

Fundamentals *of*  
Multiagent Systems  
with NetLogo Examples

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**Fundamentals of Multiagent Systems**

by José M. Vidal

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## Contents

|   |           |
|---|-----------|
| <b>Preface</b>  | <b>7</b>  |
| 0.1 Usage . . . . .   | 7         |
| <b>1 Multiagent Problem Formulation</b>                         | <b>9</b>  |
| 1.1 Utility . . . . .   | 9         |
| 1.1.1 Utility is Not Money . . . . .                            | 10        |
| 1.1.2 Expected Utility . . . . .                                | 11        |
| 1.2 Markov Decision Processes . . . . .                         | 11        |
| 1.2.1 Multiagent Markov Decision Processes . . . . .            | 14        |
| 1.2.2 Partially Observable MDPs . . . . .                       | 14        |
| 1.3 Planning . . . . .  | 15        |
| 1.3.1 Hierarchical Planning . . . . .                           | 16        |
| 1.4 Summary . . . . .   | 16        |
| Exercises . . . . .   | 17        |
| <b>2 Distributed Constraints</b>                                | <b>19</b> |
| 2.1 Distributed Constraint Satisfaction . . . . .               | 19        |
| 2.1.1 Filtering Algorithm . . . . .                             | 20        |
| 2.1.2 Hyper-Resolution Based Consistency Algorithm . . . . .    | 22        |
| 2.1.3 Asynchronous Backtracking . . . . .                       | 24        |
| 2.1.4 Asynchronous Weak-Commitment Search . . . . .             | 27        |
| 2.1.5 Distributed Breakout . . . . .                            | 29        |
| 2.2 Distributed Constraint Optimization . . . . .               | 31        |
| 2.2.1 Adopt . . . . .   | 32        |
| 2.2.2 OptAPO . . . . .  | 37        |
| 2.3 Recent Advances . . . . .                                   | 38        |
| Exercises . . . . .   | 38        |
| <b>3 Standard and Extended Form Games</b>                       | <b>39</b> |
| 3.1 Games in Normal Form . . . . .                              | 39        |
| 3.1.1 Solution Concepts . . . . .                               | 40        |
| 3.1.2 Famous Games . . . . .                                    | 41        |
| 3.1.3 Repeated Games . . . . .                                  | 43        |
| 3.2 Games in Extended Form . . . . .                            | 44        |
| 3.2.1 Solution Concepts . . . . .                               | 45        |
| 3.3 Why Do We Care? . . . . .                                   | 46        |
| Exercises . . . . .   | 46        |
| <b>4 Characteristic Form Games and Coalition Formation</b>      | <b>49</b> |
| 4.1 Characteristic Form Games . . . . .                         | 49        |
| 4.1.1 Solution Concepts . . . . .                               | 50        |
| 4.1.2 Algorithms for Finding the Utilitarian Solution . . . . . | 54        |
| 4.2 Coalition Formation . . . . .                               | 56        |
| 4.3 Recent Advances . . . . .                                   | 57        |

|          |  |            |
|----------|--|------------|
| <b>5</b> | <b>Learning in Multiagent Systems</b>          | <b>59</b>  |
| 5.1      | The Machine Learning Problem . . . . .         | 59         |
| 5.2      | Cooperative Learning . . . . .                 | 60         |
| 5.3      | Repeated Games . . . . .                       | 61         |
| 5.3.1    | Fictitious Play . . . . .                      | 61         |
| 5.3.2    | Replicator Dynamics . . . . .                  | 63         |
| 5.3.3    | The AWESOME Algorithm . . . . .                | 65         |
| 5.4      | Stochastic Games . . . . .                     | 67         |
| 5.4.1    | Reinforcement Learning . . . . .               | 67         |
| 5.5      | General Theories for Learning Agents . . . . . | 70         |
| 5.5.1    | CLRI Model . . . . .                           | 70         |
| 5.5.2    | N-Level Agents . . . . .                       | 72         |
| 5.6      | Collective Intelligence . . . . .              | 73         |
| 5.7      | Summary . . . . .                              | 75         |
| 5.8      | Recent Advances . . . . .                      | 75         |
| <b>6</b> | <b>Negotiation</b>                             | <b>77</b>  |
| 6.1      | The Bargaining Problem . . . . .               | 77         |
| 6.1.1    | Axiomatic Solution Concepts . . . . .          | 77         |
| 6.1.2    | Strategic Solution Concepts . . . . .          | 81         |
| 6.2      | Monotonic Concession Protocol . . . . .        | 83         |
| 6.2.1    | The Zeuthen Strategy . . . . .                 | 84         |
| 6.2.2    | One-Step Protocol . . . . .                    | 86         |
| 6.3      | Negotiation as Distributed Search . . . . .    | 86         |
| 6.4      | Ad-hoc Negotiation Strategies . . . . .        | 87         |
| 6.5      | The Task Allocation Problem . . . . .          | 87         |
| 6.5.1    | Payments . . . . .                             | 89         |
| 6.5.2    | Lying About Tasks . . . . .                    | 91         |
| 6.5.3    | Contracts . . . . .                            | 92         |
| 6.6      | Complex Deals . . . . .                        | 93         |
| 6.6.1    | Annealing Over Complex Deals . . . . .         | 95         |
| 6.7      | Argumentation-Based Negotiation . . . . .      | 96         |
| 6.8      | Negotiation Networks . . . . .                 | 97         |
| 6.8.1    | Network Exchange Theory . . . . .              | 98         |
|          | Exercises . . . . .                            | 100        |
| <b>7</b> | <b>Auctions</b>                                | <b>103</b> |
| 7.1      | Valuations . . . . .                           | 103        |
| 7.2      | Simple Auctions . . . . .                      | 103        |
| 7.2.1    | Analysis . . . . .                             | 105        |
| 7.2.2    | Auction Design . . . . .                       | 106        |
| 7.3      | Combinatorial Auctions . . . . .               | 107        |
| 7.3.1    | Centralized Winner Determination . . . . .     | 107        |
| 7.3.2    | Distributed Winner Determination . . . . .     | 112        |
| 7.3.3    | Bidding Languages . . . . .                    | 115        |
| 7.3.4    | Preference Elicitation . . . . .               | 116        |
| 7.3.5    | VCG Payments . . . . .                         | 118        |
|          | Exercises . . . . .                            | 119        |
| <b>8</b> | <b>Voting and Mechanism Design</b>             | <b>121</b> |
| 8.1      | The Voting Problem . . . . .                   | 121        |
| 8.1.1    | Possible Solutions . . . . .                   | 122        |
| 8.1.2    | Voting Summary . . . . .                       | 124        |
| 8.2      | Mechanism Design . . . . .                     | 124        |
| 8.2.1    | Problem Description . . . . .                  | 124        |
| 8.2.2    | Distributed Mechanism Design . . . . .         | 131        |
| 8.2.3    | Mechanism Design Summary . . . . .             | 134        |

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|           |   |            |
|-----------|---|------------|
| 8.2.4     | Recent Advances in Mechanism Design . . . . .       | 134        |
| 8.3       | History . . . . .                                   | 134        |
| <b>9</b>  | <b>Coordination Using Goal and Plan Hierarchies</b> | <b>135</b> |
| 9.1       | TÆMS . . . . .                                      | 135        |
| 9.2       | GPGP . . . . .                                      | 137        |
| 9.2.1     | Agent Architecture . . . . .                        | 137        |
| 9.2.2     | Coordination . . . . .                              | 138        |
| 9.2.3     | Design-to-Criteria Scheduler . . . . .              | 139        |
| 9.2.4     | GPGP/TÆMS Summary . . . . .                         | 139        |
| <b>10</b> | <b>Nature-Inspired Approaches</b>                   | <b>141</b> |
| 10.1      | Ants and Termites . . . . .                         | 141        |
| 10.2      | Immune System . . . . .                             | 141        |
| 10.3      | Physics . . . . .                                   | 141        |
|           | <b>Bibliography</b>                                 | <b>143</b> |
|           | <b>Index</b>  | <b>151</b> |



## Preface

The goal of this book is to cover all the material that a competent multiagent practitioner or researcher should be familiar with. Of course, since this is a relatively new field the list of required material is still growing and there is some uncertainty as to what are the most important ideas. I have made some choices in putting this book. I have concentrated on the theoretical aspects of multiagent systems since these ideas have been around for a long time and are important for a wide variety of applications. I have stayed away from technological issues because these are evolving very fast. Also, we would require another textbook to describe all the distributed programming tools available.

The book is also linked to a large number of sample NetLogo programs (Wilensky, 1999). These programs are meant to be used by the reader as an aid in understanding emergent decentralized behaviors. It is hard for people, especially those new to distributed systems, to fully grasp the order that can arise from seemingly chaotic simple interactions. As Resnick notices:

*“But even as the influence of decentralized ideas grows, there is a deep-seated resistance to such ideas. At some deep level, people seem to have strong attachments to centralized ways of thinking. When people see patterns in the world (like a flock of birds), they often assume that there is some type of centralized control (a leader of the flock). According to this way of thinking, a pattern can exist only if someone (or something) creates and orchestrates the pattern. Everything must have a single cause, and ultimate controlling factor. The continuing resistance to evolutionary theories is an example: many people still insist that someone or something must have explicitly designed the complex, orderly structures that we call Life.”* (Resnick, 1994)

The book assumes the reader is familiar with basic Artificial Intelligence techniques (Russell and Norvig, 2003). The reader should also be comfortable with mathematical notation and basic computer science algorithms. The book is written for a graduate or advanced undergraduate audience. I also recommend (Mas-Colell et al., 1995; Osborne and Rubinstein, 1999) as reference books.

### 0.1 Usage

If you are using the PDF version of this document you can click on any of the citations and your PDF reader will take you to the appropriate place in the bibliography. From there you can click on the title of the paper and your web browser will take you to a page with a full description of the paper. You can also get the full paper if you have the user-name and password I provide in class. The password is only available to my students due to licensing restrictions on some of the papers.

Whenever you see an icon such as the one on this margin it means that we have NetLogo implementation of a relevant problem. If you are using the PDF version of this document you can just click on the name and your browser will take you to the appropriate applet. Otherwise, the URL is formed by pre-pending <http://jmvidal.cse.sc.edu/netlogomas/> to the name and appending `.html` at the end. So the URL for this icon is <http://jmvidal.cse.sc.edu/netlogomas/ABTgc.html>.

Resnick points to examples such as surveys of 8–15 year old kids, half of which believe that the government sets all prices and salaries.

Resnick created StarLogo in order to teach the *decentralized* mindset. Wilensky, one of his students, later extended StarLogo and created NetLogo.



ABTgc





## Chapter 1

### Multiagent Problem Formulation

The goal of multiagent systems' research is to find methods that allow us to build complex systems composed of autonomous agents who, while operating on local knowledge and possessing only limited abilities, are nonetheless capable of enacting the desired global behaviors. We want to know how to take a description of what a *system of agents* should do and break it down into individual agent behaviors. At its most ambitious, multiagent systems aims at reverse-engineering emergent phenomena as typified by ant colonies, the economy, and the immune system. Multiagent systems approaches the problem using the well proven tools from game theory, Economics, and Biology. It supplements these with ideas and algorithms from artificial intelligence research, namely planning, reasoning methods, search methods, and machine learning.

Having such an ambitious goal has led to the development of many different approaches, some of which end up being incompatible with each other. That is, it is sometimes not clear if two researchers are studying the same problem or variations of it. Still, the model that has thus far gained most attention, probably due to its flexibility as well as its well established roots in game theory and artificial intelligence, is that of modeling agents as utility maximizers who inhabit some kind of Markov decision process. This is the model we present in this chapter and the one we will use most often throughout the book. However, while this is a very general model, it is not always the best way to describe the problem. We will also examine the traditional artificial intelligence planning model and, in later chapters, models inspired by Biology.

A more relaxed introduction to the topics in this chapter is given in (Russell and Norvig, 2003, Chapters 16–17).

#### 1.1 Utility

We generally assume that an agent's preferences are captured by a **utility function**. This function provides a map from the states of the world or outcome of game to a real number. The bigger the number the more the agent likes that particular state. Specifically, given that  $S$  is the set of states in the world the agent can perceive then agent  $i$ 's utility function is of the form

$$u_i : S \rightarrow \mathbb{R}. \quad (1.1)$$

Notice that the states are defined as those states of the world that the agent can perceive. For example, if a robot has only one sensor that feeds him a binary input, say 1 if it is bright and 0 if it is dark, then that robot has a utility function defined over only two states regardless of how complicated the real world might be. In practice, agents have sophisticated inputs and it is impractical to define a different output for each input. Thus, most agents also end up mapping their raw inputs to a smaller set of world states. Creating this mapping function can be challenging as it requires a deep understanding of the problem setting.

Given an agent's utility function we can define a **preference ordering** for the agent over the states of the world. By comparing the utility values of two states we can determine which one is preferred by the agent. This ordering has the following properties.

- *reflexive*:  $u_i(s) \geq u_i(s)$

#### UTILITY FUNCTION

In game theory it is known as the von Neumann-Morgenstern utility function.

#### PREFERENCE ORDERING

- *transitive*: If  $u_i(a) \geq u_i(b)$  and  $u_i(b) \geq u_i(c)$  then  $u_i(a) \geq u_i(c)$ .
- *comparable*:  $\forall_{a,b}$  either  $u_i(a) \geq u_i(b)$  or  $u_i(b) \geq u_i(a)$ .

We can use utility functions to describe the behavior of almost every agent. Utility functions are also useful for capturing the various tradeoffs that an agent must make, along with the value or expected value of its actions. For example, we can say that a robot receives a certain payment for delivering a package but also incurs a cost in terms of the electricity used as well as the opportunity cost—he could have been delivering other packages. If we translate all these payments and costs into utility numbers then we can easily study the tradeoffs among them.

Once we have defined a utility function for all the agents then all they have to do is take actions which maximize their utility. As in Economics, we use word **selfish** to refer to a rational agent that wants to maximize its utility. Notice that this use is slightly different from the everyday usage of the word which often implies a desire to harm others, a true selfish agent simply cares exclusively about its utility. The use of selfish agents does not preclude the implementation of cooperative multiagent systems. We can view a cooperative multiagent system as one where the agents' utility functions have been defined in such a way so that the agents seem to cooperate. For example, if an agent receives a higher utility for helping other agents then the resulting behavior will seem cooperative to an observer even though the agent is acting selfishly. Since we are concerned with building agents we need not be distracted by the question of whether or not humans are really selfish or not.

We use utility functions as a succinct way of representing an agent's behavior. In the actual implementations these functions will be sometimes be probabilistic, sometimes they will be learned as the agent acts in the world, sometimes they will be incomplete. Still, by assuming that the function can exist we can more easily design and study a multiagent system.

### 1.1.1 Utility is Not Money

Note that while utility represents an agent's preferences it is not necessarily equated with money. In fact, the utility of money has been found to be roughly logarithmic. For example, say Bill has \$100 million while Tim has \$0 in the bank. They both contemplate the possibility of winning one million dollars. Clearly, that extra million will make a significant difference in Tim's lifestyle while Bill's lifestyle will remain largely unchanged, thus Tim's utility for the same million dollars is much larger than Bill's. There is experimental evidence that shows this holds for most people. Very roughly, people's utility for smaller amounts of money is linear but for larger amounts it becomes logarithmic. Note that here we are considering the **marginal utility** or money, that is, it is the utility for the next million dollars. We assume that both Bill and Tim have the same utility for their first million dollars.

Recent results in behavioral economics have shown that the true utility function is more complicated than what can be captured by a mathematical function. One experiment shows that people are less likely to take a gamble when the wording of the question is such that the person must give back some money, even if gamble is mathematically identical to another one with different wording. The experiment in question is the following: which option would you prefer (a) I give you \$10,000 and a 50/50 chance at winning another \$10,000, or (b) I give you \$20,000 and then flip a coin and if it comes out heads you have to give me \$10,000. Both of these are equivalent but a large majority of people prefer option (a).

The fact that people are not entirely rational when making choices about money becomes important to us when we start to think about building agents that will buy, sell, or negotiate for humans. It is reasonable to assume that in these systems people will demand agents that behave like people. Otherwise, the agent would be making decisions the user finds disagreeable.

SELFISH

**selfish**, adj. Devoid of consideration for the selfishness of others. — *The Devil's Dictionary*

MARGINAL UTILITY

You can determine your own curve by repeatedly asking yourself for increasing values of  $x$ : which one would you rather have,  $x$  dollars, or a 50/50 chance at winning  $2 \cdot x$  dollars? When you start preferring the first choice you are dropping below the  $y = x$  line.

### 1.1.2 Expected Utility

Once we have utility functions we must then determine how the agents will use them. We assume each agent has some sensors which it can use to determine the state of the world and some effectors which it can use to take action. These actions can lead to new states of the world. For example, an agent senses its location and decides to move forward one foot. Of course, these sensors and effectors might not operate perfectly: the agent might not move exactly one foot or its sensors might be noisy. Let's assume, however, that the agent does know the probability of reaching state  $s'$  given that it is in state  $s$  and takes action  $a$  is given by  $T(s, a, s')$  which we call the **transition function**. Note that the transition function returns a probability. As such, the sum of  $T(s, a, s')$  over all possible  $a$  and  $s'$  must be equal to one.

Using this transition function the agent can then calculate its **expected utility** for taking action  $a$  in state  $s$  as

$$E[u_i, s, a] = \sum_{s' \in S} T(s, a, s') u_i(s'), \quad (1.2)$$

where  $S$  is the set of all possible states.

An agent can use the expected utility function to determine the value of a piece of added information it might acquire such as an extra sensor reading or a message from another agent. For example, consider a piece of new information that leads the agent to determine that it is not really in state  $s$  but it is instead in state  $t$ . Perhaps a second agent tells him that he is near a precipice and not in the middle of a plateau as the agent originally thought. In this case the agent can compare the expected utility it would have received under the old state against the new one it will now receive. That is, the **value of information** for that piece of information is given by

$$E[u_i, t, \pi_i(t)] - E[u_i, t, \pi_i(s)]. \quad (1.3)$$

The value of the piece of information is the expected utility the agent receives now that he knows he is in  $t$  and takes action accordingly minus the utility it would have received if he had instead mistakenly assumed it was in  $s$  and taken action accordingly. This equation provides a simple and robust way for agents to make meta-level decisions about what knowledge to seek: what messages to send, which sensors to turn on, how much deliberation to perform, etc.

Specifically, an agent can use it to play “*what if*” games to determine its best action in the same way a person might. For example, say a doctor has made a preliminary decision about which drug to prescribe for a patient but is uncertain about the patient's specific disease. She can calculate the value of information acquired from performing various tests which could identify the specific disease. If a test, regardless of its outcome, would still lead the doctor to prescribe the same drug then the value of the information from that test's result is zero: it does not lead to a change in action so (1.3) is zero. On the other hand, a test that is likely to change the doctor's action will likely have a high value of information.

## 1.2 Markov Decision Processes

So far we have considered only a fixed state of the world. But, the reality in most cases is that agents inhabit an environment whose state changes either because of the agent's action or due to some external event. We can think of the agent sensing the state of the world then taking an action which leads it to a new state. We also make the further simplifying assumption that the choice of the new state therefore depends only on the agent's current state and the agent's action. This idea is formally captured by a **Markov decision process** or MDP.

**Definition 1.1** (Markov Decision Processes). *An MDP consists of an initial state  $s_1$  taken from a set of states  $S$ , a transition function  $T(s, a, s')$  and a reward function  $r : S \rightarrow \mathbb{R}$ .*

TRANSITION FUNCTION

EXPECTED UTILITY

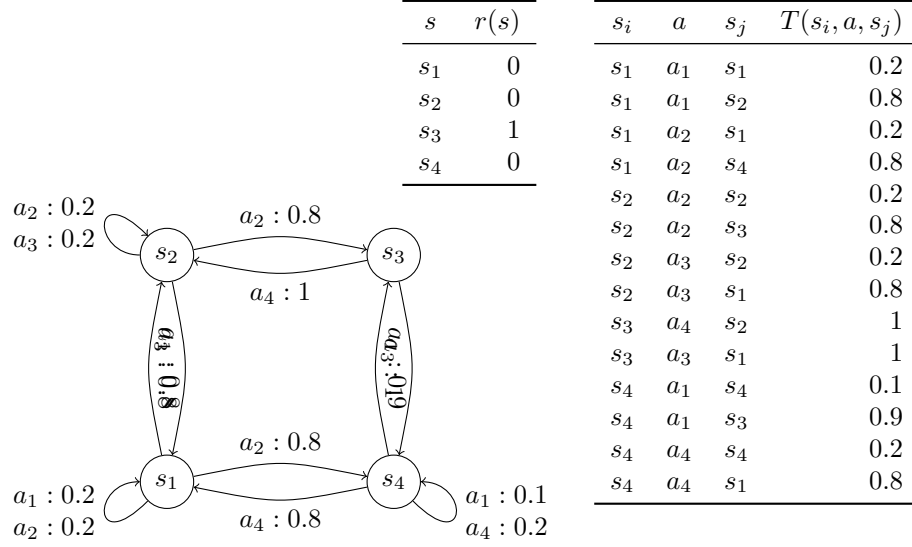
VALUE OF INFORMATION



Andrey Markov. 1856–1922.

MARKOV DECISION PROCESS

**Figure 1.1:** Graphical representation of a sample Markov decision process along with values for the transition and reward functions. We let the start state be  $s_1$ .



The transition function  $T(s, a, s')$  gives us the probability that an agent in state  $s$  who takes action  $a$  will end up in state  $s'$ . For a purely **deterministic** world we have that this function will be zero for all  $s'$  of each given  $s, a$  pair except one, for which it will be one. That is, in a deterministic world an agent's action has a completely predictable effect. This is not the case for a **non-deterministic** environment where an agent's action could lead to a number of different states, depending on the value this fixed probability function.

figure 1.1 shows a typical visualization of an MDP, this one with four states. Notice that in this example the agent's actions are limited based on the state, for example, in state  $s_1$  the agent can only take actions  $a_1$  and  $a_2$ . Also note how when the agent is in state  $s_1$  and it takes action  $a_1$  there is a 0.2 probability that it will state in the same state. In this example the agent receives a reward of 1 only when it reaches  $s_3$ , at which point its only option is to leave  $s_3$ .

The agent's behavior is captured by a **policy** which is a mapping from states to actions. We will use  $\pi$  to denote the agent's policy. The agent's goal then become that of finding its **optimal policy** which is the one that maximizes its expected utility, as we might expect a selfish agent to do. This strategy is known as the principle of **maximum expected utility**. Agent  $i$ 's optimal policy can thus be defined as

$$\pi_i^*(s) = \arg \max_{a \in A} E[u_i, s, a]. \quad (1.4)$$

But, in order to expand the expected value within that equation we must first determine how to handle future rewards. That is, is it better to take 100 actions with no reward just to get to a state which gives the agent a reward of 100, or is it better to take 100 actions each of which gives the agent a reward of 1? Since no agent is likely to live forever we will not want to wait forever for that big reward. On the other hand, it seems reasonable to give up a small reward in the next couple of step if we know there is a very big reward afterwards. As such, we generally prefer to use **discounted rewards** which allow us to smoothly reduce the impact of rewards that are farther off in the future. We do this by multiplying the agent's future rewards by an ever-decreasing **discount factor** represented by  $\gamma$ , which is a number between zero and one.

For example, if an agent using policy  $\pi$ , starts out in state  $s_1$  and visits states  $s_1, s_2, s_3, \dots$  then we say that its discounted reward is given by

$$\gamma^0 r(s_1) + \gamma^1 r(s_2) + \gamma^2 r(s_3) + \dots \quad (1.5)$$

VALUE-ITERATION( $T, r, \gamma, \epsilon$ )

```

1      do
2           $u \leftarrow u'$ 
3           $\delta \leftarrow 0$ 
4          for  $s \in S$ 
5              do  $u'(s) \leftarrow r(s) + \gamma \max_a \sum_{s'} T(s, a, s') u(s')$ 
6                  if  $|u'(s) - u(s)| > \delta$ 
7                      then  $\delta \leftarrow |u'(s) - u(s)|$ 
8          until  $\delta < \epsilon(1 - \gamma)/\gamma$ 
9      return  $u$ 

```

**Figure 1.2:** The VALUE-ITERATION algorithm. It takes as input an MDP and an error  $\epsilon$  and returns a  $u(s)$  for that MDP that is guaranteed to have an error less than  $\epsilon$ .

Note, however, that we only know  $s_1$ . The rest of the states,  $s_2, s_3, \dots$  depend on the transition function  $T$ . That is, from state  $s_1$  we know that the probability of arriving at any other state  $s'$  given that we took action  $a$  is  $T(s, a, s')$  but we do not know which specific state the agent will reach. If we assume that the agent is an utility maximizing agent then we know that it is going to take the action which maximizes its expected utility. So, when in state  $s$  the agent will take action given by (1.4), which when expanded using (1.2) give us

$$\pi^*(s) = \arg \max_a \sum_{s'} T(s, a, s') u(s'), \quad (1.6)$$

where  $u(s')$  is the utility the agent can expect from reaching  $s'$  and then continuing on to get more rewards for successive states while using  $\pi^*$ . We can now work backwards and determine what is the value of the value of  $u(s)$ . We know that when the agent arrives in  $s$  it receives a reward of  $r(s)$ , but we now also know that because it is in  $s$  it can take its action based on  $\pi^*(s)$  and will get a new reward at the next time. Of course, that reward will be discounted by  $\gamma$ . As such, we can define the real utility the agent receives for being in state  $s$  as

$$u(s) = r(s) + \gamma \max_a \sum_{s'} T(s, a, s') u(s'). \quad (1.7)$$

This is known as the **Bellman equation** and it captures the fact that an agent's utility depends not only on its immediate rewards but also on its future discounted rewards. Notice that, once we have this function defined for all  $s$  then we also have the agent's true optimal policy  $\pi^*(s)$ , namely, the agent should take the action which maximizes its expected utility, as given in (1.6).

The problem we now face is how to calculate  $u(s)$  given the MDP definition. One way is to solve the set of equations formed. Given  $n$  states we have  $n$  Bellman equations each one with a different variable. We thus have a system of  $n$  equations with  $n$  variables so, theoretically, we can find values for all these variables. In practice, however, solving this set of equations is not easy because of the max operator which makes the equations non-linear.

