

THEOREM 2.1 *Let $U, V \subset \mathbb{R}^n$ and assume that $U \sim V \neq \emptyset$. Then the following results hold.*

- (i) $U \sim V = \bigcap_{v \in V} (U - \{v\})$.
- (ii) $(U \sim V) + V \subset U$.
- (iii) $0 \in V$ implies $U \sim V \subset U$.
- (iv) $U = \{\bar{u}\} + \alpha \bar{U}, V = \{\bar{v}\} + \alpha \bar{V}$ and $\alpha \in \mathbb{R}$ implies $U \sim V = \{\bar{u} - \bar{v}\} + \alpha(\bar{U} \sim \bar{V})$.
- (v) $V = V_1 + V_2$ implies $U \sim V = (U \sim V_1) \sim V_2 = (U \sim V_2) \sim V_1$.
- (vi) $U = U_1 \cap U_2$ implies $U \sim V = (U_1 \sim V) \cap (U_2 \sim V)$.
- (vii) $V = V_1 \cup V_2$ implies $U \sim V = (U \sim V_1) \cap (U \sim V_2)$.
- (viii) If $G \in \mathbb{R}^{m \times n}$ has rank n , then $GU \sim GV = G(U \sim V)$.
- (ix) If U, V are symmetric, $U \sim V$ is symmetric.
- (x) If U is (bounded) [closed] {convex}, $U \sim V$ is (bounded) [closed] {convex}.
- (xi) If U, V are symmetric and convex, then $0 \in u \sim V$.
- (xii) If U is convex, then $U \sim V = U \sim \text{co}V$.
- (xiii) $V = \text{co}\{v_i, i=1, \dots, N\}$ implies $U \sim V = \bigcap_{i=1, \dots, N} (U \sim \{v_i\})$.
- (xiv) If V is compact, then $U \sim V = U \sim \text{ex}V$.