FRA 331 : Basic Control Theory
Homework Assignment 1: Modelling

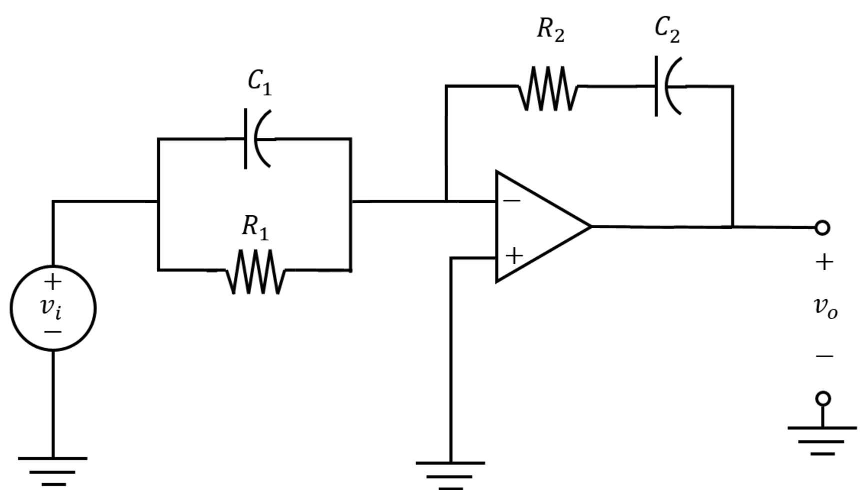
Written

1 : Integrator Circuit

The following circuit is an analog inverting PID controller . Using Krichoff's Voltage and Current laws, derive the expression of v_o in term of t, R_1, R_2, C_1, C_2, and v_i.

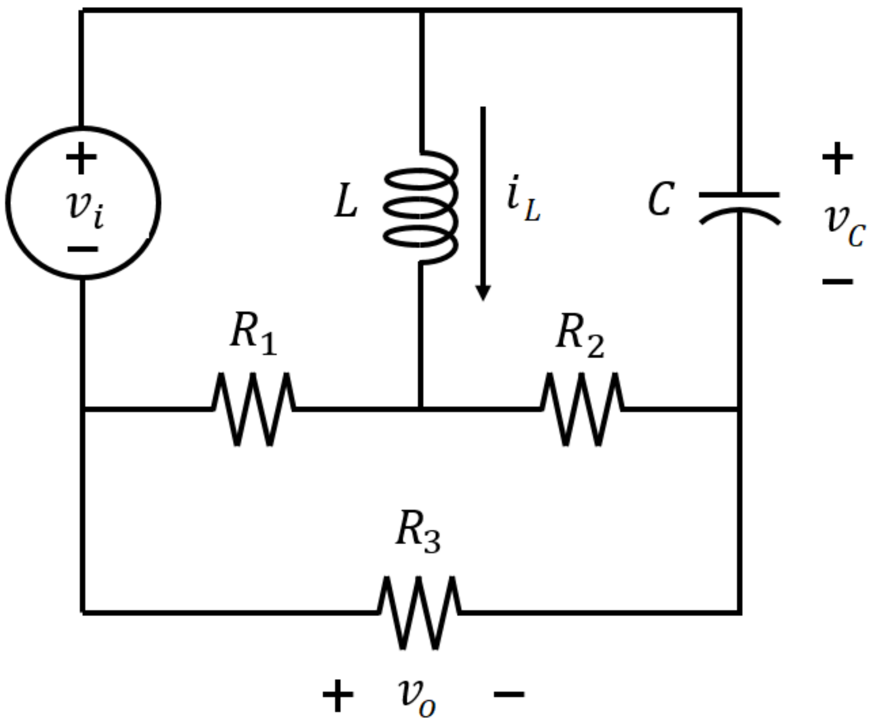
In order to receive full credits, determine the gains of inverting PID controller K_P, K_I, and K_D in term of R_1, R_2, C_1,and C_2 based on the following form.