Another approach for solving the problem is to use **value iteration**. In this method we start by setting the values of  $u(s)$  to some arbitrary numbers and then iteratively improve these numbers using the **Bellman update** equation:

$$u^{t+1}(s) \leftarrow r(s) + \gamma \max_a \sum_{s'} T(s, a, s') u^t(s'). \quad (1.8)$$

The reader will notice that this equation is nearly the same as (1.7), except that we are now updating the  $u$  values over time. It has been shown that this process will eventually, and often rapidly, converge to the real values of  $u(s)$ . We also know that if the maximum change in utility for a particular time step is less than  $\epsilon(1 - \gamma)/\gamma$  then the error is less than  $\epsilon$ . This fact can be used as a stopping condition.



Richard Bellman. 1920–1984.  
Norbert Wiener prize, IEEE  
Medal of Honor.

BELLMAN EQUATION

VALUE ITERATION

BELLMAN UPDATE



**Figure 1.3:** Example application of the VALUE-ITERATION algorithm to the MDP from figure 1.1, assuming  $\gamma = .5$  and  $\epsilon = .15$ . The algorithm stops after  $t = 4$ . The bottom table shows the optimal policy given the utility values found by the algorithm.

|          | Time ( $t$ ) |   |                 |                       |                        |
|----------|--------------|---|-----------------|-----------------------|------------------------|
|          | 0            | 1 | 2               | 3                     | 4                      |
| $u(s_1)$ | 0            | 0 | 0               | $.5(.8).45 = .18$     | $.5(.09 + .378) = .23$ |
| $u(s_2)$ | 0            | 0 | $.5(.8)1 = .4$  | $.5(.88)1 = .44$      | $.5(.18 + .98) = .57$  |
| $u(s_3)$ | 0            | 1 | 1               | $1 + .5(1).45 = 1.2$  | $1 + .5(.47) = 1.2$    |
| $u(s_4)$ | 0            | 0 | $.5(.9)1 = .45$ | $.5(.9 + .045) = .47$ | $.5(1.1 + .047) = .57$ |

| $s$   | $\pi^*(s)$ |
|-------|------------|
| $s_1$ | $a_2$      |
| $s_2$ | $a_2$      |
| $s_3$ | $a_3$      |
| $s_4$ | $a_1$      |

The VALUE-ITERATION algorithm is shown in figure 1.2. This algorithm is an example of the more general **dynamic programming** approach where the optimal solution to a problem is found by first finding the optimal solutions to sub-problems. In our case, the sub-problems are the variables themselves. Finding a value for one variable helps us find values for other variables.

For example, figure 1.3 shows how the utility values change as the VALUE-ITERATION algorithm is applied to the example from figure 1.1. The utilities start at 0 but  $s_3$  is quickly upgrade to 1 because it receives a reward of 1. This reward then propagates back to its immediate neighbors  $s_2$  and  $s_4$  at time 2 and then at time 3 it reaches  $s_1$ . At time 4 the algorithm stops because the biggest change in utility from time 3 to time 4 was 0.13, for  $s_4$ , which is less than or equal to  $\epsilon(1 - \gamma)/\gamma = .15$ .

### 1.2.1 Multiagent Markov Decision Processes

The MDP model represents the problems of only one agent, not of a multiagent system. There several ways of transforming an MDP into a multiagent MDP. The easiest way is to simply place all the other agents' effects into the transition function. That is, assume they don't really exist as entities and are merely part of the environment. This technique can work for simple cases where the agents are not changing their behavior since the transition function in an MDP must be fixed. Unfortunately, agents that change their policies over time, either because of their own learning or because of input from the users, are very common.

A better method is to extend the definition of MDP to include multiple agents all of which can take an action at each time step. As such, instead of having a transition function  $T(s, a, s')$  we have a transition function  $T(s, \vec{a}, s')$ , where  $\vec{a}$  is a vector of size equal to the number of agents where each element is an agent's action, or a symbol representing non-action by that agent. We also need to determine how the reward  $r(s)$  is to be doled out amongst the agents. One possibility is to divide it evenly among the agents. Unfortunately, such a simplistic division can mislead agents by rewarding them for bad actions. A better method is to give each agent a reward proportional to his contribution to the system's reward. We will see how this can be done in chapter 5.6.

### 1.2.2 Partially Observable MDPs

In many situations it is not possible for the agent to sense the full state of the world. Also, an agent's observations are often subject to noise. For example, a robot has only limited sensors and might not be able to see behind walls or hear soft sounds and its microphone might sometimes fail to pick up sounds. For this scenarios we would like to be able to describe the fact that the agent does not know in which state it is in but, instead, believes that it can be in any number of states with certain probability. For example, a robot might believe that it is in any one of a number of

rooms, each with equal probability, but that it is definitely not outdoors. We can capture this problem by modeling the agent's **belief state**  $\vec{b}$  instead of the world state. This belief state is merely a probability distribution over the set of possible states and it indicates the agent's belief that it is in that state. For the case with four states, the vector  $\vec{b} = \langle \frac{1}{2}, \frac{1}{2}, 0, 0 \rangle$  indicates that the agent believes it is either in  $s_1$  or  $s_2$ , with equal probability, and that it is definitely not in  $s_3$  or  $s_4$ .

BELIEF STATE

We also need an **observation model**  $O(s, o)$  which tells the agent the probability that it will perceive observation  $o$  when in state  $s$ . The agent can then use the observations it receives to update its current belief  $\vec{b}$ . Specifically, if the agent's current belief is  $\vec{b}$  and it takes action  $a$  then its new belief vector  $\vec{b}'$  can be determined using

OBSERVATION MODEL

$$\forall_{s'} \vec{b}'(s') = \alpha O(s', o) \sum_s T(s, a, s') \vec{b}(s), \quad (1.9)$$

where  $\vec{b}(s)$  is the value of  $\vec{b}$  for  $s$  and  $\alpha$  is a normalizing constant that makes the belief state sum to 1. When we put all these requirements together we have a **partially observable MDP** or POMDP which are a very natural way of describing the problems faced by an agent with limited sensors. Of course, since the agent does not know the state then it cannot use value iteration to solve this problem.

PARTIALLY OBSERVABLE  
MDP

Luckily, it turns out that we can use (1.9) to define a new transition function

$$\tau(\vec{b}, a, \vec{b}') = \begin{cases} \sum_{s'} O(s', o) \sum_s T(s, a, s') \vec{b}(s) & \text{if (1.9) is true for } \vec{b}, a, \vec{b}' \\ 0 & \text{otherwise,} \end{cases} \quad (1.10)$$

and a new reward function

$$\rho(\vec{b}) = \sum_s \vec{b}(s) r(s). \quad (1.11)$$

Solving a POMDP on a physical state amounts to solving this MDP on the belief state. Unfortunately, this MDP can have an infinite number of states since the beliefs are continuous values. Luckily, there do exist algorithms for solving these type of MDPs. These algorithms work by grouping together beliefs into regions and associating actions with each region. In general, however, when faced with a POMDP problem it is usually easier to use a dynamic decision networks to represent and solve the problem.

See (Russell and Norvig, 2003, Chapter 17.5) for introduction to dynamic decision networks.

### 1.3 Planning

In the **AI planning** problem an agent is also given a set of possible states and is asked for a sequence of actions that will lead it to a desirable world state. However, instead of a transition function the agent is given a set of operators. Each operator has pre-requisites that specify when it can be used—in which states—and effects which specify the changes the operator will cause in the state. The planning problem is to find a sequence of operators that take the agent from the start state to the goal state.

AI PLANNING

It should be clear that this problem is a special case of an MDP, one where only one state provides a reward and all the transitions have probability of 1 or 0. The transitions are generated by applying the operators to the current state. Of course, by describing the problem using operators we achieve a much more succinct definition of the problem but our goal is solving the problem, not defining it with as few bits as possible. The more detailed description of the states does, however, provide an advantage over the simple state listing used in MDP, namely, it provides added information about the composition of the state. For example, if we want to reach a state that has properties of *is-blue?* and *is-wet?* then we can use this knowledge to set about getting the state of the world blue and, once that is done, to get it wet without changing its color. This kind of information is opaque in a MDP description which simply describes a wet blue state as  $s_{11}$  and a dry blue state

as  $s_{23}$ . Operators and their corresponding state descriptions provide the agent with more knowledge about the problem domain.

There exists any planning algorithms which solve the basic planning problem and variations of it, including cases where there is a possibility that the operators don't have the desired effect (making it even more like an MDP), cases where the agent must start taking actions before the plan can be finished, and cases where there is uncertainty as to what state the agent is in (like POMDP). Most modern planning algorithms use a graphical representation of the problem. That is, they end up turning the problem into an MDP-like problem and solving that. These algorithms are sophisticated and are available as libraries to use with your programs. If the need arises to solve a planning problem or an MDP it is advisable to use one of the many available libraries or, at worst, implement one of the published algorithms rather than trying to implement your own.

See (Russell and Norvig, 2003, Chapters 11–12) for pointers to these programs.

### 1.3.1 Hierarchical Planning

A very successful technique for handling complexity is to recursively divide a problem into smaller ones until we find problems that are small enough that they can be solved in a reasonable amount of time. This general idea is known as the **divide and conquer** approach in AI. Within planning, the idea can be applied by developing plans whose primitive operators are other plans. For example, in order to achieve your goal of a holiday at the beach you must first arrange transportation, arrange lodging, and arrange meals. Each one of these can be further decomposed into smaller actions, and there might be different ways of performing this decomposition. Within the planning model this amounts to building virtual operators which are planning problems themselves, instead of atomic actions. The problem then becomes one of deciding which virtual operators to build. In fact, the problem is very similar to that of deciding which functions or classes to write when implementing a large software project.

Within the MDP model we can imagine building a hierarchy of policies, each one taking off from states that fit a particular description (say, dry states) to other types of states (say, red and wet). We can then define new MDP problems which use sets of states as atomic states and the new policies as transition functions, similar to how we built POMDPs. These techniques are studied in **hierarchical learning**.

Hierarchical planning and learning are usually studied as ways of making the planning and learning problems easier, that is, as ways to develop algorithms that will find solutions faster. However, once developed these hierarchies provide added benefits in a multiagent system. Specifically, they can be used to enable coordination among agents (Durfee, 1999). By exchanging top-level plan names (or policies) the agents have a general idea of what each other is doing without the need for exchanging detailed instructions of what they are doing at each step. For example, with two robots moving boxes in a room one might tell the other that its moving a box from the South corner to the East corner, the other agent then knows that if it stays in the Northwest part of the room it does not need to worry about the exact location of its partner—the robots stay out of each other's way without detailed coordination. This technique of using goal/plan/policy hierarchies for multiagent coordination has been very successful. We will examine it in more detail in Chapter 9.

## 1.4 Summary

The view of an autonomous agent as an utility-maximizing agent that inhabits an MDP, or variation thereof, is most popular because of its flexibility, applicability to disparate domains, and amenability to formal analysis with mathematical or computational tools. As such, this book will largely adopt this model as the basis for formulating the various multiagent problems. Note, however, that the MDP is a tool for describing the problem faced by an agent. In practice, it is rare that a practical

DIVIDE AND CONQUER

HIERARCHICAL LEARNING



solution to a real world problem is also implemented as an algorithm to solve the raw MDP. More commonly we find algorithms which use a much more succinct, and therefore practical, method for representing the problem. Unfortunately, these representations tend to be very domain specific; they cannot be used in different domains.

### Exercises

- 1.1 Marvin is a robot that inhabits a 3 by 3 grid and can only move North, South, East, and West. Marvin's wheels sometimes spin so that on each action there is a .2 probability that Marvin remains stationary. Marvin receives a reward of 1 whenever it visits the center square.
  1. Draw an MDP which describes Marvin's problem.
  2. What is the optimal policy?



## Chapter 2

### Distributed Constraints

In this chapter we look at some algorithms for performing distributed search in cooperative multiagent systems where each agent has some local information and where the goal is to get all the agents to set themselves to a state such that the set of states in the system is optimal. For example, imagine a group of small sensors in a field. Each sensor can communicate only with those that are near him and has to decide which of its modalities to use (sense temperature, point radar North, point radar South, etc.) so that the group of sensor gets a complete view of the important events in the field. Or, imagine a group of kids who have been told to stand in a circle, each one must move find a spot to stand on but the position of that spot depends on everyone else's location. In these examples the agents have local information, can take any one of their available actions at any time, but the utility of their actions depends on the actions of others. We find that many multiagent problems can be reduced to a distributed constraints problem. Thus, the algorithms we present in this chapter have a lot of different applications.

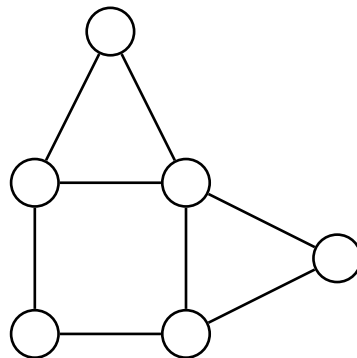
#### 2.1 Distributed Constraint Satisfaction

In a **constraint satisfaction problem** (CSP) we are given a set of variables, each with its own domain, and a set of constraint. The goal is to set every variable to a value from its domain such that no constraints are violated. Formally,

CONSTRAINT SATISFACTION  
PROBLEM

**Definition 2.1** (Constraint Satisfaction Problem). *Given a set of variables  $x_1, x_2, \dots, x_n$  with domains  $D_1, D_2, \dots, D_n$  and a set of boolean constraints  $P$  of the form  $p_k(x_{k1}, x_{k2}, \dots, x_{kj}) \rightarrow \{0, 1\}$ , find assignments for all the variables such that no constraints are violated.*

The most widely studied instance of a constraint satisfaction problem is the graph coloring problem, shown in figure 2.1. In this problem we are given a graph and a set of colors. The problem is to find out if there is a way to color each node, with one of the given colors, such that no two nodes that are connected by an edge have the same color. We can easily map the graph coloring problem to the formal definition of a CSP by considering each node to be a variable, the domains to be the set of colors, and each edge becomes a constraint between its two nodes that is true



**Figure 2.1:** Sample graph coloring problem. You must color each node either black, white, or gray so that no two nodes connected by an edge have the same color. Can you find a coloring?

**Figure 2.2:** A centralized depth first search algorithm for the CSP. Variable  $g$  holds the partial assignment of variable values. The algorithm is called with  $\text{DEPTH-FIRST-SEARCH-CSP}(1, \emptyset)$ .

```

DEPTH-FIRST-SEARCH-CSP( $i, g$ )
1  if  $i > n$ 
2    then return  $g$ 
3  for  $v \in D_i$ 
4    do if setting  $x_i \leftarrow v$  does not violate any constraint in  $P$  given  $g$ 
5      then  $g' \leftarrow \text{DEPTH-FIRST-SEARCH-CSP}(i + 1, g + \{x_i \leftarrow v\})$ 
6        if  $g' \neq \emptyset$ 
7          then return  $g'$ 
8
9  return  $\emptyset$ 

```

only if their values are different. Notice that, in graph coloring, constraints are only over two variables, instead of over any set of variables as in the general definition of a CSP.

The constraint satisfaction problem is NP-complete. As such, we must use search algorithms to find a solution and hope that the actual running time will be less than the worst-case exponential. In practice we find that there are many special cases where we can solve the problem really fast. For example, in the **n-queens problem** we are given an  $8 \times 8$  chess board with 8 queens for which we must find places on the board such that no queen can take another queen. This problem is a special case of the CSP but it is one for which there are algorithms that can find a solution quickly.

A simple but effective method for solving the centralized constraint satisfaction problem is with the use of a **depth first search** algorithm, as shown in figure 2.2. The algorithm simply chooses an arbitrary ordering for the variables. On each call it tries to set the current variable  $x_i$  to all its possible values to see which ones do not violate any of the constraints in  $P$  given the partial variable assignment  $g$ . Whenever a value  $v$  works, the algorithm adds that assignment to  $g$  and recursively tries to assign the next variable in the list. This algorithm performs a complete search and is thus guaranteed to find a solution if one exists. Depth first search is also practical because it only requires an amount of memory that is linear on the number of variables, that is, it has  $O(n)$  memory usage. As we shall see later in this chapter, some of the distributed algorithms try to parallelize this basic algorithm.

In the **distributed constraint satisfaction** (DCSP) problem we give each agent one variable. The agent is responsible for setting the value of its own variable. The agents do not know the values of any other variable but can communicate with other agents. Formally, we define:

**Definition 2.2** (Distributed Constraint Satisfaction Problem). *Give each agent one of the variables in a CSP. Agents are responsible for finding a value for their variable and can find out the values of their neighbors' via communication*

Usually the agents are limited so that they can only communicate with their neighbors—the agents with whom they share a constraint. Some algorithms, however, allow agents to communicate with any other agent. Also, most algorithms assume that the agent's know about all the constraints that involve their variable.

The DCSP is pervasive in multiagent research and many real-world problems can be reduced to a DCSP, or a variation thereof. As such, many DCSP algorithms have been developed, each one with its own set of strengths and weaknesses. In the next sections we examine some of the most popular algorithms.

### 2.1.1 Filtering Algorithm

Picture yourself as one of the agents. You know your constraints, the domain of your variable, and the domain of the variables of your neighbors. You can use

N-QUEENS PROBLEM

DEPTH FIRST SEARCH

A common modification is to improve the variables' ordering.

DISTRIBUTED CONSTRAINT SATISFACTION

```

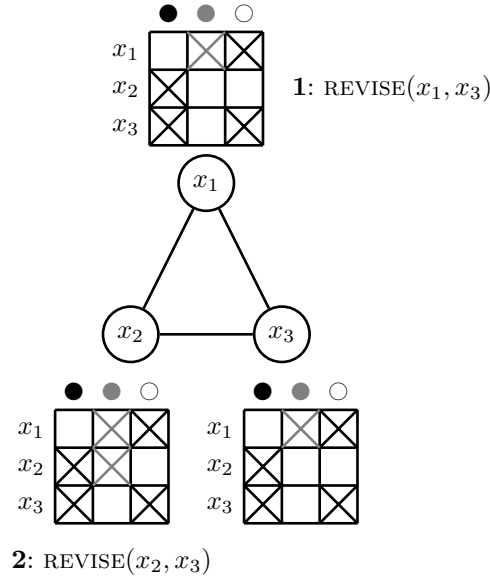
FILTERING()
1  for  $j \in \{\text{neighbors of } i\}$        $\triangleright i$  is this agent.
2      do REVISE( $x_i, x_j$ )

HANDLE-NEW-DOMAIN( $j, D'$ )
1   $D_j \leftarrow D'$ 
2  REVISE( $x_i, x_j$ )

REVISE( $x_i, x_j$ )
1   $\text{old-domain} \leftarrow D_i$ 
2  for  $v_i \in D_i$ 
3      do if there is no  $v_j \in D_j$  consistent with  $v_i$ 
4          then  $D_i \leftarrow D_i - v_i$ 
5  if  $\text{old-domain} \neq D_i$ 
6      then  $\forall k \in \{\text{neighbors of } i\} k.\text{HANDLE-NEW-DOMAIN}(i, D_i)$ 

```

**Figure 2.3:** The filtering algorithm. Each agent  $i$  executes FILTERING().



**Figure 2.4:** Filtering example. The agents start out with some prohibited colors as indicated by the black crosses. On the first step  $x_1$  does his REVISE and eliminates the color gray from consideration. It then tells everyone else about this. Then  $x_2$  does its revise and eliminates the color gray from its domain.

this information to determine if any of the values from your variable's domain is impossible. That is, if there is a value which violates some constraint no matter what value one of your neighbors uses then that value will clearly not be part of the solution so you can get rid of it. Similarly, all other agents might do the same.

This idea is implemented by the **filtering algorithm** (Waltz, 1975) shown in figure 2.3. Every agent  $i$  executes the FILTERING procedure which leads it to do a REVISE with every one of its neighbors' variables  $x_j$ . If the agent does succeed in eliminating one or more values from its domain it then tells its neighbors its new domain. When an agent receives new domains from its neighbors it again executes REVISE for those neighbors. This process repeats itself until all agents finish their computations and no more messages are sent.

The filtering algorithm is guaranteed to stop since there are only a finite number of values in all the domains and the algorithm only sends a message when a value is eliminated from a domain. Once it stops, the filtering algorithm might have reduced the size of some domains. Unfortunately, it might stop at a point where one or more of the agents still has many values in its domain.

FILTERING ALGORITHM

**Figure 2.5:** Example of a problem that does not have a solution and the filtering algorithm cannot that fact.

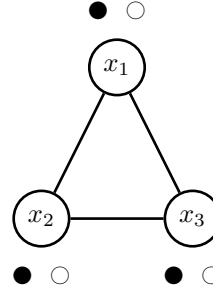


Figure 2.4 shows an example of the filtering algorithm as applied to the graph coloring problem. In this problem the agents have three possible colors but start out with already limited domains as indicated by the black crosses. For example, agent  $x_1$  can only be either black or gray. At the first step  $x_1$  executes  $\text{REVISE}(x_1, x_3)$  and realizes that since  $x_3$  must be gray that it cannot be gray. As such, it eliminates gray from its domain and tells all the other agents about this. The other agents incorporate this knowledge. Then, agent  $x_2$  does a  $\text{REVISE}(x_2, x_3)$  and realizes that it also can't be gray and thus eliminates gray from its domain. At this point  $x_2$  knows the final configuration. When it tells the other agents about its new domain then all the agents will also know the final solution.

In this example the filtering algorithm found a specific solution. Unfortunately, this is a rare occurrence. More often we find that the filtering algorithm only slightly reduces the size of the domains. For example, if we run the example from figure 2.4 but start with all possible colors in all domains then the filtering algorithm will not be able to do anything. That is, none of the agents are able to eliminate any color from their domain because many colorings are possible.

The filtering algorithm might also fail to eliminate values from the domains even when no coloring exists. That is, the filtering algorithm cannot reliably detect problems that do not have a solution. This problem can be illustrated with the example in figure 2.5. Here the nodes have to choose between either white or black. We know that there is no possible way to color these nodes without violating a constraint. However, the filtering algorithm will get stuck at the first step in this example and will not be able to reduce the size of the domains. Namely, each agent considers that if he is black then the other two could be white and if he is white then the other two could be black. As such, no agent eliminates colors from its domain.

### 2.1.2 Hyper-Resolution Based Consistency Algorithm

The reason the filtering algorithm does not work is because the  $\text{REVISE}$  function considers only the agent's own value and one other agent's value at a time. More generally, we need to consider the agent's value against all possible combinations of the values of two of its neighbors, of three of its neighbors, and so on all the way to considering the values of all of its neighbors at the same time.

More formally, we say that if an agent can find a suitable value regardless of the values of  $k$  of its neighbors then it can be consistent with anything they might do. If we can say this for all agents then we have achieved  **$k$ -consistency**.

**Definition 2.3** ( $k$ -consistency). *Given any instantiation of any  $k-1$  variables that satisfy all constraints it is possible to find an instantiation of any  $k^{\text{th}}$  variable such that all  $k$  variable values satisfy all constraints.*

If we can prove that problem is  $j$ -consistent for  $j = 2, 3, \dots$  up to some number  $k$  then we could say that the problem is strongly  $k$ -consistent.

**Definition 2.4** (Strongly  $k$ -consistent). *A problem is strongly  $k$ -consistent if it is  $j$ -consistent for all  $j \leq k$ .*

We now face the problem of proving that a particular DCSP is strongly  $k$ -consistent. Notice that the filtering algorithm guarantees 2-consistency since it makes sure that

|      | $x_1$  | $x_2$  | $x_3$  |
|------|--|--|--|
|      | $x_1 = \bigcirc \vee x_1 = \bullet$          | $x_2 = \bigcirc \vee x_2 = \bullet$          | $x_3 = \bigcirc \vee x_3 = \bullet$          |
|      | $\neg(x_1 = \bigcirc \wedge x_2 = \bigcirc)$ | $\neg(x_1 = \bigcirc \wedge x_2 = \bigcirc)$ | $\neg(x_1 = \bigcirc \wedge x_2 = \bigcirc)$ |
| Time | $\neg(x_1 = \bullet \wedge x_2 = \bullet)$   | $\neg(x_1 = \bullet \wedge x_2 = \bullet)$   | $\neg(x_1 = \bullet \wedge x_2 = \bullet)$   |
| 1    | $\neg(x_2 = \bigcirc \wedge x_3 = \bullet)$  | $\neg(x_2 = \bigcirc \wedge x_3 = \bullet)$  | $\neg(x_2 = \bigcirc \wedge x_3 = \bullet)$  |
| 2    | $\neg(x_2 = \bullet \wedge x_3 = \bigcirc)$  | $\neg(x_3 = \bullet)$                        | $\neg(x_2 = \bullet \wedge x_3 = \bigcirc)$  |
| 3    |  | $\neg(x_2 = \bullet \wedge x_3 = \bigcirc)$  | $\neg(x_3 = \bigcirc)$                       |
| 4    |  | $\neg(x_3 = \bigcirc)$                       | $\neg(x_3 = \bullet)$                        |

**Table 2.1:** Example databases for a sample run of the hyper-resolution based consistency algorithm as applied to the graph of figure 2.5. Only a few of the nogoods produced by the graph's constraints are shown due to space constraints. You can infer the rest. New statements are added starting at time 1.

all agents can find a value regardless of the value any one of their neighbors might set. We need to extend this procedure to more than one neighbor. This can be done using the **hyper-resolution rule**.

HYPER-RESOLUTION RULE

**Definition 2.5** (Hyper-Resolution Rule).

$$\begin{array}{c}
 A_1 \vee A_2 \vee \dots \vee A_m \\
 \neg(A_1 \wedge A_{11} \wedge \dots) \\
 \neg(A_2 \wedge A_{21} \wedge \dots) \\
 \vdots \\
 \neg(A_m \wedge A_{m1} \wedge \dots) \\
 \hline
 \neg(A_{11} \wedge \dots \wedge A_{21} \wedge \dots \wedge A_{m1} \wedge \dots)
 \end{array}$$

The hyper-resolution rule is a simple deduction rule from formal logic whose form just so happens to correspond exactly to what we need. The idea is that the agents' domains can be represented as an OR statement (the first one in the rule). Namely, the agent can be either gray, or black, or white. The constraints are represented as the negations (the rest of the statement). These represented the fact that it is not good if agent 1 is white and agent 2 is white, for example. Once we have represented both the domain and the constraints in this logical form then we can use the hyper-resolution rule to generate new constraints (nogoods) which are added to the agent's knowledge and communicated to the other agents.

