Technical University of Denmark

Constrained Optimization

Course 02612

Assignment 1

Authors:
Jakub Czerny, s99999

Oskar Hint, s161559

Joachim Finn Jensen, s134052

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this is some report

2 Assignment

2.1 Problem 1 - Quadratic Optimization

From the problem 1 it is seen that the system can be written as:

$$H = \begin{bmatrix} 6 & 2 & 1 \\ 2 & 5 & 2 \\ 1 & 2 & 4 \end{bmatrix}$$

$$g = \begin{bmatrix} -8 \\ -3 \\ -3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$b = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

Then the general KKT system can be written as:

$$\begin{bmatrix} H & -A \\ A^T & 0 \end{bmatrix} \times \begin{bmatrix} x \\ \lambda \end{bmatrix} = \begin{bmatrix} -g \\ b \end{bmatrix}$$

The system is solve with a function in MatLab where use the backslash command, see the code in ?X?X. The result shows that:

$$x = \begin{bmatrix} 0.5 \\ 0.25 \\ 4 \end{bmatrix}$$

$$\lambda = \begin{bmatrix} 0.7 \\ 0.5 \end{bmatrix}$$

To test the solver random values for x and λ are generated, from these the H, g, A and b are determined. With these new matrix the system are solved again to see if the x and lambda. The solve works as expected and gives the same values for x and lambda as there were generated. To see the sensitivity of the problem the parameter sensitivity analyse is made. The general function is as followed.

$$F(x, \lambda; p) = \begin{bmatrix} \nabla_x f(x,p) - \nabla_x c(x,p)\lambda \\ c(x,p) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Where the implicit function theorem gives parameter sensitivities

$$[\nabla_x(p)\lambda(p)] = -[W_{xp} - \nabla_p c(x,p) \begin{bmatrix} W_{xx} & -\nabla_x c(x,p) \\ -\nabla_x c(x,p)^T & 0 \end{bmatrix}^{-1}$$

$$W_{xx} = \nabla^2_{xx} f(x,p) - \sum_{i \in \varepsilon} \lambda_i \nabla^2_{xx} c_i(x,p)$$

$$W_{xp} = \nabla^2_{xp} f(x,p) - \sum_{i \in \varepsilon} \lambda_i \nabla^2_{xp} c_i xp)$$

From this the sensitivity of this problem can be written as:

$$f(x,p) = \frac{1}{2} x^T H x + (g + \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix})^T x$$

$$c(x,p) = A^T x - b - \begin{bmatrix} p_5 \\ p_6 \end{bmatrix}$$

$$w_{xp} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$W_{xx} = H$$

$$[\nabla x(p) \lambda(p)] = - \begin{bmatrix} H & -A^T \\ -A & 0 \end{bmatrix}$$

This are added to the solver and the sensitivity are found as:

$$\nabla x(p) = \begin{bmatrix} -0.0909 & -0.0909 & 0.0909 \\ -0.0909 & -0.0909 & 0.0909 \\ 0.0909 & 0.0909 & -0.0909 \\ 0.2727 & -0.7273 & 0.7273 \\ -0.4545 & 0.5455 & 0.4545 \end{bmatrix}$$

$$\lambda(p) = \begin{bmatrix} 0.2727 & -0.4545 \\ -0.7273 & 0.5455 \\ 0.7273 & 0.4545 \\ 0.1818 & 0.3636 \\ 0.3636 & 2.7273 \end{bmatrix}$$

Why is this correct??? When have such a quadratic problem it's always also have a dual problem.

2.2 Problem 2 - Equality Constrained Quadratic Optimization

blablablablbal

2.3 Problem 3 - Inequality Constrained Quadratic Programming

From page 475 in Nocedal and Wright the following system is given.

$$\begin{aligned} \min_x q(x) = (x_1 - 1)^2 + (x_2 - 2.5)^2 \\ s.t. x_1 - 2x_2 + 2 >= 0, \\ -x_1 - 2x_2 + 6 >= 0, \\ -x_1 + 2x_2 + 2 >= 0, \\ x_1 >= 0, \\ x_2 >= 0. \end{aligned} \tag{1}$$

in MatLab a contour plot of this is made and seen in figure 1.

