9.2-1 Show that for a real $x[n]$, Eq. (9.29) can be expressed as

$$x[n] = \frac{1}{\pi}\int_0^{\pi} |X(\Omega)| \cos(\Omega n + \angle X(\Omega))\, d\Omega$$

This is the trigonometric form of the DTFT.

9.2-2 A signal $x[n]$ can be expressed as the sum of even and odd components (Section 1.5-1):

$$x[n] = x_e[n] + x_o[n]$$

(a) If $x[n] \Longleftrightarrow X(\Omega)$, show that for real $x[n]$,

$$x_e[n] \Longleftrightarrow \text{Re}[X(\Omega)]$$

and

$$x_o[n] \Longleftrightarrow j\,\text{Im}[X(\Omega)]$$

(b) Verify these results by finding the DTFT of the even and odd components of the signal $(0.8)^n u[n]$.

9.2-3 For the following signals, find the DTFT directly, using the definition in Eq. (9.30). Assume $|\gamma| < 1$.

(a) $\delta[n]$
(b) $\delta[n-k]$
(c) $\gamma^n u[n-1]$
(d) $\gamma^n u[n+1]$
(e) $(-\gamma)^n u[n]$
(f) $\gamma^{|n|}$

9.2-4 Use Eq. (9.29) to find the inverse DTFT for the following spectra, given only over the interval $|\Omega| \le \pi$. Assume Ω_c and $\Omega_0 < \pi$.

(a) $e^{jk\Omega}$ integer k
(b) $\cos k\Omega$ integer k
(c) $\cos^2(\Omega/2)$
(d) $\Delta\left(\dfrac{\Omega}{2\Omega_c}\right)$
(e) $2\pi\delta(\Omega - \Omega_0)$
(f) $\pi[\delta(\Omega - \Omega_0) + \delta(\Omega + \Omega_0)]$

9.2-5 Using Eq. (9.29), show that the inverse DTFT of rect $((\Omega - \pi/4)/\pi)$ is 0.5 sinc $(\pi n/2)\, e^{j\pi n/4}$.

9.2-6 From definition (9.30), find the DTFT of the signals $x[n]$ in Fig. P9.2-6.

9.2-7 From definition (9.30), find the DTFT of the signals depicted in Fig. P9.2-7.

9.2-8 Use Eq. (9.29) to find the inverse DTFT of the spectra (shown only for $|\Omega| \le \pi$) in Fig. P9.2-8.