We can use the hyper-resolution rule to implement a distributed constraint satisfaction algorithm that, unlike the filtering algorithm, will detect all cases where there is no solution and will find a solution when there is one. The algorithm is basically the same as the filtering algorithm but we modify the REVISE function to instead use the hyper-resolution to generate new nogoods. These nogoods are then sent to the other agents. When the agent receives nogoods from other agents it adds them to its set of nogoods. The process continues until no new nogoods can be derived by any agent or until a contradiction is reached. When a contradiction is found it means that no solution exists to this problem.

We now show how the hyper-resolution algorithm would solve the problem from figure 2.5. Table 2.1 shows, in black, a few of the initial statements in the agents' databases. Note that many of the nogoods are missing due to space limitations but you can easily infer them as they correspond to the constraints in the problem. Once possible run is given by the following series of actions:

Let  $x_1$  start out by doing

$$\begin{array}{c}
 x_1 = \bigcirc \vee x_1 = \bullet \\
 \neg(x_1 = \bigcirc \wedge x_2 = \bigcirc) \\
 \neg(x_1 = \bullet \wedge x_3 = \bullet) \\
 \hline
 \neg(x_2 = \bigcirc \wedge x_3 = \bullet)
 \end{array}$$

at time 1. It then sends  $\neg(x_2 = \bigcirc \wedge x_3 = \bullet)$  to  $x_2$  and  $x_3$ .  $x_2$  then can do

$$\begin{array}{c}
x_2 = \circ \vee x_2 = \bullet \\
\neg(x_2 = \circ \wedge x_3 = \bullet) \\
\neg(x_2 = \bullet \wedge x_3 = \bullet) \\
\hline
\neg(x_3 = \bullet)
\end{array}$$

Meanwhile,  $x_1$  can do

$$\begin{array}{c}
x_1 = \circ \vee x_1 = \bullet \\
\neg(x_1 = \bullet \wedge x_2 = \bullet) \\
\neg(x_1 = \circ \wedge x_3 = \circ) \\
\hline
\neg(x_2 = \bullet \wedge x_3 = \circ)
\end{array}$$

$x_1$  then sends  $\neg(x_2 = \bullet \wedge x_3 = \circ)$  to  $x_2$  and  $x_3$ . Using this message,  $x_2$  can now do

$$\begin{array}{c}
x_2 = \circ \vee x_2 = \bullet \\
\neg(x_2 = \circ \wedge x_3 = \circ) \\
\neg(x_2 = \bullet \wedge x_3 = \circ) \\
\hline
\neg(x_3 = \circ)
\end{array}$$

$x_2$  then sends  $\neg(x_3 = \circ)$  and  $\neg(x_3 = \bullet)$  to  $x_3$ . Using this message,  $x_3$  can then do

$$\begin{array}{c}
x_3 = \circ \vee x_3 = \bullet \\
\neg(x_3 = \circ) \\
\neg(x_3 = \bullet) \\
\hline
\text{Contradiction}
\end{array}$$

This last step derives a contradiction which means that the algorithm has been able to prove that no solution exists.

In practice, the hyper-resolution algorithm is very slow because there is often little guidance as to which nogoods should be derived first. That is, at any one time each agent can derive a large number of different nogoods. Most of these are useless to the other agents and there is no easy way for the agent to determine which nogoods might be most useful since that would require that it also know about the other agents' databases. As such, we find that in practice the hyper-resolution algorithm spends a lot of time deriving true but useless nogoods. This is especially true as the size of the nogoods—their number of terms—increases. Also, if the problem does have a solution then the algorithm only stops when all the agents prove to themselves that they cannot generate any new nogood, which can take a very long time. In summary, hyper-resolution is impractical for most cases with the possible exception of highly over-constrained problems.

### 2.1.3 Asynchronous Backtracking

Earlier, in figure 2.2, we saw a depth first search algorithm for solving a constraint satisfaction problem. The algorithm sets a variable to a specific value and, recursively, tries to set a new variable to some value which would not violate any of the constraints with existing values. If no such value exists the algorithm backs up and re-sets one of the variables it has already set. This type of depth first search is also known as a **backtracking** algorithm because we set the variables to some values and then if there is a conflict we backtrack and unset variables and try again with different values.

The **asynchronous backtracking** algorithm (ABT), is a distributed asynchronous version of the centralized depth first search algorithm for constraint satisfaction (Yokoo and Hirayama, 2000). ABT performs the same kind of search but it does so via the use of messages between agents and under the understanding that only the owner of the variable can change its value.

The ABT algorithm also implements some performance improvements on the basic depth first search. As we mentioned earlier, there is often much to be gained

BACKTRACKING

ASYNCHRONOUS  
BACKTRACKING



by properly ordering the variables. In general, we want to place variables that have more constraints towards the top of the tree and search them first so as to avoid having to do some backtracking later on. This heuristic has been shown to work well and is employed by ABT.

Each agent  $i$  in ABT is responsible for variable  $x_i$ . All agents are given a fixed *priority* ordering, from 1 to  $n$ . Each agent  $i$  can change the value of its variable  $x_i$ . The agents keep track of the others' variables in *local-view*. The list of *neighbors* is initially set to be just the other agents with whom the agent shares a constraint but, as we will see, this list can grow. The agents find out other agents' variable values by sending messages. There are only three types of messages that ABT sends. Thus, only three remote procedures need to be implemented.

1. `HANDLE-OK?( $j, x_j$ )` where  $j$  is the name of another agent and  $x_j$  is its current variable value. This message asks the receiver if that assignment does not violate any of his constraints.
2. `HANDLE-NOGOOD( $j, nogood$ )` which means that  $j$  is reporting that it can't find a value for his variable because of *nogood*.
3. `HANDLE-ADD-NEIGHBOR( $j$ )` which requests the agent to add some other agent  $j$  as its neighbor. This message is handled by simply adding that agent to *neighbors*.

The full algorithm can be seen in figure 2.6. All agents start out by setting their  $x_i$  variable to some value and then asking all their neighbors to `HANDLE-OK?` After that they become event-driven, handling messages as they arrive. ABT assumes that all messages will arrive and in the same order they were sent.

The procedure `CHECK-LOCAL-VIEW` checks that the agent's current value  $x_i$  is consistent with the values it thinks its neighbors have. If not then it checks to see if it can find some other value that will work. If it can't find one then this means it must `BACKTRACK`. If it can then it sets  $x_i$  to that value and informs its neighbors.

Nogoods are handled by `HANDLE-NOGOOD`. The first thing this procedure does is to see if any of the agents in the nogood is not in *neighbors* so it can add it. This step is needed to ensure the algorithm will arrive at a correct answer. The nogood is added to the set of constraints and `CHECK-LOCAL-VIEW` is called to ensure there are no conflicts.

Finally, the `BACKTRACK` procedure is responsible for generating and sending new nogoods. As we saw, it is only called when there is no way for the agent to set its value so that no constraints are violated. This means that someone else must change their value. That someone else is the agent with the lowest priority in the nogood. `BACKTRACK` first uses the hyper-resolution rule (or similar) to generate one or more nogoods. Each one of these nogoods is then sent to the agent in that nogood that has the lowest priority. Also, if the hyper-resolution generated a contradiction, represented as an empty set then this means that no solution exists and thus the algorithm is terminated.

Notice that, since the nogoods are always sent to the lowest priority agent and the agents have linear priorities, the nogoods always flow from the high priority agents to the low priority agents. Namely, the highest priority agent will never receive a nogood, the second highest priority agent will only receive nogoods from the highest priority agent, and so on. This suggests to us that the lower priority agents will end up doing more work.

An example of ABT at work can be seen in figure 2.7. Here agents  $x_2$  and  $x_3$  start by setting their colors to be gray and black, respectively, and asking  $x_1$  to `HANDLE-OK?`. Agent  $x_1$  then generates a new nogood and sends a *nogood* to  $x_3$ .  $x_3$  notices that  $x_2$  is in the nogood but not in his *neighbors* and so adds it as his neighbor and asks  $x_2$  to add him as a neighbor. In response to this  $x_2$  asks  $x_3$  to `HANDLE-OK?` with his new color. Agent  $x_3$  uses  $x_2$ 's new color to generate a new nogood that it sends to  $x_2$ . The figure ends at this point but the algorithm would continue with  $x_2$

Some versions of ABT use one nogood in line 1 of `BACKTRACK`, others use more.



```

HANDLE-OK?( $j, x_j$ )
1   $local-view \leftarrow local-view + (j, x_j)$ 
2  CHECK-LOCAL-VIEW()

CHECK-LOCAL-VIEW()
1  if  $local-view$  and  $x_i$  are not consistent
2      then if no value in  $D_i$  is consistent with  $local-view$ 
3          then BACKTRACK()
4          else select  $d \in D_i$  consistent with  $local-view$ 
5               $x_i \leftarrow d$ 
6               $\forall_{k \in neighbors} k.HANDLE-OK?(i, x_i)$ 

HANDLE-ADD-NEIGHBOR( $j$ )
1   $neighbors \leftarrow neighbors + j$ 

HANDLE-NOGOOD( $j, nogood$ )
1  record  $nogood$  as a new constraint
2  for  $(k, x_k) \in nogood$  where  $k \notin neighbors$ 
3      do  $k.HANDLE-ADD-NEIGHBOR(i)$ 
4           $neighbors \leftarrow neighbors + k$ 
5           $local-view \leftarrow local-view + (k, x_k)$ 
6   $old-value \leftarrow x_i$ 
7  CHECK-LOCAL-VIEW()
8  if  $old-value \neq x_i$ 
9      then  $j.HANDLE-OK?(i, x_i)$ 

BACKTRACK()
1   $nogoods \leftarrow \{V \mid V = \text{inconsistent subset of } local-view \text{ using hyper-resolution rule}\}$ 
2  if an empty set is an element of  $nogoods$ 
3      then broadcast that there is no solution
4      terminate this algorithm
5  for  $V \in nogoods$ 
6      do select  $(j, x_j)$  where  $j$  has lowest priority in  $V$ 
7           $j.HANDLE-NOGOOD(i, V)$ 
8           $local-view \leftarrow local-view - (j, x_j)$ 
9  CHECK-LOCAL-VIEW()

```

**Figure 2.6:** ABT algorithm. All agents start by setting themselves to some color and invoking a HANDLE-OK? on all their neighbors. After this they become event driven.

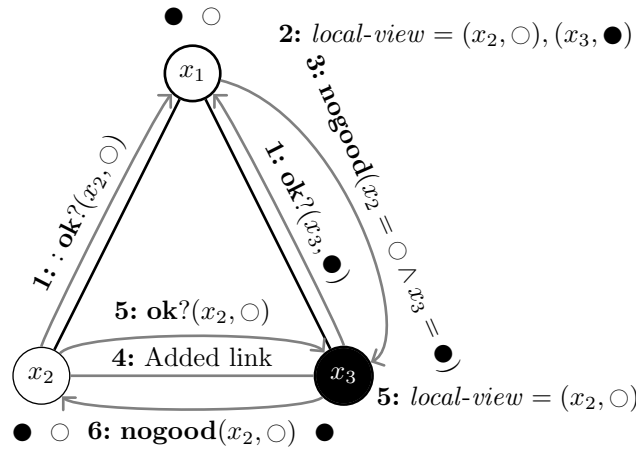
realizing that it must set itself to black and sending the appropriate HANDLE-OK? messages, and so on.

The ABT algorithm has been shown to be complete.

**Theorem 2.1** (ABT is Complete). *The ABT algorithm always finds a solution if one exists and terminates with the appropriate message if there is no solution (Yokoo et al., 1998).*

*Proof.* The proof works by induction. In order to prove it we must first show that the agent with the highest priority never enters an infinite loop. Then show that given that all the agents with lower priority than  $k$  never fall into an infinite loop then  $k$  will not fall into an infinite loop.  $\square$

Unfortunately, as you will notice from running the NetLogo model, this algo-



**Figure 2.7:** ABT example. The numbers preceding each message indicate the time at which the particular action took place. Only the first six steps are shown.

gorithms has some problems. The most important is the uneven division of labor. In ABT the lowest priority agent ends up doing many times more work than the highest priority agents, which is reflected by the number of messages each agent receives. The difference only gets worse as the number of agents increases. Another drawback is the fact that in practice ABT can take very long as it also suffers from the same problem as regular backtracking. That is, if the high priority agents make bad decisions (choose colors that will not work) then it takes a long time to fix these as all the other agents have to prove that those colors will not work by trying all possible combinations of their colors. You can check this in our NetLogo model by running it with increasing edge ratios. You will notice that the time to solve a problem increases exponentially as the ratio increases. This means that as the problem becomes more constrained the time to find a solution explodes.

Another possible improvement to ABT can be found in line 1 of BACKTRACKING which tells us to find new nogoods. As you can see, the line does not specify which subset of nogoods to generate. One way to improve ABT is by managing which nogoods get generated such that only those that are likely to lead to a solution get generated (Jiang and Vidal, 2005; Jiang and Vidal, 2006). This strategy has the effect of greatly reducing the number of cycles needed to find a solution.

We can also extend ABT by allowing agents to solve the problem without the need to add new links, thus limiting the agent to agent communication. This enhancement has been implemented by the kernel ABT algorithm (Bessière et al., 2005). This algorithm could be used in applications where it is impossible to change the original communication network.



#### 2.1.4 Asynchronous Weak-Commitment Search

As we mentioned, ABT suffers from the fact that the agents have a fixed priority so that if the highest priority agent makes a bad decision then all the other agents must perform an exhaustive search to prove him wrong. The **Asynchronous Weak-Commitment** (AWC) algorithm tries to avoid this problem by giving the agents dynamic priorities. The agents all start out with the same priority but whenever an agent is forced to backtrack and resolve a new nogood then it raises its priority to be higher than anyone else's. The agents inform each other about their priorities by including them in the HANDLE-OK? and HANDLE-NOGOOD remote procedure calls.

AWC also uses the min-conflict heuristic (Minton et al., 1992) in order to reduce the risk of making a bad decision. The min-conflict heuristic tells the agents to choose the value for their variable which minimizes the number of constraint violations with other agents given the current values.

The AWC algorithm is the same as ABT except that it re-defines both CHECK-LOCAL-VIEW and BACKTRACK. The re-defined functions from the AWC algorithm can be seen in figure 2.8. As you can see, it amounts to only a small modification to

ASYNCHRONOUS  
WEAK-COMMITMENT



## CHECK-LOCAL-VIEW

```

1  if  $x_i$  is consistent with local-view
2    then return
3  if no value in  $D_i$  is consistent with local-view
4    then BACKTRACK()
5  else select  $d \in D_i$  consistent with local-view and which minimizes constraint
                                     violations with lower priority agents.
6       $x_i \leftarrow d$ 
7       $\forall_{k \in \text{neighbors}} k.\text{HANDLE-OK?}(i, x_i, \text{priority})$ 

```

## BACKTRACK

```

1  generate a nogood  $V$ 
2  if  $V$  is empty nogood
3    then broadcast that there is no solution
4    terminate this algorithm
5  if  $V$  is a new nogood
6    then  $\forall_{(k, x_k) \in V} k.\text{HANDLE-NOGOOD}(i, j, \text{priority})$ 
7         $\text{priority} \leftarrow 1 + \max\{\text{neighbors' priorities}\}$ 
8        select  $d \in D_i$  consistent with local-view and which minimizes constraint
                                     violations with lower priority agents.
9       $x_i \leftarrow d$ 
10      $\forall_{k \in \text{neighbors}} k.\text{HANDLE-OK?}(i, x_i, \text{priority})$ 

```

**Figure 2.8:** Asynchronous weak-commitment search algorithm. The rest of the procedures are the same as in ABT, from figure 2.6 except that they now also record the calling agent's priority.

ABT. The agents now must keep track of all their neighbors' priorities. Whenever an agent detects a nogood it sets its priority to be higher than all its neighbors. This amounts to having the most constrained agent pick its value first.

Note that the algorithm as presented in figure 2.8 glosses over some important implementation details. Line 5 of BACKTRACK asks us to check if  $V$  is a new nogood. This means that the nogoods need to be stored in a data structure with a fast search function, perhaps a hash table or a database. Line 5 in CHECK-LOCAL-VIEW, as well as line 8 in BACKTRACK, ask us to solve a minimization problem which could also take a long time for agents with high priority. Finally, line 1 of BACKTRACK does not specify which nogood we should generate and, as we mentioned in the previous section, is thus amenable to improvements.

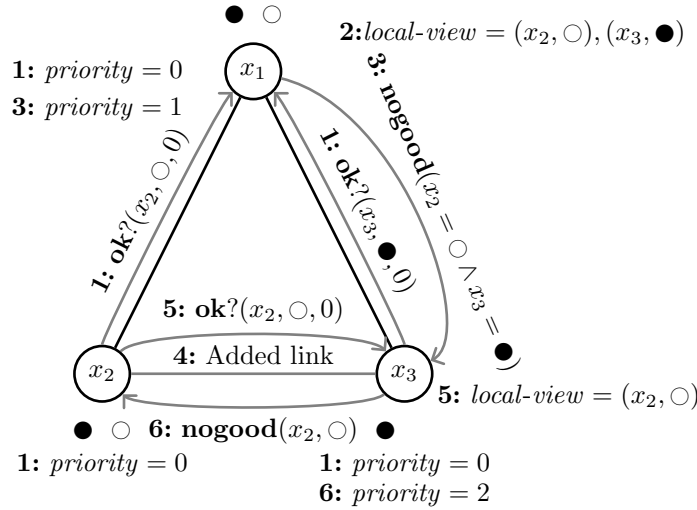
In practice, a problem that plagues both AWC and ABT is the fact that the implementation of a good hyper-resolution function is not easy. Even if implemented correctly, finding a new nogood could take a very long time. In fact, one would expect that the time required to generate new nogoods would increase linearly, perhaps exponentially, as the number of known nogoods increases.

Just like its predecessor, AWC has been shown to be complete. The only tricky part to proving its completeness lies in those changing priorities. Luckily, we note that these priorities cannot be forever changing as there are only a finite number of impossible value assignments. Thus, at some point we will have tried all possible value combinations and the priorities will stop increasing.

**Theorem 2.2** (AWC is complete). *The AWC algorithm always finds a solution if one exists and terminates with the appropriate message if there is no solution (Yokoo and Hirayama, 2000).*

*Proof.* The priority values are changed if and only if a new nogood is found. Since the number of possible nogoods is finite the priority values cannot be changed indefinitely. When the priority values are stable AWC becomes ABT, which is complete.  $\square$

It has been shown that AWC outperforms ABT in that it finds the solution much faster in randomly generated graphs (Yokoo and Hirayama, 2000). Neither ABT nor



**Figure 2.9:** AWC example. The numbers indicate the time at which the particular action took place. Only the first six steps are shown. Note how the priorities of the agents change.

AWC provide us with guidance as to which nogood should be generated. That is, at any step there are many different nogoods that can be generated and these nogoods get bigger as the number of neighbors increases.

### 2.1.5 Distributed Breakout

Another way to approach this problem, different from the backtracking idea, is to use a hill-climbing method. In hill-climbing we start by randomly giving all variables a value then we change the values so as to minimize the constraint violations. We repeat the process until we cannot improve on the current solution. In a distributed version of hill-climbing each agent is a variable and all agents change their color in parallel, sending messages to their neighbors which indicate their new color.

All hill-climbing algorithms suffer from the problem of local minima. That is, it is possible for the system to reach a state where no one agent can find a better value for its value and yet that is not the best possible value assignment for the system. A local minimum can be recognized by the fact that no one agent can reduce the total number of constraint violations by changing its color but that number could be reduced by having two or more agents change their color simultaneously. As such, identifying the fact that the system is in a local minimum requires all agents to broadcast the fact that they are in a local minimum and these broadcasts must be further annotated to ensure that the agents' local views are coherent with each other, that is, they were all in the same global state when they sent out their broadcasts. All these would require a complex implementation, large amounts of communications, and would result in longer run times.

The **distributed breakout** (Yokoo and Hirayama, 1996; Hirayama and Yokoo, 2005; Yokoo and Hirayama, 2000) algorithm tries to bypass this problem by recognizing a **quasi-local minimum** instead of a the full local minimum. The quasi-local minimum has the advantage that it can be recognized by an individual agent.

**Definition 2.6** (Quasi-local minimum). *An agent  $x_i$  is in a quasi-local minimum if it is violating some constraint and neither it nor any of its neighbors can make a change that results in lower total cost for the system.*

Unfortunately, the fact that an agent is in a quasi-local minimum does not necessarily imply that it is in a local minimum since it is possible that some non-neighboring agents could make changes that would lower the total cost. On the other hand, if the system is in a local minimum then at least one agent will find itself in a quasi-local minimum. Thus, to escape local minimum the algorithm identifies quasi-local minima and increases the cost of the constraint violations.

That is, the breakout algorithm also maintains a weight associated with constraint. These weights are initially set to one. Whenever an agent finds itself in a

A local minimum is akin to a Nash equilibrium while the optimal solution is akin to the utilitarian solution in game theory.

DISTRIBUTED BREAKOUT

QUASI-LOCAL MINIMUM

```

HANDLE-OK?( $j, x_j$ )
1   $received-ok[j] \leftarrow \text{TRUE}$ 
2   $agent-view \leftarrow agent-view + (j, x_j)$ 
3  if  $\forall_{k \in neighbors} received-ok[k] = \text{TRUE}$ 
4    then SEND-IMPROVE()
5     $\forall_{k \in neighbors} received-ok[k] \leftarrow \text{FALSE}$ 

SEND-IMPROVE()
1   $cost \leftarrow$  evaluation of  $x_i$  given current weights and values.
2   $my-improve \leftarrow$  possible maximal improvement
3   $new-value \leftarrow$  value that gives maximal improvement
4   $\forall_{k \in neighbors} k.HANDLE-IMPROVE(i, my-improve, cost)$ 

HANDLE-IMPROVE( $j, improve, eval$ )
1   $received-improve[j] \leftarrow improve$ 
2  if  $\forall_{k \in neighbors} received-improve[k] \neq \text{NONE}$ 
3    then SEND-OK
4     $agent-view \leftarrow \emptyset$ 
5     $\forall_{k \in neighbors} received-improve[k] \leftarrow \text{NONE}$ 

SEND-OK()
1  if  $\forall_{k \in neighbors} my-improve \geq received-improve[k]$ 
2    then  $x_i \leftarrow new-value$ 
3  if  $cost > 0 \wedge \forall_{k \in neighbors} received-improve[k] \leq 0 \triangleright$  quasi-local optimum
4    then increase weight of constraint violations
5   $\forall_{k \in neighbors} k.HANDLE-OK?(i, x_i)$ 

```

**Figure 2.10:** The distributed breakout algorithm. Agents start by setting themselves to some color and then calling HANDLE-OK? on all their neighbors. Line 1 of SEND-IMPROVE sets the *cost* to be the weighed sum of all the current constraint violations involving the agent.

quasi-local-minimum it increases the weights of the constraints that are being violated. The weights in turn are used to calculate the cost of a constraint violation. That is, rather than simply counting the number of violated constraints, distributed breakout uses the weighted sum of the violated constraints. In this way, a constraint that was violated increases in weight so it becomes more important to satisfy that constraint the next time around.

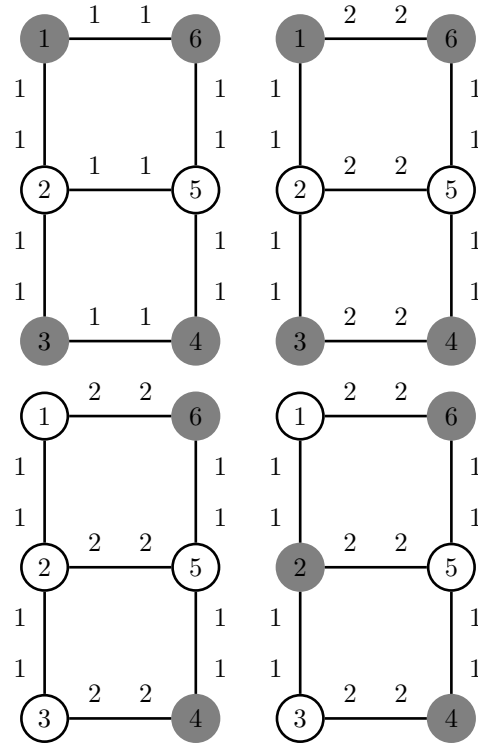
Distributed breakout implements two remote procedures.

- HANDLE-OK?( $i, x_i$ ) where  $i$  is the agent and  $x_i$  is its current value,
- HANDLE-IMPROVE( $i, improve, eval$ ) where *improve* is the maximum  $i$  could gain by changing to some other color and *eval* is its current cost.

The full algorithm can be seen in figure 2.10. In it we can see that the agents spend their time either waiting to HANDLE-OK? or HANDLE-IMPROVE from all their neighbors. Once an agent gets all the HANDLE-OK? invocations it calculates its new cost and possible improvement and sends an HANDLE-IMPROVE to all its neighbors. Once an agent gets all the HANDLE-IMPROVE messages then the agent determines if it is in a quasi-local-minimum. If so then it increases the weights of the constraint violations and sends a HANDLE-OK? to its neighbors.

Figure 2.11 shows an example application of the distributed breakout algorithm. We start out with a random coloring for all the nodes as seen on the first picture on the left. Once the agents receive all the HANDLE-OK? from their neighbors they check to see how much they could improve by changing and send the HANDLE-IMPROVE messages. In this case, it turns out that for all agents  $my-improve \leq 0$ .





**Figure 2.11:** Example of distributed breakout. The only colors allowed are white and gray. The numbers represent the weights that the agents give to each of their constraints. We show four steps, starting at the top left.

Therefore, the agents increase weights to that shown on the second picture. At this point nodes 1, 2, 4 and 6 have a *my-improve* = 1 while 2, 5 are at  $-3$ . We break ties using the agent number where smaller number wins. As such, agents 1 and 3 are the ones that get to change their color. They do so in the third picture. They then tell their neighbors their new color. At this point agent 2 now has a *my-improve* = 4 (that is  $1 + 1 + 2$ ) which is higher than anyone else's so it gets to change its color which moves us to the last picture.

The performance of distributed breakout is, on average, superior to AWC which is in turn superior to ABT. However, we should always remember that distributed breakout is not complete.

**Theorem 2.3** (Distributed Breakout is **not** Complete). *Distributed breakout can get stuck in local minimum. Therefore, there are cases where a solution exists and it cannot find it.*

Distributed breakout will also not be able to identify that a particular problem lacks a solution. Specifically, the algorithm will either continue running or get stuck in a local minimum when given a problem without a solution. As such, breakout cannot be used when there is any chance that the problem lacks a solution.

