for $i = 1, \ldots, N$. *Then,*

$$U \sim V = \{z \in \mathbb{R}^n\colon s_i^{\mathrm{T}} z \le r_i - h_V(s_i), \quad i = 1, \ldots, N\}. \tag{2.4}$$

Proof The result (2.4) is valid if $N = 1$ since then by result (i) of Theorem 2.1, $U \sim V = \bigcap_{v \in V}\{z\colon s_1^{\mathrm{T}}(z + v) \le r_1\} = \{z\colon s_1^{\mathrm{T}} z + h_v(s_1) \le r_1\}$. Recursive application of result (vi) in Theorem 2.1 proves (2.4) for $N > 1$.

Remark 2.2 If $h_V(s_i)$ is not defined ($s_i^{\mathrm{T}} v$ is unbounded from above on V) for some $i = 1, \ldots, N$, then $U \sim V$ is empty. If $h_V(s_i)$ is defined for $i = 1, \ldots, N$ it is still possible that $U \sim V = \emptyset$. In this case emptyness can be checked by the usual linear programming test for feasibility: maximize α over those $(z, \alpha) \in \mathbb{R}^{n+1}$ which satisfy $s_i^{\mathrm{T}} z + \alpha \le r_i - h_V(s_i)$, $i = 1, \ldots, N$; $U \sim V \ne \emptyset$ if and only if $\max \alpha \ge 0$.

Remark 2.3 Suppose (2.3) is a nonredundant characterization of U, i.e., the removal of any one of the N inequalities changes U. It is still possible that (2.4) is a redundant characterization of $U \sim V$. Redundant inequalities can be sequentially eliminated by applying linear programming. For example, if $\max s_1^{\mathrm{T}} z < r_1 - h_V(s_1)$ for all z such that $s_i^{\mathrm{T}} z < r_i - h_V(s_i)$, $i = 2, \ldots, N$, the first inequality may be removed.

Remark 2.4 It is not assumed that either U or V is bounded. Moreover, (2.4) can be applied numerically to a wide class of V. It is only necessary to have a procedure for computing the $h_V(s_i)$; see, for instance, the next to the last paragraph in Section 1.