Project: Cooperative Load Transport with group of Quadrotors

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Overview

We address the problem of cooperative transportation of a cable-suspended payload by multiple quadrotors. This work based on the work which is maded by Kumar and Sreenath[1]. In the beginning, we determine the equations describing the dynamics of the system. Then we define the differential flatness of a dynamic system and determine the flat output vector of our system. Finally we write a program with matlab to implement the motion of N quadrotors with a mass point.

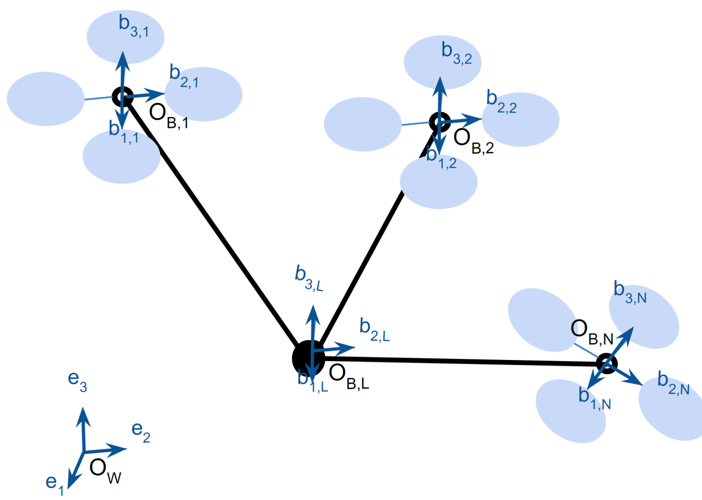

Figure 1: Three quadrotors transport a mass point load

Dynamic Model of a Group of Quadrotors with a Mass Punkt Load

First we define the used notations: The i^{th} quadrotor flies in 3-D Space. To describe its motion we

Symbol	Definition
$N \in \mathbb{Z}^+$	The number of Quadrotors
$g \in \mathbb{R}$	The acceleration of the gravity
$m_L \in \mathbb{R}$	The Mass of point mass load L
$m_i \in \mathbb{R}$	The Mass of i^{th} quadrotor $i \in \{1, \ldots, N\}$
$J_i \in \mathbb{R}^{3\times3}$	Inertia matrix of the i^{th} quadrotor with respect to the body-fixed frame $i \in \{1, \ldots, N\}$
$l_i \in \mathbb{R}^+$	Length of the cable between the i^{th} quadrotor and the load
$\mathbf{p}_L \in \mathbb{R}^3$	Position vector of the point mass L in the inertial frame
$\mathbf{q}_i \in S^2$	Unit vector from the i^{th} quadrotor to the load in body-xed frame of the load
$\boldsymbol{\omega}_i \in \mathbb{R}^3$	Angular velocity of $\mathbf{q}_i$ defined as $\mathbf{q}_i \times \dot{\mathbf{q}}_i$
$T_i \in \mathbb{R}$	Tension in the the cable between the i^{th} quadrotor and the load
$\mathbf{p}_i \in \mathbb{R}^3$	Position vector of the center of mass the i^{th} quadrotor in the inertial frame
${}^W\mathbf{R}_{B,i} \in \mathcal{SO}(3)$	The rotation matrix of the i^{th} quadrotor from the body-xed frame to the inertial frame
$\psi_i \in \mathcal{SO}(3)$	Yaw angle of the i^{th} quadrotor
$\mathbf{\Omega}_{B,i} \in \mathbb{R}^3$	Angular velocity of the i^{th} quadrotor in the body-fixed frame
$F_i \in \mathbb{R}$	Force produced by the i^{th} quadrotor
$\mathbf{M}_i \in \mathbb{R}^3$	Moment produced by th i^{th} quadrotor

Table 1: The used Symbols in our model

need six paramters, three to describe the position and three to describe its orientation. On the other hand the mass point load doesn't have orientation that is why only three paramenters is needed to describe its motion. The configuration space of the system is given by $Q = \mathbb{R}^3 \times (S^2 \times SO(3))^N$ and the position of the i^{th} quadrotor given by the following kinematic relation,

$$\mathbf{p}_i = \mathbf{p}_L - l_i \mathbf{q}_i \tag{1}$$

According to [1] the Euler dynamics of the N quadrotors and the load can be easily written down as follows:
First the translational motion of i^{th} quadrotor:

$$m_i \ddot{\mathbf{p}}_i = F^W \mathbf{R}_B e_3 - m_i g e_3 + T_i \mathbf{q}_i, \ i \in \{1, \ldots, N\} \tag{2}$$

the rotational motion of i^{th} quadrotor:

$$J_i \dot{\mathbf{\Omega}}_{B,i} = \mathbf{M}_i - \mathbf{\Omega}_{B,i} \times J_i \mathbf{\Omega}_{B,i} \tag{3}$$

finally the translational motion of the point mass load:

$$m_L \ddot{\mathbf{p}}_L = -m_L g e_3 - \sum_{i=i}^{N} T_i \mathbf{q}_i \tag{4}$$