Tests have also shown that the load distribution on distributed breakout is very uneven. That is, some agents end up handling several times more messages than other agents. There is probably a simple correlation between the number of edges an agent has and the number of messages it must process.

Still, distributed breakout has been shown to, in practice, find the local optima a very large percentage of the time, and variations of the basic algorithm perform even better. Specifically, **Multi-DB<sup>++</sup>** has been shown to, in practice, always find the solution for 3SAT problems (Hirayama and Yokoo, 2005).

Exercise: Give an example which proves the theorem.

MULTI-DB<sup>++</sup>

## 2.2 Distributed Constraint Optimization

The **distributed constraint optimization** problem is similar to the satisfaction problem except that the constraints return a real number and the goal is to minimize the value of these constraint violations.

DISTRIBUTED CONSTRAINT  
OPTIMIZATION

```

BRANCH-AND-BOUND-COP()
1   $c^* \leftarrow \infty$     ▷ Minimum cost found. Global variable.
2   $g^* \leftarrow \emptyset$     ▷ Best solution found. Global variable.
3  BRANCH-AND-BOUND-COP-HELPER(1,  $\emptyset$ )
4  return  $g^*$ 

BRANCH-AND-BOUND-COP-HELPER( $i, g$ )
1  if  $i = n$ 
2      then if  $P(g) < c^*$ 
3          then  $g^* \leftarrow g$ 
4               $c^* \leftarrow P(g)$ 
5          return
6  for  $v \in D_i$ 
7      do  $g' \leftarrow g + \{x_i \leftarrow v\}$ 
8          if  $P(g) < c^*$ 
9              then BRANCH-AND-BOUND-COP-HELPER( $i + 1, g'$ )

```

**Figure 2.12:** A centralized branch and bound algorithm for constraint optimization. It assumes there are  $n$  variables,  $x_1, \dots, x_n$ , and that  $P(g)$  returns the total of any constraint violations given partial assignment  $g$  and constraints  $P$ .

**Definition 2.7** (Constraint Optimization Problem (COP)). *Given a set of variables  $x_1, x_2, \dots, x_n$  with domains  $D_1, D_2, \dots, D_n$  and a set of constraints  $P$  of the form  $pk(x_{k1}, x_{k2}, \dots, x_{kj}) \rightarrow \mathbb{R}$ , find assignments for all the variables such that the sum of the constraint values is minimized.*

COP is an NP-Complete problem. That means that, at worse, all algorithms will take an exponential time (in the number of variables) to finish. Even parallel algorithms will, at worst, need exponential time. However, those are worst case scenarios. In practice we find some algorithms that do very well on most problems.

The most common approach to solving COPs is to use a **branch and bound** algorithm such as the one shown in figure 2.12. Branch and bound algorithms perform a depth first search of the variable assignments but also maintain a bound,  $c^*$ , which is the cost of the best solution found thus far. If a partial variable assignment already has a cost that is equal or higher than  $c^*$  then we know that there is no need to find values for the rest of the variables as the cost will only increase. The algorithm is complete so it will find the best possible solution to the problem, but doing so might require a lot of time. Since it is based on a depth first search its memory requirements are linear on the number of variables.

The distributed constraint optimization problem distributes the variables to the agents.

**Definition 2.8** (Distributed Constraint Optimization Problem (DCOP)). *Give each agent one of the variables in a COP. Agents are responsible for finding a value for their variable and can find out the values of their neighbors' via communication*

We now look at some algorithms for solving the DCOP problem.

### 2.2.1 Adopt

The **Adopt** algorithm (Modi et al., 2005) is a recent addition to the family of DCOP algorithms. It roughly depth-first search on the set of possible value assignments, but with a lot of improvements on the basic search strategy. Namely, each agent in the depth-first tree keeps a lower and upper bound on the cost for the sub-problem below him (given assignments from above) and on the sub-problems for each one of his children. It then tells the children to look for a solution but ignore any partial solution whose cost is above the lower bound because, we already know that we can get that lower cost, so why bother another.



RESET-VARIABLES( $d, c$ )

```

1  lower-bound[ $d, c$ ]  $\leftarrow 0$ 
2   $t[d, c] \leftarrow 0$ 
3  upper-bound[ $d, c$ ]  $\leftarrow \infty$ 
4  context[ $d, c$ ]  $\leftarrow \{\}$ 

```

INITIALIZE()

```

1  threshold  $\leftarrow 0$ 
2  received-terminate  $\leftarrow \text{FALSE}$ 
3  current-context  $\leftarrow \{\}$ 
4   $\forall d \in D_i, c \in \text{children}$  RESET-VARIABLES( $d, c$ )
5   $x_i \leftarrow d \in D_i$  which minimizes: my cost plus  $\sum_{c \in \text{children}} \text{lower-bound}[d, c]$ 
6  BACKTRACK()

```

HANDLE-THRESHOLD( $t, \text{context}$ )

```

1  if context is compatible with current-context
2      then threshold  $\leftarrow t$ 
3          MAINTAIN-THRESHOLD-INVARIANT()
4          BACKTRACK()

```

HANDLE-TERMINATE(*context*)

```

1  received-terminate  $\leftarrow \text{TRUE}$ 
2  current-context  $\leftarrow \text{context}$ 
3  BACKTRACK()

```

HANDLE-VALUE( $j, x_j$ )

```

1  if  $\neg \text{received-terminate}$ 
2      then current-context[ $j$ ]  $\leftarrow x_j$ 
3          for  $d \in D_i, c \in \text{children}$  such that context[ $d, c$ ] is
              incompatible with current-context
4              do RESET-VARIABLES( $d, c$ )
5              MAINTAIN-THRESHOLD-INVARIANT()
6              BACKTRACK()

```

HANDLE-COST( $k, \text{context}, lb, ub$ )

```

1   $d \leftarrow \text{context}[i]$ 
2  delete context[ $i$ ]
3  if  $\neg \text{received-terminate}$ 
4      then for  $(j, x_j) \in \text{context}$  and  $j$  is not my neighbor
5          do current-context[ $j$ ]  $\leftarrow x_j$ 
6          for  $d' \in D_i, c \in \text{children}$  such that context[ $d, c$ ] is
              incompatible with current-context
7              do RESET-VARIABLES( $d', c$ )
8          if context compatible with current-context
9              then lower-bound[ $d, k$ ]  $\leftarrow lb$ 
10                 upper-bound[ $d, k$ ]  $\leftarrow ub$ 
11                 context[ $d, k$ ]  $\leftarrow \text{context}$ 
12                 MAINTAIN-CHILD-THRESHOLD-INVARIANT()
13                 MAINTAIN-THRESHOLD-INVARIANT()
14                 BACKTRACK()

```

**Figure 2.13:** The Adopt algorithm. Before the algorithm is even called the agents must form a depth first search tree. Each agent has a *parent* variable which points to its parent or, in the case of the root, in null.

```

BACKTRACK()
1  if  $threshold = \min_{d \in D_i} \text{cost}(d) + \sum_{c \in \text{children}} \text{upper-bound}[d, c]$ 
2    then  $x_i \leftarrow \arg \min_{d \in D_i} \text{cost}(d) + \sum_{c \in \text{children}} \text{upper-bound}[d, c]$ 
3  elseif  $threshold < \text{cost}(x_i) + \sum_{c \in \text{children}} \text{lower-bound}[x_i, c]$ 
4    then  $x_i \leftarrow \arg \min_{d \in D_i} \text{cost}(d) + \sum_{c \in \text{children}} \text{lower-bound}[d, c]$ 
5   $\forall k \in \text{neighbors} \wedge k \text{ has lower priority } k.\text{HANDLE-VALUE}(i, x_i)$ 
6  MAINTAIN-ALLOCATION-INVARIANT()
7  if  $threshold = \min_{d \in D_i} \text{cost}(d) + \sum_{c \in \text{children}} \text{upper-bound}[d, c]$  and
    (received-terminate or I am root)
8    then  $\text{current-context}[i] \leftarrow x_i$ 
9     $\forall c \in \text{children } c.\text{HANDLE-TERMINATE}(\text{current-context})$ 
10   exit
11   $\text{parent}.\text{HANDLE-COST}(\text{current-context},$ 
     $\min_{d \in D_i} \text{cost}(d) + \sum_{c \in \text{children}} \text{lower-bound}[d, c],$ 
     $\min_{d \in D_i} \text{cost}(d) + \sum_{c \in \text{children}} \text{upper-bound}[d, c])$ 

MAINTAIN-THRESHOLD-INVARIANT()
1  if  $threshold < \min_{d \in D_i} \text{cost}(d) + \sum_{c \in \text{children}} \text{lower-bound}[d, c]$ 
2    then  $threshold \leftarrow \min_{d \in D_i} \text{cost}(d) + \sum_{c \in \text{children}} \text{lower-bound}[d, c]$ 
3  if  $threshold > \min_{d \in D_i} \text{cost}(d) + \sum_{c \in \text{children}} \text{upper-bound}[d, c]$ 
4    then  $threshold \leftarrow \min_{d \in D_i} \text{cost}(d) + \sum_{c \in \text{children}} \text{upper-bound}[d, c]$ 

MAINTAIN-ALLOCATION-INVARIANT()
1  while  $threshold > \text{cost}(x_i) + \sum_{c \in \text{children}} t[x_i, c]$ 
2    do  $\text{chosen} \leftarrow c' \in \text{children}$  such that  $\text{upper-bound}[x_i, c'] > t[x_i, c']$ 
3     $t[x_i, \text{chosen}] \leftarrow t[x_i, \text{chosen}] + 1$ 
4  while  $threshold < \text{cost}(x_i) + \sum_{c \in \text{children}} t[x_i, c]$ 
5    do  $\text{chosen} \leftarrow c' \in \text{children}$  such that  $\text{lower-bound}[x_i, c'] < t[x_i, c']$ 
6     $t[x_i, \text{chosen}] \leftarrow t[x_i, \text{chosen}] - 1$ 
7   $\forall c \in \text{children } c.\text{HANDLE-THRESHOLD}(t[x_i, \text{chosen}], \text{current-context})$ 

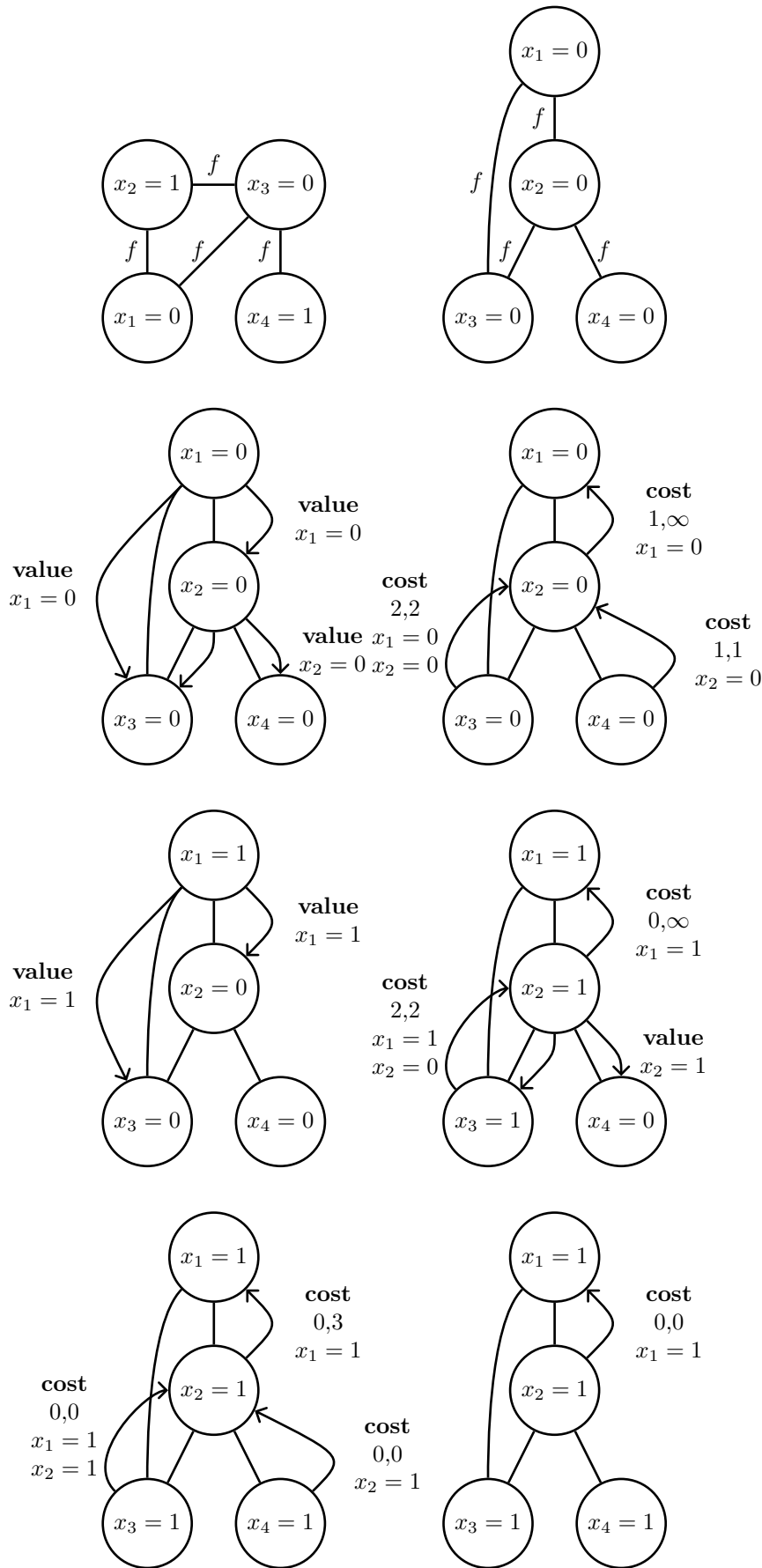
MAINTAIN-CHILD-THRESHOLD-INVARIANT()
1  for  $d \in D_i, c \in \text{children}$ 
2    do if  $\text{lower-bound}[d, c] > t[d, c]$ 
3      then  $t[d, c] \leftarrow \text{lower-bound}[d, c]$ 
4  for  $d \in D_i, c \in \text{children}$ 
5    do if  $\text{upper-bound}[d, c] < t[d, c]$ 
6      then  $t[d, c] \leftarrow \text{upper-bound}[d, c]$ 

```

**Figure 2.14:** The Adopt algorithm, continued.

The general idea in Adopt is that each node in tree calculates upper and lower bounds on the cost for all the constraints that involve all the nodes in the subtree rooted at that node, given an assignment to the nodes on the path to the root. Each node changes to the value (color) whose lower bound is smaller at each time and then asks its children to calculate new bounds given the new value. Upon receiving this message each node again changes its value to the one with the lowest lower bound and the process is repeated. Figures 2.13 and 2.14 show the full algorithm.

The basic workings of Adopt are best shown via an example. Table 2.2 shows the cost function we will be using in our example. Figure 2.15 shows a trace of an application of Adopt to a simple problem. As you can see, all the constraints are binary, as Adopt can only handle binary constraints. All the agents keep track of the



**Figure 2.15:** Adopt example. Start at the top left and read left to right. The domain for all nodes is the set  $\{0, 1\}$ ,

**Table 2.2:** Cost function for Adopt example in figure 2.15.

| $d_i$ | $d_j$ | $p(d_i, d_j)$ |
|-------|-------|---------------|
| 0     | 0     | 1             |
| 0     | 1     | 2             |
| 1     | 0     | 2             |
| 1     | 1     | 0             |

upper and lower bounds sent by their children along with the associated context. They use these bounds to calculate bounds on their own costs. Initially, all lower bounds are set to 0 and all upper bounds to infinity.

The first diagram is the graph itself. The first thing we must do before even starting Adopt is to form a depth-first search graph. That is, a graph such that all constraints emanating from a node go to either one of its descendants—the tree below it—or one of its ancestors—agents on the path from the node to the root. There exists several algorithms for finding such a tree. The second diagram shows a possible depth-first search tree for our example.

The third diagram shows the first step in the algorithm. All the nodes invoke HANDLE-VALUE on all the descendants with whom they share a constraint. That is why  $x_1$  sends a message to  $x_3$  but not  $x_4$ . These HANDLE-VALUE are similar to the HANDLE-OK? functions we saw before. They simply inform the other agents of the new value, in this case the original values.

The agents use these values to calculate both upper and lower bounds on their cost and report these back up to their parents, as shown in the fourth diagram. For example,  $x_3$  calculated that, given that  $x_1 = 0$  and  $x_2 = 0$  if it set itself to 0 the cost would be 2 and if set itself to 1 the cost would be 4, remember that we are using the cost function from Table 2.2, as such it stays at 0 and sends a HANDLE-COST to its parent saying that its lower and upper bounds, given  $x_1 = 0$  and  $x_2 = 0$ , are 2. This message means that the cost of the constraints for the subtree rooted at  $x_3$  (since  $x_3$  is a leaf this then means just all the constraints in which  $x_3$  is involved) given  $x_1 = 0$  and  $x_2 = 0$ , are no lower or no higher than 2.  $x_3$  can set both its upper and lower bounds because it has values for all its constraints. On the other hand,  $x_2$  sends an upper bound of  $\infty$  because it does not know the upper bounds of its children, so it does not know how much those costs could add up to.

In the fifth diagram the agents calculate new best values and so  $x_1$  sends HANDLE-VALUE messages. In the sixth diagram  $x_2$  receives the cost messages from before and sets himself to 1 because it has a lower bound (because all lower bounds are initially set to 0). It then sends the appropriate HANDLE-VALUE messages.

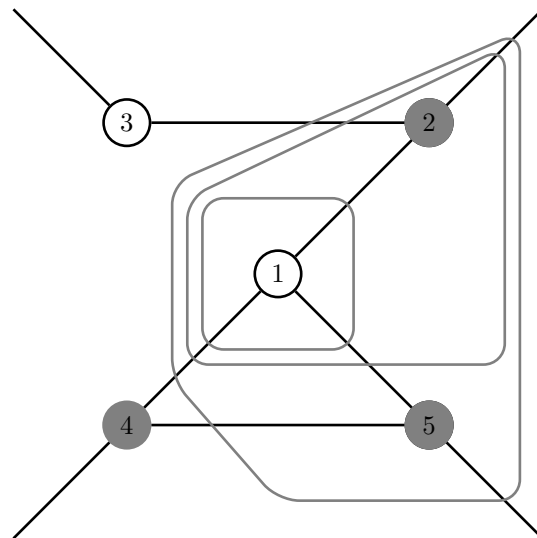
In the seventh diagram both  $x_3$  and  $x_4$  calculate that both their upper and lower bounds are 0 given that  $x_1 = 0$  and  $x_2 = 0$  and send this message up to  $x_2$ . In the eighth diagram  $x_2$  uses this message to calculate its new bounds, also 0 and 0, and sends it up to the root. Upon receiving this message the root knows it can stop the algorithm because it has identical upper and lower bounds. Finally, note that the Adopt algorithm is actually a bit more complicated than this example shows as we ignored any HANDLE-THRESHOLD invocations.

Test results on Adopt show that there is a very wide discrepancy in the number of messages each agent must handle and, therefore, the individual workload. Namely, agents at the leafs of the tree end up doing all of the work while the root sits idle. This gets worse as the tree becomes thinner (like a line). Since the tree must be a depth-first search graph, this means that problems with a lot of constraints will be doubly bad because they will need a thinner tree and the extra constraints means more work for the agents.

Also, our tests reveal that if each agent handles each message as it comes (and calls the BACKTRACK routine) then Adopt will take forever to run. For Adopt to work the agents need to accumulate a number of HANDLE-VALUE and HANDLE-COST invocations before running their BACKTRACK procedure.



ADOPTgc



**Figure 2.16:** Trace of APO algorithm showing the growing size of node 1's *good-list*. Initially only 1 is in the *good-list*, then 2 joins, finally 5 joins.

### 2.2.2 OptAPO

A different approach to solving the distributed constraint optimization problem was taken by the Optimizing Asynchronous Partial Overlay (**optapo**) algorithm (Mailler and Lesser, 2004a), which is the cousin of the **apo** algorithm (Mailler and Lesser, 2004b) for distributed constraint satisfaction.

Both of these algorithms use the same basic idea. The agents start out doing individual hill-climbing, that is, each one changes its color so as to minimize constraints violations with its neighbors. When an agent realizes that it cannot find a value with no violations it tries to find a coloring for a (growing) set of agents which includes itself. More specifically, each agent maintains a variable called its *good-list* which initially contains only the agent. As the agent finds conflicts with other agents that it cannot resolve it adds these agents to its *good-list*. Then, the conflict is resolved by electing one of the agents in the conflict to be the *mediator* of the conflict. The mediator agent then finds a coloring for all the agents in its *good-list* and which minimizes violations with agents outside the *good-list* using any centralized algorithm, such as the branch and bound algorithm from figure 2.12. The agent then informs the other agents of their new color. The agents change to their new color as instructed by the mediator.

Figure 2.16 gives an idea of how APO works. Initially, node 1 has only itself in its *good-list*. If it finds that it cannot resolve a conflict with agent 2 then they enter into a negotiation phase to elect a mediator. In this case 1 becomes the mediator and then finds colors for both 1 and 2. Later on another conflict between 1 and 3 might arise which would lead to 1 becoming mediator for 1,2,3.

In practice, APO and optAPO can be very fast, but their theoretical worst-case bound is still exponential.

**Theorem 2.4** (APO worst case is centralized search). *In the worst case APO (optAPO) will make one agent do a completely centralized search of the complete problem space.*

optAPO requires fewer number of synchronous cycles (that is, steps) than Adopt but performs a lot more computations as measured by number of constraints checks (Davin and Modi, 2005). That is, optAPO centralizes the problem much more than Adopt. When an agent becomes a mediator it centrally solves a large part of the problem and tells the other agents what to do. As such, that one agent ends up doing a lot of work while everyone else does nothing (generally there can be more than one mediator, each mediating over non-overlapping sets of agents, but this is rare). In this way optAPO can finish in a much smaller number of steps simply because some of those steps take a long time.

OPTAPO  
APO

△  
APOgc

Exercise: give an example which proves the theorem.

In general we find that Adopt is better when agent to agent communications are fast and optAPO is better when communications are slow. Both of them have exponential worst-case running times but perform reasonable well for small problems.

### 2.3 Recent Advances

A new algorithm for DCOP based on dynamic programming is presented in (Petcu and Faltings, 2005). This algorithm is then improved by the ODPOP algorithm presented in (Petcu and Faltings, 2006).

#### Exercises

- 2.1 In both ABT and AWC the load distribution among the agents is very uneven, as demonstrated in our NetLogo implementations. Can you come up with an equation that predicts how many messages, proportionally, each agent receives given the initial problem structure and priorities?
- 2.2 The Sudoku puzzle is an instance of a constraint satisfaction problem. We can view it as a distributed constraint satisfaction problem by simply assuming that each empty square on the board corresponds to an agent. Implement any one of the distributed constraint satisfaction algorithms in this chapter to solve Sudoku puzzles.



DBsudoku

## Chapter 3

### Standard and Extended Form Games

Game theory was first formally introduced in the book “*The Theory of Games and Economic Behavior*” (Neumann and Morgenstern, 1944). The book introduced a mathematical way of analyzing certain types of decisions where two or more parties take actions that impact all. In this chapter we present the two most basic forms of games: normal and extended form games. These games are known as **non-cooperative games** because the agents’ preferred sets of actions can be in conflict with each other. That is, what is good for one agent might be bad for the others.

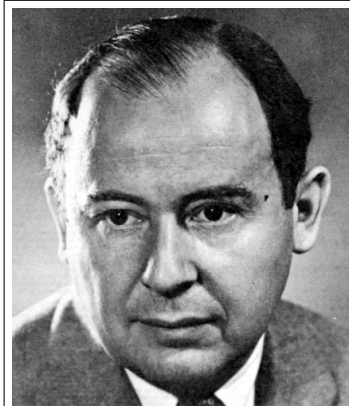
#### 3.1 Games in Normal Form

The most commonly used problem representation in game theory is the **payoff matrix** which shows the utility the agents will receive given their actions. Figure 3.1 shows a sample game matrix in **normal form**, also known as **strategic form**, a phrase introduced by Shapley in 1965. In this game if Bob takes action *a* and Alice takes action *c* then Bob will receive a utility of 1 and Alice a utility of 2. We can extend the payoff matrix to any number of players and actions. In these games we assume that the players take their actions simultaneously.

Normal form games assume that the players have **common knowledge** of the utilities that all players can receive. That is, everybody knows that everybody knows that everybody knows, and so on, the values in the payoff matrix. This situation is different from having the agents know the values in the matrix but not know that the others know those values. For example, if you know that I am giving you a surprise party then you might go along and act surprised when we all jump and yell “surprise!”. However, if you know that I know that you know I will be giving you a surprise party then the surprise has been ruined.

It is also interesting to note that in message-passing multiagent systems where messages can be lost it is impossible for agents to ever achieve common knowledge about anything (Fagin et al., 1995). The problem is historically described as the **Byzantine generals problem** where two generals from the same army are poised on opposite sides of a valley which is occupied by the enemy. The generals must both attack at the same time in order to defeat the enemy. However, their only method of communication is by sending a messenger who could be captured by the enemy. We can see that, if one general sends a message to the other saying “We attack at dawn” it has no way of confirming whether the other general received this message. Similarly, if a general does receive the message and sends another messenger back acknowledging receipt then it has no way of confirming whether the other general received the acknowledgment. Thus, it is impossible to agree on a time. In practice, however, multiagent systems that need common knowledge either give their agents

|     |          | Alice    |          |
|-----|----------|----------|----------|
|     |          | <i>c</i> | <i>d</i> |
| Bob | <i>a</i> | 1,2      | 2,3      |
|     | <i>b</i> | 4,5      | 6,7      |



John Von Neumann.  
1903–1957. Pioneer of the  
digital computer, game theory  
and cellular automata.

NON-COOPERATIVE GAMES

PAYOFF MATRIX  
NORMAL FORM  
STRATEGIC FORM

COMMON KNOWLEDGE

(Fagin et al., 1995) describes a  
logic for representing agents’  
knowledge about others’  
knowledge.

BYZANTINE GENERALS  
PROBLEM

**Figure 3.1:** Sample game matrix in normal form.

STRATEGY

PURE STRATEGY  
MIXED STRATEGY

RATIONAL

ZERO-SUM GAME

MAXMIN STRATEGY

MINIMAX THEOREM

DOMINANT

ITERATED DOMINANCE

SOCIAL WELFARE

this common knowledge from the beginning or assume that communications are reliable enough that it is safe to assume that all messages are delivered.

