

Introduction to Parallel Distributed Processing models

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002-Constraint Satisfaction

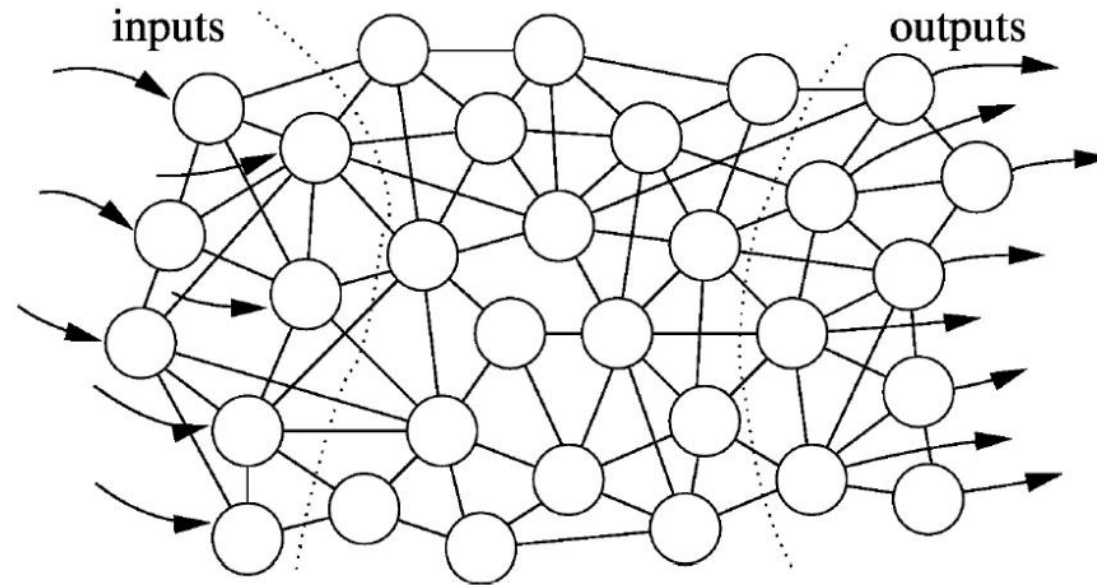
Constraint satisfaction

Constraint satisfaction

- PDP networks can be thought of as a kind of constraint satisfaction network

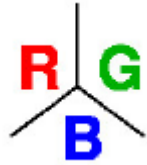
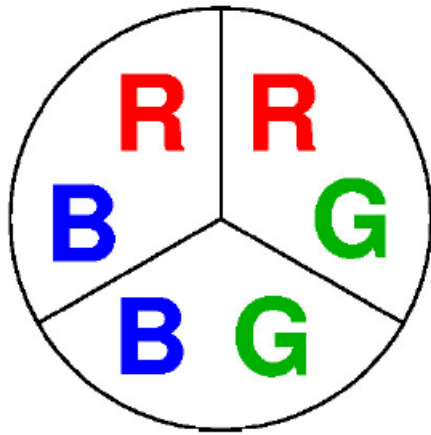
GOAL: Satisfy as many constraints as possible

Constraint satisfaction

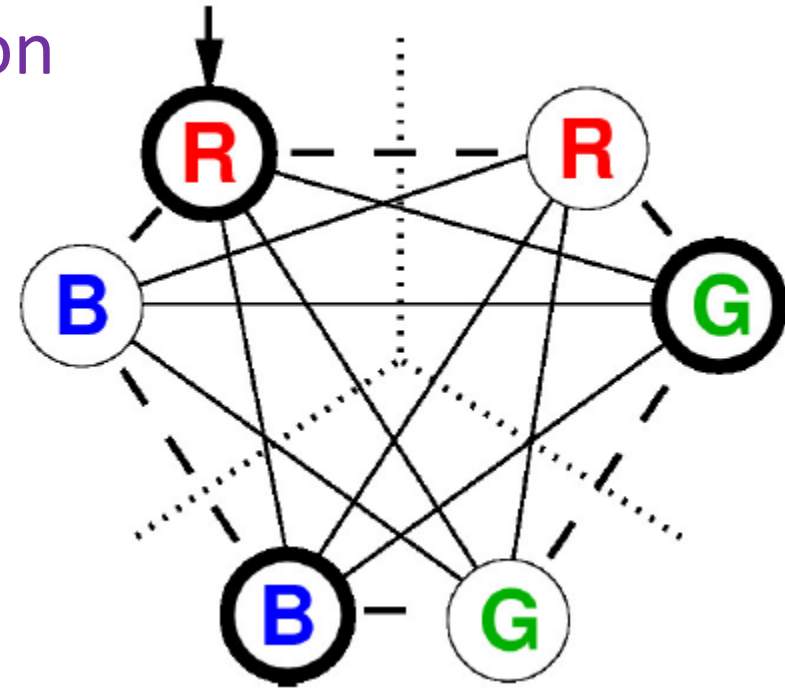


- Units represent **hypotheses** about parts of a problem
- Weights code **constraints** on how hypotheses can combine (i.e., the degree to which they are consistent or inconsistent)
- Possible **solutions** correspond to particular patterns of active units
- External input introduces **bias** to favor one possible solution over others

Color mapping

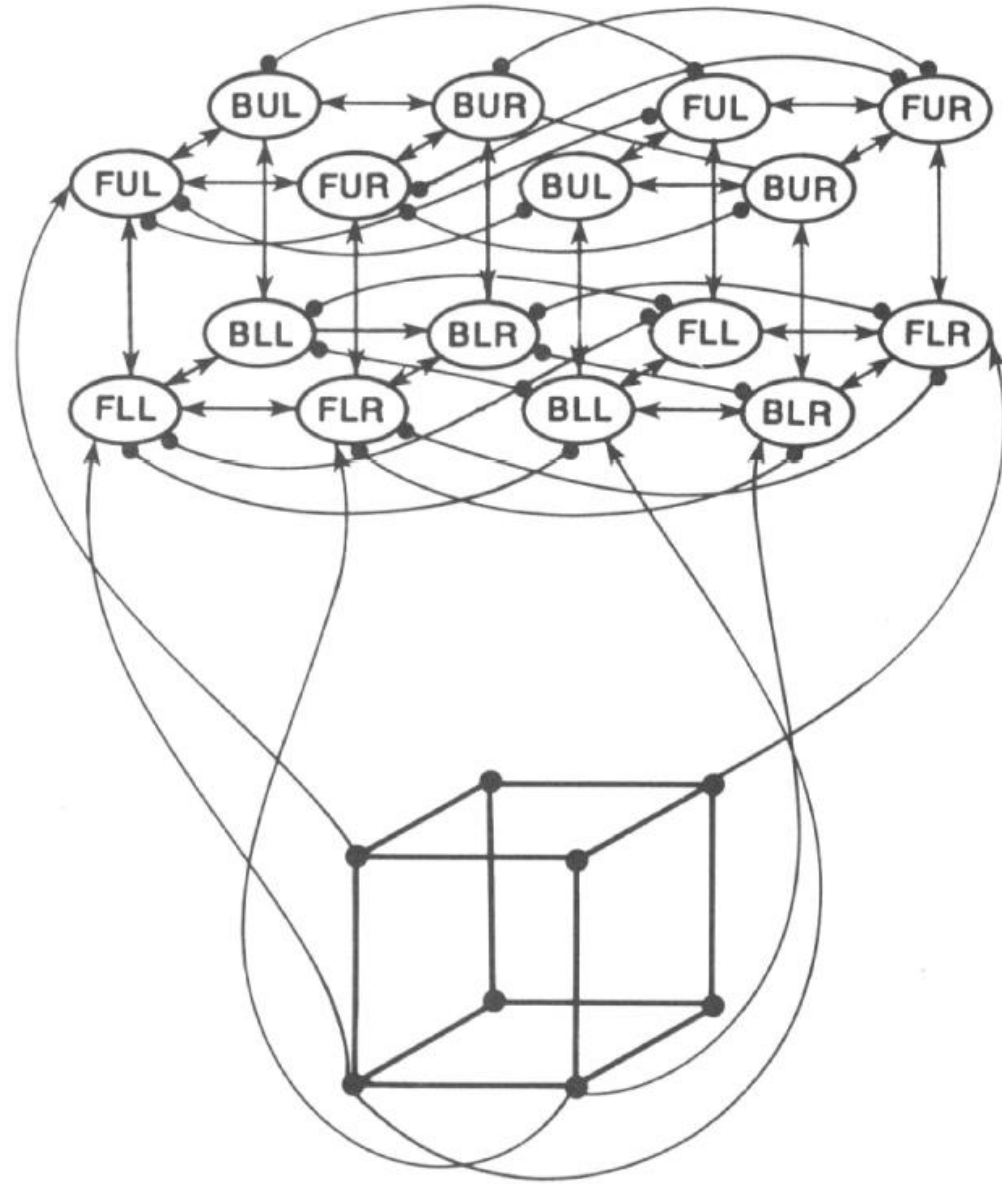


Relaxation
!