We can put these equations in the form $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$ with state vector $\mathbf{x} \in \mathbb{R}^n$

$$\mathbf{X} = \begin{bmatrix} \mathbf{p}_L & \dot{\mathbf{p}}_L & \mathbf{q} & \boldsymbol{\omega} & {}^W\mathbf{R}_B & \mathbf{\Omega}_B \end{bmatrix}^T$$

and the input vector $\mathbf{u} \in \mathbb{R}^m$

$$\mathbf{U} = \begin{bmatrix} \mathbf{F} & \mathbf{M} \end{bmatrix}^T$$

where:

$$\begin{aligned}
\mathbf{q} &= \begin{bmatrix} \mathbf{q}_1 \dots \mathbf{q}_N \end{bmatrix} \\
\boldsymbol{\omega} &= \begin{bmatrix} \omega_1 \dots \omega_N \end{bmatrix} \\
{}^W\mathbf{R}_B &= \begin{bmatrix} {}^W\mathbf{R}_{B,1} \dots {}^W\mathbf{R}_{B,N} \end{bmatrix} \\
\mathbf{\Omega}_B &= \begin{bmatrix} \mathbf{\Omega}_{B,1} \dots \mathbf{\Omega}_{B,N} \end{bmatrix} \\
\mathbf{F} &= \begin{bmatrix} F_1 \dots F_N \end{bmatrix} \\
\mathbf{M} &= \begin{bmatrix} \mathbf{M}_1 \dots \mathbf{M}_N \end{bmatrix}
\end{aligned}$$

Differential Flatness

First we define which does Differential Flatness mean, then we want to explain why do we use this property to control our system. Let us start with the definition, according to [2]: The states and the inputs differentially flat system can be written as algebraic functions of carefully selected at outputs and their derivatives. This facilitates the automated generation of trajectories since any smooth trajectory (with reasonably bounded derivatives) in the space of at outputs can be followed by our system. Our choice of at outputs is given by:

$$\mathbf{y} = \begin{bmatrix} \mathbf{p}_L & \mathbf{Tq}_{-i} & \boldsymbol{\psi} \end{bmatrix}^T$$

where:

$$\begin{aligned}
\mathbf{Tq}_{-i} &= \begin{bmatrix} T_1\mathbf{q}_1 & \dots & T_{i-1}\mathbf{q}_{i-1} & T_{i+1}\mathbf{q}_{i+1} & \dots & T_N\mathbf{q}_N \end{bmatrix} \\
\boldsymbol{\psi} &= \begin{bmatrix} \psi_1 & \dots & \psi_N \end{bmatrix}
\end{aligned}$$

The used equations to calculate the input vector from the output vector and its derivatives are

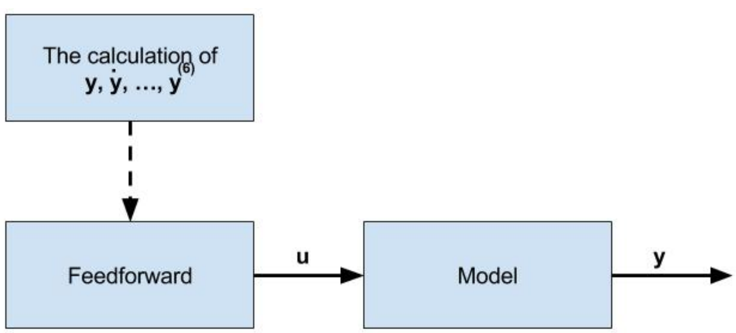

Figure 2: Feedforward: First we determine the output vector and its first six derivatives. From the results we can find the input vector of the System

represented in [3]. In [3] the case of 2 Quadrotors was considered. The figure 3 shows us a clarification of the differential flatness principle. We can plan in the flat output space, rather than planning trajectories for the full state vector, which is usually high dimensional and has dynamically coupled variables. To plan the output vector $\mathbf{y}$ we follow the following steps: is to that we should plane the following things:

1. find the optimal trajectory for the mass point Load L, The position vector of the point mass L in the inertial frame $\mathbf{p}_L \in \mathbb{R}^3$ to calculate it we used the same method which is described in [2].

