FRA 333 : Intro to Robotics
Assignment 1: Coordinate Frame & Rotation

1 : Rotation about x-axis (Written)

Show that the rotation about standard x-axis can be represented by the following rotation matrix.

$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

where θ is an angle of rotation about the x-axis.
Hint: The derivation is similar to the one from the class.

2 : Sequence of rotations (Written)

Write the matrix product that will give the resulting rotation matrix (do not perform the matrix multiplication). Consider the following sequence of rotations:

1. Rotate by θ about the current x-axis
2. Rotate by ϕ about the fixed z-axis
3. Rotate by α about the current z-axis
4. Rotate by β about the current y-axis
5. Rotate by δ about the fixed x-axis

3 : Sequence of rotations (Written)

If coordinate frame F_1 is obtained from the coordinate frame F_0 by a rotation of $\frac{\pi}{2}$ about the x-axis followed by a rotation of $\frac{\pi}{2}$ about the fixed y-axis, find the rotation matrix R representing the composite transformation. Sketch the initial and final frames. (Do the multiplication).

4 : Inverse of rotation (Written)

Suppose that three coordinate frames F_1, F_2, and F_3 are given, and suppose

$$R_2^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} : R_3^1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

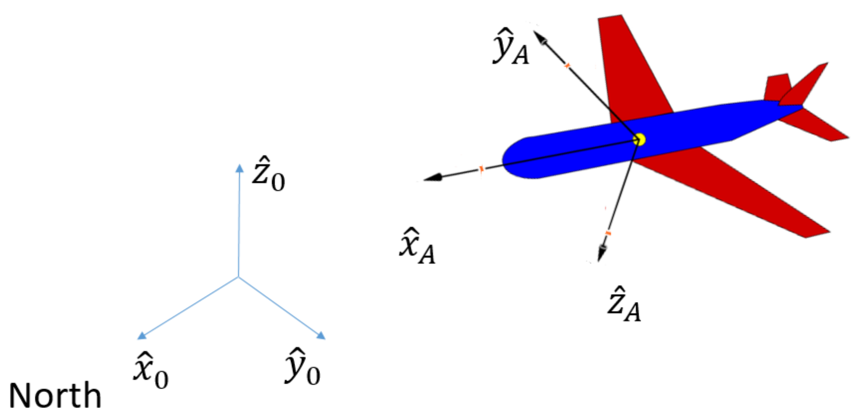
Find the matrix R_3^2

5 : Aircraft Orientation

In aerospace application, a rotation matrix can be used to represent an orientation of an aircraft. You and your team are asked to write a program in MATLAB to determine the state of an aircraft at different orientation. Apply what you learned from the class, reading assignment, and individual homework.

Let F_0 denote a coordinate frame which has the same orientation as the observer, where $\hat{z}_0$ points upward from the ground (to the sky), and $\hat{x}_0$ points toward north.

Let F_A denote a coordinate frame which is rigidly attached to the center of mass of an aircraft, where $\hat{z}_A$ points toward the underbelly of the aircraft, and $\hat{x}_A$ points toward the nose of the aircraft.



Coordinate Frames

To make sure that an aircraft is in good flight condition, $\hat{z}_A^0$ has to be bounded in a downward cone. The equation of the surface of a downward cone is the following.

$$z = -\frac{1}{\tan \alpha}\sqrt{x^2 + y^2} \tag{1}$$

where α is an angle of deviation. In the program, THIS SHOULD BE SET TO 10 degrees.

The program must take an orientation of an aircraft, which can be given in the following representations.

1. Roll, Pitch, Yaw (RPY)
2. ZYZ-Euler angles
3. 3x3 rotation matrix

Therefore, you might have to considered input a string indicating the type of representations.
The program has to output the followings.

1. $\hat{x}_A^0$ represented as 3×1 column vector
2. $\hat{y}_A^0$ represented as 3×1 column vector
3. $\hat{z}_A^0$ represented as 3×1 column vector
4. a string stating "Yes" or "No" depending on the flight condition (Yes if it's in a good flight condition)