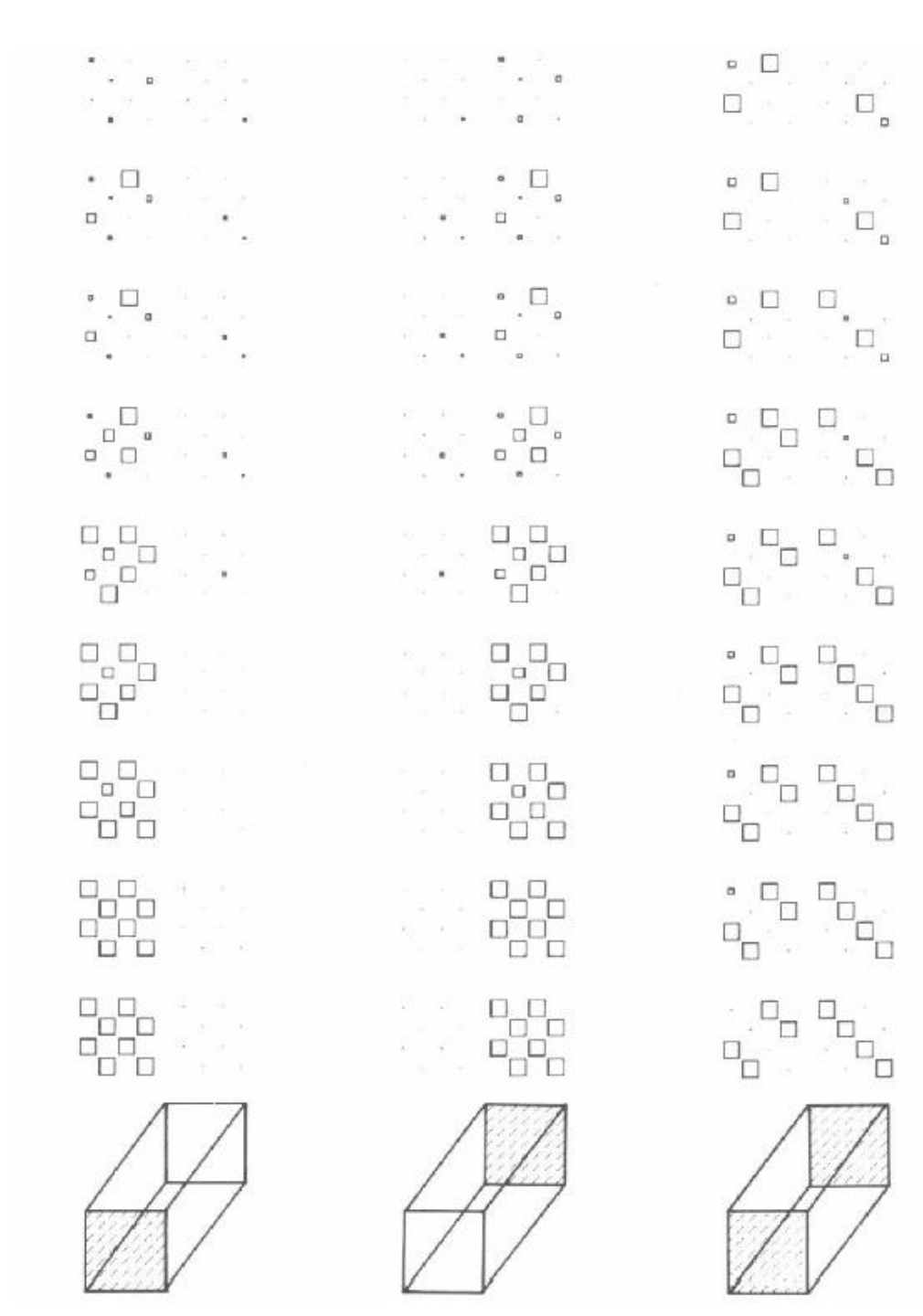
Goal: Assign colors to regions so that no adjacent regions have the same color

Necker cube



Necker cube “relaxation”

2^{16} possible states, BUT...



Maximizing Goodness (= minimizing Energy)

Global measure of degree to which activations satisfy weight constraints

$$G \text{ (Goodness = -Energy)} = \sum_{i < j} a_i a_j w_{ij}$$

How should unit k behave locally so as to increase global Goodness?

- Set $a_k = 1$ if $G_{a_k=1} > G_{a_k=0}$ (or, equivalently, $G_{a_k=1} - G_{a_k=0} > 0$)

$$G_{a_k=1} = \sum_i a_i w_{ik} + \sum_{i < j \neq k} a_i a_j w_{ij}$$

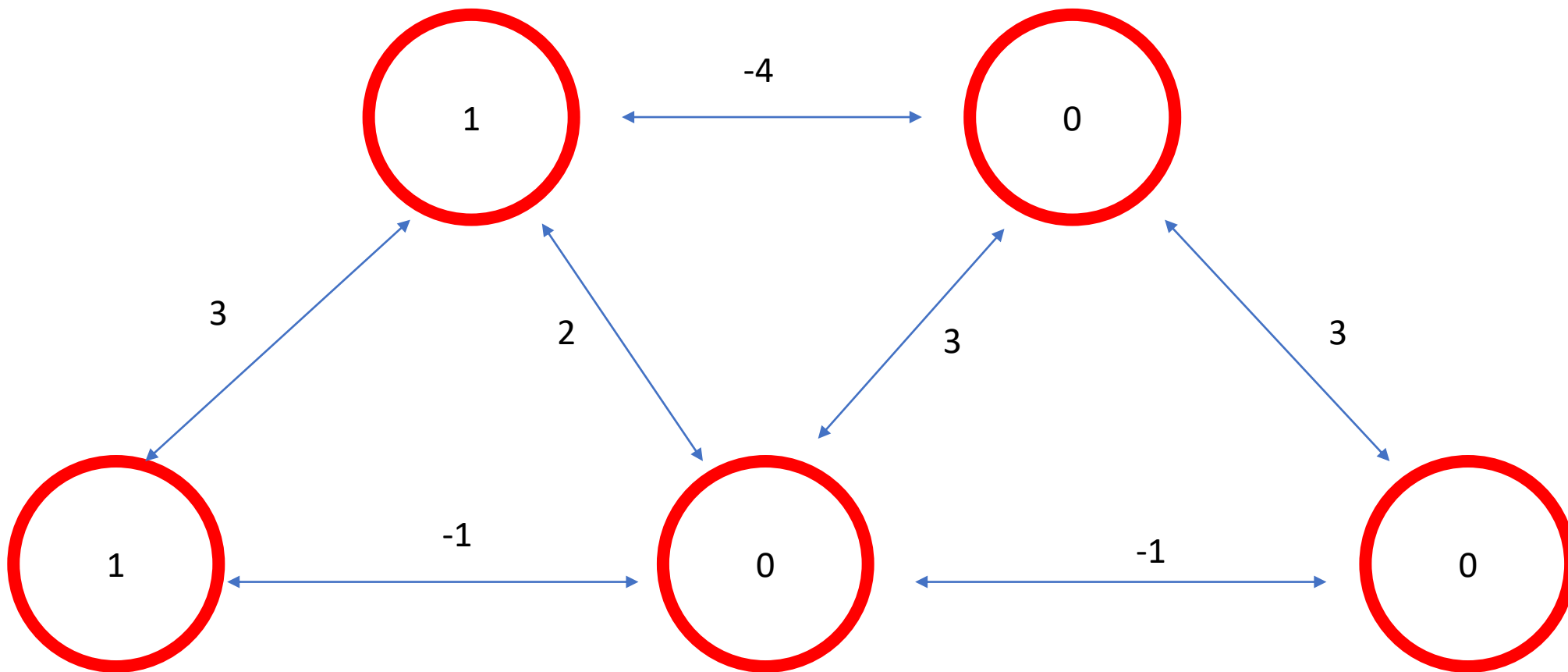
$$G_{a_k=0} = \sum_{i < j \neq k} a_i a_j w_{ij}$$

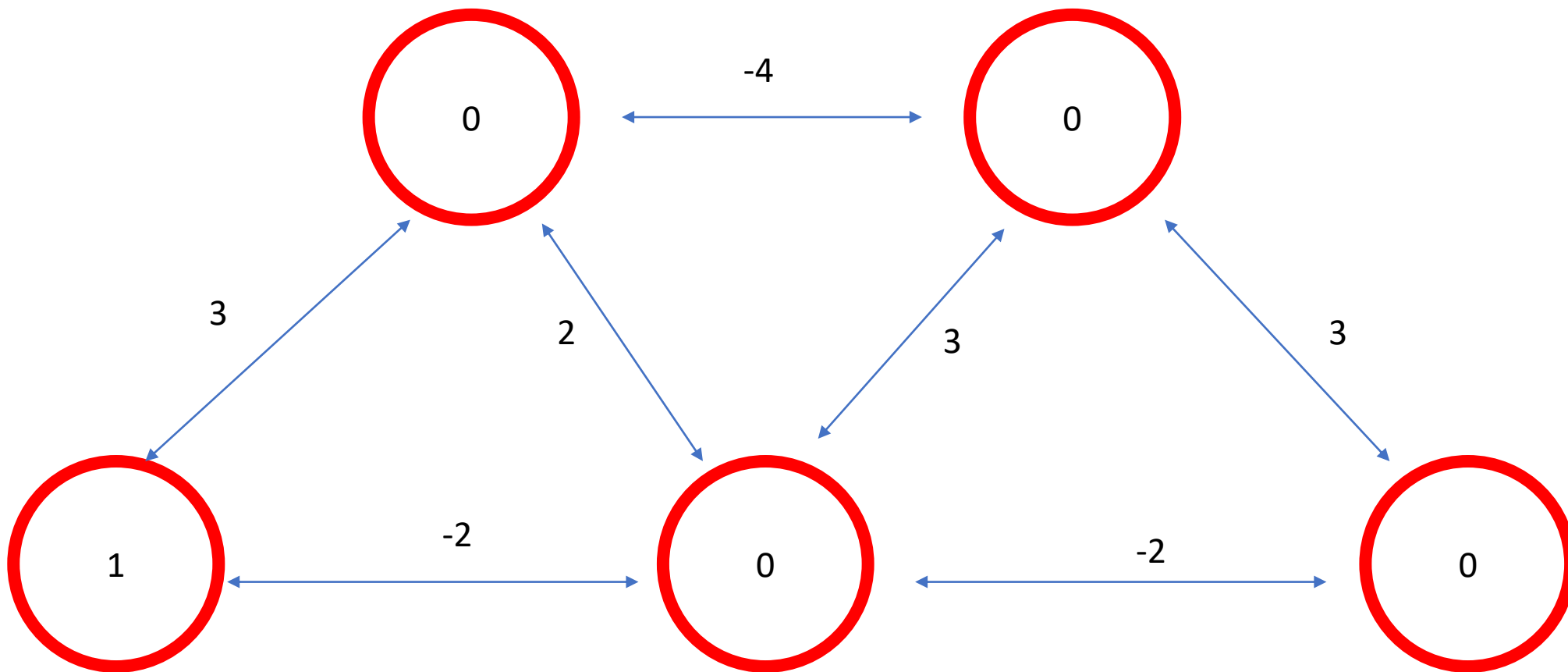
$$G_{a_k=1} - G_{a_k=0} = \sum_i a_i w_{ik}$$