We define a **strategy**  $s$  to be the set of actions all players take. In this case a strategy of  $s = (a, c)$  would give Bob a utility of 1 and Alice a utility of 2, in other words,  $u_{\text{Bob}}(s) = 1$  and  $u_{\text{Alice}}(s) = 2$ . We also refer to Bob's strategy in  $s$  as  $s_{\text{Bob}}$ , which is  $a$  in this case. This strategy is also an example of a **pure strategy**: one where the agents take a specific action. In contrast, a **mixed strategy** is one where the agents take different actions, each with some fixed probabilities. For example, a mixed strategy for Bob is to take action  $a$  with probability of .3 and action  $b$  with a probability of .7. Note that in a mixed strategy the probabilities for all actions have to add up to 1.

Typically, game theory further assumes that player are **rational**, which we use as a synonym for selfish. That is, a rational player always acts so as to maximize its utility. Finally, we say that a game is a **zero-sum game** if the values in each one of the boxes add up to 0. For example, in a zero-sum game with two players if for a particular strategy  $s$  one player gets a utility of 5 then the other player must receive a utility of  $-5$ . In zero-sum games one player's gain is the other player's loss.

### 3.1.1 Solution Concepts

Given a game matrix we can't help but ask: what strategy should they use? Which is best the best strategy in any given game? The problem, of course, is that there is no simple way to define what's best since what is best for one agent might not be good for another. As such, different solution concepts have been proposed.

The earliest solution concepts were proposed by Von Neumann. He realized that in any given game an agent could always take the action which maximized the worst possible utility it could get. This is known as the **maxmin strategy**, or minmax, if we are talking about losses instead of gains. Specifically, in a game with two agents,  $i$  and  $j$ , agent  $i$ 's maxmin strategy is given by

$$s_i^* = \max_{s_i} \min_{s_j} u_i(s_i, s_j). \quad (3.1)$$

Unfortunately, the strategy where both players play their maxmin strategy might not be stable in the general case. In some payoff matrices it can happen that if  $i$  knows that  $j$  will play its maxmin strategy then  $i$  will prefer a strategy different from its maxmin strategy. Still, the **minimax theorem** (Neumann, 1928) states that a strategy that minimizes the maximum loss, a minmax strategy, can always be found for all two-person zero-sum games.

On a similar note, we say that  $s$  is the **dominant** strategy for agent  $i$  if the agent is better off doing  $s$  no matter what strategies the others play. Formally, we say that action  $a_i$  is dominant for agent  $i$  if

$$\forall_{a_{-i}} \forall_{b_i \neq a_i} u_i(a_{-i}, a_i) \geq u_i(a_{-i}, b_i) \quad (3.2)$$

This idea can be expanded to the **iterated dominance** solution where dominated strategies are eliminated in succession. Unfortunately, the iterated dominance algorithm almost always end before a solution is found. That is, in most games none of the players has a dominant strategy.

We can take a step back and look at the problem from a system designer's perspective. With our new enlightened outlook we might think that the right solution is to maximize overall welfare for all players. The **social welfare** strategy is the one that maximizes the sum of everyone's payoffs. That is

$$s^* = \arg \max_s \sum_i u_i(s). \quad (3.3)$$

However, once again we have the problem that a social welfare strategy might not be stable. Each selfish agent cares only about his own utility and will thus play a different strategy if it can get a higher utility, even if it means everyone else is



|   |              | A                               |                                 |
|---|--------------|---------------------------------|---------------------------------|
|   |              | Stays Silent                    | Betrays                         |
| B | Stays Silent | Both serve six months.          | B serves 10 years; A goes free. |
|   | Betrays      | A serves 10 years; B goes free. | Both serve two years.           |

**Figure 3.2:** Payoff matrix for original prisoner's dilemma problem.

much worse. Also, the social welfare strategy might not seem fair as it could be one where one agent gets an extremely high utility and everyone else gets nothing.

We can solve the unfairness problem by defining a solution concept that does not take into account the agents' absolute utility values. A strategy  $s$  is said to be **Pareto optimal** if there is no other strategy  $s'$  such that at least one agent is better off in  $s'$  and no agent is worse off in  $s'$  than in  $s$ . There can be more than one Pareto optimal solutions for a given problem. The set of all Pareto strategies for a given problem is given by

$$\{s \mid \neg \exists s' \neq s \exists i u_i(s') > u_i(s) \wedge \neg \exists j \in -i u_j(s) > u_j(s')\} \quad (3.4)$$

where  $-i$  represents the set of all agents except  $i$ . Unfortunately, Pareto solutions might also be unstable in that one player might have an incentive to play a different action because he gets higher utility, which means others get lower utility.

The problem of stability was solved by Nash. We say that a strategy  $s$  is a **Nash equilibrium** if for all agents  $i$ ,  $s(i)$  is  $i$ 's best strategy given that all the other players will play the strategies in  $s$ . That is, if everyone else is playing the Nash equilibrium then the best thing for everyone to do is to play the Nash equilibrium. Formally, the set of all Nash equilibrium strategies for a given game is given by

$$\{s \mid \forall i \forall a_i \neq s_i u_i(s_{-i}, s_i) \geq u_i(s_{-i}, a_i)\}. \quad (3.5)$$

Nash showed that all game matrices have at least one equilibrium strategy, but this strategy might be mixed. The only problem with the Nash equilibrium is the fact that in many games there are many such equilibria. Also, some of those Nash equilibria might be better for some players than other. Thus, the players might end up arguing about which specific one to adopt. Still, once the agents have agreed on one it is a very stable solution.

All these various solution concepts make it clear that it is not a simple matter to define what is the best answer. With multiple players, what is best depends on how much we are willing to respect the agents' selfishness, how we trade-off the various agents utilities, and how fair we wish to be.

The most common problem we face when designing multiagent systems is the existence of multiple equilibria. That is, we can generally design a system so that selfish will converge to an equilibrium solution if there is one. But, if there are more than one solution then we need to add extra coordination mechanism to ensure that the agents converge to the same strategy.

### 3.1.2 Famous Games

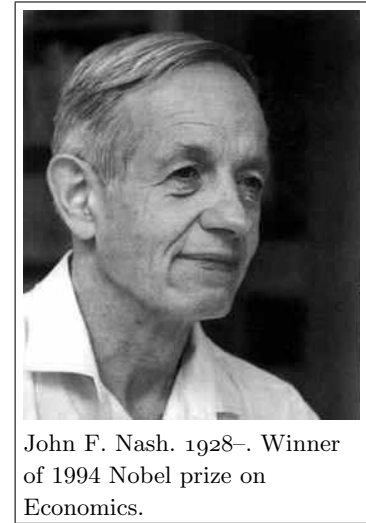
The most famous game of all is the **Prisoner's Dilemma**. Its story typically goes something like the following.

Two suspects A, B are arrested by the police. The police have insufficient evidence for a conviction, and having separated both prisoners, visit each of them and offer the same deal: if one testifies for the prosecution against the other and the other remains silent, the silent accomplice



Vilfredo Pareto. 1848–1923.

PARETO OPTIMAL  
NASH EQUILIBRIUM



John F. Nash. 1928–. Winner of 1994 Nobel prize on Economics.

PRISONER'S DILEMMA

**Figure 3.3:** Prisoner’s dilemma with standard payoff values.

|   |           | A         |        |
|---|-----------|-----------|--------|
|   |           | Cooperate | Defect |
| B | Cooperate | 3,3       | 0,5    |
|   | Defect    | 5,0       | 1,1    |

**Figure 3.4:** Battle of the sexes game.

|     |            | Alice      |          |
|-----|------------|------------|----------|
|     |            | Ice Hockey | Football |
| Bob | Ice Hockey | 4,7        | 0,0      |
|     | Football   | 3,3        | 7,4      |

receives the full 10-year sentence and the betrayer goes free. If both stay silent, the police can only give both prisoners 6 months for a minor charge. If both betray each other, they receive a 2-year sentence each.

From this story we can generate a payoff matrix as shown in figure 3.2. We can replace the prison sentences with utility values and arrive at the standard prisoner’s dilemma payoff matrix shown in figure 3.3, note that longer prison terms translate into lower utility.

Analysis of this matrix reveals that the social welfare solution is (C,C), the set of Pareto optimal strategies is {(C,C) (D,C) (C,D)}, that the dominant strategy for both players is Defection, and that the Nash equilibrium strategy is (D,D).

The Prisoner’s dilemma is interesting because the best choice is not the rational choice. That is, even though the (C,C) solution Pareto dominates the (D,D) solution, a rational player will only play D as that strategy is dominant. The game is widely studied because it applies to many real-world situations like nuclear disarmament and getting kids to clean up their room. Some object to the fact that defection is the equilibrium and claim that real people do not act like that.

BATTLE OF THE SEXES

The **battle of the sexes** is another popular game. It is shown in figure 3.4. In this game Alice and Bob like each other and would like to spend time together but must decide, without communicating with each other, where to go. Alice likes ice hockey while Bob likes football. As such, they have a coordination problem where each prefers to go to a different place but they would like to be together. This type of problem arises frequently in multiagent systems where we often have agents that want to cooperate with each other to achieve some larger goal but have conflicting priorities about how to achieve that goal. For example, the agents in a company want to get a new contract so that they can all get paid but each one wants to do as little work as possible while getting as much money as possible. Or, agents want to deliver a package but have different preferences on where the handoff from one agent to the other should occur.

After some analysis of this game we can determine that the social welfare solutions are (I,I) (F,F), the Pareto optimal solutions are also (I,I) (F,F), there is no dominant strategy, and the Nash equilibrium solutions are (I,I) (F,F). As you can see, the problem here is that there are two strategies both of which are equally attractive. This is the type of problem that we could fix easily if we just had a little communication.

GAME OF CHICKEN

The **game of chicken**, shown in figure 3.5, is also common. In this story, two maladjusted teenagers drive their cars towards each other at high speed. The one who swerves first is a chicken and thus loses the game. But, if neither of them swerves then they both die in a horrible crash. After some analysis we can see that this game is very similar to the battle of the sexes. Its social welfare, Pareto optimal, and Nash strategies are all the same, namely (C,S) (S,C), and there is no dominant strategy.

|     |          | Alice    |        |
|-----|----------|----------|--------|
|     |          | Continue | Swerve |
| Bob | Continue | -1,-1    | 5,1    |
|     | Swerve   | 1,5      | 1,1    |

**Figure 3.5:** The game of chicken.

|        |             | Pig     |             |
|--------|-------------|---------|-------------|
|        |             | Nothing | Press Lever |
| Piglet | Nothing     | 0,0     | 5,1         |
|        | Press Lever | -1,6    | 1,5         |

**Figure 3.6:** The pig and the piglet.

One way out of the problem of multiple Nash equilibria is for one of the players to eliminate one of his choices, say by soldering the steering wheel of the car so that it does not turn. This would make it so that the other player's rational choice is to swerve. Sometimes, an agent increases its bargaining power by limiting its set of possible actions.

The final game is taken from the world of zoology. Imagine there is a big pen with a food dispenser at one end and the food comes out at the other end. The pen is occupied by one big, but slow, pig and one small and fast piglet. Which one of them will press the lever and which one will eat. The situation can be summarized by the matrix in figure 3.6. Namely, if the pig presses the lever then the piglet can stay standing by where the food comes out and eat some of it before the pig comes and shoves him out. If the piglet presses it then it will not get any food because the pig will be standing by the food dispenser and will not let the piglet get close to the food. If both of them press it then the piglet will be able to eat a little bit since it can run faster than the pig.

An analysis of this game shows that the social welfare solutions are (N,P) (P,P), the Pareto optimal solutions are (N,P) (P,P) (P,N), the piglet has a dominant strategy of N, and the Nash equilibrium is (N,P). Since the piglet has a dominant strategy, it is clear that it will chose that, as such the pig will have to press the lever.

Animal behaviorists have actually conducted experiments similar to this one and found that the big pig usually ends up pushing the lever. Of course, the pigs do not actually think about their options. They simply try the different actions until they find the one that works best. This tells us that agents can arrive at an equilibrium solution without having much, if any, intelligence and instead using adaptation. We will learn about learning later in a later chapter.

### 3.1.3 Repeated Games

We saw how in the prisoner's dilemma the dominant strategy was to defect but both players could have received higher utility by cooperating. One way to try to get out of this conundrum, and to better simulate real-world interactions, is to let two players play the same game some number of times. This new game is known as the **iterated prisoner's dilemma** and, in general, we refer to these type of games as repeated games. Repeated games are a special case of extended form games, which we will cover in the next section. They have, however, sometimes been studied independently.

One way to analyze repeated games that last for a finite number of periods is to backtrack from the end. Let's say you are playing an iterated prisoner's dilemma which will last for 50 rounds. You know that at the last round you will defect because that is the dominant strategy and there is no need to be nice to the other player as that is the last game. Of course, since you realized this then you also know that

ITERATED PRISONER'S  
DILEMMA

the other player realized the same thing and so he will also defect at the last round. As such, you know that you have nothing to gain by cooperating at round 49 so you will defect at 49, and so will he. This backward induction can be repeated any number of times leading us to the conclusion that for any finite number of games the rational strategy is to always defect. However, people don't act like this.

We can also formally prove a cooperative equilibrium for the iterated prisoner's dilemma if instead of a fixed known number of interactions there is always a small probability that every interaction will be the last interaction. That is, the players never know if this will be their last interaction or not. In this scenario we can show that the dominant strategy for the iterated prisoner's dilemma is to cooperate, with a certain probability.

FOLK THEOREM

More generally, however, the **folk theorem** tells us that in a repeated game any strategy where every agent gets a utility that is higher than his maxmin utility (3.1) and the strategy is not Pareto-dominated by another is a feasible equilibrium strategy. That is, each player knows that he can get his maxmin utility by playing the appropriate action, regardless of what the others do. Thus, no player will be satisfied with less than his maxmin utility. On the other hand, if the agents agree to a strategy that gives everyone at least their maxmin and is not Pareto-dominated then any agent that diverges from it could be penalized by the other agents so that any gains he incurred from the deviation would be erased. In this way the agents can police any chosen strategy and thus ensure that it is a stable strategy.

Such analysis still leaves open the question of which, in practice, is the optimal strategy for the iterated prisoner's dilemma. In the early 1980's Robert Axelrod performed some experiments on the iterated prisoner's dilemma (Axelrod, 1984). He sent out an email asking people to submit fortran programs that played the prisoner's dilemma against each other for 200 rounds. The winner was the one that accumulated more points. Many entries were submitted. They included the following strategies.

- *ALL-D*: always play defect.
- *RANDOM*: pick action randomly.
- *TIT-FOR-TAT*: cooperate in the first round, then do whatever the other player did last time.
- *TESTER*: defect on the first round. If other player defects then play tit-for-tat. If he cooperated then cooperate for two rounds then defect.
- *JOSS*: play tit-for-tat but 10% of the time defect instead of cooperating.

TIT-FOR-TAT

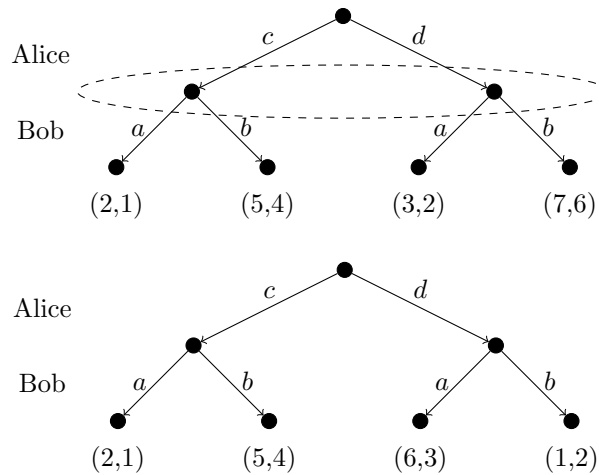
The **tit-for-tat** strategy won the tournament. It still made less than ALL-D when playing against it but, overall, it won more than any other strategy. It was successful because it had the opportunity to play against other programs that were inclined to cooperate. It cooperated with those that could cooperate and defected against the rest. This result, while intriguing, is not theoretically robust. For example a tit-for-tat strategy would lose against a strategy that plays tit-for-tat but defects on the last round.

Another method for analyzing repeated games is by assuming that the players use some learning algorithm and then trying to determine which strategy their learning will converge to. This research area is known as learning in games, which we present in section 5.3.

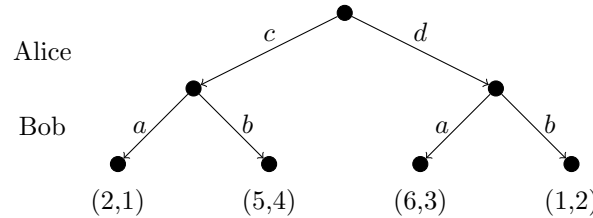
### 3.2 Games in Extended Form

EXTENDED FORM GAMES

In **extended form games** the players take sequential actions. These games are represented using a tree where the branches at each level correspond to a different player's actions and the payoffs to all agents are given at the leafs. Extended form



**Figure 3.7:** Game in extended form that corresponds to the normal form game from Figure 3.1.



**Figure 3.8:** Game in extended form.

games can also represent simultaneous actions by using dotted ellipses to group together nodes which are equivalent to the rest of the agents because, for example, because they could not see the action taken by the opponent.

Figure 3.7 shows the extended form version of the payoff matrix from figure 3.1. In it, the first number inside the parentheses is the payoff to Bob, and the second number is the payoff to Alice. The dotted line groups together two states which are invisible to Bob: it shows that Bob does not know the action Alice took. If we eliminated the dotted line then in that new game Alice takes an action, which Bob can see, and then Bob takes his action.

In extended game a player's strategy  $s_i$  is no longer just an action but can now be a series of actions if the player gets to take more than one action in the tree, or if the player can have different actions depending on which node in the tree it is in. That is, if a player can see others' actions that come before him then his action can vary. For example, in figure 3.8 a rational Bob would choose  $b$  if Alice has chosen  $c$  but would choose  $a$  if Alice has chosen  $d$ . As before, agent  $i$ 's utility from strategy  $s$  is given by  $u_i(s)$  and corresponds to the values on the leaf node reached when all players play  $s$ .

Extended form games, without the dotted ellipses, are nearly identical those built by the minimax algorithm (Russell and Norvig, 2003, Chapter 6).

### 3.2.1 Solution Concepts

We can apply the Nash equilibrium idea from normal form games to extended games. Namely, we say that an extended game has a Nash equilibrium strategy  $s^*$  if for all agents  $i$  it is true that they can't gain any more utility by playing a strategy different from  $s_i^*$  given that everyone else is playing  $s_{-i}^*$ . For example, for the game in figure 3.8 the Nash equilibrium is  $(c, b)$ —Alice plays  $c$  and Bob plays  $b$ . While this strategy is in equilibrium notice how, if there was some problem or noise in the system and Alice ends up playing  $d$  then Bob's strategy of playing  $b$  is no longer his best response. That is, the basic Nash equilibrium solution might not be stable under noisy conditions.

A stronger solution concept is the **subgame perfect equilibrium** strategy  $s^*$  which is defined as one where for all agents  $i$  and all subgames it is true that  $i$  can't gain any more utility by playing a strategy different from  $s_i^*$ . We define a subgame to be any subtree of the extended game. That is, a subgame perfect equilibrium gives all the agents their best response strategies for every node in the tree. For example, the subgame perfect equilibrium for figure 3.8 is for Alice to play  $c$  and Bob to play  $b$  if Alice plays  $c$  and  $a$  if Alice plays  $d$ .

SUBGAME PERFECT  
EQUILIBRIUM

Extended form games are a useful way to represent more complex multiagent interactions. Notice, however, that they are a special case of a multiagent Markov decision process. Most researchers have opted to use the more complete Markov model over the simpler extended form game for modeling multiagent systems.

3.3 Why Do We Care?

These solution concepts provide us with solutions to shoot for when building multiagent systems, even when these solutions are impossible to find. For example, say you want to build a team of robot soccer players. Each robot will have a specific behavior that depends on its current perception of the world and its internal state. Each robot can also be thought to receive a payoff whenever it, or someone in its team, scores a goal. If you consider the set of all possible agent behaviors for all agents in the team as the set of actions in a game then we can talk about the players being at a Nash equilibrium, or about the players in one team being in the Pareto optima given the other team’s behaviors. Of course, such equilibria are impossible to calculate but they do give us a precise goal, of the intellectual kind, to shoot for. Also, as many researchers have done, we can try to break up that immense problem into smaller problems for which the agents can find equilibrium strategies.

There is an extensive literature on centralized algorithms for finding the various equilibrium strategies for a given game they are, however, generally considered to be outside the purview of multiagent research. Generally, the algorithms involve a complete search using heuristics for pruning. The **Gambit** software program (McKelvey et al., 2006) is an open source implementation of several of these algorithms.

However, note that some of these solutions can be found by more multiagent-friendly distributed algorithms. In chapter 5 we show multiagent learning algorithms which allow learning agents to converge to Nash and other equilibria.

Exercises

- 3.1 Show a 2-person standard form game with 3 actions for each player in which iterated dominance leads to a unique equilibrium strategy.
- 3.2 Assume a 2-person standard form game with  $x$  actions for each player.

1. What is the maximum number of Pareto optimal pure strategies that such a game could have?

2. Assuming that all payoff values are different, what is the maximum number of Pareto optimal pure strategies that such a game could have?
- 3.3 Find the pure Nash equilibria, and the Pareto optimal solutions of the following game:

|     |     | Alice |     |     |
|-----|-----|-------|-----|-----|
|     |     | $d$   | $e$ | $f$ |
| Bob | $a$ | 1,2   | 2,3 | 2,3 |
|     | $b$ | 4,5   | 6,7 | 3,4 |
|     | $c$ | 5,4   | 6,5 | 5,6 |

- 3.4 The battle of the sexes game, figure 3.4, is a classic coordination game in which the players must somehow coordinate to agree upon one one of the Nash equilibria. These problems are solved by populations who adopt a social law after some trial and error adaptation phase. For example, in the US people drive on the right side of the rode while in England they drive on the left. Both solutions are equally valid.

Implement a NetLogo program where each patch repeatedly engages in a battle of the sexes game with one of its neighbors, chosen at random. Then try to come up with some simple adaptation strategy which the agents could use so that the population will quickly converge to an equilibrium.

GAMBIT



Simple adaptation strategies for solving this problem exist (Shoham and Tennenholtz, 1997), as well as for more complex neighborhood definitions (Delgado, 2002).

- 3.5 Lets say that in a repeated version of the game of chicken, figure 3.5, Alice and Bob decided to converge to the strategy (Swerve, Swerve). Since this strategy satisfies the maxmin criteria (check this) the Folk theorem tells us that it will be stable since both players could play an iterated strategy that penalizes the other so that any gains from defection are erased after some rounds. Write the short algorithm which describes the player's iterated strategy for this situation, that is, the iterated strategy they must use to guarantee that (Swerve, Swerve) is the equilibrium strategy on every step of the iterated game.
- 3.6 A generalization of the Tit-for-Tat strategy is to, instead of simply doing exactly what the other agent did last time, make a stochastic decision based on the other agent's action. That is, if the other agent defected last time then you will be very likely, with a given probability, to defect and vice versa. Write a NetLogo program where you add these type of agents to a tournament similar to Axelrod's. Which strategy triumphs in the end?

This general strategy is known as reciprocity and often occurs in human interaction: you are more likely, but not entirely sure, to be nice to those that were nice to you in the past. Simulations of populations with various numbers of reciprocating agents have shown that populations of reciprocating agents tend to do better as a whole (since they help each other) but can be exploited by selfish agents. This exploitation can be curbed by having the reciprocating agents share their opinions of other (Sen, 2002).



packages





## Chapter 4

### Characteristic Form Games and Coalition Formation

There is another type of game studied in game theory: the **characteristic form game** or **coalition game** (Osborne and Rubinstein, 1999). In these games the agents decide how to form coalitions among themselves and each coalition receives some utility. For example, a group of people, each with different skills, all want to start new companies. The problem they face is deciding how to divide themselves into subgroups such that each subgroup has the needed set of skills to succeed in the marketplace. Similarly, a group of agents with different skills must decide how to divide itself into subgroups so as handle as many tasks as possible in the most efficient manner. Because the agents must cooperate with each other in order to form coalitions these games are known as **cooperative games**. The basic characteristic form game has also been extended to the more general coalition formation, which we also present in this chapter.

CHARACTERISTIC FORM  
GAME  
COALITION GAME

COOPERATIVE GAMES

#### 4.1 Characteristic Form Games

Formally, a game in characteristic form includes a set  $A = \{1, \dots, |A|\}$  of agents. The final result of the deliberation is an **outcome**  $\vec{u} = (u_1, \dots, u_{|A|}) \in \mathbb{R}^{|A|}$  which is just a vector of utilities, one for each agent. There is also a rule  $V(\cdot)$  that maps every coalition  $S \subset A$  to a utility possibility set, that is  $V(S) \subset \mathbb{R}^{|S|}$ . Notice that  $V(S)$  returns a set of utility vectors, not a single utility vector. As such,  $V(\cdot)$  provides us the set of payoffs that players in  $S$  can achieve if they form a coalition. For example, for the players  $\{1, 2, 3\}$  we might have that  $V(\{1, 2\}) = \{(5, 4), (3, 6)\}$  meaning that if agents 1 and 2 formed a coalition they could either get 5 for agent 1 and 4 for agent 2, or they could get 3 for agent 1 and 6 for agent 2. The function  $V$  must be defined for all subsets of  $A$ .

OUTCOME

A special case of the characteristic form game—the one nearly all multiagent research focuses on—is the **transferable utility** game in characteristic form. This game assumes that the players can exchange utilities among themselves as they see fit. For example, if the utility payments are in the form of money then we only need to specify the total amount of money the coalition will receive and decide later how this money will be distributed among the agents in the coalition. More formally, we define a transferable utility game

TRANSFERABLE UTILITY

**Definition 4.1** (Transferable utility characteristic form game). *These games consist of a set of agents  $A = \{1, \dots, A\}$  and **characteristic function**  $v(S) \rightarrow \mathbb{R}$  defined for every  $S \subseteq A$ .*

CHARACTERISTIC FUNCTION

The characteristic function  $v(S)$  is also sometimes simply referred to as the **value function** for the coalitions. Characteristic form games with transferable utility can represent many multiagent scenarios. For example, they can represent a task allocation problem where a set of tasks has to be performed by a set of agents subsets of whom can sometimes improve their performance by joining together to perform a task, sensor network problems where the sensors must join together in subgroups to further refine their readings or relay important information, or workflow scheduling systems where agents must form groups to handle incoming workflows.

