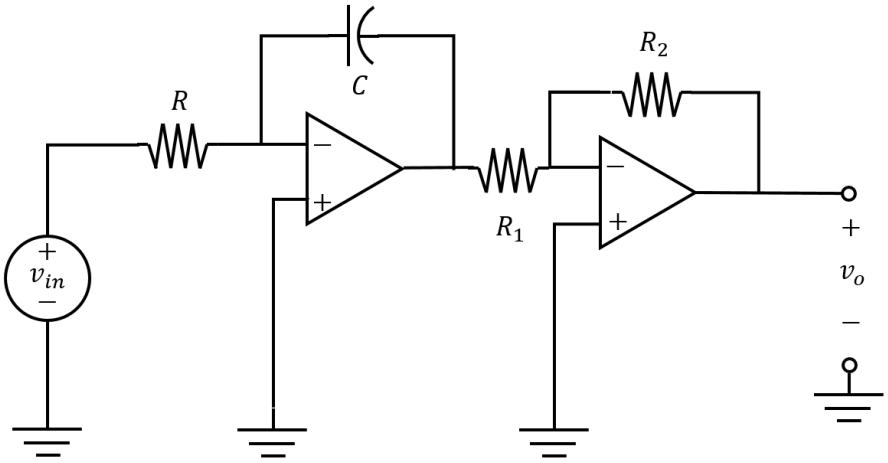
FRA 331 : Basic Control Theory
Homework Assignment 1: Modelling

Written

1 : Integrator Circuit

The following circuit is an integrator. Using Krichoff's Voltage and Current laws, derive the expression of v_o in term of t, R, R_1, R_2, C, and v_in.

In order to receive full credits, determine the condition of R_1 and R_2 such that the circuit is a direct integrator without any gain.



Problem 1

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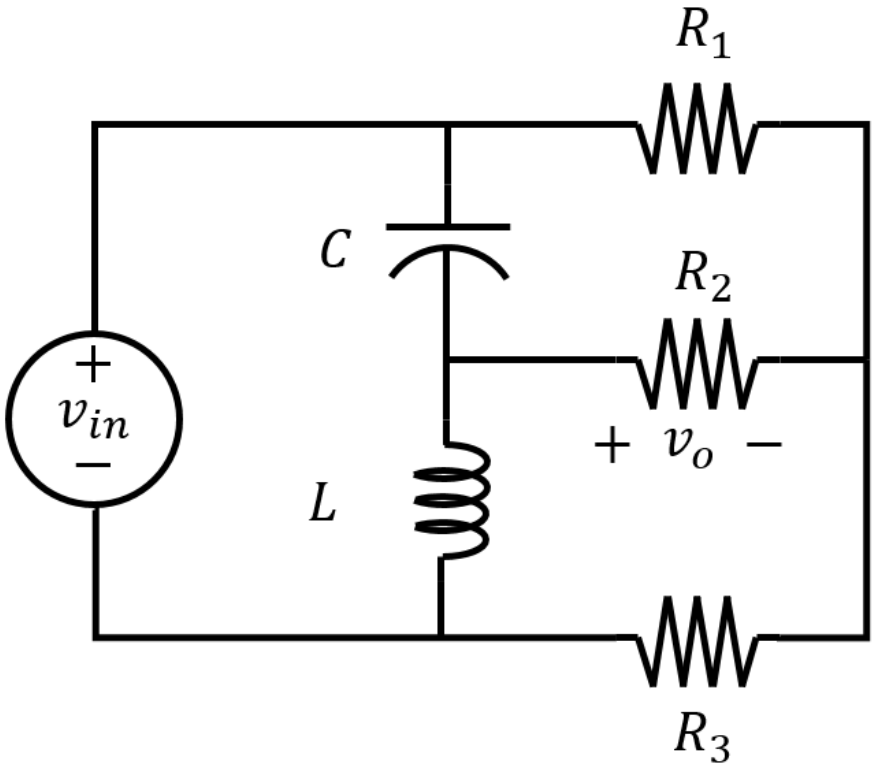
2 : second-order circuit



