

# MECE-743 Digital Control Systems

## Take Home Mid-Term:

Due Date: Monday March 28, 2016

It is assumed you will be doing this exam alone without any outside aid.

**Given:** The plant transfer function represents the heating of a room in a house is as follows where  $R$  is the thermal resistance in the walls and  $C$  is the thermal capacitance of the air in the room.

$$G_P = \frac{R}{RCs + 1}$$

The input  $u(t)$  is the heat flow coming from a space heater. Assume  $R=150$  and  $C=0.1$ . The room heating control system is as follows. The sensor transfer function  $H(s)=1$ .

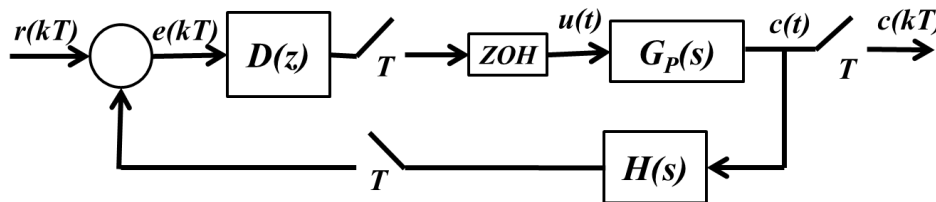


Figure 1 - Room Heating Control System

### Goal #1:

1. Derive the discrete-time closed-loop transfer function symbolically.
2. Determine the stability region if  $D(z)$  is a proportional controller,  $K$  for a sampling time of  $T=1$  and  $T=10$  seconds
3. Simulate both sampling time cases in Matlab when the reference input is  $r(kt)=40$ . Select two different  $K$  values. (1) on the edge of stability ( $z=\pm 1$ ), (2) the poles are at the origin ( $z=0$ ).
4. For the  $T=10$  seconds case determine the output value at  $t=0, 5, 10, 15$ , etc seconds up to 50 seconds using the method of modified Z-transforms.
5. In Simulink simulated this control system using a fixed sample discrete time integrator (ode5) at  $T=0.01$  seconds. Using a triggered subsystem simulate the controller only updating at  $T=1$  and  $T=10$ . This will provide plant output values at  $T=0.01$  to verify your modified Z-transform calculations. (this will require you to learn triggered subsystems)

### Goal #2:

Prove the effect of sampling by curve fitting two sets of data with sine waves of various frequencies. Create a loop that increments the frequency range  $f$  of the following equation  $y = A \sin(2\pi f * t)$  from 0 to 50 Hz. Each iteration least squares fit (or polyfit) the function with the data and determine the RMS error for each iteration. A perfect fit of the data will result in a near zero RMS indicating a possible frequency component. It might be helpful to plot each fit for each iteration to see the quality visually. Plot to corresponding RMS values versus frequency to identify the possible solutions to the data. The data was created with a single frequency. Can you determine what it was? What is your guess?

**Deliverable:** A concise report on the approach with a discussion of the final results is required. The number of plots should be kept to a minimum but be of high quality and description (legends, captions). Your lengthy theoretical derivations can be neatly hand written. Please submit all of your Matlab/Simulink code separately and consolidate it to as few pages as possible.