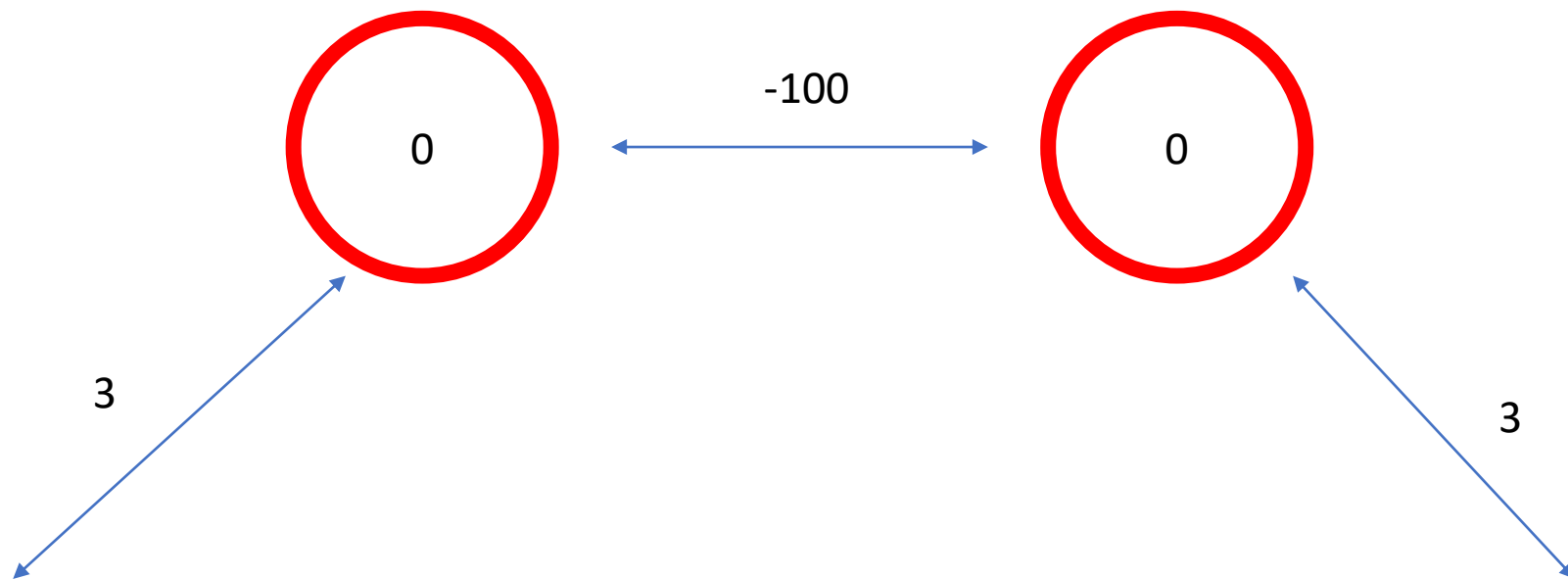
We're also guaranteed a locally optimal solution (at minimum) if we go down hill in Energy

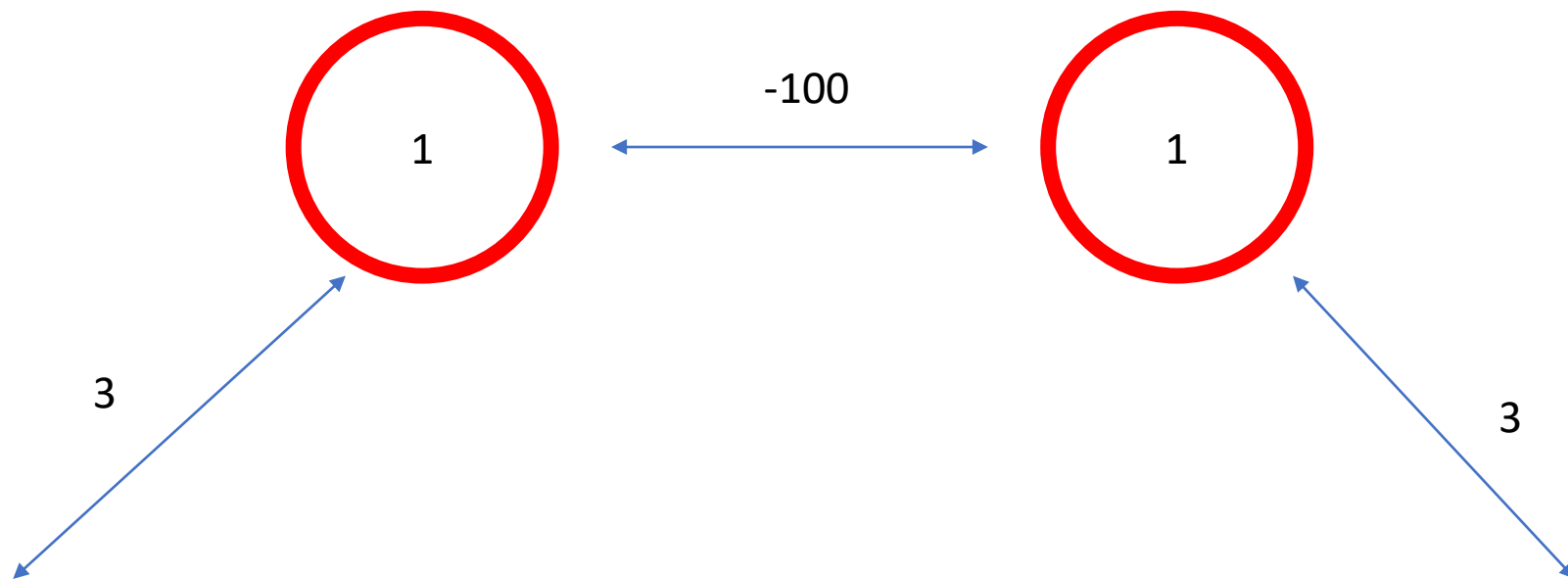
$$E = -\sum_i a_i b_i - \sum_{i < j} a_i a_j w_{ij}$$

$$\frac{\partial E}{\partial a_i} = b_i + \sum_j a_j w_{ij}$$









Maximizing Goodness (= minimizing Energy)

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How should unit k behave locally so as to increase global Goodness?

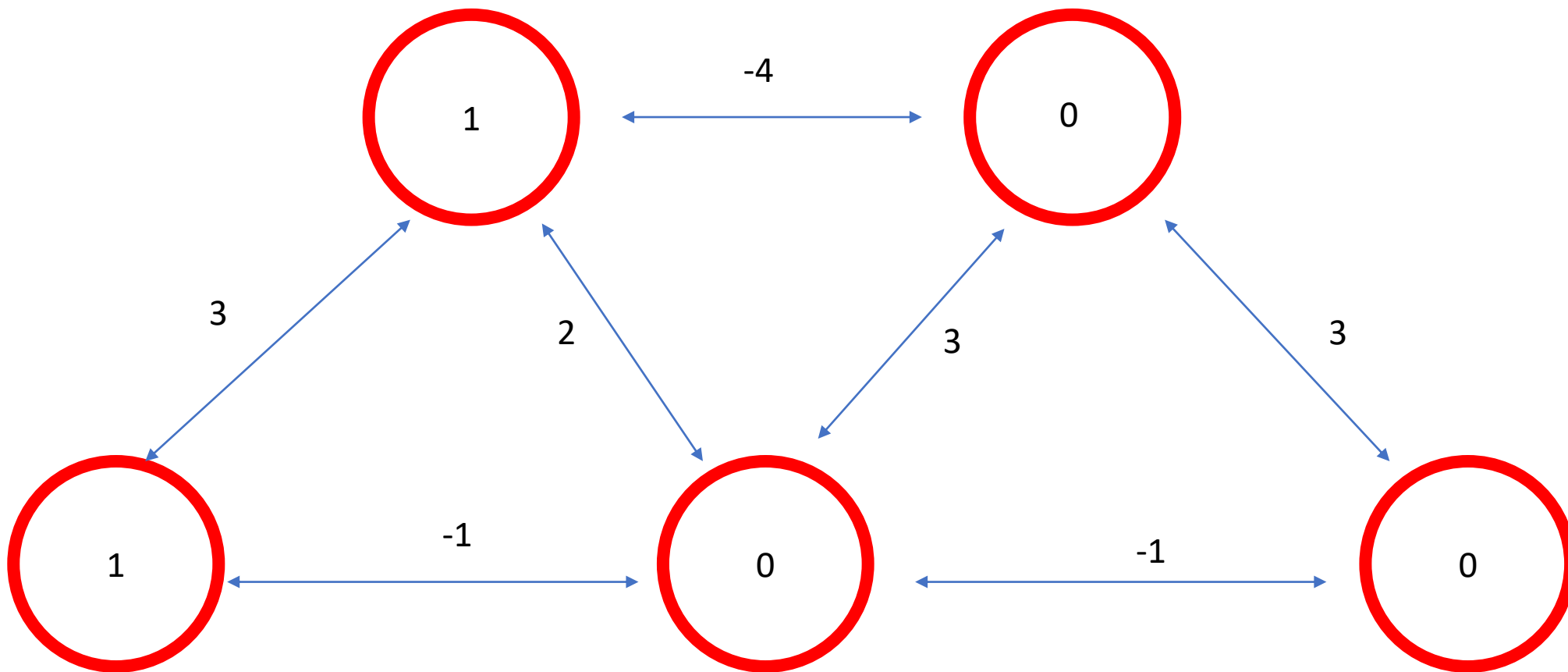
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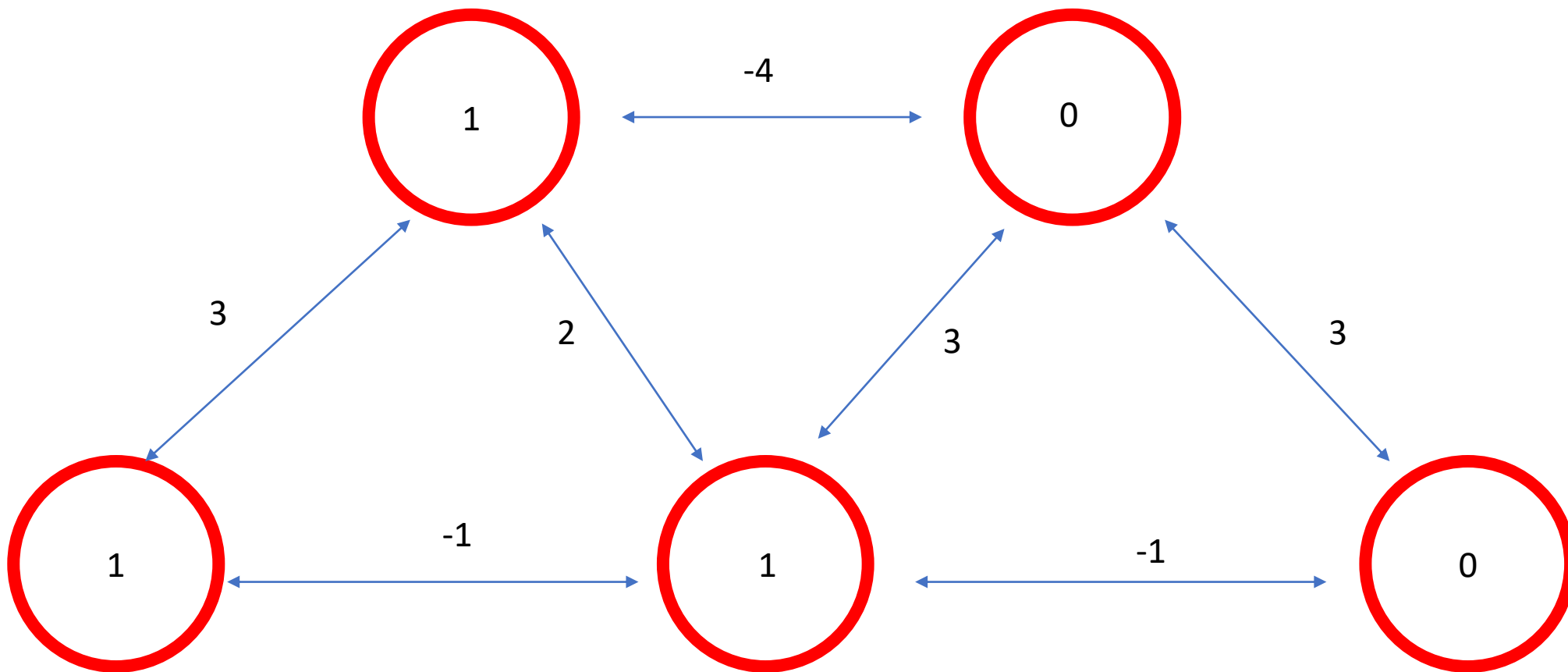
$$G_{a_k=1} = \sum_i a_i w_{ik} + \sum_{i < j \neq k} a_i a_j w_{ij}$$