$$v_o = -[K_P(v_i) + K_I(\int v_i dt) + K_D(\frac{dv_i}{dt})] \tag{1}$$



Problem 1

2 : second-order circuit



Problem 2

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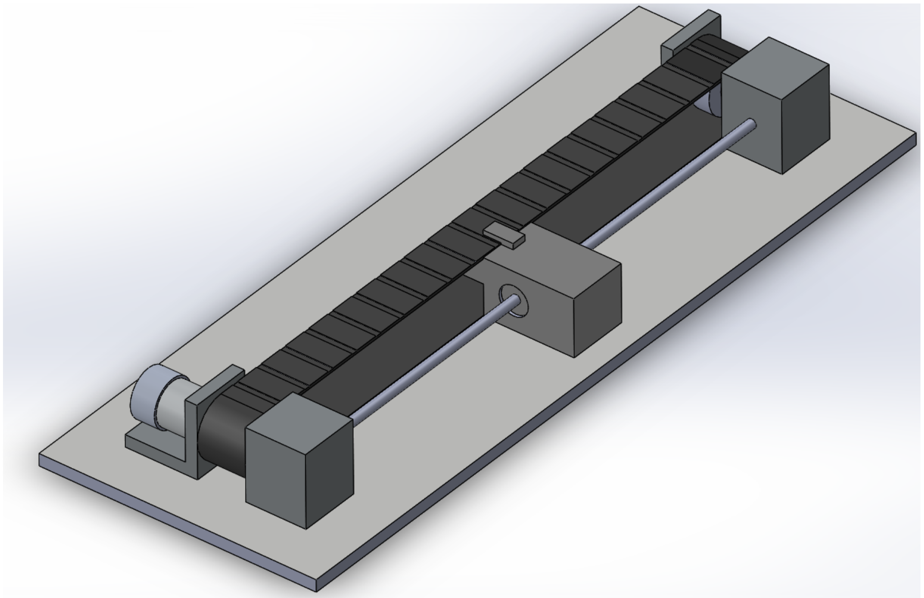
Using Kirchoff's Voltage and Current laws, show that the dynamics of the circuit is the following. You do not need to box the answer for this problem.

$$L\frac{di_L}{dt} = -\Big(\frac{R_1}{R_1+R_2}\Big)\Big(R_2 i_L + v_c - \frac{R_2}{R_1}v_i\Big)$$

$$C\frac{dv_c}{dt} = -\Big(\frac{R_1}{R_1+R_2}\Big)i_L - \Big(\frac{1}{R_1+R_2} + \frac{1}{R_3}\Big)(v_c - v_i)$$

$$v_o = v_c - v_i$$

3 : Belt-Driven Mechanism



Problem 3

The given belt-driven mechanism consists of a brushless DC motor with the following specification.

- R : internal resistance $[\Omega]$
- L : internal inductance $[H]$
- K_m : motor-torque constant $[\frac{N \cdot m}{A}]$
- K_b : internal inductance $[\frac{V \cdots}{rad}$
- r : gearbox ratio $[]$
- J_m : total motor inertia $[kg \cdot m^2]$
- B_m : total motor mechanical damping $[\frac{kg \cdot m^2}{s}]$

Given an input voltage of v_{in} and external load the output shaft τ_l, the current of the motor i_a and the angular displacement of output shaft θ_s change due to its dynamics.

The belt mechanism can be modeled as a mass-damper-spring system with a mass of m, mechanical damping of b , and a stiffness of k. The radius of the pulley is c. Let the linear displacement z of the belt mechanism.

Derive the equation which describes the dynamics of the entire system, where v_{in} is the input. DO NOT LEAVE THE ANSWER IN TERM OF θ (Note: the transmission formula for belt is $\tau_l = cf$,where f is the transmitted force by the belt. And $z = c\theta$, where θ is the angular displacement of the output shaft at the pulley)