VALUE FUNCTION

**Figure 4.1:** Sample characteristic form game with transferable utility. The table on the left shows the values of each coalition. On the right are the coalition structures. Below each one we calculate its value.

| $S$   | $v(S)$ |              |                 |             |
|-------|--------|--------------|-----------------|-------------|
| (1)   | 2      |              | (1)(2)(3)       |             |
| (2)   | 2      |              | $2 + 2 + 4 = 8$ |             |
| (3)   | 4      | (1)(23)      | (2)(13)         | (3)(12)     |
| (12)  | 5      | $2 + 8 = 10$ | $2 + 7 = 9$     | $4 + 5 = 9$ |
| (13)  | 7      |              |                 |             |
| (23)  | 8      |              | (123)           |             |
| (123) | 9      |              | 9               |             |

#### 4.1.1 Solution Concepts

As is often the case in game theory, there is no clear best solution to all characteristic form games. Instead, various solutions concepts have been proposed each one having its own advantages and disadvantages.

Before defining the solution concepts, we must first notice that the outcome as we have defined it allows for impossible utility values in the transferable utility game. Specifically, there might not be a set of coalitions such that, given  $v$ , the agents can all get their utility as promised by  $\vec{u}$ . In order to rectify this problem we first specify that we are only interested in **feasible** outcomes, that is, those that can be implemented given  $v$ .

**Definition 4.2** (Feasible). *An outcome  $\vec{u}$  is feasible if there exists a set of coalitions  $T = S_1, \dots, S_k$  where  $\bigcup_{S \in T} S = A$  such that  $\sum_{S \in T} v(S) \geq \sum_{i \in A} \vec{u}_i$ .*

That is, an outcome  $\vec{u}$  is feasible if we can find a disjoint set of coalitions whose values are as much as that in  $\vec{u}$ , so we can payoff  $\vec{u}$  with  $v$ . The set of disjoint coalitions  $T$  defined above is also often referred to as a **coalition structure**, also sometimes represented with the symbol  $CS$ .

Notice that, if the characteristic function is super-additive then we can check if an outcome  $\vec{u}$  is feasible by simply ensuring that

$$\sum_{i \in A} \vec{u}_i = v(A). \quad (4.1)$$

We define a **super-additive** domain as one where for all disjoint  $S \subset A$  and  $T \subset A$  we have that  $v(S \cup T) \geq v(S) + v(T)$ . That is, there is nothing to be lost by merging into a bigger coalition. Unfortunately, multiagent systems are rarely super-additive since agents have a habit of getting into each others' way, so that a team is not always better than letting each agent work on a task separately.

The problem of finding feasible solutions can best be illustrated with an example. Figure 4.1 shows a sample transferable utility game for three agents along with the definition of the  $v$  function and all possible coalition formations. In this game the outcome  $\vec{u} = \{5, 5, 5\}$  is not feasible since there is no way to divide the agents into subsets such that they can all get their utility. If we tried the coalition (123) then we only have a value of 9 to distribute and we need a total of 15. On the other hand, the outcome  $\vec{u} = \{2, 4, 3\}$ , is feasible because the coalition structures (1)(23), (2)(13), and (123) can satisfy it (but not the other ones). However,  $\vec{u} = \{2, 2, 2\}$  does have another problem in that the agent 3 is getting an utility of 3 while we have that  $v(\{3\}) = 4$ . That is, agent 3 could defect any one of the three coalition structures we found, join the coalition (3), and get a higher utility than he currently has. This outcome thus seems unstable.

#### The Core

In general, we say that an outcome  $\vec{u}$  is stable if no subset of agents gets paid more, as a whole, than what they get paid in  $\vec{u}$ . Stability is a nice property because it means that the agents do not have an incentive to go off into their own coalition. Our first solution concept, the **core**, refers to all the outcomes that are stable.

Talk about Pareto solutions, Shubik figure 6.4.

| $S$   | $v(S)$ | (1)(2)(3)<br>$1 + 2 + 2 = 5$ |             |             |
|-------|--------|------------------------------|-------------|-------------|
| (1)   | 1      |                              |             |             |
| (2)   | 2      |                              |             |             |
| (3)   | 2      | (1)(23)                      | (2)(13)     | (3)(12)     |
| (12)  | 4      | $1 + 4 = 5$                  | $2 + 3 = 5$ | $2 + 4 = 6$ |
| (13)  | 3      |                              |             |             |
| (23)  | 4      |                              |             |             |
| (123) | 6      |                              |             |             |

| $\vec{u}$     | in Core? | (123) |
|---------------|----------|-------|
| $\{2, 2, 2\}$ | yes      | 6     |
| $\{2, 2, 3\}$ | no       |       |
| $\{1, 2, 2\}$ | no       |       |

**Figure 4.2:** Sample game showing some outcomes that are in the core and some that are not. The characteristic function  $v$  is super-additive.

| $S$   | $v(S)$ |
|-------|--------|
| (1)   | 0      |
| (2)   | 0      |
| (3)   | 0      |
| (12)  | 10     |
| (13)  | 10     |
| (23)  | 10     |
| (123) | 10     |

**Figure 4.3:** Set of payments for a game with an empty core

**Definition 4.3** (Core). *An outcome  $\vec{u}$  is in the core if*

1.

$$\forall_{S \subset A} : \sum_{i \in S} \vec{u}_i \geq v(S)$$

2. *it is feasible.*

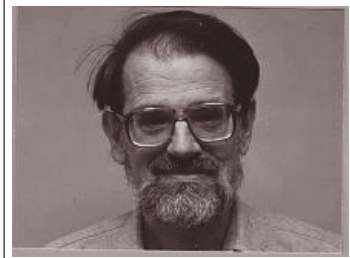
The first condition in this definition tells us that the utility the agents receive in outcome  $\vec{u}$  is bigger than those of any coalition, for the agents in the coalition. In other words, that there is no coalition  $S$  whose  $v(S)$  is bigger than the sum of payments the agents in  $S$  get under  $\vec{u}$ . The second condition merely checks that the total utility we are giving out is not more than what is coming in via  $v(\cdot)$ .

Figure 4.2 shows a new game with different payments and a list of outcomes some of which are in the core and some which are not.  $\{2, 2, 2\}$  is in the core because it is feasible and there is no subset of agents  $S$  with a  $v(S)$  that is bigger than what they could get in this outcome.  $\{2, 2, 3\}$  is not in the core because it is not feasible. This outcome adds up to 7 and there is no coalition structure that adds up to 7. Note that, since the  $v$  is super-additive, all we need to check is the grand coalition. Finally,  $\{1, 2, 2\}$  is not in the core because agents 1 and 2 are getting a total of 2 while if they formed the coalition (12) they would get a utility of 4.

We like the core because we know that any solution that is in the core cannot be improved by having any of the  $2^A$  subsets of agents form a coalition of higher value than they are getting now. It is a very stable solution. Unfortunately, there are many games with empty cores. Figure 4.3 shows one such example. Try to find an outcome in the core for this example. You will see that every outcome is blocked by some other outcome.

### The Shapley Value

While the core gives us one possible solution, it suffers from the fact that many games don't have any solutions in the core, and from its lack of guidance in fairly distributing the payments from a coalition to its members. The Shapley value gives us one specific set of payments for coalition members, which are deemed fair.



Lloyd F. Shapley. 1923-. Responsible for the core and Shapley value solution concepts.

| $S$    | $v(S)$ |
|--------|--------|
| $()$   | 0      |
| $(1)$  | 1      |
| $(2)$  | 3      |
| $(12)$ | 6      |

Figure 4.4: Example game.

The problem with identifying fairness in characteristic form games is best illustrated by an example. Figure 4.4 shows a game for two players. Clearly, we should choose the coalition  $(12)$  as it has the highest value. Now we must decide how much each agent should get. The simplest solution is to divide the total of 6 evenly amongst the coalition members, so that each agent gets 3. This seems unfair to agent 2 because agent 2 could have gotten 3 by simply staying on its own coalition  $(2)$ . It seems like the fair thing to do is to give each agent a payment that is proportional to what it would have gotten by itself. But, how do we extend this idea to cases with more than 2 agents?

Shapley was able to extend this idea by realizing that each agent should get a payment that corresponds to its marginal contribution to the final value. An agent's marginal contribution is the difference between the value before the agent joined the coalition and after he joined. For example, if before you join Initech their annual profits are \$10M but after you are there for a year they increase to \$11M then you can claim that your marginal contribution is \$1M assuming, of course, that everything else stays the same.

The one remaining problem is that there are many different orderings in which  $n$  agents could have joined the coalition, namely  $n!$ . The **Shapley value** simply averages over all possible orderings. That is, the Shapley value gives each agent a utility proportional to its average marginal contribution to every possible coalition, in every possible order it could have been formed.

## SHAPLEY VALUE

**Definition 4.4** (Shapley Value). *Let  $B(\pi, i)$  be the set of agents in the agent ordering  $\pi$  which appear before agent  $i$ . The Shapley value for agent  $i$  given  $A$  agents is given by*

$$\phi(i, A) = \frac{1}{A!} \sum_{\pi \in \Pi_A} v(B(\pi, i) \cup i) - v(B(\pi, i)),$$

where  $\Pi_A$  is the set of all possible orderings of the set  $A$ . Another way to express the same formula is

$$\phi(i, A) = \sum_{S \subseteq A} \frac{(|A| - |S|)! (|S| - 1)!}{|A|!} [v(S) - v(S - \{i\})].$$

That the Shapley values are calculated for a particular coalition,  $A$  in the definition above. They are not meant as a way of determining which is the best coalition structures. They can only be used to distribute the payments of a coalition once it is formed.

Lets calculate the Shapley values for the game in figure 4.4 and the grand coalition  $(12)$ . Since there are only two agents it means that there are only two possible orderings:  $(12)$  and  $(21)$ . As such we have that

$$\begin{aligned} \phi(\{1, 2\}, 1) &= \frac{1}{2} \cdot (v(1) - v()) + v(21) - v(2)) \\ &= \frac{1}{2} \cdot (1 - 0 + 6 - 3) = 2 \\ \phi(\{1, 2\}, 2) &= \frac{1}{2} \cdot (v(12) - v(1) + v(2) - v()) \\ &= \frac{1}{2} \cdot (6 - 1 + 3 - 0) = 4 \end{aligned}$$

A somewhat surprising but very good characteristic of the Shapley value is that it is always feasible. In our example the payments of 4 and 2 add up to 6 which is the same value we get in the grand coalition (12). Another nice feature of the Shapley value is that it always exists and is unique. One other interesting result is that the Shapley value might not be in the core, even for cases where the core exists.

Unfortunately, while the Shapley value has some very attractive theoretical properties, it does have some serious drawbacks when we try to use it for building multi-agent systems. The biggest problem is computational. The Shapley value requires us to calculate at least  $2^{|A|}$  orderings, that is only possible for very small sets  $A$ . It also requires that we know the value of  $v$  for every single subset  $S$ . In many real-world applications the calculation of  $v$  is complex. For example, it might require simulating how a particular coalition of agents would work together. These complex calculations could dramatically increase the total time. Finally, the Shapley value does not give us the actual coalition structure. In some cases, especially with super-additive  $v$ , this can be done very quickly but in other cases this might take longer.

### The Nucleolus

Since the core is often empty, researchers started looking for ways of relaxing it and find a new solution concept that would exist for every game. The problem with the core is that it says that there is no subset of agents that could get paid more than what they are currently getting paid in  $\vec{u}$ , because then they would be tempted to defect and form a new coalition. If it is impossible to find such an  $\vec{u}$  then the next best thing would be to find the  $\vec{u}$  that minimizes the total temptation felt by the agents. That is what the **nucleolus** aims to do.

We start by clarifying what we mean by temptation. Specifically, a coalition  $S$  is more tempting the higher its value is over the what the agents get in  $\vec{u}$ . This is known as the **excess**.

NUCLEOLUS

EXCESS

**Definition 4.5** (excess). *The excess of coalition  $S$  given outcome  $\vec{u}$  is given by*

$$e(S, \vec{u}) = v(S) - \vec{u}(S),$$

where

$$\vec{u}(S) = \sum_{i \in S} \vec{u}_i.$$

That is, a coalition  $S$  has a positive excess, given  $\vec{u}$ , if the agents in  $S$  can get more from  $v(S)$  than they can from  $\vec{u}$ . The more they can get from  $S$  the higher the excess. Note that, by definition, given an outcome  $\vec{u}$  that is in the core all coalitions have a excess that is less than or equal to 0 with respect to that outcome. But, since we are now concerned with outcomes that are not in the core we will instead look for those with minimal excess. Since excess is defined for all possible subsets  $S$  we first need a way to compare the excesses of two outcomes. We do this by putting them in a sorted list and comparing the list. The one with the higher excess first is declared more excessive. More formally, for each  $\vec{u}$  we find its excess for all subsets  $S$  and order these in a list; the higher excesses come first, as such

$$\theta(\vec{u}) = \langle e(S_1^{\vec{u}}, \vec{u}), e(S_2^{\vec{u}}, \vec{u}), \dots, e(S_{2^{|A|}}^{\vec{u}}, \vec{u}) \rangle. \quad (4.2)$$

We then define a lexicographical ordering  $\succ$  over these lists where  $\theta(\vec{u}) \succ \theta(\vec{v})$  is true when there is some number  $q \in 1 \dots 2^{|A|}$  such for all  $p < q$  we have that  $e(S_p^{\vec{u}}, \vec{u}) = e(S_p^{\vec{v}}, \vec{v})$  and  $e(S_q^{\vec{u}}, \vec{u}) > e(S_q^{\vec{v}}, \vec{v})$  where the  $S_i$  have been sorted as per  $\theta$ . That is, if  $\theta(\vec{u}) \succ \theta(\vec{v})$  then that means that when we sort their excesses for all subsets their first, and greatest, excesses are all the same and on the first set for which they have a discrepancy  $\vec{u}$  has the highest excess. We can now define the **nucleolus** as the  $\vec{u}$  which is not lexicographically bigger than anyone else.

NUCLEOLUS

**Definition 4.6** (nucleolus). *The nucleolus is the set*

$$\{\vec{u} \mid \theta(\vec{u}) \not\prec \theta(\vec{v}) \text{ for all } \vec{v}, \text{ given that } \vec{u} \text{ and } \vec{v} \text{ are feasible.}\}$$

In other words, the outcomes in the nucleolus are those where the excesses for all possible sets are, in a way, not greater than those of any other outcome. One nice thing about the nucleolus is that it is always unique for each coalition structure. That is, given a coalition structure there is only one nucleolus.

The nucleolus captures, to some degree, the idea of an outcome that minimizes the temptation the agents face. However, notice that the lexicographic order it defines only cares about the first subset that has a higher excess, it does not care about the ones after that. This could lead to a situation where the sum of the excess from the nucleolus is actually larger than that of some other outcome. As such, the nucleolus does not seem to minimize the sum of temptations.

Can you come up with an example of this case?

### Equal Excess

Another technique for calculating the agents' payoff, besides the Shapley value, is called **equal excess**. It is an iterative algorithm where we adjust the payments that the agents expected they will receive from each coalition that includes them. At each time step  $t$  we let  $E^t(i, S)$  be agent  $i$ 's expected payoff for each coalition  $S$  which includes him. Initially these are set to 0. We let

$$A^t(i, S) = \max_{T \neq S} E^t(i, T) \quad (4.3)$$

be agent  $i$ 's expected payment from not choosing  $S$  and instead choosing the best alternative coalition. Then, at each time step we update the players' expected payments using

$$E^{t+1}(i, S) = A^t(i, S) + \frac{v(S) - \sum_{j \in S} A^t(j, S)}{|S|}. \quad (4.4)$$

For example, for the value function given in figure 4.4 we start with  $E^0(1, *) = E^0(2, *) = 0$ . Then, at time 1 agent 1 can update  $E^1(1, (1)) = 1$  and  $E^1(1, (1, 2)) = 3$  and then  $A^1(1, (1)) = 3$  and  $A^1(1, (1, 2)) = 1$ , while agent 2 updates  $E^1(2, (2)) = 3$  and  $E^1(2, (1, 2)) = 3$  and then  $A^1(2, (1)) = 3$  and  $A^1(2, (1, 2)) = 3$ .

Note that, since the equal excess procedure requires us to know the  $A$  values of all agents it is a centralized procedure. However, it does not seem impossible that we could parallelize this procedure by having the agents exchange  $A$  values. Unfortunately, in such a scenario it would not be in the agents' best interest to tell the truth about its current  $A$ .

Exercise: Give an example when an agent would want to lie.

### 4.1.2 Algorithms for Finding the Utilitarian Solution

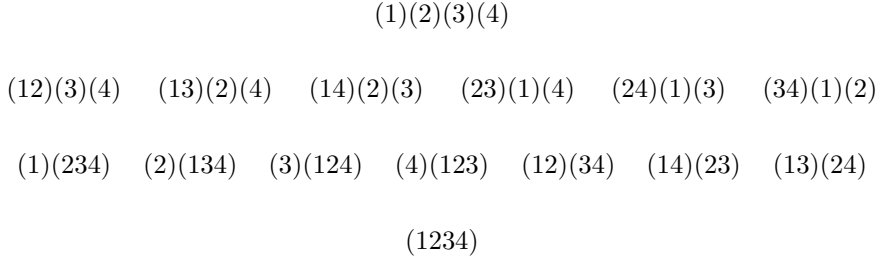
As a multiagent system designer we often simply want to find the outcome that maximizes the sum of values. That is, we want to find the utilitarian solution. When the characteristic function is super-additive then the grand coalition will have the highest value and thus finding the utilitarian solution is trivial. However, if the characteristic is not super-additive—as is often the case in multiagent systems—then we will want an algorithm for finding it.. Notice that under this formulation we are no longer interested in the specific outcome (that is, individual payments to agents) we are now only interested in finding best coalition structure, ignoring the problem of dividing up the value of each coalition among its participants.

#### Centralized Algorithm

One proposed approach is to perform a complete search of the complete set of possible coalition structures, but in a specified order. Figure 4.5 shows all the possible coalition structures for four agents. Notice that the bottom two rows contain all possible coalitions. This means that after searching those two rows we have seen

EQUAL EXCESS

I *think* they are set to 0. Must check with the coalition book.



**Figure 4.5:** Coalition formation possibilities for four agents.

| Level    | Bound    |
|----------|----------|
| $A$      | $A/2$    |
| $A - 1$  | $A/2$    |
| $A - 2$  | $A/3$    |
| $A - 3$  | $A/3$    |
| $A - 4$  | $A/4$    |
| $A - 5$  | $A/4$    |
| $\vdots$ | $\vdots$ |
| 2        | $A$      |
| 1        | none     |

**Figure 4.6:** Bounds on optimality after searching various levels.

all possible coalitions. If we let  $S^*$  be the value of the highest valued coalition (*not* coalition structure) found after searching those two rows then we know that the best coalition structure cannot be more than  $A \cdot S^*$ . As such, after searching the bottom two levels we can say that the optimal solution is no more than  $A$  times better than the best solution we have found thus far.

Figure 4.6 shows the bounds that can be calculated after examining each of the levels in the graph (Sandholm et al., 1999). The algorithm the authors propose consists of first searching the bottom two levels then continue searching down from the top level. In this way, the bound from optimal is reduced as indicated in the figure. Note that searching the levels in some other order will not guarantee these bounds. Notice also that the size of the second level is

$$\text{Stirling}(A, 2) = \frac{1}{2} \sum_{i=0}^1 (-1)^i \binom{2}{i} (2-i)^A = 2^{A-1} - 1.$$

So it takes exponential time just to search the second level.

### Distributed Algorithm

While the previous algorithm found the optimal coalition structure, it did so at the expense of a lot of computation and in a centralized manner. We now look at one possible way of finding a good, but possibly not optimal, coalition structure in a decentralized manner.

Figure 4.7 shows a distributed algorithm for coalition formation (Shehory and Kraus, 1998). The agents order all their possible coalitions based on how much each will get in that coalition, where each agent  $i$  gets  $v_i(S) = v(S)/|S|$  if it joins coalition  $S$ . The agents then broadcast their best coalition. If there are one or more coalitions that are the best for all the agents in them, that is, all the agents in the coalition think that is the best coalition for them, then those agents join the coalition and drop out of the algorithm. The remaining agents take note of the missing agents by eliminating from consideration all coalitions that include them. The process is then repeated again with the new set of coalitions.

This is a classic example of a greedy or hill-climbing algorithm. As such, we know that it might get stuck on one of the local optima, which might or might not be the global optimum. Still, the algorithm should execute very fast as there are



find-coalition

```

FIND-COALITION( $i$ )
1   $L_i \leftarrow$  set of all coalitions that include  $i$ .
2   $S_i^* \leftarrow \arg \max_{S \in L_i} v_i(S)$ 
3   $w_i^* \leftarrow v_i(S_i^*)$ 
4  Broadcast  $(w_i^*, S_i^*)$  and wait for all other broadcasts.
   Put into  $W^*, S^*$  sets.
5   $w_{\max} = \max W^*$  and  $S_{\max}$  is the corresponding coalition.
6  if  $i \in S_{\max}$ 
7      then join  $S_{\max}$ 
8  Delete  $S_{\max}$  from  $L_i$ .
9  Delete all  $S \in L_i$  which include agents from  $S_{\max}$ .
10 if  $L_i$  is not empty
11     then goto 2
12 return

```

**Figure 4.7:** Distributed algorithm for coalition formation. Each agent  $i$  must execute this function. We let  $v_i(S) = \frac{v_S}{|S|}$

at most  $A$  steps and each step involves having each agent examine all the possible coalitions that it can be participate in.

### Reduction to Constraint Optimization

We note that the problem of finding the optimal coalition structure can be reduced to a constraint optimization problem. Specifically, given a characteristic function  $v$  and over  $n$  agents, we let there be one variable for each agent and set the variable's domain to the set  $\{1, \dots, n\}$ . That is, we have a placeholder, say, a room, for each of the possible coalitions and the agents decide which room to join. We define a constraint for each room. Its value is simply  $-v(S)$  where  $S$  is the number of agents that have chosen the bucket—the agents in the coalition, and  $v(\emptyset) = 0$ . We then then have a constraint optimization problem where we are trying to the set of values which minimizes the sum of the constraint violations, thus maximizing the sum of the valuations.

Note that this is a degenerate case of the constraint optimization problem in that all the  $n$  constraints are over all agents. Most constraint problems exhibit some degree of locality in that constraints are only over a small subset of the variables. Having all constraints be over all variables makes this problem harder than average.

## 4.2 Coalition Formation

The **coalition formation** problem, as studied in multiagent systems, extends the basic characteristic form game in an effort to make it a better match for real world problems. A coalition formation problem consists of three steps.

1. Agents generate values for the  $v(\cdot)$  function.
2. Agents solve the characteristic form game by finding a suitable set of coalitions.
3. Agents distribute the payments from these coalitions to themselves in a suitable manner.

Steps 2 and 3 can be thought of as part of the traditional characteristic form game. The coalition formation definition simply chooses to split the problem of finding a suitable outcome  $\vec{u}$  into two parts: finding the coalitions and then dividing the payments. The split mirrors the requirements of many application domains. Step 1 is completely new. It is there because in many domains it is computationally expensive to determine the value of  $v(S)$  for a given  $S$ . For example, if the agents are trying to form groups that solve particular tasks then calculating  $v(S)$  requires



them to determine out how well they can perform the task as a group, which requires considering how all their different skills can be brought to bear and, in some common scenarios, requires the development of a full plan—an exponential problem. The approaches at solving step 1 are thus generally dependent on the domain and do not generalize well.

Find a good example.

### 4.3 Recent Advances

In (Dang and Jennings, 2004) the authors give an algorithm for finding the optimal coalition structure which has slightly better bounds than the one presented here.

(Yokoo et al., 2005) extend the problem of coalition formation by defining the value function over abilities and then giving the agents sets of abilities. They then look at the possibility of agents pretending to be more than one agent (each with a different ability), omitting to mention the skills they have (lying), or colluding with each other. They define the concept of an anonymity-proof core. However, they don't provide any algorithms or mechanisms for finding outcomes in this core.



## Chapter 5

### Learning in Multiagent Systems

Machine learning algorithms have achieved impressive results. We can write software that processes larger amounts of data than any human can and which can learn to find patterns that escape even the best experts in the field. As such, it is only reasonable that at some point we will want to add learning agents to our multiagent system. There are several reasons why one might want to add these learning agents.

Many multiagent systems have as their goal the exploration or monitoring of a given space, where each agent has only a local view of their own area. In these scenarios we can envision that each agent learns a map of its world and the agents further share this map in order to aggregate a global view of the field and cooperatively decide which areas need further exploration. This is a form of cooperative learning.

In competitive environments each selfish agent tries to maximize its own utility by learning the other agents' behaviors and weaknesses. In these scenarios we are interested in the dynamics of the system and in determining if the agents will reach a stable equilibrium. At their simplest these scenarios are repeated games with learning agents.

To summarize, agents might learn because they don't know everything about their environment or because they don't know how the other agents behave. Furthermore, the learning can happen in a cooperative environment where we also want the agents to share their learned knowledge, or in a competitive environment where we want them to best each other. We present analysis and algorithms for learning agents in these various environment.

#### 5.1 The Machine Learning Problem

Before delving into multiagent learning we first present a high level view of what we mean by **machine learning**. The word "learning" as used casually can have many different meanings, from remembering to deduction, but machine learning researchers have a very specific meaning for the machine learning problem.

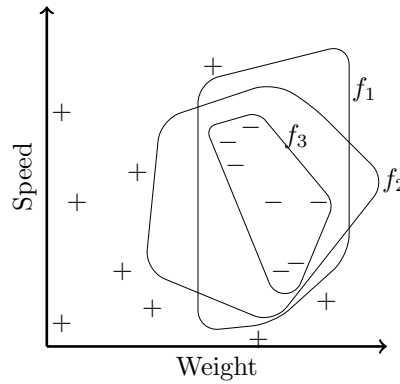
MACHINE LEARNING

The goal of machine learning research is the development of algorithms that increase the ability of an agent to match a set of inputs to their corresponding outputs (Mitchell, 1997). That is, we assume the existence of a large set of examples  $E$ . Each example  $e \in E$  is a pair  $e = \{a, b\}$  where  $a \in A$  represents the input the agent receives and  $b \in B$  is the output the agent should produce when receiving this input. The agent must find a function  $f$  which maps  $A \rightarrow B$  for as many examples of  $A$  as possible.