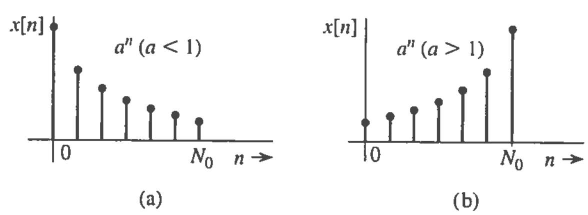

Figure P9.2-6

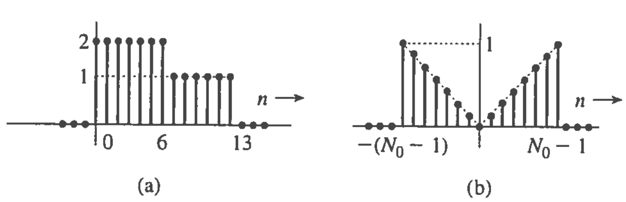

Figure P9.2-7

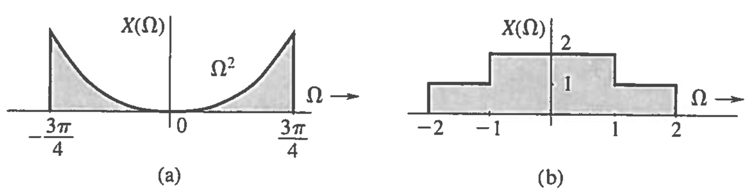

Figure P9.2-8

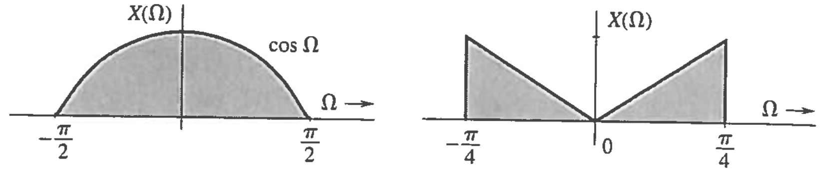

Figure P9.2-9

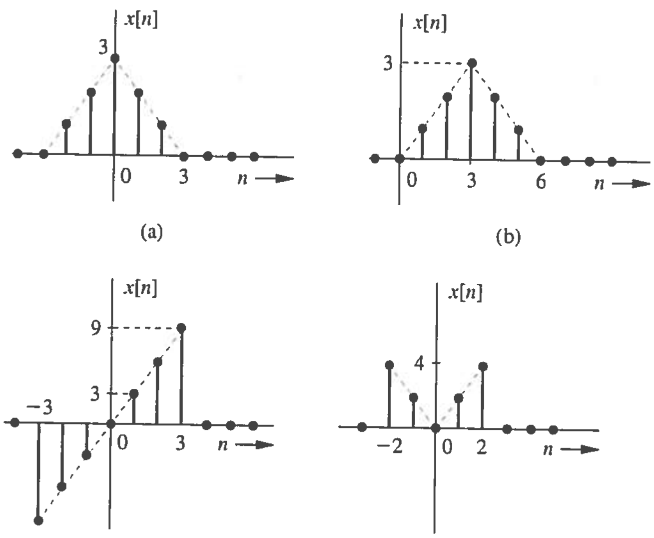

Figure P9.2-10

9.2-9 Use Eq. (9.29) to find the inverse DTFT of the spectra (shown only for $|\Omega| \le \pi$) in Fig. P9.2-9.

9.2-10 Find the DTFT for the signals shown in Fig. P9.2-10.

9.2-11 Find the inverse DTFT of $X(\Omega)$ (shown only for $|\Omega| \le \pi$) for the spectra illustrated in Fig. P9.2-11. [Hint: $X(\Omega) = |X(\Omega)|e^{j\angle X(\Omega)}$. This problem illustrates how different phase spectra (both with the same amplitude spectrum) represent entirely different signals.]

9.2-12 (a) Show that time-expanded signal $x_e[n]$ in Eq. (3.4) can also be expressed as

$$x_e[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n - Lk]$$

(b) Find the DTFT of $x_e[n]$ by finding the DTFT of the right-hand side of the equation in part (a).

(c) Use the result in part (b) and Table 9.1 to find the DTFT of $z[n]$, shown in Fig. P9.2-12.

9.2-13 (a) A glance at Eq. (9.29) shows that the inverse DTFT equation is identical to the inverse (continuous-time) Fourier transform Eq. (7.8b) for a signal $x(t)$ bandlimited to π rad/s. Hence, we should be able to use the continuous-time Fourier transform Table 7.1 to find DTFT pairs that correspond to continuous-time transform pairs for bandlimited signals. Use this fact to derive DTFT pairs 8, 9, 11, 12, 13, and 14 in Table 9.1 by means of the appropriate pairs in Table 7.1.

(b) Can this method be used to derive pairs 2, 3, 4, 5, 6, 7, 10, 15, and 16 in Table 9.1? Justify your answer with specific reason(s).

Figu

F

9.2-14 Are the followir valid DTFT's? A your answers.

(a) $X(\Omega) = \Omega +$
(b) $X(\Omega) = j +$
(c) $X(\Omega) = \sin$
(d) $X(\Omega) = \sin$
(e) $X(\Omega) = \delta(\Omega$

9.3-1 Using only pairs time-shifting pro of the following s

(a) $u[n] - u[n -$
(b) $a^{n-m}u[n - m$
(c) $a^{n-3}(u[n] -$
(d) $a^{n-m}u[n]$
(e) $a^n u[n - m]$
(f) $(n - m)a^{n-m}$
(g) $(n - m)a^n u[$
(h) $na^{n-m}u[n -$