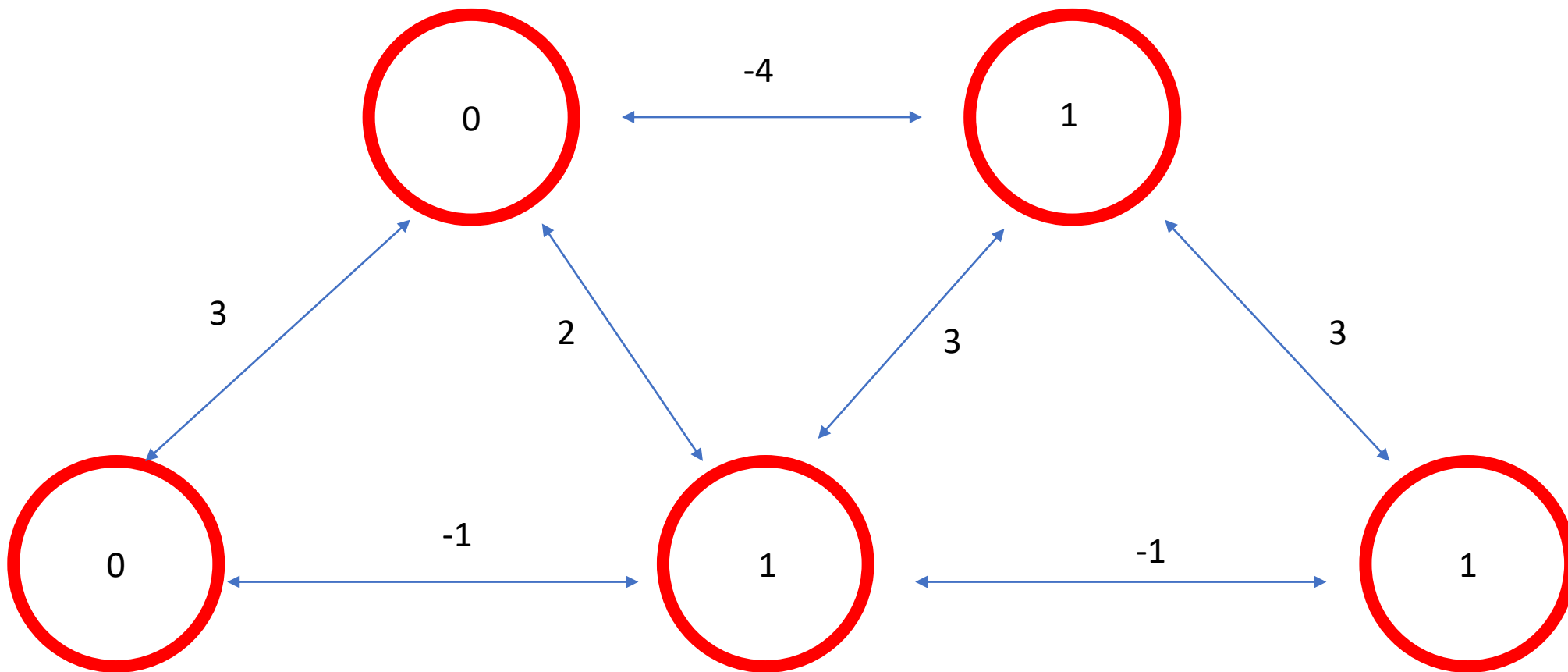
$$G_{a_k=0} = \sum_{i < j \neq k} a_i a_j w_{ij}$$

$$G_{a_k=1} - G_{a_k=0} = \sum_i a_i w_{ik}$$

- Set $a_k = 1$ if $\sum_i a_i w_{ik} > 0$ (= **binary threshold unit**)







Goodness surface in state space

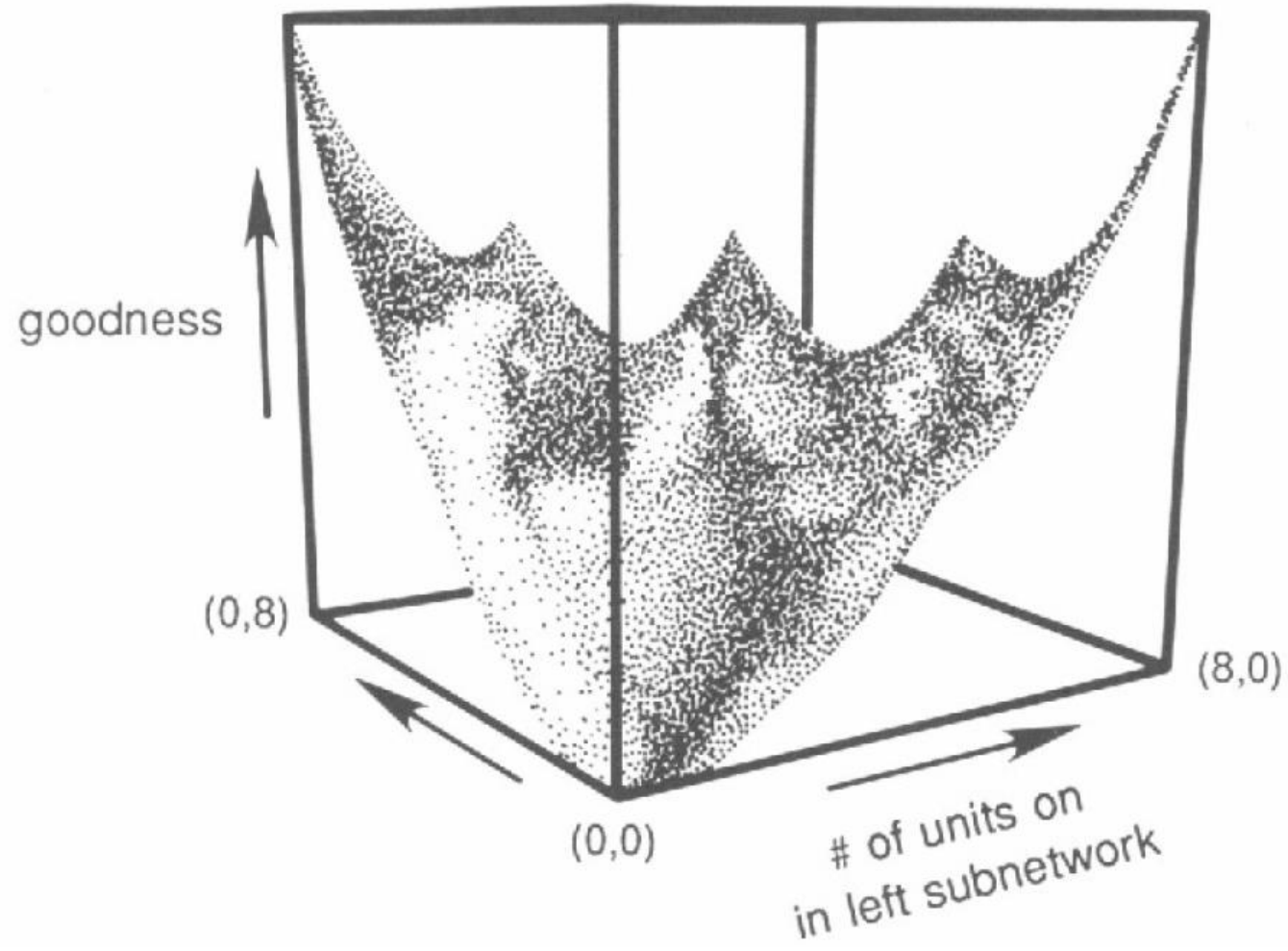
State space

- A high-dimensional space with a dimension for each of n units in the network
- Each unit's activity (state) can be interpreted as a coordinate along its corresponding axis/dimension
- At any instant in time, the current pattern of activity over the entire network corresponds to a particular n -dimensional point in the space
- As units update their states, the point moves in state space

Goodness surface

- Each instantaneous pattern of activity has a corresponding “goodness” value
- Add an additional ($n+1$ st) dimension to state space so that the goodness of each point (activity pattern) can be plotted “above” it
- The set of goodness values corresponding to all possible activity patterns forms a continuous surface “above” state space

Goodness surface (Necker cube)