In a controlled test the set  $E$  is usually first divided into a training set which is used for training the agent, and a testing set which is used for testing the performance of the agent. In some scenarios it is impossible to first train the agent and then test it. In these cases the training and testing examples are interleaved. The agent's performance is assessed on an ongoing manner.

Figure 5.1 shows a graphical representation of the machine learning problem. The learning problem is coming up with a function that maps all the points in the space to either  $+$  or  $-$  such that it will also correctly categorize any new examples that we have not seen. For example, imagine that the points are really photos of

**Figure 5.1:** The machine learning problem. The input set  $A$  corresponds to the two axis: Weight and Speed. The outputs  $B$  are the set  $\{+, -\}$ . The lines represent three possible functions  $f$  which, in this case, map anything within the lines as a  $-$  and anything outside as a  $+$ . Note that all lines have correctly solved the learning problem but in different ways.



INDUCTION

people's faces and you must identify them as either male or female. Our function must, therefore, extrapolate from the examples it has seen and generalize to all possible instances. That is, machine learning performs **induction** over the set of examples it has seen in order to categorize all possible examples.

Figure 5.1 shows three different functions,  $f_1$ ,  $f_2$ , and  $f_3$ , each one of which correctly solves the learning problem. That is, they are correct inductions since they correctly categorize all the examples. In fact, there are an infinite number of such functions. But, which function is the best one to use? We don't have a general answer to that question. Each learning algorithm, from reinforcement learning to support vector machines, arrives at one learned function but the choice is arbitrary. That is, given the same examples two learning algorithms can learn to perfectly classify them but still have learned different functions. This effect is known as the **induction bias** of a learning algorithm. It is because of this induction bias that some learning algorithms appear to perform better than others in certain domains—their bias coincides with the implicit structure of the problem. The fact that no learning algorithm outperforms all others in all possible problem domains has been formalized by the **no free lunch theorem** (Wolpert and Macready, 1995).

INDUCTION BIAS

NO FREE LUNCH THEOREM

When a learning agent is placed in a multiagent scenario these fundamental assumptions of machine learning are violated. The agent is no longer learning to extrapolate from the examples it has seen of fixed set  $E$ , instead it's target concept keeps changing (the points in figure 5.1 keep moving), leading to a **moving target function** problem (Vidal and Durfee, 1998b). In general, however, the target concept does not change randomly; it changes based on the learning dynamics of the other agents in the system. Since these agents also learn using machine learning algorithms we are left with some hope that we might someday be able to understand the complex dynamics of these type of systems.

MOVING TARGET FUNCTION

Learning agents are most often selfish utility maximizers. These agents often face each other in encounters where the simultaneous actions of a set of agents leads to different utility payoffs for all the participants. For example, in a market-based setting a set of agents might submit their bids to a first-price sealed-bid auction. The outcome of this auction will result in a utility gain or loss for all the agents. In a robotic setting two agents headed in a collision course towards each other have to decide whether to stay the course or to swerve. The results of their combined actions have direct results in the utilities the agents receive from their actions. However, even agents that are trying to learn for themselves might find utility in sharing this knowledge.

## 5.2 Cooperative Learning

Imagine two robots equipped with wireless communication capabilities and trying to map an unknown environment. One of the robots could learn that the red rocks can be moved but the black rocks are too heavy to move. The robot could communicate this information to the other one so that it does not have to re-learn it. Similarly,

|     |   | $j$  |     |
|-----|---|------|-----|
|     |   | C    | D   |
| $i$ | A | 0,0  | 5,1 |
|     | B | -1,6 | 1,5 |

**Figure 5.2:** Sample two-player game matrix. Agent  $i$  chooses from the rows and agent  $j$  chooses from the columns.

once one robot has build a map of one area it could send this map to the other robot. Of course, this scenario assumes that the two robots are cooperating with each other in order to build the map.

This type of problem is easy to solve when the robots are identical. In this case they can simply tell each other everything that they learn knowing that it will be equally applicable to the other one. One challenge is trying to prevent the robots from telling each other things the other already knows. The problem gets much harder when the robots are heterogenous. For example, one robot might have learned that the black rocks can be moved using its large arm but the other robot might not have an arm that large so this knowledge is useless to him. To solve this problem we need to somehow model the agents' capabilities so as to allow one agent to determine which parts of his learned knowledge will be useful to an agent with a different set of capabilities. To date, there is scant research on general approaches to the problem of sharing learned knowledge. Most systems that share learned knowledge among agents, such as (Stone, 2000), simply assume that all agents have the same capabilities.

### 5.3 Repeated Games

We now focus on the problem of learning in repeated games (Fudenberg and Levine, 1998). In these problems we have two players that face each other repeatedly on the same game matrix, like the one shown in figure 5.2, and each one tries to maximize the sum of its payoffs over time. You will remember that we already saw a specific version of this problem called the iterated prisoner's dilemma.

The theory of learning in games studies the equilibrium concepts dictated by various simple learning mechanisms. That is, while the Nash equilibrium is based on the assumption of perfectly rational players, in learning in games the assumption is that the agents use some kind of algorithm. The theory determines the equilibrium strategy that will be arrived at by the various learning mechanisms and maps these equilibria to the standard solution concepts, if possible. Many learning mechanisms have been studied. The most common of them are explained in the next few subsections.

#### 5.3.1 Fictitious Play

A widely studied model of learning in games is the process of **fictitious play**. In it agents assume that their opponents are playing a fixed strategy. The agents use their past experiences to build a model of the opponent's strategy and use this model to choose their own action. Mathematicians have studied these types of games in order to determine when and whether the system converges to a stable strategy.

Fictitious play uses a simple form of learning where an agent remembers everything the other agents have done and uses this information to build a probability distribution for the other agents' expected strategy. Formally, for the two agent ( $i$  and  $j$ ) case we say that  $i$  maintains a weight function  $k_i : S_j \rightarrow \mathcal{R}^+$ . The weight function changes over time as the agent learns. The weight function at time  $t$  is represented by  $k_i^t$  which keeps a count of how many times each strategy has been played. When at time  $t - 1$  opponent  $j$  plays strategy  $s_j^{t-1}$  then  $i$  updates its weight

FICTITIOUS PLAY

**Figure 5.3:** Example of fictitious play. The matrix is shown above and the values at successive times, each on a different row, are shown on the table below. The first row corresponds to time 0. Note that only  $i$  is using fictitious play,  $j$  plays the values as in the  $s_j$  column.  $i$ 's first two actions are stochastically chosen.

|     |     | $j$ |       | $s_i$ | $s_j$ | $k_i(C)$ | $k_i(D)$ | $\Pr_i[C]$ | $\Pr_i[D]$ |
|-----|-----|-----|-------|-------|-------|----------|----------|------------|------------|
| $i$ | $A$ | $C$ | $0,0$ | $A$   | $C$   | 1        | 0        | 1          | 0          |
|     |     | $D$ | $1,2$ | $B$   | $D$   | 1        | 1        | .5         | .5         |
|     | $B$ | $C$ | $0,0$ | $A$   | $D$   | 1        | 2        | 1/3        | 2/3        |
|     |     | $D$ | $1,2$ | $A$   | $D$   | 1        | 3        | 1/4        | 3/4        |
|     |     |     |       | $A$   | $D$   | 1        | 4        | 1/5        | 4/5        |

function with

$$k_i^t(s_j) = k_i^{t-1}(s_j) + \begin{cases} 1 & \text{if } s_j^{t-1} = s_j, \\ 0 & \text{if } s_j^{t-1} \neq s_j. \end{cases} \quad (5.1)$$

Using this weight function, agent  $i$  can now assign a probability to  $j$  playing any of its  $s_j \in S_j$  strategies with

$$\Pr_i^t[s_j] = \frac{k_i^t(s_j)}{\sum_{\tilde{s}_j \in S_j} k_i^t(\tilde{s}_j)}. \quad (5.2)$$

Player  $i$  then determines the strategy that will give it the highest expected utility given that  $j$  will play each of its  $s_j \in S_j$  with probability  $\Pr_i^t[s_j]$ . That is,  $i$  determines its best response to a probability distribution over  $j$ 's possible strategies. This amounts to  $i$  assuming that  $j$ 's strategy at each time is taken from some fixed but unknown probability distribution.

An example of the best response dynamic at work is shown in figure 5.3. Here we see the values for agent  $i$  and its best responses to agent  $j$ 's action. Note that in this example agent  $j$  is not using best response. Agent  $i$  first notices that  $j$  played  $C$  and thus sets  $k_i^1(C) = 1$ . It, therefore predicts that  $j$  will play  $C$  with probability 1 so its best response at time 2 is to play  $B$ , as seen in the second row. Agent  $j$  then plays  $D$  which makes  $i$  have  $\Pr_i^1[C] = \Pr_i^1[D] = .5$ , so both of  $i$ 's actions have the same expected payoff (1) so it randomly chooses to play  $B$ . After that, when  $j$  plays  $D$  again then  $i$ 's best response is unequivocally  $B$ .

Several interesting results have been derived by researchers in this area. These results assume that all players are using fictitious play. In (Fudenberg and Kreps, 1990) it was shown that the following two theorems hold.

**Theorem 5.1** (Nash Equilibrium is Attractor to Fictitious Play). *If  $s$  is a strict Nash equilibrium and it is played at time  $t$  then it will be played at all times greater than  $t$ .*

Intuitively we can see that if the fictitious play algorithm leads to all players to play the same Nash equilibrium then, afterward, they will increase the probability that all others are playing the equilibrium. Since, by definition, the best response of a player when everyone else is playing a strict Nash equilibrium is to play the same equilibrium, all players will play the same strategy and the next time. The same holds true for every time after that.

**Theorem 5.2** (Fictitious Play Converges to Nash). *If fictitious play converges to a pure strategy then that strategy must be a Nash equilibrium.*

We can show this by contradiction. If fictitious play converges to a strategy that is not a Nash equilibrium then this means that the best response for at least one of the players is not the same as the convergent strategy. Therefore, that player will take that action at the next time, taking the system away from the strategy profile it was supposed to have converged to.

|     |     | $j$ |     | $s_i$ | $s_j$ | $k_i(C)$ | $k_i(D)$ | $k_j(A)$ | $k_j(B)$ |
|-----|-----|-----|-----|-------|-------|----------|----------|----------|----------|
|     |     | $C$ | $D$ |       |       | 1        | 1.5      | 1        | 1.5      |
| $i$ | $A$ | 0,0 | 1,1 | $A$   | $C$   | 2        | 1.5      | 2        | 1.5      |
|     | $B$ | 1,1 | 0,0 | $B$   | $D$   | 2        | 2.5      | 2        | 2.5      |
|     |     |     |     |       |       | $A$      | $C$      | $3$      | $2.5$    |
|     |     |     |     |       |       | $B$      | $D$      | $3$      | $3.5$    |

**Figure 5.4:** A game matrix with an infinite cycle.

An obvious problem with the solutions provided by fictitious play can be seen in the existence of infinite cycles of behaviors. An example is illustrated by the game matrix in figure 5.4. If the players start with initial weights of  $k_i^1(C) = 1$ ,  $k_i^1(D) = 1.5$ ,  $k_j^1(A) = 1$ , and  $k_j^1(B) = 1.5$  they will both believe that the other will play  $B$  or  $D$  and will, therefore, play  $A$  or  $C$  respectively. The weights will then be updated to  $k_i^2(C) = 2$ ,  $k_i^2(D) = 1.5$ ,  $k_j^2(A) = 2$ , and  $k_j^2(B) = 1.5$ . Next time, both agents will believe that the other will play  $A$  or  $C$  so both will play  $B$  or  $D$ . The agents will engage in an endless cycle where they alternatively play  $(A, C)$  and  $(B, D)$ . The agents end up receiving the worst possible payoff.

This example illustrates the type of problems we encounter when adding learning to multiagent systems. Most learning algorithms can easily fall into cycles such as this one. One common strategy for avoiding this problem is the use of randomness. Agents will sometimes take a random action in an effort to exit possible loops and to explore the search space. It is interesting to note that, as in the example from figure 5.4, it is often the case that the loops the agents fall in often reflect one of the mixed strategy Nash equilibria for the game. That is,  $(.5, .5)$  is a Nash equilibrium for this game. Unfortunately, if the agents are synchronized, as in this case, the implementation of a mixed strategy could lead to a lower payoff.

Games with more than two players require that we decide whether the agent should learn individual models of each of the other agents independently or a joint probability distribution over their combined strategies. Individual models assume that each agent operates independently while the joint distributions capture the possibility that the others agents' strategies are correlated. Unfortunately, for any interesting system the set of all possible strategy profiles is too large to explore—it grows exponentially with the number of agents. Therefore, most learning systems assume that all agents operate independently so they need to maintain only one model per agent.

Which game matrices have cycles for fictitious play?

### 5.3.2 Replicator Dynamics

Another widely studied model is **replicator dynamics**. This model assumes that the percentage of agents playing a particular strategy will grow in proportion to how well that strategy performs in the population. A homogeneous population of agents is assumed. The agents are randomly paired in order to play a symmetric game, that is, a game where both agents have the same set of possible strategies and receive the same payoffs for the same actions. The replicator dynamics model is meant to capture situations where agents reproduce in proportion to how well they are doing.

Formally, we let  $\phi^t(s)$  be the number of agents using strategy  $s$  at time  $t$ . We can then define

$$\theta^t(s) = \frac{\phi^t(s)}{\sum_{s' \in S} \phi^t(s')} \quad (5.3)$$

to be the fraction of agents playing  $s$  at time  $t$ . The expected utility for an agent playing strategy  $s$  at time  $t$  is defined as

$$u^t(s) \equiv \sum_{s' \in S} \theta^t(s') u(s, s'), \quad (5.4)$$

REPLICATOR DYNAMICS

where  $u(s, s')$  is the utility than an agent playing  $s$  receives against an agent playing  $s'$ . Notice that this expected utility assumes that the agents face each other in pairs and choose their opponents randomly. In the replicator dynamics the reproduction rate for each agent is proportional to how well it did on the previous step, that is,

$$\phi^{t+1}(s) = \phi^t(s)(1 + u^t(s)). \quad (5.5)$$

Notice that the number of agents playing a particular strategy will continue to increase as long as the expected utility for that strategy is greater than zero. Only strategies whose expected utility is negative will decrease in population. It is also true that under these dynamics the size of a population will constantly fluctuate. However, when studying replicator dynamics we ignore the absolute size of the population and focus on the fraction of the population playing a particular strategy, i.e.,  $\theta^t(s)$ , as time goes on. We are also interested in determining if the system's dynamics will converge to some strategy and, if so, which one.

In order to study these systems using the standard solution concepts we view the fraction of agents playing each strategy as a mixed strategy for the game. Since the game is symmetric we can use that strategy as the strategy for both players, so it becomes a strategy profile. We say that the system is in a Nash equilibrium if the fraction of players playing each strategy is the same as the probability that the strategy will be played on a Nash equilibrium. In the case of a pure strategy Nash equilibrium this means that all players are playing the same strategy.

An examination of these systems quickly leads to the conclusion that

**Theorem 5.3** (Nash equilibrium is a Steady State). *Every Nash equilibrium is a steady state for the replicator dynamics.*

*Proof. By contradiction.* If an agent had a pure strategy that would return a higher utility than any other strategy then this strategy would be a best response to the Nash equilibrium. If this strategy was different from the Nash equilibrium then we would have a best response to the equilibrium which is not the equilibrium, so the system could not be at a Nash equilibrium.  $\square$

It has also been shown (Fudenberg and Levine, 1998) that

**Theorem 5.4** (Stable Steady State is a Nash Equilibrium). *A stable steady state of the replicator dynamics is a Nash equilibrium. A stable steady state is one that, after suffering from a small perturbation, is pushed back to the same steady state by the system's dynamics.*

These states are necessarily Nash equilibria because if they were not then there would exist some particular small perturbation which would take the system away from the steady state. This correspondence was further refined by Bomze (Bomze, 1986) who showed that

**Theorem 5.5** (Asymptotically Stable is Trembling-Hand Nash). *An asymptotically stable steady state corresponds to a Nash equilibrium that is trembling-hand perfect and isolated. That is, the stable steady states are a refinement on Nash equilibria—only a few Nash equilibria are stable steady states.*

On the other hand, it is also possible that a replicator dynamics system will never converge. In fact, there are many examples of simple games with no asymptotically stable steady states.

While replicator dynamics reflect some of the most troublesome aspects of learning in multiagent systems some differences are evident. These differences are mainly due to the replication assumption. Agents are not usually expected to replicate, instead they acquire the strategies of others. For example, in a real multiagent system all the agents might choose to play the strategy that performed best in the last round instead of choosing their next strategy in proportion to how well it did last time. As





such, we cannot directly apply the results from replicator dynamics to multiagent systems. However, the convergence of the systems' dynamics to a Nash equilibrium does illustrate the importance of this solution concept as an attractor of learning agent's dynamics.

An Evolutionary Stable Strategy (ESS) is an equilibrium concept applied to dynamic systems such as the replicator dynamics system of the previous section.

**Definition 5.1** (Evolutionary Stable Strategy). *An ESS is an equilibrium strategy that can overcome the presence of a small number of invaders. That is, if the equilibrium strategy profile is  $\omega$  and small number  $\epsilon$  of invaders start playing  $\omega'$  then ESS states that the existing population should get a higher payoff against the new mixture  $(\epsilon\omega' + (1 - \epsilon)\omega)$  than the invaders.*

It has been shown (Taylor and Jonker, 1978) that

**Theorem 5.6** (ESS is Steady State of Replicator Dynamics). *ESS is an asymptotically stable steady state of the replicator dynamics. However, the converse need not be true—a stable state in the replicator dynamics does not need to be an ESS.*

This means that ESS is a further refinement of the solution concept provided by the replicator dynamics. ESS can be used when we need a very stable equilibrium concept.

### 5.3.3 The AWESOME Algorithm

While fictitious play does not always converge to an equilibrium, we can guarantee that agents will converge to an equilibrium by having them calculate and play the same equilibrium strategy upon starting. Then, if an agent notices that the others are not playing the agreed upon equilibrium strategy it can play fictitious play instead. This is the basic idea of the **AWESOME** (Adapt When Every is Stationary, Otherwise Move to Equilibrium) algorithm (Conitzer and Sandholm, 2003; Conitzer and Sandholm, 2006). That is, if the other agents appear to be stationary then an AWESOME agent plays the best response to their strategy. On the other hand, if they appear to be adapting then the agents retreats to the equilibrium strategy.

AWESOME

The AWESOME algorithm starts by playing the equilibrium strategy  $\pi_i$  and keeping track of the actions each other player  $j$  has played. Every  $N$  rounds (an epoch) it uses these actions to build a, possibly mixed, strategy  $s_j$  for each player  $j$ . If  $s_j$  is the equilibrium strategy  $\pi_j$  for all players  $j$  then the algorithm keeps playing the equilibrium strategy. Otherwise, the algorithm plays the best response to  $s_j$ . It is easy to see that if all the other players are AWESOME players then they will play their equilibrium strategies and will never diverge from it. If, on the other hand, an AWESOME player is playing against some players who are, eventually, playing some other fixed strategy then it will play a best response strategy against those fixed strategies.

While the basic idea is simple, in order to prove that the algorithm will converge some extra complexity had to be added to it. The algorithm, shown in figure 5.5, has two boolean state variables *playing-equilibrium* which is true when all other agents played the equilibrium strategy during the last epoch and *playing-stationary* which is true when all the other agents played a stationary strategy during the last epoch. Also, *playing-equilibrium-just-rejected* is true when *playing-equilibrium* has just been set to false during the last check. The algorithm plays the same strategy  $\phi$  for a whole epoch and then assesses the situation. If it turns out that either the players are not stationary or not in equilibrium the algorithm makes a note of this and changes its state. If the stationarity hypothesis is rejected then the whole algorithm is re-started again (back to line 2).

In order for the algorithm to always converge,  $\epsilon_e$  and  $\epsilon_s$  must be decreased and  $N$  must be increased over time using a schedule where

1.  $\epsilon_s$  and  $\epsilon_e$  decrease monotonically to 0,

AWESOME

```

1   $\pi \leftarrow$  equilibrium strategy
2  repeat
3       $playing\_equilibrium \leftarrow$  TRUE
4       $playing\_stationary \leftarrow$  TRUE
5       $playing\_equilibrium\_just\_rejected \leftarrow$  FALSE
6       $\phi \leftarrow \pi_i$ 
7       $t \leftarrow 0$ 
8      while  $playing\_stationary$ 
9          do play  $\phi$  for  $N^t$  times in a row (an epoch)
10          $\forall_j$  update  $s_j$  given what they played in these  $N^t$  rounds.
11         if  $playing\_equilibrium$ 
12             then if some player  $j$  has  $\max_a(s_j(a), \pi_j(a)) > \epsilon_e$ 
13                 then  $playing\_equilibrium\_just\_rejected \leftarrow$  TRUE
14                      $\phi \leftarrow$  random action
15             else if  $playing\_equilibrium\_just\_rejected =$  FALSE
16                 and some  $j$  has  $\max_a(s_j^{old}(a), s_j(a)) > \epsilon_s$ 
17                     then  $playing\_stationary \leftarrow$  FALSE
18                      $playing\_equilibrium\_just\_rejected \leftarrow$  FALSE
19                      $b \leftarrow \arg \max_a u_i(a, s_{-i})$ 
20                     if  $u_i(b, s_{-i}) > u_i(\phi, s_{-i}) + n|A_i|\epsilon_s^{t+1}\mu$ 
21                         then  $\phi \leftarrow b$ 
22          $\forall_j s_j^{old} \leftarrow s_j$ 
23          $t \leftarrow t + 1$ 

```

**Figure 5.5:** The AWESOME algorithm. Here  $\pi$  is the equilibrium strategy which has been calculated before the algorithm starts,  $n$  is the number of agents,  $|A_i|$  is the number of actions the agent can take,  $\mu$  is the difference between the player's best and worst possible utility values, and  $s_j(a)$  gives the probability with which  $j$  will play action  $a$  under strategy  $s_j$ . Also,  $\epsilon_e$  and  $\epsilon_s$  must be decreased and  $N$  must be increased over time using a valid schedule.

2.  $N$  increases to infinity,

$$3. \prod_{t \leftarrow 1, \dots, \infty} 1 - \frac{\sum_i |A_i|}{N^t (\epsilon_e^t)^2} > 0$$

$$4. \prod_{t \leftarrow 1, \dots, \infty} 1 - \frac{\sum_i |A_i|}{N^t (\epsilon_s^t)^2} > 0.$$

Given such a valid schedule, it can be shown that

**Theorem 5.7** (AWESOME converges). *With a valid schedule, the AWESOME algorithm converges to best response if all the other players play fixed strategies and to a Nash equilibrium if all the other players are AWESOME players.*

Notice that the theorem says that, in self play, it converges to a Nash equilibrium which might be different from the originally agreed upon equilibrium strategy  $\pi$ . For example, say the agents agree on a mixed equilibrium strategy  $\pi$  but some of the actions played in  $\pi$  constitute a pure Nash equilibrium. In this case it could happen that the players end up, by chance, repeatedly playing the actions in the pure Nash equilibrium during the first epoch. They might then decide that everybody else is playing a stationary strategy which constitutes the pure Nash equilibrium. They will then play the best response to that pure Nash which, we know, is the same pure Nash equilibrium. As such, they will get stuck in that equilibrium.

The AWESOME algorithm is a simple way to force agents to converge to a Nash equilibrium while not letting them be exploited by other agents that are not using the same algorithm. In those cases where all agents are AWESOME agents then it converges from the first step. However, it remains to be seen exactly how much it can be exploited by clever opponents who know the equilibrium it wants to play but would rather play something else.

## Q-LEARNING

```

1   $\forall_s \forall_a Q(s, a) \leftarrow 0; \lambda \leftarrow 1; \epsilon \leftarrow 1$ 
2   $s \leftarrow$  current state
3  if  $\text{RAND}() < \epsilon$   $\triangleright$  Exploration rate
4      then  $a \leftarrow$  random action
5      else  $a \leftarrow \arg \max_a Q(s, a)$ 
6  Take action  $a$ 
7  Receive reward  $r$ 
8   $s' \leftarrow$  current state
9   $Q(s, a) \leftarrow \lambda(r + \gamma \max_{a'} Q(s', a')) + (1 - \lambda)Q(s, a)$ 
10  $\lambda \leftarrow .99\lambda$ 
11  $\epsilon \leftarrow .98\epsilon$ 
12 goto 2

```

**Figure 5.6:** Q-LEARNING algorithm. Note that the .99 and .98 numbers are domain-dependent and need to be changed for each problem to ensure that the algorithm works. With  $\epsilon \leftarrow 0$  the algorithm is still guaranteed to work, but in practice it might take longer to converge.

## 5.4 Stochastic Games

In many multiagent applications the agents do not know the payoffs they will receive for their action. Instead, they must take random actions in order to first explore the world so that they may then determine which actions lead them to the best payoffs. That is, the agents inhabit a multiagent Markov decision problem.

## 5.4.1 Reinforcement Learning

A very popular machine learning technique for solving these types of problems is called **reinforcement learning** (Sutton and Barto, 1998), a specific instance of it is known as **Q-learning** (Watkins and Dayan, 1992). Reinforcement learning assumes that the agent lives in a Markov process and receives a reward in certain states. The goal is to find the right action to take in each state that maximizes the agent's discounted future reward.

More formally, we define the reinforcement learning problem as given by an MDP (section 1.2) where the rewards are given on the edges instead of in the states. That is, the reward function is  $r(s_t, a_t) \rightarrow \mathbb{R}$ . A reinforcement learning agent must find the policy  $\pi^*$  which maximizes his future discounted rewards (1.6).

The reinforcement learning problem can be solved using the Q-LEARNING algorithm shown in figure 5.6. Here  $\lambda$  is the **learning rate** and  $\epsilon$  is the **exploration rate**. Both are always between 0 and 1. The learning rate guides how heavily we consider new rewards versus old values we have learned. When  $\lambda = 1$  the algorithm completely re-writes the old  $Q(s, a)$  value. The exploration rate ensures that we do not converge too quickly to a solution.

