Assignment 5
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1. Prove $\sum_{i=1}^{n} ba_i = b \sum_{i=1}^{n} a_i$

Base case: If n = 1, $\sum_{i=1}^{1} ba_i = ba_i = b \sum_{i=1}^{1} a_i$

General case:

Inductive Hypothesis: $\sum_{i=1}^{n} ba_i = b \sum_{i=1}^{n} a_i$

Show: $\sum_{i=1}^{n+1} ba_i = b \sum_{i=1}^{n+1} a_i$

$$\sum_{i=1}^{n+1} ba_i = \sum_{i=1}^{n} ba_i + ba_{n+1}$$
$$= b \sum_{i=1}^{n} a_i + ba_{n+1}$$
$$= b(\sum_{i=1}^{n} a_i + a_{n+1})$$
$$= b \sum_{i=1}^{n+1} a_i$$

Therefore $\sum_{i=1}^{n} ba_i = b \sum_{i=1}^{n} a_i$

Prove $\sum_{i=1}^{n}(a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$

Base case: If n = 1, $\sum_{i=1}^{1}(a_i + b_i) = a_i + b_i = \sum_{i=1}^{1} a_i + \sum_{i=1}^{1} b_i$

General case:

Inductive Hypothesis: $\sum_{i=1}^{n}(a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$

Show: $\sum_{i=1}^{n+1}(a_i + b_i) = \sum_{i=1}^{n+1} a_i + \sum_{i=1}^{n+1} b_i$

$$\sum_{i=1}^{n+1}(a_i + b_i) = \sum_{i=1}^{n}(a_i + b_i) + (a_{n+i} + b_{n+i})$$
$$= \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i + (a_{n+i} + b_{n+i})$$

$$= (\sum_{i=1}^{n} a_i + a_{n+i}) + (\sum_{i=1}^{n} b_i + b_{n+i})$$

$$= \sum_{i=1}^{n+1} a_i + \sum_{i=1}^{n+1} b_i$$

Therefore $\sum_{i=1}^{n}(a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$

2. $F_n = F_{n-1} + F_{n-2}$, $F_0 = 0$, $F_1 = 1$
     Find m that $F_m > (3/2)^m$

n	F_n	$(3/2)^n$
0	0	1
1	1	1.5
2	1	2.25
3	2	3.375
4	3	5.0625
5	5	7.59375
6	8	11.390625
7	13	17.0859375
8	21	25.62890625
9	34	38.44335938
10	55	57.66503906
11	89	86.49755859
12	144	129.7463379
13	233	194.6195068

From the table we can get when n = 11, the value of $F_n > (3/2)^n$, so 11 is the smallest non-negative integer.

Prove $F_n > (3/2)^n$ for all values of n > 11

Base case: If n = 11, $F_n > (3/2)^n$ is true by last step.

General case:

Inductive hypothesis: $F_n > (3/2)^n$, $F_{n-1} > (3/2)^{n-1}$

Show: $F_{n+1} > (3/2)^{n+1}$

$$
\begin{aligned}
F_{n+1} = F_n + F_{n-1} &> (3/2)^n + (3/2)^{n-1} \\
= F_n + F_{n-1} &> (3/2)^n + (3/2)^n * (3/2)^{-1} \\
= F_n + F_{n-1} &> (3/2)^n * (1 + 2/3) \\
= F_n + F_{n-1} &> (3/2)^n * (5/3) \\
= F_n + F_{n-1} &> (3/2)^{n+1} * (2/3) * (5/3) \\
= F_n + F_{n-1} &> (3/2)^{n+1} * (10/9)
\end{aligned}
$$

Since 10/9 > 1, so $(3/2)^{n+1} * (10/9) > (3/2)^{n+1}$, and $F_{n+1} > (3/2)^{n+1} * (10/9) > (3/2)^{n+1}$.

Therefore $F_n > (3/2)^n$ for all values of n > 11

3.   Prove a truth table with n variables has 2^n rows

Base case: If n = 1, the truth table has 2 rows(T and F). which is 2^1.

General case:

Inductive hypothesis: a truth table with n variables has 2^n rows.

Show: a truth table with n+1 variables has 2^{n+1} rows

Since a truth table with n variables has 2^n rows. The extra variable in n+1 has two copies of the table of with n variables(T for one copy and F for another copy). Therefore the total number of rows of n+1 variables table is $2 * 2^n = 2^{n+1}$.

4.   Let $x_1...x_n$ be binary variables. Prove parity($x_1...x_n$)=$x_1 \oplus ... \oplus x_n$

Base case: If n = 1, parity(0) = 0, parity(1) = 1, therefore parity(x) = x.

General case:

Inductive hypothesis: $x_1 \oplus ... \oplus x_n = parity(x_1...x_n)$

Show: $x_1 \oplus ... \oplus x_{n+1} = parity(x_1...x_n) \oplus x_{n+1}$

If $x_{n+1} = 0$, $parity(x_1...x_n) \oplus 0$, *Since* $x \oplus 0 = x$, $parity(x_1...x_n, 0) = parity(x_1...x_n)$

$\qquad parity(x_1...x_n) \oplus x_{n+1} = parity(x_1...x_{n+1})$

If $x_{n+1} = 1$, $parity(x_1...x_n) \oplus 1$, *Since* $x \oplus 1 = c$ *(complement of x)*

$\qquad parity(x_1...x_n, 1) = \overline{parity(x_1...x_n)}$ *(complement of parity($x_1...x_n$))*

5. Prove $G_n = [0G_{n-1}, 1G'_{n-1}]$ is gray code

Base case: If $n = 1$, $[0..2^1-1] = [0..1]$, which is a gray code.
General case:
Inductive hypothesis: G_{n-1} is a gray code
Show: $G_n = [0G_{n-1}, 1G'_{n-1}]$ is a gray code

Since G_{n-1} is a gray code, $0G_{n-1}$, $1G_{n-1}$ and G'_{n-1} are gray code. Since the last element of G_{n-1} is the same as the first element of G'_{n-1}, so the last element of $0G_{n-1}$ and the first element of $1G'_{n-1}$ differ by exactly one bit.
Therefore $G_n = [0G_{n-1}, 1G'_{n-1}]$ is a gray code.

6. Prove $G_n(i)$, $i=0,...,2^n-1 = [G_n(0)...G_n(2^n-1)]$ is a binary-reflected gray code

Base case: If $n = 1$, $G_1(i)=[G_1(0),G_1(1)] = [0,1]$, which is a gray code.
General case:
Inductive hypothesis: $G_n = [G_n(0)...G_n(2^n-1)]$ is a binary-reflected gray code
Show: $G_{n+1} = [G_{n+1}(0)...G_{n+1}(2^{n+1}-1)]$ is a binary-reflected gray code

$[G_{n+1}(0)...G_{n+1}(2^{n+1}-1)] =$