ECE 726 Project Report Vinod K. Singla (# 200149761) Fall ’17

“Investigating variation in performance with sparsity in feedback gains for a string of moving vehicles”

Problem Description

With self-driving vehicles on the horizon, the control of vehicular formations is becoming increasingly lucrative. Of particular interest is a 1D formation wherein the vehicles move with a constant speed at fixed relative distances. For a small number of vehicles in the formation, all vehicles can easily communicate with one another and the problem is relatively easy to solve. However, as the number of vehicles in the formation increases, communications between individual vehicles becomes more difficult. Figure 1 shows a string of moving vehicles where m_k is the mass, y_k is the velocity and z_k is the position of the k^{th} vehicle in the formation. The desired separation between k^{th} & $(k+1)^{th}$ vehicle and the variation from it is denoted by Δ_k & δ_k respectively.

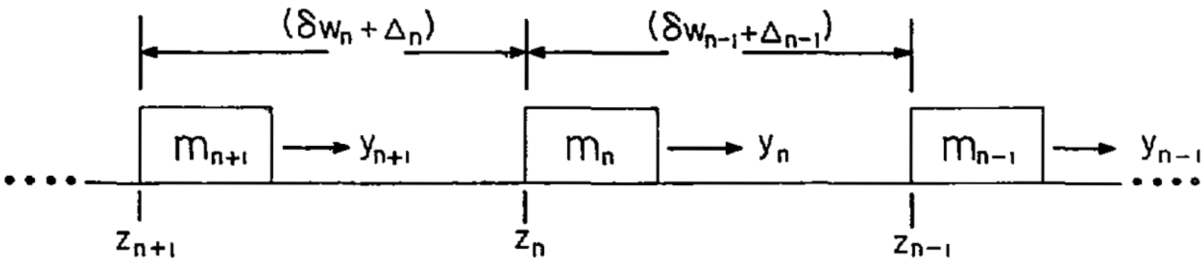


Figure 1. String of moving vehicles [1]

The plant (state variable model) as given by Levine [1] is:

$$
\frac{d}{dt}\underbrace{\begin{bmatrix}\delta y_1(t)\\ \delta w_1(t)\\ \delta y_2(t)\\ \delta w_2(t)\\ \delta y_3(t)\\ \delta w_3(t)\\ \vdots\end{bmatrix}}_{x(t)} = \underbrace{\begin{bmatrix} -\frac{\alpha_1}{m_1} & 0 & 0 & 0 & 0 & 0 & \\ 1 & 0 & -1 & 0 & 0 & 0 & \cdots \\ 0 & 0 & -\frac{\alpha_2}{m_2} & 0 & 0 & 0 & \\ 0 & 0 & 1 & 0 & -1 & 0 & \cdots \\ 0 & 0 & 0 & 0 & -\frac{\alpha_3}{m_3} & 0 & \\ 0 & 0 & 0 & 0 & 1 & 0 & \cdots \\ & \vdots & & \vdots & & \vdots & \vdots \end{bmatrix}}_{A}\underbrace{\begin{bmatrix}\delta y_1(t)\\ \delta w_1(t)\\ \delta y_2(t)\\ \delta w_2(t)\\ \delta y_3(t)\\ \delta w_3(t)\\ \vdots\end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix} \frac{1}{m_1} & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & \cdots \\ 0 & \frac{1}{m_2} & 0 & 0 & \\ 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & \frac{1}{m_3} & 0 & \\ 0 & 0 & 0 & 0 & \cdots \\ & & & \vdots & \vdots \end{bmatrix}}_{B}\underbrace{\begin{bmatrix}\delta f_1(t)\\ \delta f_2(t)\\ \delta f_3(t)\\ \vdots\end{bmatrix}}_{u(t)}
$$

Here, α_k is the steady state drag force and f_k is the applied force on the k^{th} vehicle

