FRA331: Basic Control Theory

Homework 4: Modelling in Frequency Domain (Transfer function)

There is no programming part for this homework Assignment

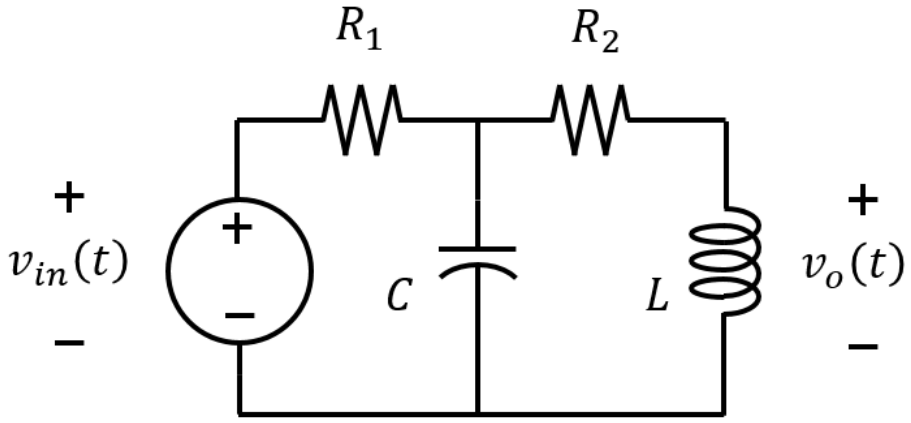
1.) Modelling Circuitry with Impedance

It is known that the impedance of electrical components can be computed in frequency domain. The impedance of 3 passive components can be described as the following.

$$Inductor: Z_L(s) = Ls$$

$$Resistor: Z_R(s) = R$$

$$Capacitor: Z_C(s) = \frac{1}{Cs}$$



The given circuit is known as a bandpass filter. Given the input v_{in}, the circuit filter any frequency outside its bound. The output is the voltage across the inductor v_o.

a.) Find the transfer function $G(s) = \frac{V_o(s)}{V_{in}(s)}$ using the above impedance relationship.

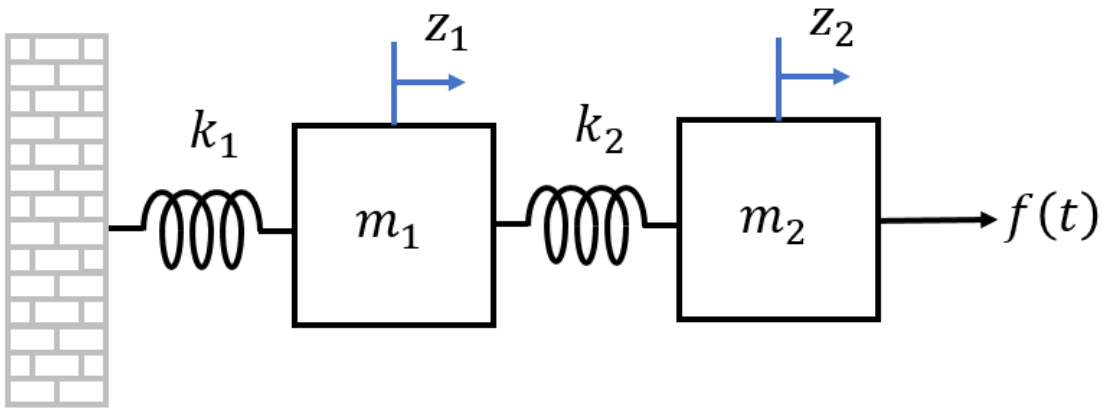
b.) The transfer function can be written in the following form.

$$G(s) = \frac{N(s)}{D(s)} = \frac{b_m s^m + b_{m-1}\, s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1}\, s^{n-1} + \cdots + a_1 s + a_0}$$

Identify maximum order of the numerator $N(s)$ and denominator $D(s)$

c.) If $a_n = 1$, what are the coefficients of numerators $N(s)$ and denominator $D(s)$

2.) Given a 2-DOF system with no damping, we can obtain the mathematic model of the system as a system of 2nd-order differential equations.



$$m_1\ddot{z}_1 = -k_1 z_1 - k_2(z_1 - z_2)$$
$$m_2\ddot{z}_2 = -k_2(z_2 - z_1) + f(t)$$

Let the output be the center of mass of the system

$$y = \frac{m_1 z_1 + m_2 z_2}{m_1 + m_2}$$

a.) Find the transfer function $G(s) = \frac{Y(s)}{F(s)}$

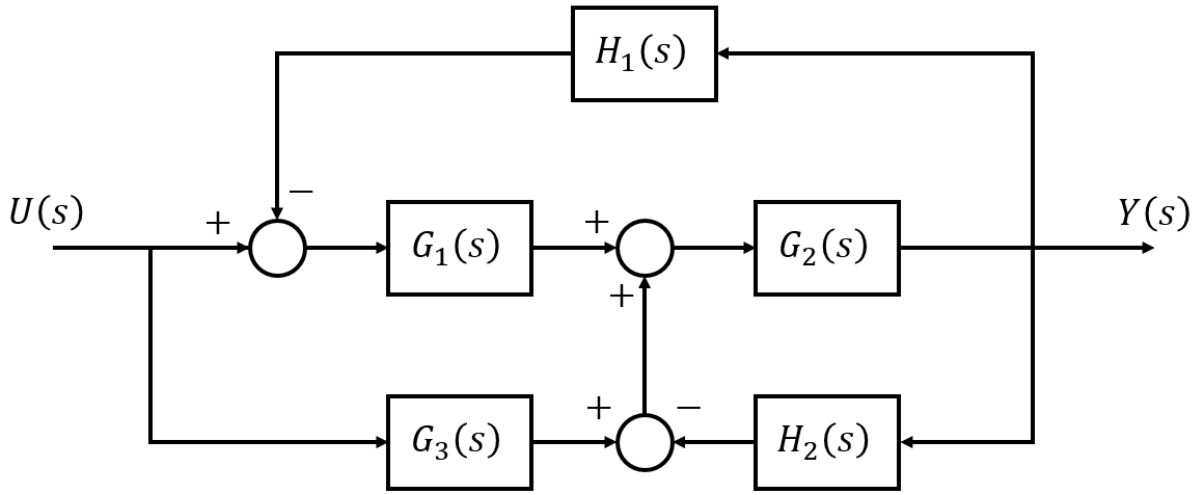
b.) The transfer function can be written in the following form.

$$G(s) = \frac{N(s)}{D(s)} = \frac{b_m s^m + b_{m-1}\, s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1}\, s^{n-1} + \cdots + a_1 s + a_0}$$

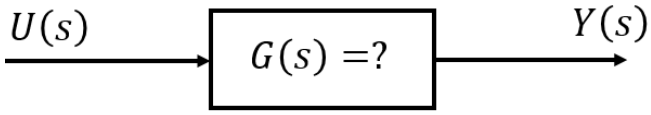
Identify maximum order of the numerator $N(s)$ and denominator $D(s)$

c.) If $a_n = 1$, what are the coefficients of numerators $N(s)$ and denominator $D(s)$

3.) Given a block diagram that represents a complex dynamic system as follows



a.) Reduce the block diagram down to a single block that represent the equivalent transfer function. Show each step in details.



NOTE: The result transfer function must be in the following form

$$G(s) = \frac{N(s)}{D(s)}$$

Where $N(s)$ and $D(s)$ **MUST NOT** be in form of fractions.