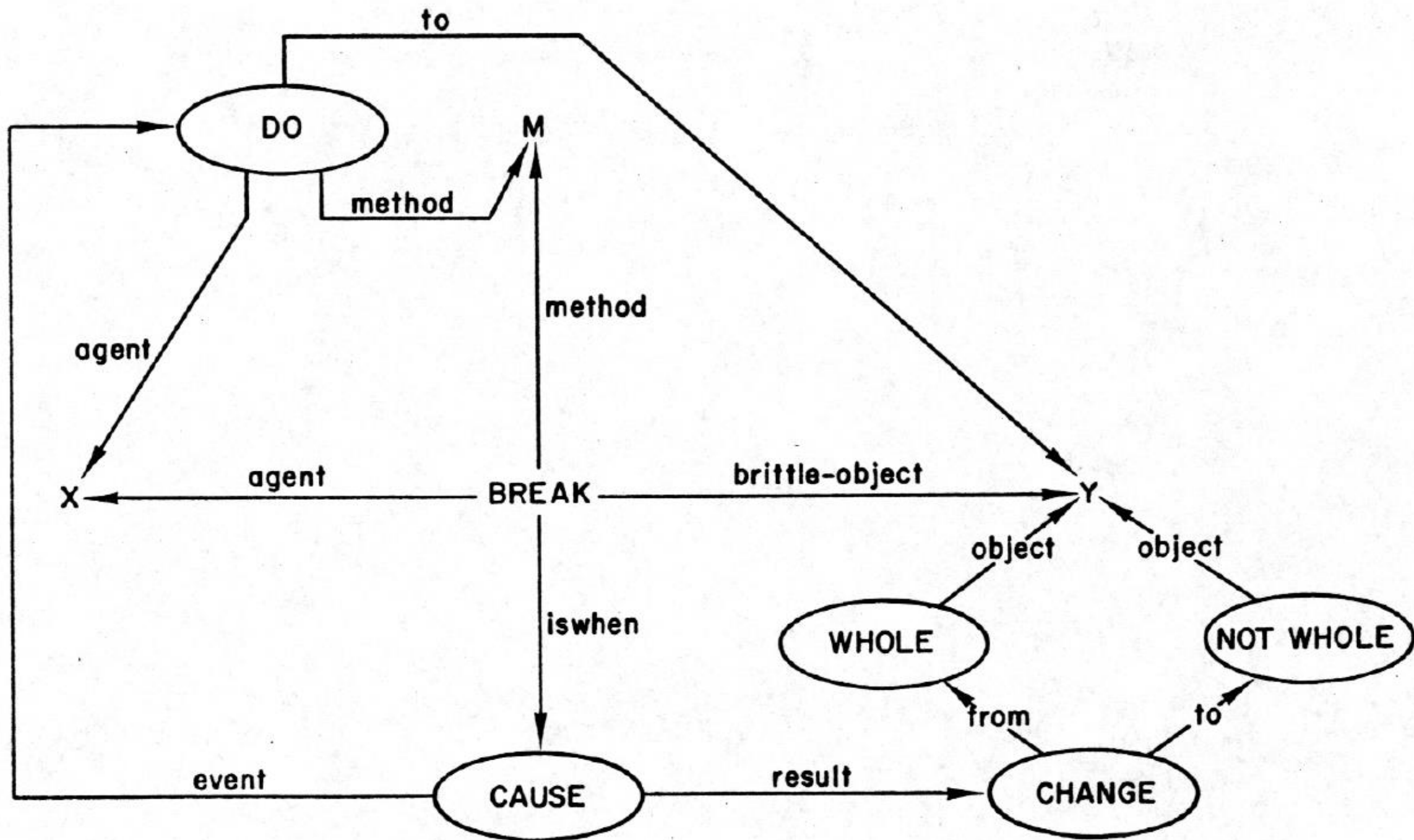


Schema and schemata

So, what is a schema?

- Minsky (1975) – the frame
- Schank and Abelson (1977) – the script
- Bobrow and Norman (1975) and Rumelhart (1975) – a more explicit notion of the schema

“Schemata are data structures for representing the generic concepts stored in memory”



Schemas in constraint satisfaction networks

- Situations composed of primitive “features”
- A schema consists of knowledge about what features go with other features (i.e. **constraints** between features)
- Certain subpatterns tend to act in concert
 - Support each other and inhibit same sets of other units (“stable coalitions”)
- Good interpretations are **goodness maxima / energy minima**
- **No structure corresponds to a schema**

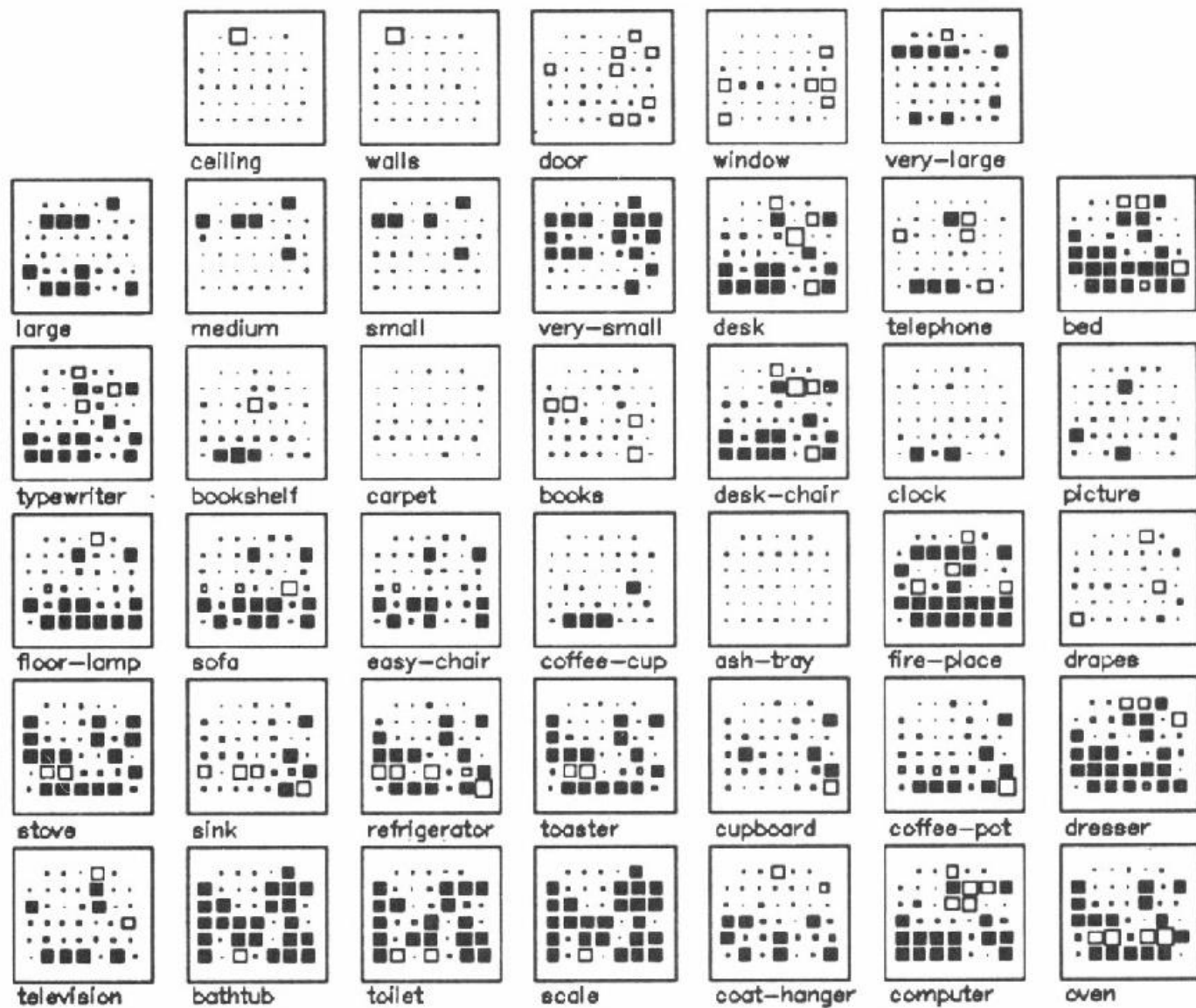
Schema model (Rumelhart et al. 1986)

- Two subjects each imagined 8 different versions of 5 room types
 - kitchen, office, bathroom, bedroom, living room
- For each imagined room, subject decided which of 40 **descriptors** applied to it
- Network has 40 **units** (one per descriptor); fully connected
- **Weights** on connections between units were set based on the likelihoods, across rooms, that the two descriptors agreed (both on or both off)
- **Biases** of units were set based on the likelihoods, across rooms, that individual descriptors were included

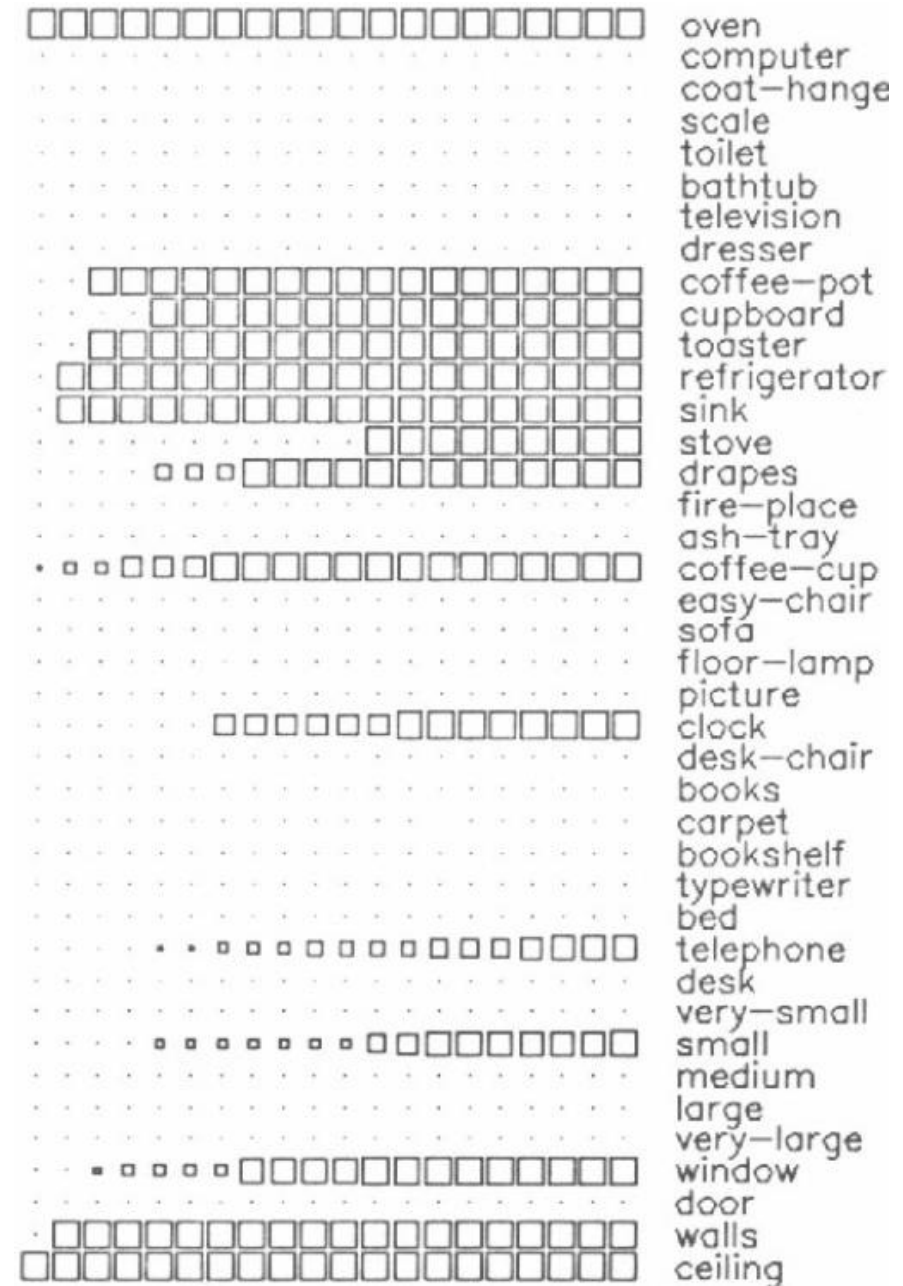
ceiling	walls	door	windows	very-large
large	medium	small	very-small	desk
telephone	bed	typewriter	bookshelf	carpet
books	desk-chair	clock	picture	floor-lamp
sofa	easy-chair	coffee-cup	ashtray	fireplace
drapes	stove	coffeepot	refrigerator	toaster
cupboard	sink	dresser	television	bathtub
toilet	scale	oven	computer	clothes-hanger

$$w_{ij} = -\ln \frac{p(x_i = 0 \ \& \ x_j = 1)p(x_i = 1 \ \& \ x_j = 0)}{p(x_i = 1 \ \& \ x_j = 1)p(x_i = 0 \ \& \ x_j = 0)}$$