Problem 2

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Using Kirchoff's Voltage and Current laws, show that the dynamics of the circuit is the following. You do not need to box the answer for this problem.

$$v_{in} = v_c + L\frac{di_L}{dt}$$

$$C\frac{dv_c}{dt} = i_L + \Big(\frac{1}{R_1R_2 + R_1R_3 + R_2R_3}\Big)\Big(R_1v_{in} - (R_1 + R_3)v_c\Big)$$

$$v_o = \Big(\frac{R_2}{R_1R_2 + R_1R_3 + R_2R_3}\Big)\Big(R_1v_{in} - (R_1 + R_3)v_c\Big)$$

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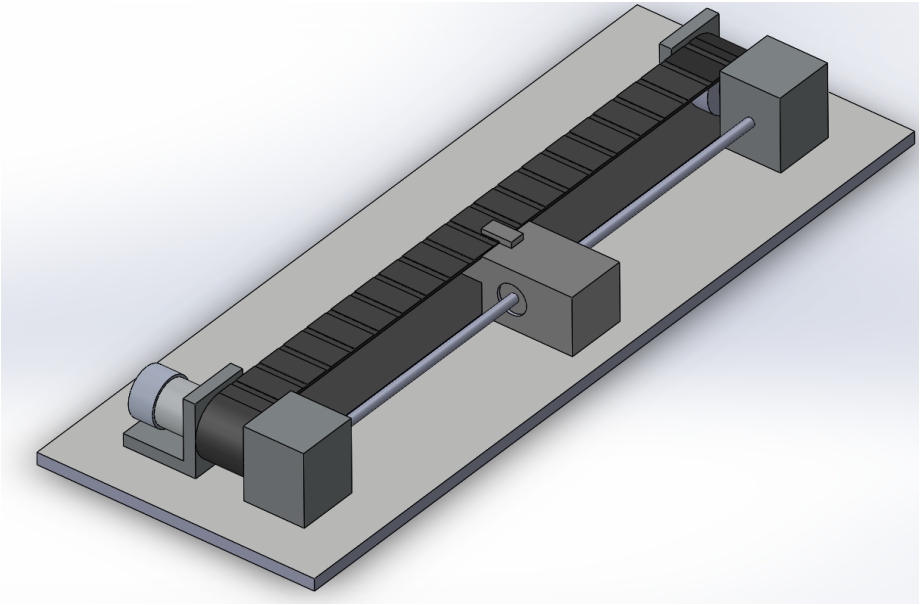
3 : Belt-Driven Mechanism



Problem 3

The given belt-driven mechanism consists of a brushless DC motor with the following specification.

- R : internal resistance $[\Omega]$
- L : internal inductance $[H]$
- K_m : motor-torque constant $[\frac{N \cdot m}{A}]$
- K_b : internal inductance $[\frac{V \cdots}{rad}$
- r : gearbox ratio $[_]$
- J_m : total motor inertia $[kg \cdot m^2]$
- B_m : total motor mechanical damping $[\frac{kg \cdot m^2}{s}]$

Given an input voltage of v_{in} and external load the output shaft τ_l, the current of the motor i_a and the angular displacement of output shaft θ_s change due to its dynamics.

The belt mechanism can be modeled as a mass-damper-spring system with a mass of m, mechanical damping of b , and a stiffness of k. The radius of the pulley is c. Let the linear displacement z of the belt mechanism.

Derive the equation which describes the dynamics of the entire system, where v_{in} is the input. DO NOT LEAVE THE ANSWER IN TERM OF θ (Note: the transmission formula for belt is $\tau_l = cf$,where f is the transmitted force by the belt. And $z = c\theta$, where θ is the angular displacement of the output shaft at the pulley)

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