed children’s information-processing abilities, they resort to less sophisticated strategies and cognitive mechanisms such as associative learning (e.g., Cohen et al., 2002).

Some potential criticisms are worth noting. First, in the present study, children’s reasoning overall was more consistent with an associative model than one that is described by Bayesian inference. Yet that does not mean that Bayesian models could not explain the data under some circumstances. For instance, in cases where the causal efficacy is shown to be rare, children might be cued not to use a counting strategy, even when faced with multiple potential causes. That is, their inferences about unambiguous data (i.e., individual objects that specifically do or do not activate the machine) should be unchanged, but other inferences about ambiguous data might be different. Although we can think of modifications to our associative model, which could theoretically consider such base rate data, the simple connectionist model that we used to simulate the data here would be less explanatory than the Bayesian model we present.

A second criticism concerns the artificial nature of the paradigm used here, which was necessitated by the COVID-19 pandemic. Testing remotely on a computer screen may have introduced a level of noise in the data that is fundamentally different than testing in person with real objects. Future studies should replicate our study. If such a study revealed that participants performed more normatively than associatively in person, this would suggest that children’s normative inferences may not be as robust as originally thought—it is present when tested in person but nearly absent when tested on a computer. Such a finding would be interesting regardless because it would add nuance to the literature on children’s causal inferences.

Third, the logic behind our model fitting is based on aggregating a group of children’s yes/no responses and fitting those averages to a model’s stochastic prediction. Previous studies on children’s causal inferences used such an approach. Studies with adults, however, asked them to make more graded inferences (e.g., rate on a scale of 1-10 how likely a particular object caused the machine to activate). Given that we investigated a slightly older sample than some other studies of retrospective reasoning in children, such a graded response measure could be used in a reproduction of these studies. This could further help distinguish between the qualitative predictions of each model and the quantitative model fits.

**Conclusion**

This study constitutes one of the first systematic attempts to examine retrospective reasoning in human children in the context of multiple candidate causes. A longstanding view has been that the cognitive mechanism by which people reason about causal events is Bayesian inference rather than associative processes. The experiments reported here support a different conclusion: children might be relying on both associative learning *and* Bayesian inference to reason about causal events.

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1. The Griffiths et al. (2011) model assumes that this can be learned through a hierarchical process; we are presenting a simpler model for the purposes of this investigation. [↑](#footnote-ref-1)