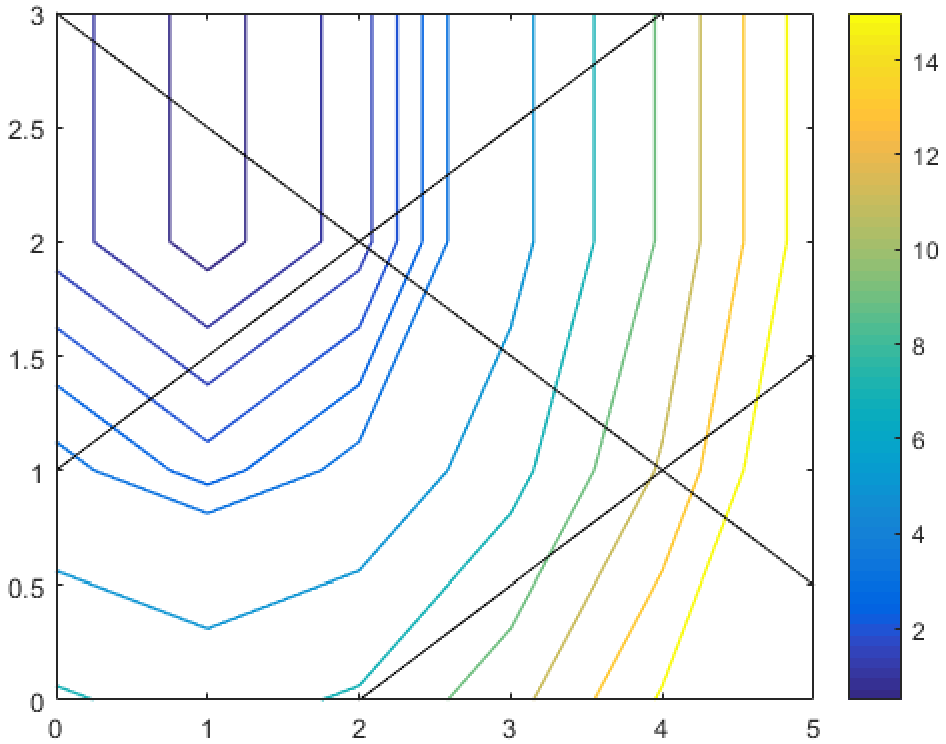

Figure 1: A contour plot of the problem.

From the line it is seen that the feasible region is a pentagon.
The problem can be written in the standart martix way as:

$$H = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$g = \begin{bmatrix} -2 \\ -5 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & -1 & 1 & 0 \\ -2 & -2 & 2 & 0 & 1 \end{bmatrix}$$

$$b = \begin{bmatrix} -2 \\ -6 \\ -2 \\ 0 \\ 0 \end{bmatrix}$$

Then the general KKT system can be written as:

$$\begin{bmatrix} H & -A \\ A^T & 0 \end{bmatrix} \times \begin{bmatrix} x \\ \lambda \end{bmatrix} = \begin{bmatrix} -g \\ b \end{bmatrix}$$

When the system can be solve with the same QP solver as used for problem 1. That gives:

$$x = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

$$\lambda = \begin{bmatrix} 0 \\ 1.8 \\ -0.3 \\ -2.1 \\ 3.0 \end{bmatrix}$$

Where all the values are multiply with 10 in the power of 16.
A way to found the optimal solution is with an active set algorithm. This is applyed as the one in the Example 16.4 in N and W. The algorithm is seen in the MatLab code and the contour plot with the four different workingset are seen in figure 2 below.

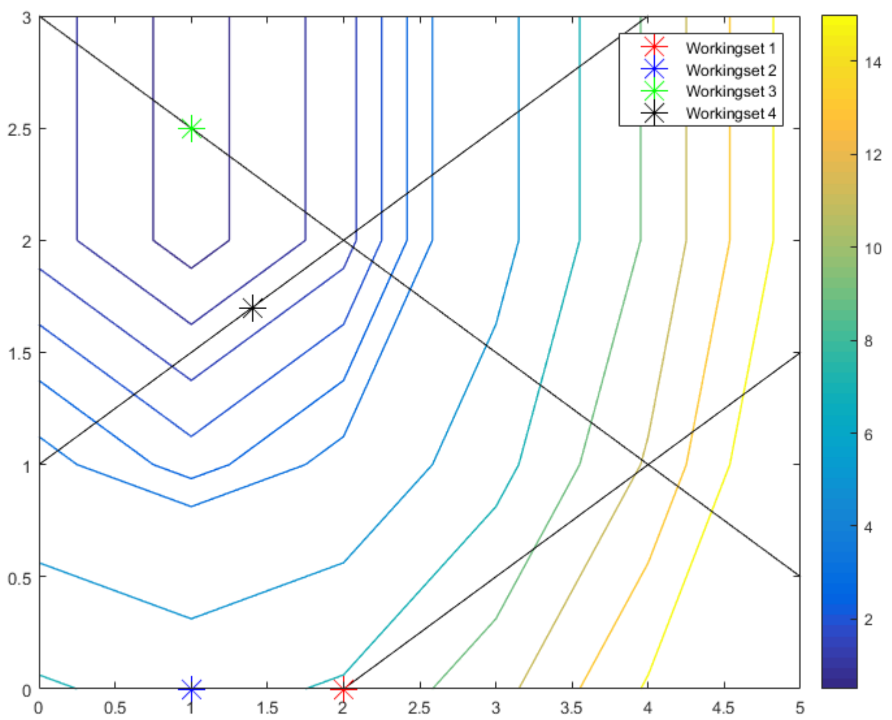

Figure 2: A contour plot of the problem, with the working set applyed .

2.4 Problem 4 - Markowitz Portfolio Optimization

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2.5 Problem 5 - Interior-Point Algorithm for Convex Quadratic Programming

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3 Conclusion

Some conclusions things