9.3-2 The triangular P9.3-2a is given b

$X(\Omega) =$

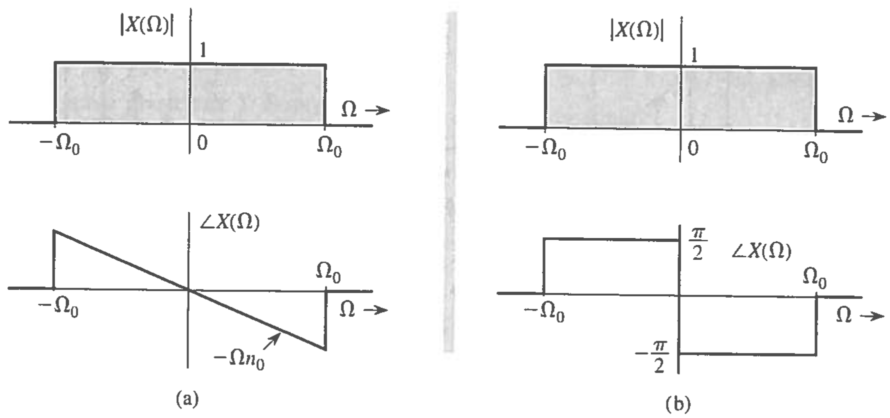

Figure P9.2-11

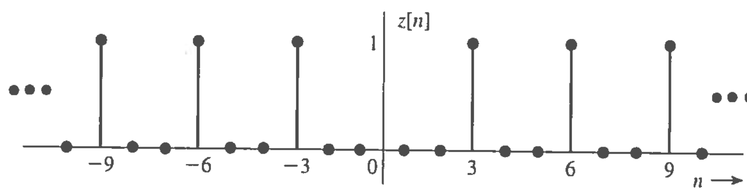

Figure P9.2-12

9.2-14 Are the following frequency domain signals valid DTFT's? Answer yes or no, and justify your answers.

(a) $X(\Omega) = \Omega + \pi$
(b) $X(\Omega) = j + \pi$
(c) $X(\Omega) = \sin(10\Omega)$
(d) $X(\Omega) = \sin(\Omega/10)$
(e) $X(\Omega) = \delta(\Omega)$

9.3-1 Using only pairs 2 and 5 (Table 9.1) and the time-shifting property (9.52), find the DTFT of the following signals, assuming $|a| < 1$

(a) $u[n] - u[n-9]$
(b) $a^{n-m}u[n-m]$
(c) $a^{n-3}(u[n] - u[n-10])$
(d) $a^{n-m}u[n]$
(e) $a^n u[n-m]$
(f) $(n-m)a^{n-m}u[n-m]$
(g) $(n-m)a^n u[n]$
(h) $na^{n-m}u[n-m]$

9.3-2 The triangular pulse $x[n]$ shown in Fig. P9.3-2a is given by

$$X(\Omega) = \frac{4e^{j6\Omega} - 5e^{j5\Omega} + e^{j\Omega}}{(e^{j\Omega} - 1)^2}$$

Use this information and the DTFT properties to find the DTFT of the signals $x_1[n]$, $x_2[n]$, $x_3[n]$, and $x_4[n]$ shown in Fig. P9.3-2b, P9.3-2c, P9.3-2d, and P9.3-2e, respectively.

9.3-3 Show that periodic convolution $X(\Omega) \circledast Y(\Omega) = 2\pi X(\Omega)$ if

$$X(\Omega) = \sum_{k=0}^{4} a_k\, e^{-jk\Omega}$$

and

$$Y(\Omega) = \frac{\sin(5\Omega/2)}{\sin(\Omega/2)}\, e^{-j2\Omega}$$

where a_k is a set of arbitrary constants.

9.3-4 Using only pair 2 (Table 9.1) and properties of DTFT, find the DTFT of the following signals, assuming $|a| < 1$ and $\Omega_0 < \pi$.

(a) $a^n \cos \Omega_0 n u[n]$
(b) $n^2 a^n u[n]$
(c) $(n-k)a^{2n}u[n-m]$

9.3-5 Use pair 10 in Table 9.1, and some suitable property or properties of the DTFT, to derive pairs 11, 12, 13, 14, 15, and 16.