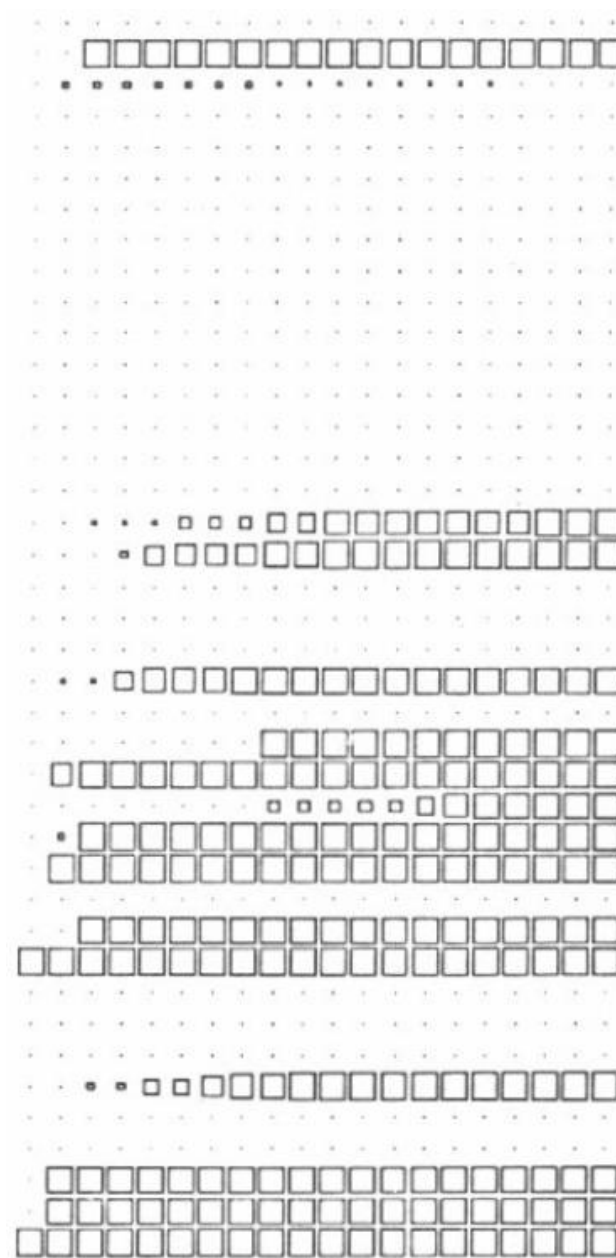
$$bias_i = -\ln \frac{p(x_i = 0)}{p(x_i = 1)}$$



Kitchen



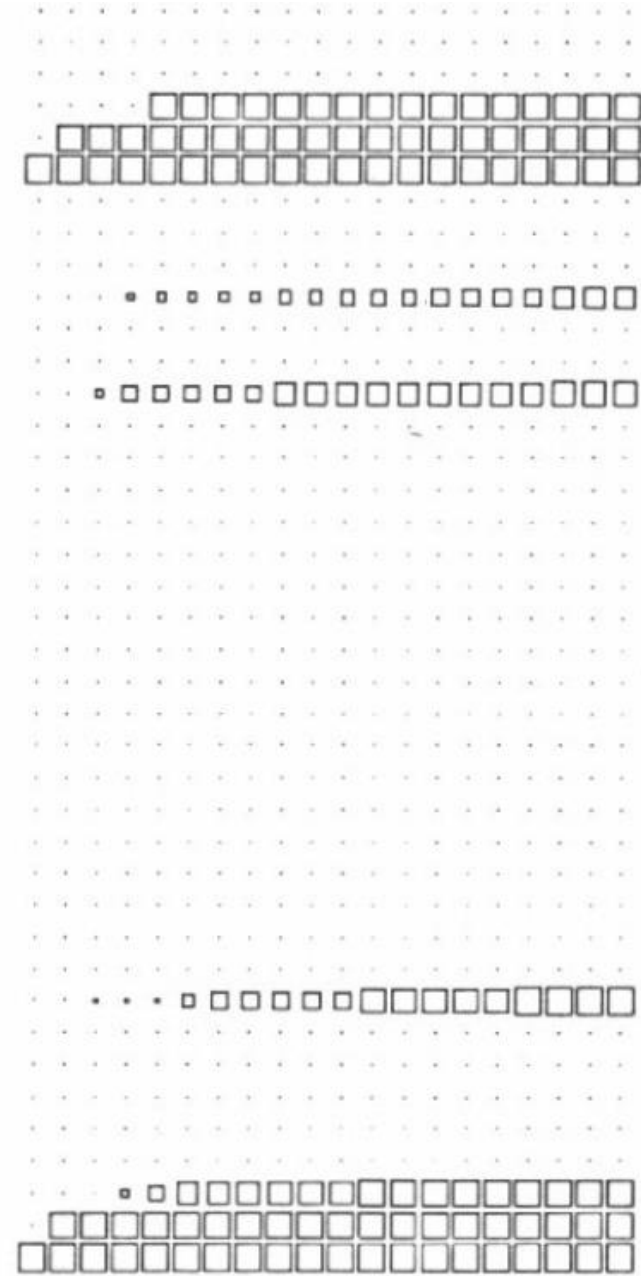
Office



oven
computer
coat-hange
scale
toilet
bathtub
television
dresser
coffee-pot
cupboard
toaster
refrigerator
sink
stove
drapes
fire-place
ash-tray
coffee-cup
easy-chair
sofa
floor-lamp
picture
clock
desk-chair
books
carpet
bookshelf
typewriter
bed
telephone
desk
very-small
small
medium
large
very-large
window
door
walls
ceiling

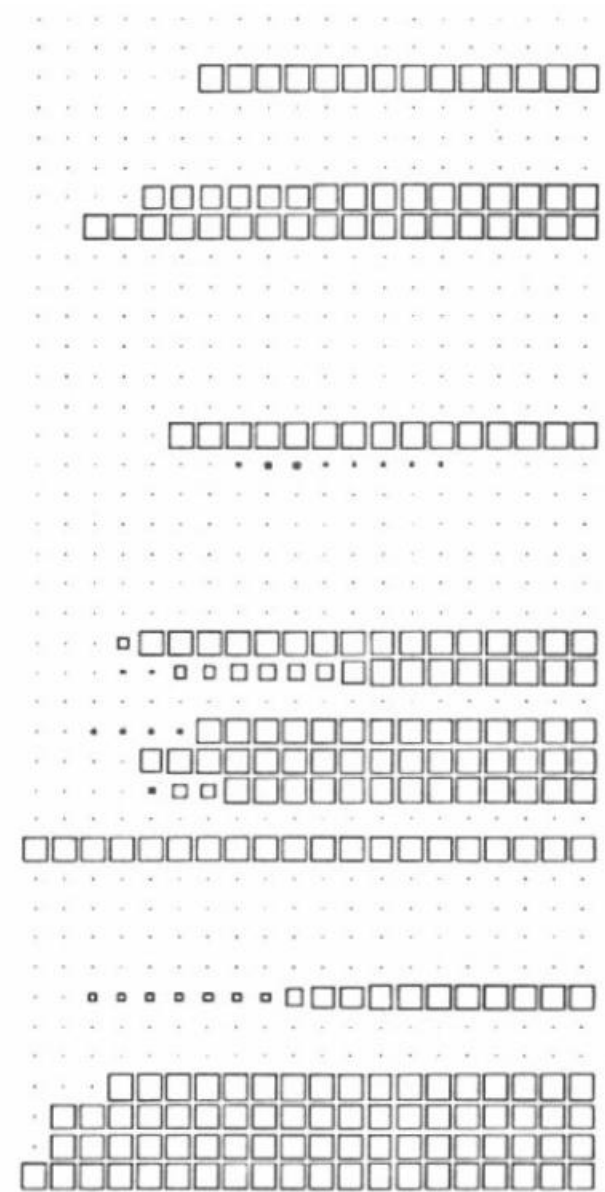
Bathroom

oven
computer
coat-hanger
scale
toilet
bathtub
television
dresser
coffee-pot
cupboard
toaster
refrigerator
sink
stove
drapes
fire-place
ash-tray
coffee-cup
easy-chair
sofa
floor-lamp
picture
clock
desk-chair
books
carpet
bookshelf
typewriter
bed
telephone
desk
very-small
small
medium
large
very-large
window
door
walls
ceiling

