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1.11:

$g_1(n) = n^2$ for even $n \geq 0$, $g_2(n) = n$ for $0 \leq n \leq 100$, $g_1(n)/g_2(n) = n^2/n = n \to \infty$ as $n \to \infty$
So, $g_1(n)$ is $\Omega(g_2(n))$ for even n in $0 \leq n \leq 100$
$g_2(n) = n^3$ for $n > 100$, $g_1(n)/g_2(n) = n^2/n^3 = 1/n \to 0$ as $n \to \infty$
So, $g_1(n)$ is $O(g_2(n))$ for even n in $n > 100$
$g_3(n) = n^{2.5}$, $g_1(n)/g_3(n) = n^2/n^{2.5} = 1/n^{0.5} \to 0$ as $n \to \infty$
So, $g_1(n)$ is $O(g_3(n))$ for even $n \geq 0$

$g_1(n) = n^3$ for odd $n \geq 1$, $g_2(n) = n$ for $0 \leq n \leq 100$, $g_1(n)/g_2(n) = n^3/n = n^2 \to \infty$ as $n \to \infty$
So, $g_1(n)$ is $\Omega(g_2(n))$ for odd n in $0 \leq n \leq 100$
$g_2(n) = n^3$ for $n > 100$, $g_1(n)/g_2(n) = n^3/n^3 = 1 \to 1$ as $n \to \infty$
So, $g_1(n)$ is $O(g_2(n))$ for odd n in $n > 100$
$g_3(n) = n^{2.5}$, $g_1(n)/g_3(n) = n^3/n^{2.5} = n^{1.5} \to \infty$ as $n \to \infty$
So, $g_1(n)$ is $\Omega(g_3(n))$ for odd $n \geq 1$

$g_2(n) = n$ for $0 \leq n \leq 100$, $g_1(n) = n^2$ for even $n \geq 0$, $g_2(n)/g_1(n) = n/n^2 = 1/n \to 0$ as $n \to \infty$
So, $g_2(n)$ is $O(g_1(n))$ for even n in $0 \leq n \leq 100$
$g_1(n) = n^3$ for odd $n \geq 1$, $g_2(n)/g_1(n) = n/n^3 = 1/n^2 \to 0$ as $n \to \infty$
So, $g_2(n)$ is $O(g_1(n))$ for odd n in $0 \leq n \leq 100$
$g_3(n) = n^{2.5}$, $g_2(n)/g_3(n) = n/n^{2.5} = 1/n^{1.5} \to 0$ as $n \to \infty$
So, $g_2(n)$ is $O(g_3(n))$ for $0 \leq n \leq 100$

$g_2(n) = n^3$ for $n > 100$, $g_1(n) = n^2$ for even $n \geq 0$, $g_2(n)/g_1(n) = n^3/n^2 = n \to \infty$ as $n \to \infty$
So, $g_2(n)$ is $\Omega(g_1(n))$ for even n in $0 \leq n \leq 100$
$g_1(n) = n^3$ for odd $n \geq 1$, $g_2(n)/g_1(n) = n^3/n^3 = 1 \to 1$ as $n \to \infty$
So, $g_2(n)$ is $O(g_1(n))$ for odd n in $0 \leq n \leq 100$
$g_3(n) = n^{2.5}$, $g_2(n)/g_3(n) = n^3/n^{2.5} = n^{0.5} \to \infty$ as $n \to \infty$
So, $g_2(n)$ is $\Omega(g_3(n))$ for $n > 100$

$g_3(n)$ is neither $O(g_1(n))$ nor $\Omega(g_1(n))$
$g_3(n)$ is neither $O(g_2(n))$ nor $\Omega(g_2(n))$

1.13:

a. $17/1 \to 17$, So 17 is $O(1)$

b. $n(n-1)/2 / n^2 = (n^2-n) / (2n^2) = ½ - 1/(2n) \to ½$, So $n(n-1)/2$ is $O(n^2)$

c. $\max(n^3, 10n^2) / n^3 = \max(1, 10/n) \to \max(1, 0) \to 1$, So $\max(n^3, 10n^2)$ is $O(n^3)$

d. If k>0, $\sum_{i=1}^{n} i^k = 1^k + 2^k + \ldots + n^k \to \infty$, $\sum_{i=1}^{n} i^k / n^{k+1} \to \infty$, So $\sum_{i=1}^{n} i^k$ is $\Omega(n^{k+1})$
If k<0, $\sum_{i=1}^{n} i^k = 1^k + 2^k + \ldots + n^k \to$ some constant $> 1^k$, $\sum_{i=1}^{n} i^k / n^{k+1} \to$ constant, So $\sum_{i=1}^{n} i^k$ is $O(n^{k+1})$

e. k polynomial $P(n) = a_k n^k + a_{k-1} n^{k-1} + \ldots + a_0$
If k>0, $(a_k n^k + a_{k-1} n^{k-1} + \ldots\ldots + a_0)/ n^k = a_k + a_{k-1}/n + a_{k-2}/n^2 \ldots + a_0/n^k \to a_k/$ as $n \to \infty$, So P(n) is $O(n^k)$
If k<0, $(a_k n^k + a_{k-1} n^{k-1} + \ldots\ldots + a_0)/ n^k = a_k + a_{k-1} n + a_{k-2} n^2 \ldots + a_0 n^k \to \infty$ as $n \to \infty$, So P(n) is $\Omega(n^k)$

1.14:
a is true. Since $T_1(n)$ is $\Omega(f(n))$, $T_1(n) \geq c_1*f(n)$ for $n>n_0$ and $T_2(n)$ is $\Omega(g(n))$, $T_2(n) \geq c_2*g(n)$.
$T_1(n) + T_2(n) \geq c_1*f(n) + c_2*g(n) \rightarrow c*(f(n) + g(n))$
Since $f(n) + g(n) = max(f(n), g(n))$ by the book.
Therefore $T_1(n) + T_2(n) \geq c*(f(n) + g(n))$ is equivalent to $T_1(n) + T_2(n)$ is $\Omega(max(f(n), g(n)))$.
b is false. Because either $T_1(n)$, $T_2(n)$ or even both of them are in the range (0,1), then the multiplication will be $T_1(n)*T_2(n) \leq f(n)*g(n)$, so it is not always true.

1.17:
n = 2, 4, 8, 16…

n = 2	count = 0
n = 4	count = 1
n = 8	count = 2
n = 16 . . . $n = 2^{count + 1}$	count = 3 . . . $count = log_2n - 1$