2. The second step is to determine $\mathbf{Tq}_{-i}$. We use the method mentioned in [3]. First we find the optimal Tension force for each quadrotor in the group by minimizing cost function:

$$J = \min_{\mathbf{T}} \|\mathbf{T}\|^2 \tag{5}$$

   wobei:

$$\mathbf{T} = \begin{bmatrix} T_1 & \dots & T_N \end{bmatrix}$$

   then we determine the unit vector $\mathbf{q}$ which assures that there is no collision between quadrotors in the group.

3. The last step is to determine the Yaw angle for all the Quadrotors in the group $\boldsymbol{\psi}$. Because the value of Yaw angel has no direct affect on the dynamic of the quadrotors we can choose this angle as:

$$\begin{aligned}
\boldsymbol{\psi} &= \begin{bmatrix} \psi_1 & \dots & \psi_N \end{bmatrix} = \mathbf{0}_{1\times N} \\
\dot{\boldsymbol{\psi}} &= \begin{bmatrix} \dot{\psi}_1 & \dots & \dot{\psi}_N \end{bmatrix} = \mathbf{0}_{1\times N} \\
\ddot{\boldsymbol{\psi}} &= \begin{bmatrix} \ddot{\psi}_1 & \dots & \ddot{\psi}_N \end{bmatrix} = \mathbf{0}_{1\times N}
\end{aligned}$$

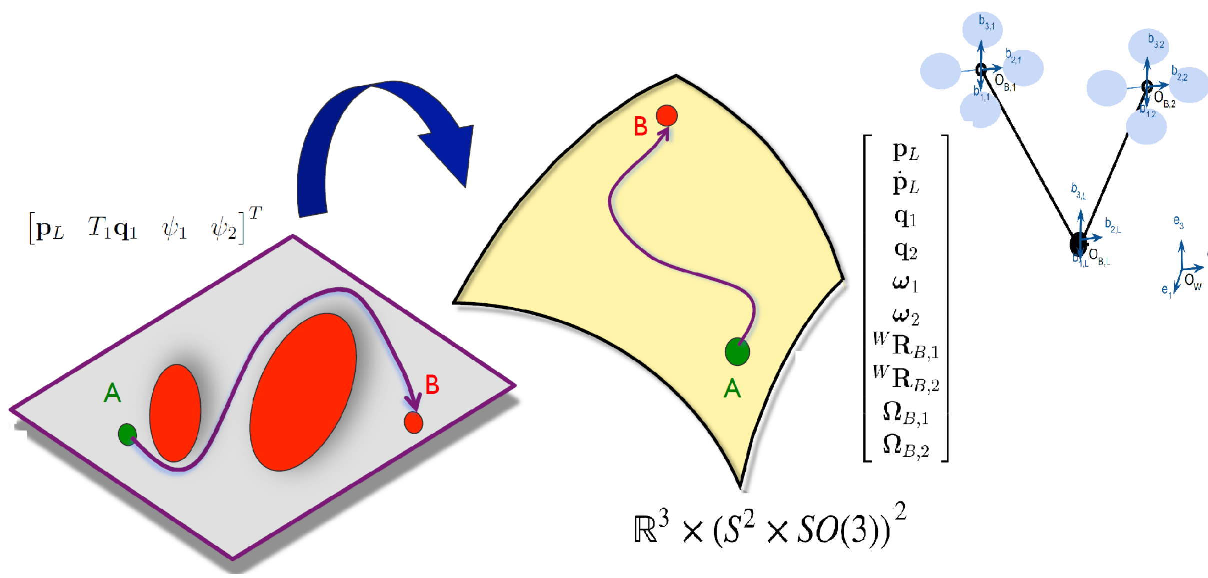

Figure 3: Differential Flatness: We can plan in the flat output space, rather than planning trajectories for the full state vector, which is usually high dimensional and has dynamically coupled variables. This allows for planning in a lower-dimensional space where all variables are independent. Any smooth trajectory for the flat outputs can then be mapped back into feasible trajectories for the full set of states and inputs of the system.

References

[1] K. Sreenath and V. Kumar. Dynamics, Control and Planning for Cooperative Manipulation of Payloads Suspended by Cables from Multiple Quadrotor Robots, 2013, http://www.kumarrobotics.org/wp-content/uploads/2014/01/p11.pdf

[2] D. Mellinger and V. Kumar. Minimum Snap Trajectory Generation and Control for Quadrotors, http://www.personal.acfr.usyd.edu.au/spns/cdm/papers/Mellinger.pdf

[3] M. K. Daaboul. Kooperative Regelung von Quadrokoptern bei Bercksichtigung von Hindernissen und Kopplung, 2017, university of Kassel