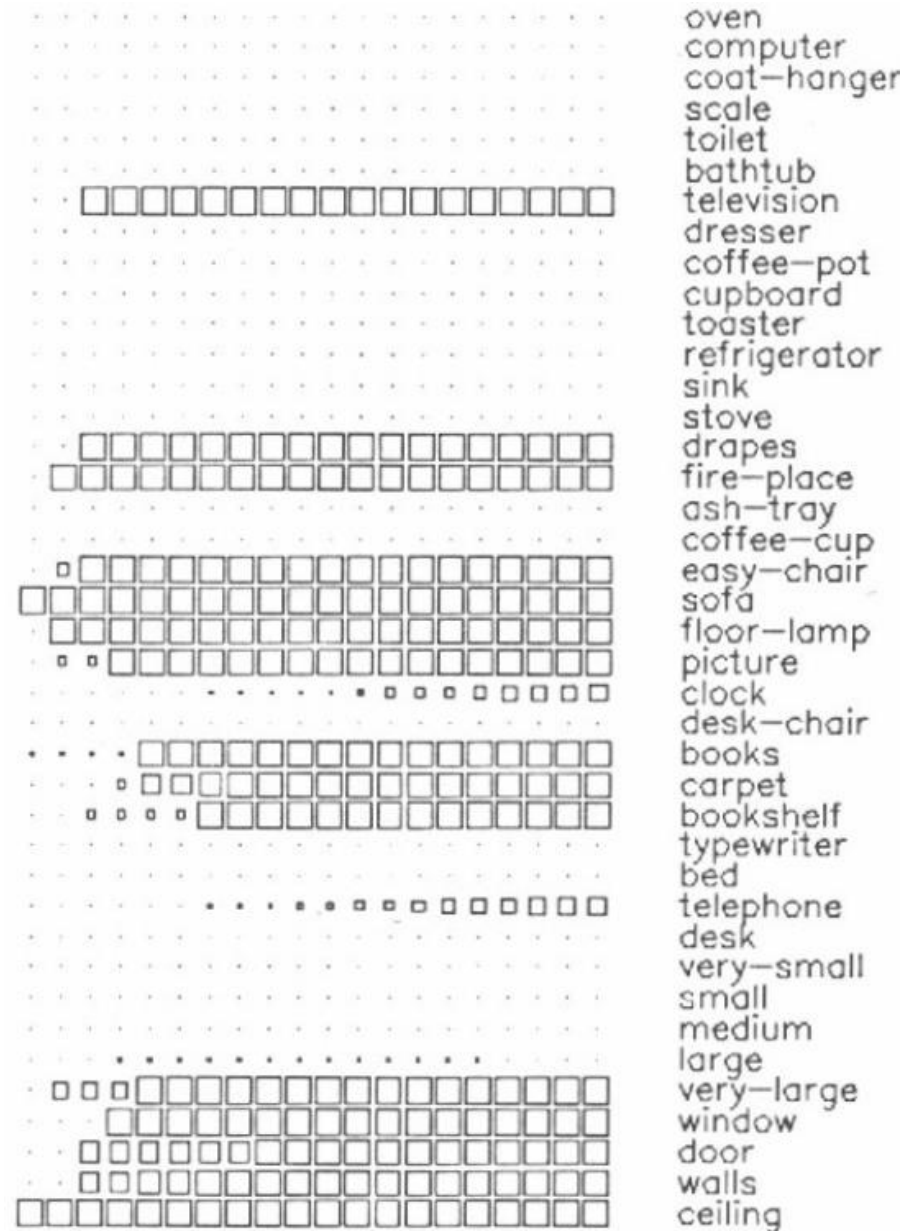


Bedroom

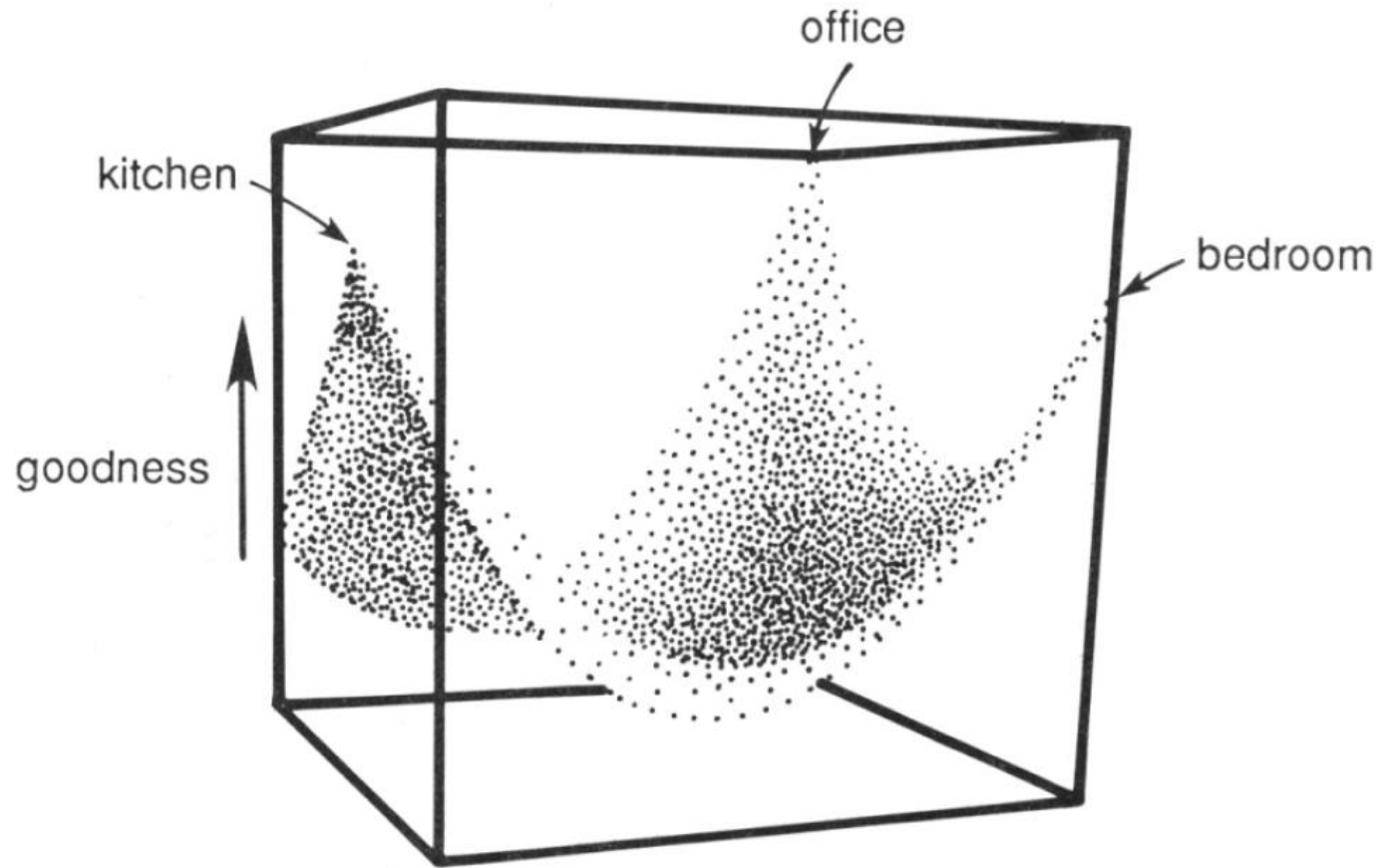
oven
computer
coat-hanger
scale
toilet
bathtub
television
dresser
coffee-pot
cupboard
toaster
refrigerator
sink
stove
drapes
fire-place
ash-tray
coffee-cup
easy-chair
sofa
floor-lamp
picture
clock
desk-chair
books
carpet
bookshelf
typewriter
bed
telephone
desk
very-small
small
medium
large
very-large
window
door
walls
ceiling



