The support function of U, evaluated at $\eta \in \mathbb{R}^n$, is

$$h_U(\eta) = \sup_{u \in U} \eta^{\mathrm{T}} u. \tag{1.11}$$

The domain, $K_U \subset \mathbb{R}^n$, on which the support function is defined is a convex cone with vertex at the origin; specifically, for $\eta \notin K_U$, $\eta^{\mathrm{T}} u$ is unbounded from above on U. If U is bounded, $K_U = \mathbb{R}^n$. Suppose U is closed and convex. Then $U = \{u\colon \eta^{\mathrm{T}} u \leq h_U(\eta),\ \eta \in K_U\}$, the intersection of its supporting half spaces; moreover, $V \subset U$ if and only if $h_V(\eta) \leq h_U(\eta)$ for all $\eta \in K$. Testing the inclusion $V \subset U$ is much easier when U is the polyhedron,

$$U = \{u\colon s_i^{\mathrm{T}} u \leq r_i, \quad i = 1, \ldots, N\}. \tag{1.12}$$

Then $V \subset U$ if and only if $h_V(s_i) \leq r_i$, $i = 1, \ldots, N$. For $\alpha \geq 0$, $u \in \mathbb{R}^n$, $\mu \in \mathbb{R}^m$, $G^{\mathrm{T}} \mu \in K_U$, the following identities are easily confirmed: $h_U(\eta) = h_{coU}(\eta)$, $h_U(\alpha\eta) = \alpha h_U(\eta)$, $h_{\{u\}+U}(\eta) = \eta^{\mathrm{T}} u + h_U(\eta)$, $h_{U+V}(\eta) = h_U(\eta) + h_V(\eta)$, $h_{GU}(\mu) = h_U(G^{\mathrm{T}} \mu)$. Furthermore, if U is compact it follows that $coU = co(exU)$ and $h_U(\eta) = h_{exU}(\eta)$.

In what follows it is necessary to both characterize and numerically evaluate support functions. In many situations this can be done using the preceding identities and simple observations such as: $U = \{u\colon u^{\mathrm{T}} P^{-1} u \leq 1\}$, $P = P^{\mathrm{T}} > 0$ implies $h_U(\eta) = \sqrt{\eta^{\mathrm{T}} P \eta}$; $U = \{u\colon |u|_p \leq 1\}$, $1 \leq p \leq \infty$ implies $h_U(\eta) = |\eta|_q$, $p^{-1} + q^{-1} = 1$; $U = co\{u_i\colon i = 1, \ldots, N\}$ implies $h_U(\eta) = \max \eta^{\mathrm{T}} u_i$, $i = 1, \ldots, N$; when U is the polyhedron (1.12) $h_U(\eta)$ is the solution of the linear program (LP), maximize $\eta^{\mathrm{T}} u$ subject to $s_i^{\mathrm{T}} u \leq r_i$, $i = 1, \ldots, N$.