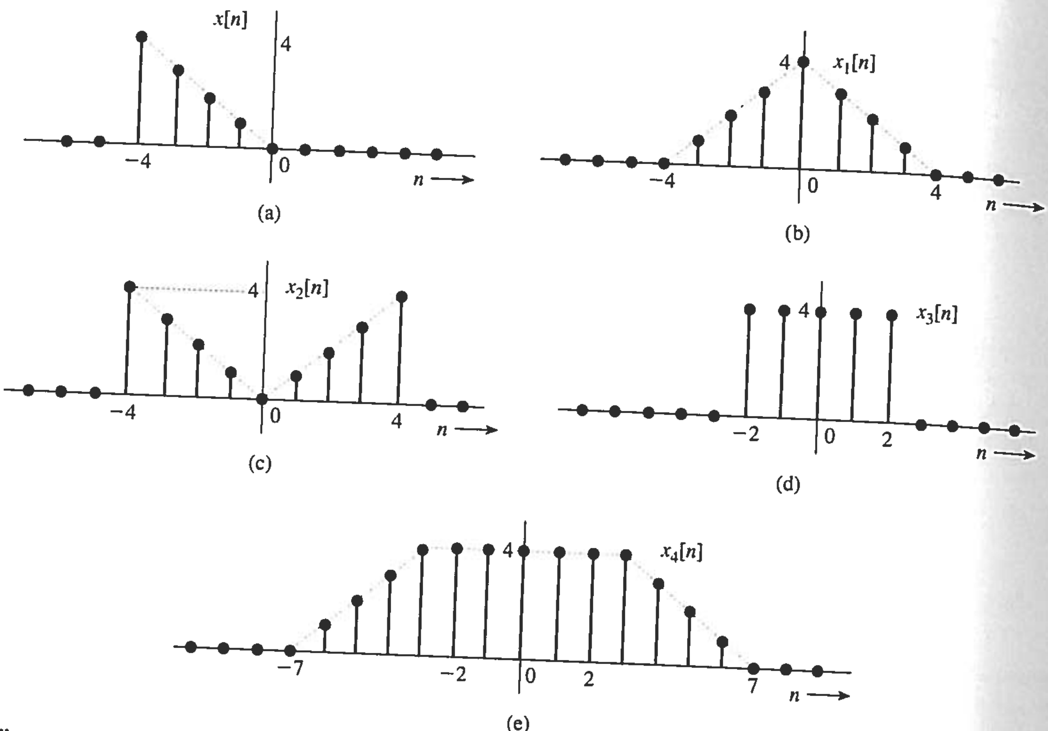

Figure P9.3-2

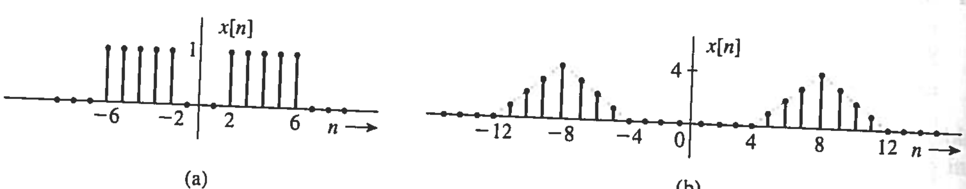

Figure P9.3-6

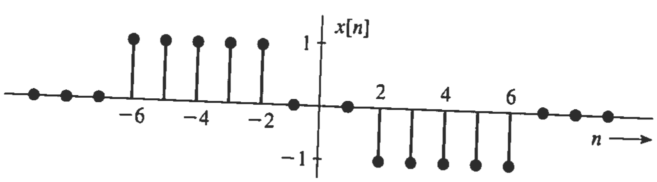

Figure P9.3-7

9.3-6 Use the time-shifting property to show that

$$x[n+k] + x[n-k] \Longleftrightarrow 2X(\Omega)\cos k\Omega$$

Use this result to find the DTFT of the signals shown in Fig. P9.3-6.

9.3-7 Use the time-shifting property to show that

$$x[n+k] - x[n-k] \Longleftrightarrow 2jX(\Omega)\sin k\Omega$$

Use this result to find the DTFT of the signal shown in Fig. P9.3-7.

9.3-8 Using onl
volution p
$X(\Omega) = e$

9.3-9 In Table 9
informatic
or properti
5, 6, and 7
with pair 1
able prope
From pairs

9.3-10 From the p
over the fu
convolution
Assume Ω_c

9.3-11 From the
DTFT, show

(a) $\sum_{n=-\infty}^{\infty}$ si

(b) $\sum_{n=-\infty}^{\infty}$ (−

(c) $\sum_{n=-\infty}^{\infty}$ sin

(d) $\sum_{n=-\infty}^{\infty}$ (−

(e) $\int_{-\pi}^{\pi}$ sin(/ sin

(f) $\sum_{n=-\infty}^{\infty}$ |sin

9.3-12 Show that the
in Eq. (9.73) i
of the discrete
is bandlimited
that sinc funct

$\int_{-\infty}^{\infty}$ sinc [

$= \begin{cases} 0 \\ 1 \end{cases}$

9.4-1 Use the DTFT
sponse $y[n]$ of

9.3-8 Using only pair 2 in Table 9.1 and the convolution property, find the inverse DTFT of $X(\Omega) = e^{2j\Omega}/(e^{j\Omega} - \gamma)^2$.