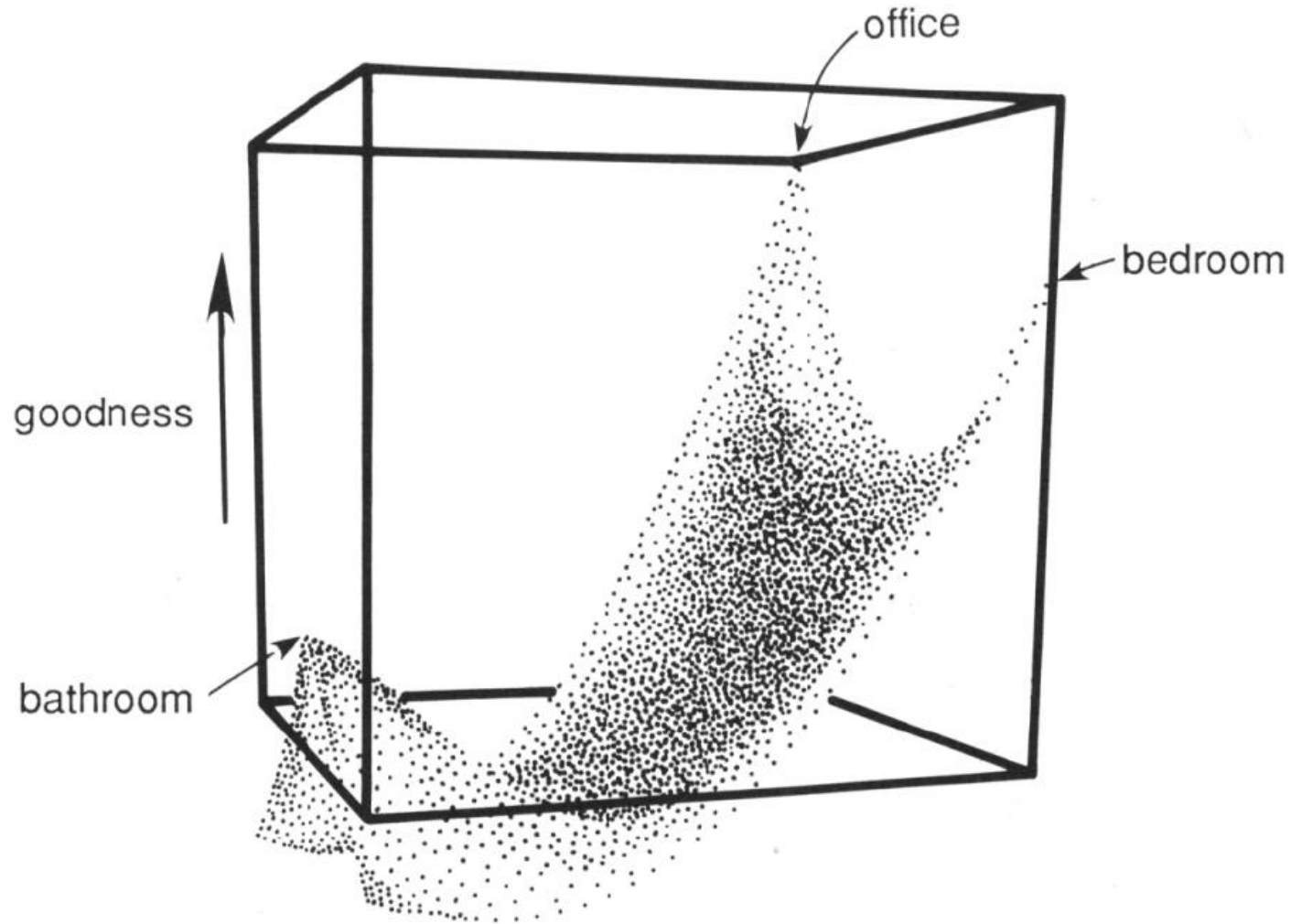
Living room



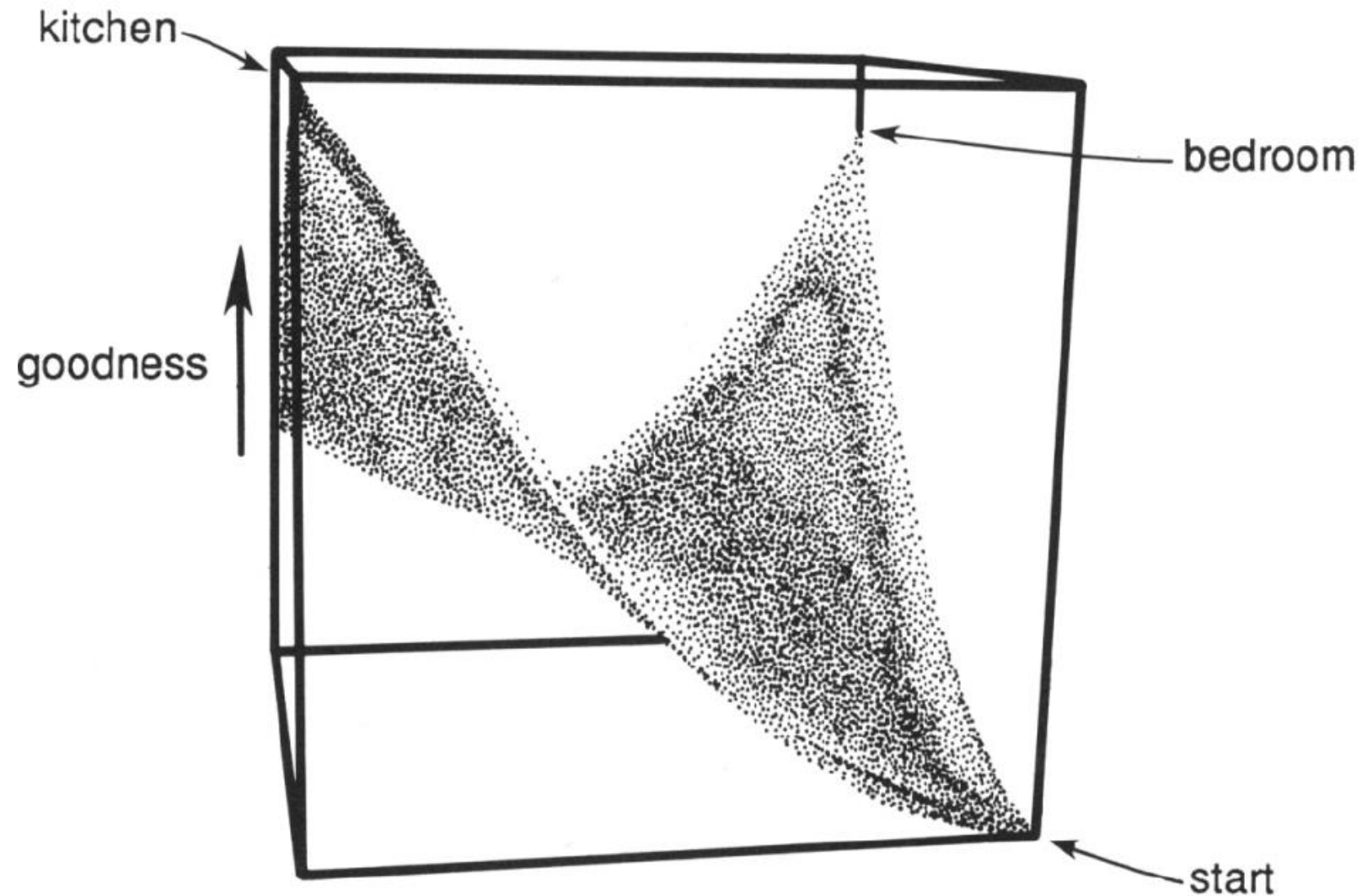
Goodness surface: Kitchen, Office, Bedroom



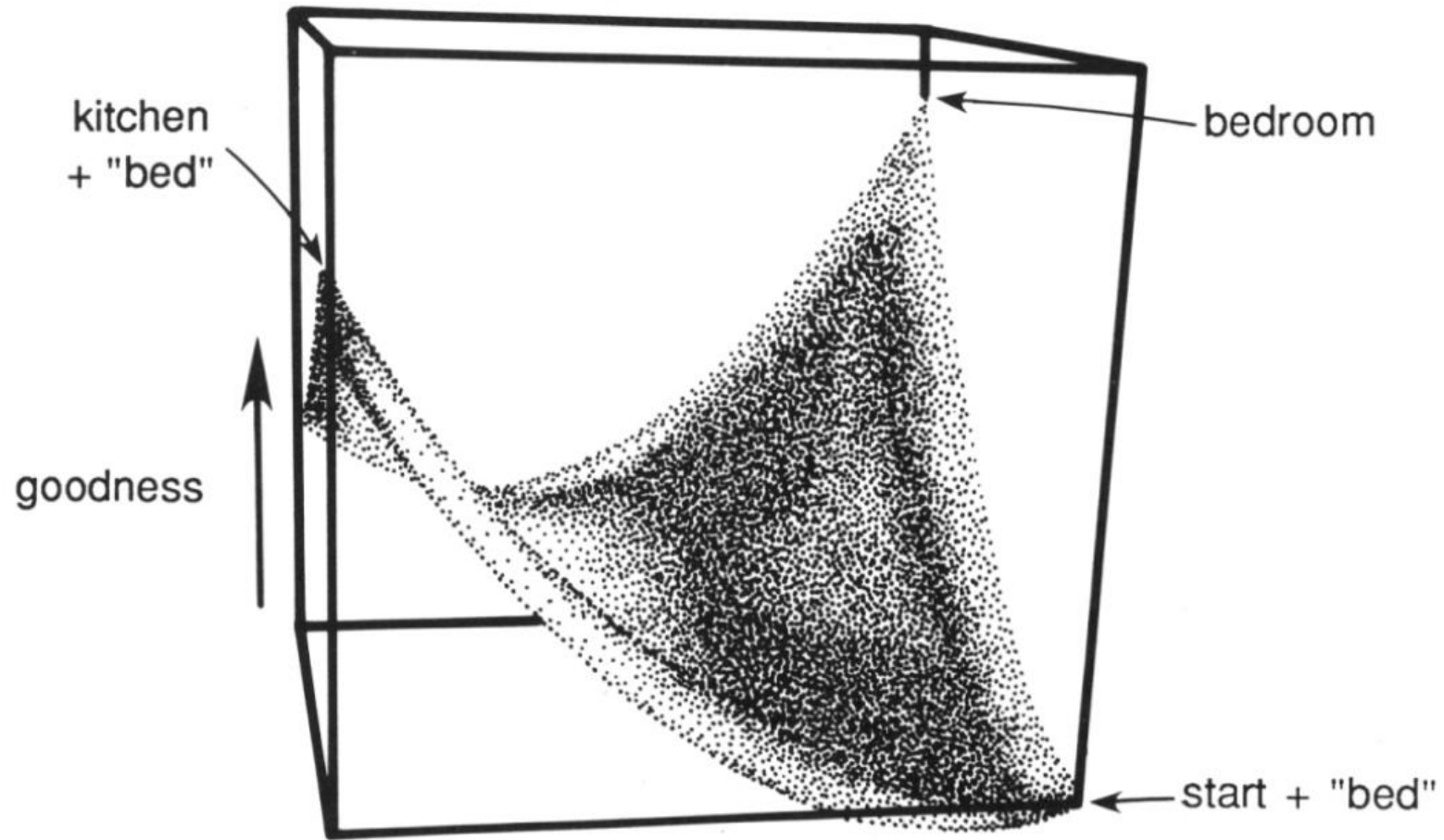
Goodness surface: Bathroom, Office, Bedroom



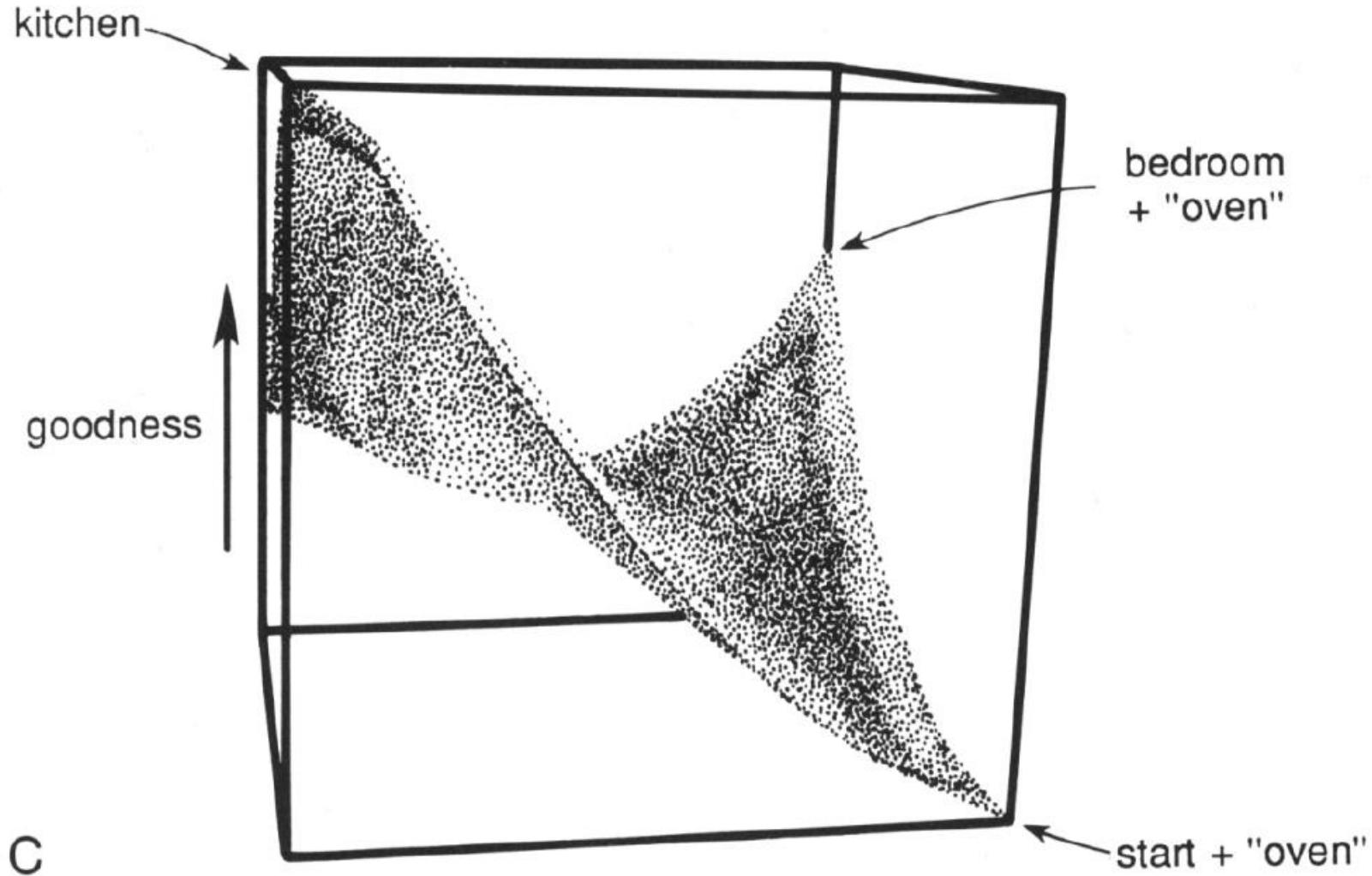
Goodness surface: Kitchen, Bedroom, (start)



Goodness surface: Kitchen+"bed", Bedroom



Goodness surface: Kitchen, Bedroom+ “oven”



Schema embedding

