TABLE 5.1 Select (Unilateral) z-Transform Pairs

No.	$x[n]$	$X[z]$
1	$\delta[n-k]$	z^{-k}
2	$u[n]$	$\dfrac{z}{z-1}$
3	$nu[n]$	$\dfrac{z}{(z-1)^2}$
4	$n^2u[n]$	$\dfrac{z(z+1)}{(z-1)^3}$
5	$n^3u[n]$	$\dfrac{z(z^2+4z+1)}{(z-1)^4}$
6	$\gamma^n u[n]$	$\dfrac{z}{z-\gamma}$
7	$\gamma^{n-1}u[n-1]$	$\dfrac{1}{z-\gamma}$
8	$n\gamma^n u[n]$	$\dfrac{\gamma z}{(z-\gamma)^2}$
9	$n^2\gamma^n u[n]$	$\dfrac{\gamma z(z+\gamma)}{(z-\gamma)^3}$
10	$\dfrac{n(n-1)(n-2)\cdots(n-m+1)}{\gamma^m m!}\gamma^n u[n]$	$\dfrac{z}{(z-\gamma)^{m+1}}$
11a	$\lvert\gamma\rvert^n \cos \beta n\, u[n]$	$\dfrac{z(z-\lvert\gamma\rvert\cos\beta)}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
11b	$\lvert\gamma\rvert^n \sin \beta n\, u[n]$	$\dfrac{z\lvert\gamma\rvert\sin\beta}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12a	$r\lvert\gamma\rvert^n \cos(\beta n+\theta)u[n]$	$\dfrac{rz[z\cos\theta-\lvert\gamma\rvert\cos(\beta-\theta)]}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12b	$r\lvert\gamma\rvert^n \cos(\beta n+\theta)u[n] \qquad \gamma=\lvert\gamma\rvert e^{j\beta}$	$\dfrac{(0.5re^{j\theta})z}{z-\gamma}+\dfrac{(0.5re^{-j\theta})z}{z-\gamma^*}$
12c	$r\lvert\gamma\rvert^n \cos(\beta n+\theta)u[n]$ $r=\sqrt{\dfrac{A^2\lvert\gamma\rvert^2+B^2-2AaB}{\lvert\gamma\rvert^2-a^2}}$ $\beta=\cos^{-1}\dfrac{-a}{\lvert\gamma\rvert}$ $\theta=\tan^{-1}\dfrac{Aa-B}{A\sqrt{\lvert\gamma\rvert^2-a^2}}$	$\dfrac{z(Az+B)}{z^2+2az+\lvert\gamma\rvert^2}$

and

$$\lim_{z\to 1}\frac{(z-1)X[z]}{z}=\lim_{z\to 1}(z-1)X[z]=\lim_{z\to 1}\lim_{N\to\infty}\sum_{n=-\infty}^{N}\{x[n]-x[n-1]\}z^{-n}=\lim_{N\to\infty}x[N]$$

All these properties of the z-transform are listed in Table 5.2.

TABLE 5.2 z-Transform Properties

Operation	$x[n]$	$X[z]$	
Addition	$x_1[n]+x_2[n]$	$X_1[z]+X_2[z]$	
Scalar multiplication	$ax[n]$	$aX[z]$	
Right shifting	$x[n-m]u[n-m]$	$\frac{1}{z^m}X[z]$	
	$x[n-m]u[n]$	$\frac{1}{z^m}X[z]+\frac{1}{z^m}\sum_{n=1}^{m}x[-n]z^n$	
	$x[n-1]u[n]$	$\frac{1}{z}X[z]+x[-1]$	
	$x[n-2]u[n]$	$\frac{1}{z^2}X[z]+\frac{1}{z}x[-1]+x[-2]$	
	$x[n-3]u[n]$	$\frac{1}{z^3}X[z]+\frac{1}{z^2}x[-1]+\frac{1}{z}x[-2]+x[-3]$	
Left shifting	$x[n+m]u[n]$	$z^mX[z]-z^m\sum_{n=0}^{m-1}x[n]z^{-n}$	
	$x[n+1]u[n]$	$zX[z]-zx[0]$	
	$x[n+2]u[n]$	$z^2X[z]-z^2x[0]-zx[1]$	
	$x[n+3]u[n]$	$z^3X[z]-z^3x[0]-z^2x[1]-zx[2]$	
Multiplication by γ^n	$\gamma^n x[n]u[n]$	$X\left[\frac{z}{\gamma}\right]$	
Multiplication by n	$nx[n]u[n]$	$-z\frac{d}{dz}X[z]$	
Time reversal	$x[-n]$	$X[1/z]$	
Time convolution	$x_1[n]*x_2[n]$	$X_1[z]X_2[z]$	
Initial value	$x[0]$	$\lim_{z\to\infty}X[z]$	
Final value	$\lim_{N\to\infty}x[N]$	$\lim_{z\to 1}(z-1)X[z]$	Poles of $(z-1)X[z]$ inside the unit circle