9.3-9 In Table 9.1, you are given pair 1. From this information and using some suitable property or properties of the DTFT, derive pairs 2, 3, 4, 5, 6, and 7 of Table 9.1. For example, starting with pair 1, derive pair 2. From pair 2, use suitable properties of the DTFT to derive pair 3. From pairs 2 and 3, derive pair 4, and so on.

9.3-10 From the pair $e^{j(\Omega_0/2)n} \Longleftrightarrow 2\pi\delta(\Omega - (\Omega_0/2))$ over the fundamental band, and the frequency-convolution property, find the DTFT of $e^{j\Omega_0 n}$. Assume $\Omega_0 < \pi/2$.

9.3-11 From the definition and properties of the DTFT, show that

(a) $$\sum_{n=-\infty}^{\infty} \text{sinc}\,(\Omega_c n) = \frac{\pi}{\Omega_c} \quad \Omega_c < \pi$$

(b) $$\sum_{n=-\infty}^{\infty} (-1)^n \text{sinc}\,(\Omega_c n) = 0 \quad \Omega_c < \pi$$

(c) $$\sum_{n=-\infty}^{\infty} \text{sinc}^2(\Omega_c n) = \frac{\pi}{\Omega_c} \quad \Omega_c < \pi/2$$

(d) $$\sum_{n=-\infty}^{\infty} (-1)^n \text{sinc}^2(\Omega_c n) = 0 \quad \Omega_c < \pi/2$$

(e) $$\int_{-\pi}^{\pi} \frac{\sin(M\Omega/2)}{\sin(\Omega/2)} = 2\pi \quad \text{odd } M$$

(f) $$\sum_{n=-\infty}^{\infty} |\text{sinc}\,(\Omega_c n)|^4 = 2\pi/3\Omega_c \quad \Omega_c < \pi/2$$

9.3-12 Show that the energy of signal $x_c(t)$ specified in Eq. (9.73) is identical to T times the energy of the discrete-time signal $x[n]$, assuming $x_c(t)$ is bandlimited to $B \le 1/2T$ Hz. [Hint: Recall that sinc functions are orthogonal, that is,

$$\int_{-\infty}^{\infty} \text{sinc}\,[\pi(t-m)]\,\text{sinc}\,[\pi(t-n)]\,dt = \begin{cases} 0 & m \ne n \\ 1 & m = n \end{cases}$$

9.4-1 Use the DTFT method to find the zero-state response $y[n]$ of a causal system with frequency response

$$H(\Omega) = \frac{e^{j\Omega} + 0.32}{e^{j2\Omega} + e^{j\Omega} + 0.16}$$

and the input

$$x[n] = (-0.5)^n u[n]$$

9.4-2 Repeat Prob. 9.4-1 for

$$H(\Omega) = \frac{e^{j\Omega} + 0.32}{e^{j2\Omega} + e^{j\Omega} + 0.16}$$

and input

$$x[n] = u[n]$$

9.4-3 Repeat Prob. 9.4-1 for

$$H(\Omega) = \frac{e^{j\Omega}}{e^{j\Omega} - 0.5}$$

and

$$x[n] = 0.8^n u[n] + 2(2)^n u[-(n+1)]$$

9.4-4 An accumulator system has the property that an input $x[n]$ results in the output

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

(a) Find the unit impulse response $h[n]$ and the frequency response $H(\Omega)$ for the accumulator.
(b) Use the results in part a to find the DTFT of $u[n]$.

9.4-5 An LTID system frequency response over $|\Omega| \le \pi$ is

$$H(\Omega) = \text{rect}\left(\frac{\Omega}{\pi}\right) e^{-j2\Omega}$$

Find the output $y[n]$ of this system, if the input $x[n]$ is given by

(a) sinc $(\pi n/2)$
(b) sinc (πn)
(c) $\text{sinc}^2\,(\pi n/4)$

9.4-6 (a) If $x[n] \Longleftrightarrow X(\Omega)$, then, show that $(-1)^n x[n] \Longleftrightarrow X(\Omega - \pi)$.
(b) Sketch $\gamma^n u[n]$ and $(-\gamma)^n u[n]$ for $\gamma = 0.8$; see the spectra for $\gamma^n u[n]$ in Fig. 9.4b and 9.4c. From these spectra, sketch the spectra for $(-\gamma)^n u[n]$.
(c) An ideal lowpass filter of cutoff frequency Ω_c is specified by the frequency response