The optimal control problem is to minimize the power spectral density of noise while ensuring stability and minimum number of communication links between vehicles. The original plant with disturbance inputs is rewritten as:

$$\dot{x} = Ax + B_1 d + B_2 u$$

$$u = -Fx$$

where *d* and *u* are disturbance and control inputs respectively. To promote sparsity in the state feedback matrix, *F*, the original objective function *J(F)* based on power spectral density of noise is modified to penalize the cost for every additional non-zero element. The modified objective function is given by:

$$\min\left(J(F) + \gamma cardinality(F)\right)$$

where

γ = weight emphasizing importance on sparsity

$$J(F) = \begin{cases} trace(B_1^T P(F) B_1), F - stabilizing \\ \infty, otherwise \end{cases}$$

$$P(F) = \int_0^\infty e^{(A-B_2F)^T t}(Q + F^T RF)e^{(A-B_2F)t}dt$$

Also, P(F) is a solution of the Lyapunov equation:

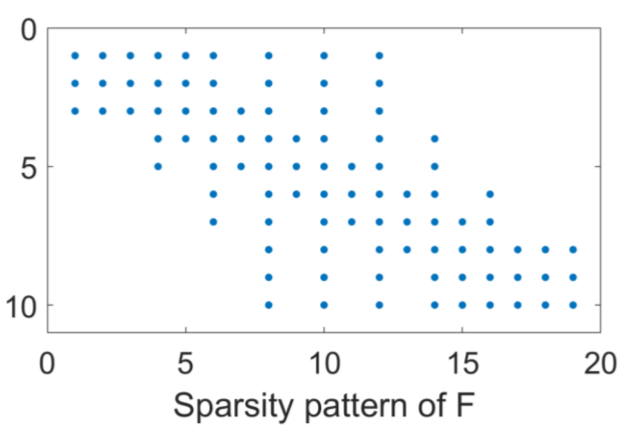
$$(A - B_2F)^T P + P(A - B_2F) = -(Q + F^T RF)$$

The sparsity structure of F, which minimizes the modified objective function while exploiting the substructures of *J(F)* and *cardinality(F)* is found by employing the Alternating Direction Method of Multipliers (ADMM) algorithm as presented by Lin [2]. The original problem to minimize objective *J(F)* subject to the identified sparsity structure is then solved with varying γ values using Newton's method. The respective number of non-zero elements and the objective function value is then recorded to compare performances.

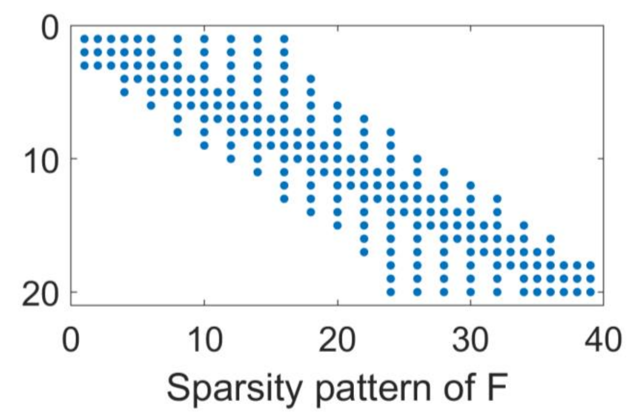
Results

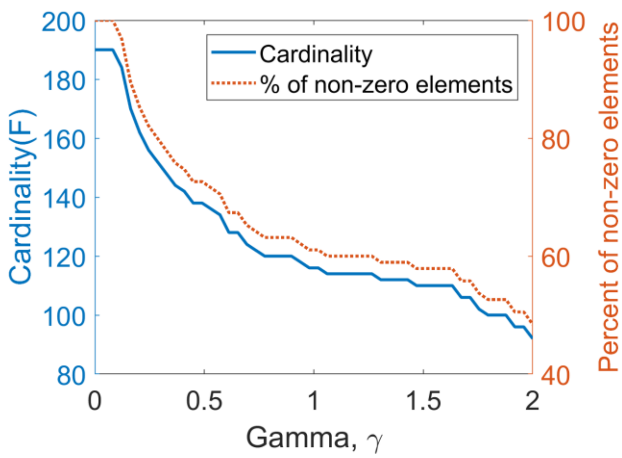
A MATLAB routine was developed to implement the strategies proposed by Lin [2]. The routine was ran for 10 and 20 vehicles in a formation for γ values ranging from 0-2 and the results are presented below.

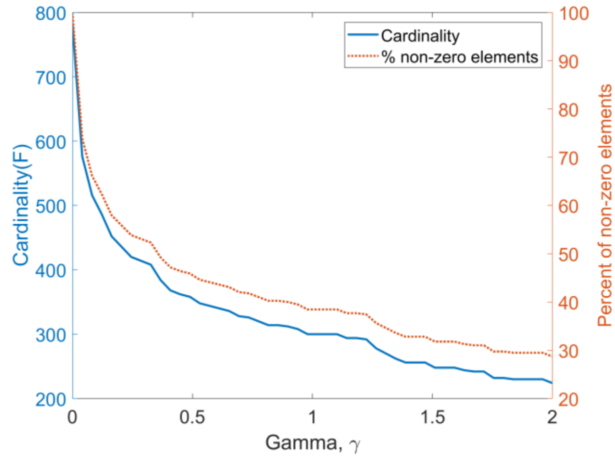
N = 10

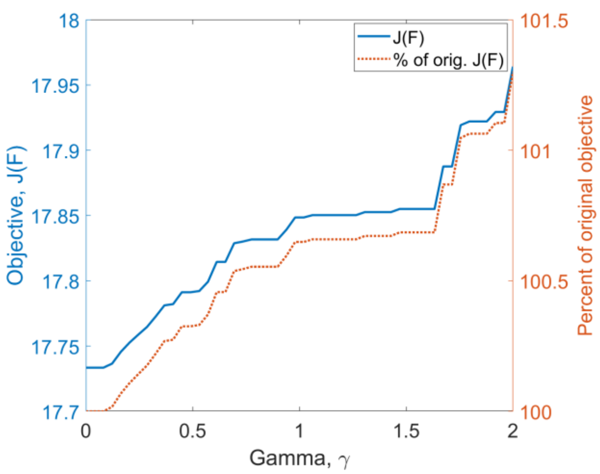


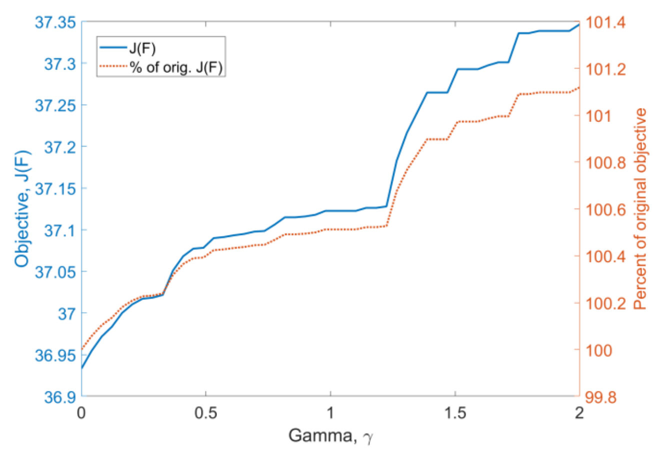
N = 20











Observations

1. The identified sparsity pattern demonstrates a diagonal structure. This makes sense as most vehicles will communicate only with vehicles that are immediate or near immediate neighbors.

2. The number of non-zero elements has been slashed significantly (over 50%) for both N = 10 and N = 20 cases resulting in significant communication savings. However, this has come at the expense of increased cost function value by nearly 1.5 %.

3. Future work can involve experimentation with different penalty schemes like sum of logs, l_1 norm or weighted l_1 norm and observing changes in performance.

References:

[1] Levine, W., and M. Athans. "On the Optimal Error Regulation of a String of Moving Vehicles." *IEEE Transactions on Automatic Control*, vol. 11, no. 3, 1966, pp. 355–361., doi:10.1109/tac.1966.1098376.

[2] Lin, Fu, et al. "Design of Optimal Sparse Feedback Gains via the Alternating Direction Method of Multipliers." *IEEE Transactions on Automatic Control*, vol. 58, no. 9, 2013, pp. 2426–2431., doi:10.1109/tac.2013.2257618.