It has been shown (Watkins and Dayan, 1992) that Q-LEARNING will converge.

**Theorem 5.8** (Q-LEARNING Converges). *Given that the learning and exploration rates decrease slowly enough, Q-LEARNING is guaranteed to converge to the optimal policy.*

Q-LEARNING differs from the value-iteration algorithm in several respects. In Q-LEARNING the agent takes actions and learns at the same time. Of course, the agent's initial actions will be completely random as it has not learned anything about its expected rewards but, as it takes more actions it learns so the actions chosen become better. Also, the value-iteration algorithm requires knowledge of the complete transition and reward functions of the MDP while the Q-LEARNING algorithm explores the parts of the MDP that it needs. However, to be sure that it has found the optimal policy, a Q-LEARNING agent will need to visit every state and take every action.

The converge results of Q-LEARNING are impressive, but they assume that only one agent is learning and we are interested in multiagent systems where multiple

REINFORCEMENT LEARNING  
Q-LEARNING

LEARNING RATE  
EXPLORATION RATE

  
qllearning

```

NASHQ-LEARNING
1   $t \leftarrow 0$ 
2   $s \leftarrow$  current state
3   $\forall s \in S \forall j \leftarrow 1, \dots, n \forall a_j \in A_j Q_j^t(s, a_1, \dots, a_n) \leftarrow 0$ 
4  Choose action  $a_i^t$ 
5   $s \leftarrow s'$ 
6  Observe  $r_1^t, \dots, r_n^t; a_1^t, \dots, a_n^t; s'$ 
7  for  $j \leftarrow 1, \dots, n$ 
8      do  $Q_j^{t+1}(s, a_1, \dots, a_n) \leftarrow$ 
            $(1 - \lambda^t)Q_j^t(s, a_1, \dots, a_n) + \lambda^t(r_j^t + \gamma \text{Nash}Q_j^t(s'))$ 
           where  $\text{Nash}Q_j^t(s') = Q_j^t(s', \pi_1(s') \cdots \pi_n(s'))$ 
           and  $\pi_1(s') \cdots \pi_n(s')$  are Nash EP calculated from  $Q$  values
9   $t \leftarrow t + 1$ 
10 goto 4

```

**Figure 5.7:** NASHQ-LEARNING algorithm

agents are learning. In these games the reward function is no longer a function of the state and the agent's actions, it is now a function of the state and all the agents' actions. That is, one agent's reward depends on the actions that other agents take, as captured by the multiagent MDP we discussed in section 1.2.1. In these multiagent MDPs it might be impossible for a Q-LEARNING agent to converge to an optimal policy.

The first thing we need to do is choose a new equilibrium. Under the single agent problem definition we were simply looking for the policy that maximizes the agent's discounted rewards. However, when we have multiple agents we might want to maximize the sum of all agents' discounted future rewards (social welfare), or we might choose a more amenable (for convergence) equilibrium such as the **Nash equilibrium point**.

#### NASH EQUILIBRIUM POINT

**Definition 5.2** (Nash Equilibrium Point). *A tuple of  $n$  policies  $(\pi_1^*, \dots, \pi_n^*)$  such that for all  $s \in S$  and  $i = 1, \dots, n$ ,*

$$\forall \pi_i \in \Pi_i v_i(s, \pi_1^*, \dots, \pi_n^*) \geq v_i(s, \pi_1^*, \dots, \pi_{i-1}^*, \pi_i, \pi_{i+1}^*, \dots, \pi_n^*),$$

where  $v_i(s, \pi_1^*, \dots, \pi_n^*)$  is the total rewards (value) that agent  $i$  can expect to receive starting from state  $s$  and assuming that agents use policies  $\pi_1^*, \dots, \pi_n^*$ .

The Nash equilibrium point is a set of policies such that no one agent  $i$  will gain anything by changing its policy from its Nash equilibrium point strategies to something else. Like with the regular Nash equilibrium, it has been shown that the Nash equilibrium point always exists.

**Theorem 5.9** (Nash Equilibrium Point Exists). *Every  $n$ -player discounted stochastic game possesses at least one Nash equilibrium point in stationary strategies.*

We can find the Nash equilibrium point in a system where all the agents use the NASHQ-LEARNING algorithm (Hu and Wellman, 2003) shown in figure 5.7. In this algorithm each agent must keep  $n$   $Q$ -tables, one for each agent in the population. These tables are updated in line 7 using a formula similar to the standard Q-LEARNING update formula but instead of using the  $Q$  values to determine future rewards it uses the *NashQ* tables. These tables hold the  $Q$  value for every agent given that all agents play the same Nash equilibrium policy on the multiagent MDP game induced by the current  $Q$  values. That is, the algorithm assumes that the MDP is defined by the  $Q$  tables it has and then calculates a Nash equilibrium point for this problem. We thus note that at each step the each agent must calculate the Nash equilibrium point given the current  $Q$  functions. This can be an extremely expensive computation—harder than finding the Nash equilibrium for a game matrix.

FRIEND-OR-FOE

```

1   $t \leftarrow 0$ 
2   $s_0 \leftarrow$  current state
3   $\forall s \in S \forall a_j \in A_j Q_i^t(s, a_1, \dots, a_n) \leftarrow 0$ 
4  Choose action  $a_i^t$ 
5   $s \leftarrow s'$ 
6  Observe  $r_1^t, \dots, r_n^t; a_1^t, \dots, a_n^t; s'$ 
7   $Q_i^{t+1}(s, a_1, \dots, a_n) \leftarrow$ 
    $(1 - \lambda^t)Q_i^t(s, a_1, \dots, a_n) + \lambda^t(r_i^t + \gamma \text{Nash}Q_i^t(s'))$ 
   where  $\text{Nash}Q_i^t(s') = \max_{\pi \in \Pi(X_1 \times \dots \times X_k)} \min_{y_i, \dots, y_l \in Y_1 \times \dots \times Y_l}$ 
    $\sum_{x_1, \dots, x_k \in X_1 \times \dots \times X_k} \pi(x_1) \dots \pi(x_k) Q_i(s, x_1, \dots, x_k, y_1, \dots, y_l)$ 
   and  $X$  are actions for  $i$ 's friends and  $Y$  are for the foes.
8   $t \leftarrow t + 1$ 
9  goto 4

```

**Figure 5.8:** FRIEND-OR-FOE algorithm. There are  $k$  friends with actions taken from  $X_1, \dots, X_k$ , and  $l$  foes with actions taken from  $Y_1, \dots, Y_l$ .

Still, the NASHQ-LEARNING algorithm is guaranteed to converge as long as enough time is given so that all state/action pairs are explored sufficiently, and the following assumptions hold.

**Assumption 5.1.** *There exists an adversarial equilibrium for the entire game and for every game defined by the  $Q$  functions encountered during learning.*

Where an adversarial equilibrium is one where no agent has anything to lose if the other agents change their policies. That is, if the other agents change their policies from equilibrium then the agent's expected reward will either stay the same or increase.

**Assumption 5.2.** *There exists a coordination equilibrium for the entire game and for every game defined by the  $Q$  functions encountered during learning.*

Where a coordination equilibrium is one where all the agents receive the highest possible value. That is, the social welfare solution.

Under these assumptions it can be shown that NASHQ-LEARNING converges.

**Theorem 5.10** (NASHQ-LEARNING Converges). *Under assumptions 5.1 and 5.2 NASHQ-LEARNING converges to a Nash equilibrium as long as all the equilibria encountered during the game are unique (Hu and Wellman, 2003).*

These assumptions can be further relaxed by assuming that we can tell the agent whether the opponent is a “friend”, in which case we are looking for a coordination equilibrium, or a “foe”, in which case we are looking for an adversarial equilibrium (Littman, 2001). With this additional information we no longer need to maintain a  $Q$  table for each opponent and can achieve convergence with only one, expanded,  $Q$  table. The algorithm is thus called the FRIEND-OR-FOE algorithm and is shown in figure 5.8.

In the FRIEND-OR-FOE the agent  $i$  has  $k$  friends with action sets represented by  $X_1, \dots, X_k$  and  $l$  foes with action sets represented by  $Y_1, \dots, Y_l$ . The equation implements the idea that  $i$ 's friends are assumed to work together to maximize  $i$ 's value and  $i$ 's foes are working to minimize it. We can show that FRIEND-OR-FOE converges to a stable policy. However, in general these do not correspond to a Nash equilibrium point. Still, we can show that it often converges to a Nash equilibrium.

**Theorem 5.11.** *FOE-Q learns values for a Nash equilibrium policy if the game has an adversarial equilibrium and FRIEND-Q learns values for a Nash equilibrium policy if the game has a coordination equilibrium. This is true regardless of opponent behavior (Littman, 2001).*

That is, if the game has one of the equilibria and we correctly classify all the other agents as either friends or foes then the FRIEND-OR-FOE algorithm is guaranteed to converge.

FRIEND-OR-FOE has several advantages over NASHQ-LEARNING. It does not require the learning of  $Q$  functions for each one of the other agents and it is easy to implement as it does not require the calculation of a Nash equilibrium point at each step. On the other hand, it does require us to know if the opponents are friends or foes, that is, whether there exists a coordination or an adversarial equilibrium.

Neither algorithm deals with the problem of finding equilibria in cases without either coordination or adversarial equilibria. Such cases are the most common and most interesting as they require some degree of cooperation among otherwise selfish agents.

## 5.5 General Theories for Learning Agents

The theory of learning in games provides the designer of multiagent systems with many useful tools for determining the possible equilibrium points of a system. Unfortunately, most multiagent systems with learning agents do not converge to an equilibrium. Designers use learning agents because they do not know, at design time, the specific circumstances that the agents will face at run time. If a designer knew the best strategy, that is, the Nash equilibrium strategy, for his agent then he would simply implement this strategy and avoid the complexities of implementing a learning algorithm. Therefore, the only times we will see a multiagent system with learning agents are when the designer cannot predict that an equilibrium solution will emerge.

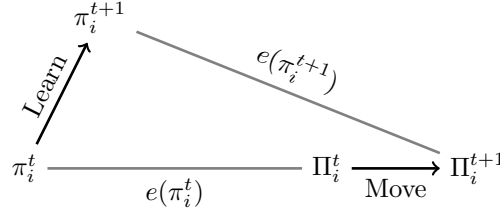
The two main reasons for this inability to predict the equilibrium solution of a system are the existence of unpredictable environmental changes that affect the agents' payoffs and the fact that on many systems an agent only has access to its own set of payoffs—it does not know the payoffs of other agents. These two reasons make it impossible for a designer to predict which equilibria, if any, the system would converge to. However, the agents in the system are still playing a game for which an equilibrium exists, even if the designer cannot predict it at design-time. But, since the actual payoffs keep changing it is often the case that the agents are constantly changing their strategy in order to accommodate the new payoffs.

As mentioned earlier, learning agents in a multiagent system are faced with a moving target function problem (Vidal and Durfee, 1998b). That is, as the agents change their behavior in an effort to maximize their utility their payoffs for those actions change, changing the expected utility of their behavior. The system will likely have non-stationary dynamics—always changing in order to match the new goal. While game theory tells us where the equilibrium points are, given that the payoffs stay fixed, multiagent systems often never get to those points. A system designer needs to know how changes in the design of the system and learning algorithms will affect the time to convergence. This type of information can be determined by using CLRI theory.

### 5.5.1 CLRI Model

The **CLRI model** (Vidal and Durfee, 2003) provides a method for analyzing a system composed of learning agents and determining how an agent's learning is expected to affect the learning of other agents in the system. It assumes a system where each agent has a decision function that governs its behavior as well as a target function that describes the agent's best possible behavior. The target function is unknown to the agent. The goal of the agent's learning is to have its decision function be an exact duplicate of its target function. Of course, the target function keeps changing as a result of other agents' learning.

Formally, the CLRI model assumes that there is a fixed set of autonomous agents in the system. The system can be described by a set of discrete states  $s \in S$  which



**Figure 5.9:** The moving target function problem.

are presented to the agent with a probability dictated by the fixed probability distribution  $\mathcal{D}(S)$ . Each agent  $i$  has a set of possible actions  $A_i$  where  $|A_i| \geq 2$ . Time is discrete and indexed by a variable  $t$ . At each time  $t$  all agents are presented with a new  $s \in \mathcal{D}(S)$ , take a simultaneous action, and receive some payoff. The scenario is similar to the one used by fictitious play except for the addition of state  $s$ .

Each agent  $i$ 's behavior is defined by a policy  $\pi_i^t(s) : S \rightarrow A$ . When  $i$  learns at time  $t$  that it is in state  $s$  it will take action  $\pi_i^t(s)$ . At any time there is an optimal policy for each agent  $i$ , also known as its target function, which we represent as  $\Pi_i^t(s)$ . Agent  $i$ 's learning algorithm will try to reduce the discrepancy between  $\pi_i$  and  $\Pi_i$  by using the payoffs it receives for each action as clues as to what it should do given that it does not have direct access to  $\Pi_i$ . The probability that an agent will take a wrong action is given by its error  $e(\pi_i^t) = \Pr[\pi_i^t(s) \neq \Pi_i^t(s) \mid s \in \mathcal{D}(S)]$ . As other agents learn and change their decision function,  $i$ 's target function will also change, leading to the moving target function problem, as depicted in figure 5.9.

An agent's error is based on a fixed probability distribution over world states and a boolean matching between the decision and target functions. These constraints are often too restrictive to properly model many multiagent systems. However, even if the system being modeled does not completely obey these two constraints, the use of the CLRI model in these cases still gives the designer valuable insight into how changes in the design will affect the dynamics of the system. This practice is akin to the use of Q-learning in non-Markovian games—while Q-learning is only guaranteed to converge if the environment is Markovian, it can still perform well on other domains.

The CLRI model allows a designer to understand the expected dynamics of the system, regardless of what learning algorithm is used, by modeling the system using four parameters: Change rate, Learning rate, Retention rate, and Impact (CLRI). A designer can determine values for these parameters and then use the CLRI difference equation to determine the expected behavior of the system.

The change rate ( $c$ ) is the probability that an agent will change at least one of its incorrect mappings in  $\delta^t(w)$  for the new  $\delta^{t+1}(w)$ . It captures the rate at which the agent's learning algorithm tries to change its erroneous mappings. The learning rate ( $l$ ) is the probability that the agent changes an incorrect mapping to the correct one. That is, the probability that  $\delta^{t+1}(w) = \Delta^t(w)$ , for all  $w$ . By definition, the learning rate must be less than or equal to the change rate, i.e.  $l \leq c$ . The retention rate ( $r$ ) represents the probability that the agent will retain its correct mapping. That is, the probability that  $\delta^{t+1}(w) = \delta^t(w)$  given that  $\delta^t(w) = \Delta^t(w)$ .

CLRI defines a volatility term ( $v$ ) to be the probability that the target function  $\Delta$  changes from time  $t$  to  $t + 1$ . That is, the probability that  $\Delta^t(w) \neq \Delta^{t+1}(w)$ . As one would expect, volatility captures the amount of change that the agent must deal with. It can also be viewed as the speed of the target function in the moving target function problem, with the learning and retention rates representing the speed of the decision function. Since the volatility is a dynamic property of the system (usually it can only be calculated by running the system) CLRI provides an impact ( $I_{ij}$ ) measure.  $I_{ij}$  represents the impact that  $i$ 's learning has on  $j$ 's target function. Specifically, it is the probability that  $\Delta_j^t(w)$  will change given that  $\delta_i^{t+1}(w) \neq \delta_i^t(w)$ .

Someone trying to build a multiagent system with learning agents would determine the appropriate values for  $c$ ,  $l$ ,  $r$ , and either  $v$  or  $I$  and then use

$$\begin{aligned}
E[e(\delta_i^{t+1})] &= 1 - r_i + v_i \left( \frac{|A_i|r_i - 1}{|A_i| - 1} \right) \\
&\quad + e(\delta_i^t) \left( r_i - l_i + v_i \left( \frac{|A_i|(l_i - r_i) + l_i - c_i}{|A_i| - 1} \right) \right) \quad (5.6)
\end{aligned}$$

in order to determine the successive expected errors for a typical agent  $i$ . This equation relies on a definition of volatility in terms of impact given by

$$\begin{aligned}
\forall_{w \in W} v_i^t &= \mathbf{Pr}[\Delta_i^{t+1}(w) \neq \Delta_i^t(w)] \\
&= 1 - \prod_{j \in N-i} (1 - I_{ji} \mathbf{Pr}[\delta_j^{t+1}(w) \neq \delta_j^t(w)]), \quad (5.7)
\end{aligned}$$

which makes the simplifying assumption that changes in agents' decision functions will not cancel each other out when calculating their impact on other agents. The difference equation (5.6) cannot, under most circumstances, be collapsed into a function of  $t$  so it must still be iterated over. On the other hand, a careful study of the function and the reasoning behind the choice of the CLRI parameter leads to an intuitive understanding of how changes in these parameters will be reflected in the function and, therefore, the system. A knowledgeable designer can simply use this added understanding to determine the expected behavior of his system under various assumptions. An example of this approach is shown in (Brooks and Durfee, 2003).

For example, it is easy to see that an agent's learning rate and the system's volatility together help to determine how fast, if ever, the agent will reach its target function. A large learning rate means that an agent will change its decision function to almost match the target function. Meanwhile, a low volatility means that the target function will not move much, so it will be easy for the agent to match it. Thus, if the agents have no impact on each other, that is,  $I_{ij} = 0$  for all  $i, j$ , then the agents will learn their target function and the system will converge. As the impact increases then it becomes more likely that the system will never converge.

Of course, this type of simple analysis ignores a common situation where the agent's high learning rate is coupled with a high impact on other agents' target function making their volatility much higher. These agents might then have to increase their learning rate and thereby increase the original agent's volatility. Equation (5.6) is most helpful in these type of feedback situations.

### 5.5.2 N-Level Agents

Another issue that arises when building learning agents is the choice of a modeling level. A designer must decide whether his agent will learn to correlate actions with rewards, or will try to learn to predict the expected actions of others and use these predictions along with knowledge of the problem domain to determine its actions, or will try to learn how other agents build models of other agents, etc. These choices are usually referred to as **n-level** modeling agents—an idea first presented in the recursive modeling method (Gmytrasiewicz and Durfee, 1995) (Gmytrasiewicz and Durfee, 2001).

A 0-level agent is one that does not recognize the existence of other agents in the world. It learns which action to take in each possible state of the world because it receives a reward after its actions. The state is usually defined as a static snapshot of the observable aspects of the agent's environment. A 1-level agent recognizes that there are other agents in the world whose actions affect its payoff. It also has some knowledge that tells it the utility it will receive given any set of joint actions. This knowledge usually takes the form of a game matrix that only has utility values for the agent. The 1-level agent observes the other agents' actions and builds probabilistic models of the other agents. It then uses these models to predict their action probability distribution and uses these distributions to determine its



best possible action. A 2-level agent believes that all other agents are 1-level agents. It, therefore, builds models of their models of other agents based on the actions it thinks they have seen others take. In essence, the 2-level agent applies the 1-level algorithm to all other agents in an effort to predict their action probability distribution and uses these distributions to determine its best possible actions. A 3-level agent believes that all other agents are 2-level, and so on. Using these guidelines we can determine that fictitious play (section 5.3.1) uses 1-level agents while the replicator dynamics (section 5.3.2) uses 0-level agents.

These categorizations help us to determine the relative computational costs of each approach and the machine-learning algorithms that are best suited for that learning problem. 0-level is usually the easiest to implement since it only requires the learning of one function and no additional knowledge. 1-level learning requires us to build a model of every agent and can only be implemented if the agent has the knowledge that tells it which action to take given the set of actions that others have taken. This knowledge must be integrated into the agents. However, recent studies in layered learning (Stone, 2000) have shown how some knowledge could be learned in a “training” situation and then fixed into the agent so that other knowledge that uses the first one can be learned, either at runtime or in another training situation. In general, a change in the level that an agent operates on implies a change on the learning problem and the knowledge built into the agent.

Studies with  $n$ -level agents have shown (Vidal and Durfee, 1998a) that an  $n$ -level agent will always perform better in a society full of  $(n-1)$ -level agents, and that the computational costs of increasing a level grow exponentially. Meanwhile, the utility gains to the agent grow smaller as the agents in the system increase their level, within an economic scenario. The reason is that an  $n$ -level agent is able to exploit the non-equilibrium dynamics of a system composed of  $(n-1)$ -level agents. However, as the agents increase their level the system reaches equilibrium faster so the advantages of strategic thinking are reduced—it is best to play the equilibrium strategy and not worry about what others might do. On the other hand, if all agents stopped learning then it would be very easy for a new learning agent to take advantage of them. As such, the research concludes that some of the agents should do some learning some of the time in order to preserve the robustness of the system, even if this learning does not have any direct results. That is, there appear to be decreasing marginal returns for strategic thinking.

## 5.6 Collective Intelligence

We can also try to use machine learning as a way to automatically build multiagent systems where each one of the agents learns to do what we want. For example, imagine a factory floor with boxes that need to be moved and robots of different abilities. Depending on the size and weight of the boxes, different robots, or combinations of robots, are able to move them. We know that our goal is for all boxes to be placed against the South wall. Then, rather than trying to come up with specific plans or rules of behavior for each robot, we instead install a reinforcement learning algorithm in each one of them and set them off to learn how best to coordinate. The question we must then ask is: what is the reward function for each agent? As we saw in the CLRI model, one agent’s actions can have an impact on another agent’s target function thus making it hard, or even impossible, for the other agent to converge in its learning. For example, if one robot keeps changing its behavior and moving around randomly then it will be impossible for another agent to learn to predict when they will both be in front of a large box so that they can move it together.

Collective intelligence (**COIN**) aims to formalize these ideas and determine what kind of rewards we should provide reinforcement-learning agents in order to achieve a desired global behavior (Wolpert and Tumer, 1999). Specifically, we are given a global utility function  $U(s, \vec{a}) \rightarrow \mathbb{R}$  which tells us the value for every vector of actions  $\vec{a} = \{a_1, a_2, \dots, a_n\}$  of the agents and state of the world  $s$ . The agents are assumed to use some reinforcement learning algorithm, such as Q-learning. Our problem is

then to define the agents' reward functions  $u_i(s, \vec{a})$  such that the agents will end up with policies  $\pi_i^*$  that maximize  $U$ . Notice that simply setting  $u_i(s, \vec{a}) = U(s, \vec{a})$  for all  $i$  can lead to agents receiving an uninformative reward. For example, if a confused agents throws itself into a wall while, at the same time, all the other agents cooperate to move the box correctly then the confused agent will received a high reward and will thus likely continue to throw itself against the wall.

Still, in general we do want to align each agent's rewards with the global utility. We can quantify this alignment by defining agent  $i$ 's preference over  $s, \vec{a}$  as

$$P_i(s, \vec{a}) = \frac{\sum_{\vec{a}' \in \vec{A}} \Theta[u_i(s, \vec{a}) - u_i(s, \vec{a}')] }{|\vec{A}|}, \quad (5.8)$$

where  $\Theta(x)$  is the Heaviside function which is 1 if  $x$  is greater than or equal to 0, otherwise it is 0. Similarly, we can define the global preference function as

$$P(s, \vec{a}) = \frac{\sum_{\vec{a}' \in \vec{A}} \Theta[U(s, \vec{a}) - U(s, \vec{a}')] }{|\vec{A}|}. \quad (5.9)$$

Both of these functions serve to rank the agents', or the global, preferences over the set of all possible states and actions vectors. For example, if a particular  $(s, \vec{a})$  is preferred over all others then  $P(s, \vec{a}) = 1$ .

In general, we might want an agent's preferences to be similar to the system's preferences so that the agent's learning mechanism will try to converge towards the desired system's preferences. A system where, for all agents  $i$  it is true that  $P_i(s, \vec{a}) = P(s, \vec{a})$  is called **factored**. These systems are nice in that they provide the correct incentives to the agents in all situations. Thus, factored systems are more likely to converge to the set of policies that maximizes  $U$ .

It is possible that factored systems might not converge because agents' action might have an impact on each other thus changing each other's target function. This idea is captured within the COIN framework as the **opacity** of  $u_i(s, \vec{a})$ . Specifically, we define the opacity  $\Omega_i$  for agent  $i$  as

$$\Omega_i(s, \vec{a}) = \sum_{\vec{a}' \in \vec{A}} \Pr[\vec{a}'] \frac{|u_i(s, \vec{a}) - u_i(s, \vec{a}'_{-i}, \vec{a}_i)|}{|u_i(s, \vec{a}) - u_i(s, \vec{a}_{-i}, \vec{a}'_i)|}. \quad (5.10)$$

The opacity is zero when the agent's rewards are the same regardless of the actions taken by other, that is, when the other agents' actions have no impact on what the agent should do. On the other hand, the opacity gets larger as the other agents' actions change the reward the agent will get. Just like with the CLRI impact, if the opacity is zero for all states and action vectors then the multiagent learning problem is reduced to the single-agent learning problem as the agents have no effect on each other's target function. More generally, a decrease in the opacity amounts to an increase in the signal-to-noise ratio that the agent receives via its reward function.

Systems that are both factored and have zero opacity for all agents are extremely rare, but would be easy to solve. Systems where the opacity is zero for all agents amount to multiple learning problems that do not interfere with each other and are easy to solve, but they are also very rare. Our goal can now be re-stated as that of finding reward functions for each agent that have as low opacity as possible while also being as highly factored as possible.

One solution COIN proposes is the use of the **wonderful life** reward function which gives each agent a reward proportional to its own contribution to the global utility. It implements the same idea as the VCG payments (Chapter 8.4). Formally, agent  $i$ 's wonderful life reward is given by

$$u_i(s, \vec{a}) = U(s, \vec{a}) - U(s, \vec{a}_{-i}), \quad (5.11)$$

where the  $U(s, \vec{a}_{-i})$  represents the utility in state  $s$  when agent  $i$  does not exist and all other agents take actions as in  $\vec{a}_{-i}$ . Notice that this equation can be derived

FACTORED

OPACITY

WONDERFUL LIFE

directly from the global utility function. Thus, it is applicable to any system where we have a pre-defined utility function  $U$ . It has been shown that this reward function performs better than using  $u_i = U$  and other seemingly appropriate reward functions (Wolpert et al., 1999; Tumer and Wolpert, 2004).

Another solution is the **aristocrat utility**, which is defined as

$$u_i(s, \vec{a}) = U(s, \vec{a}) - \sum_{\vec{a}' \in \vec{A}} \mathbf{Pr}[\vec{a}'] U(s, \vec{a}_{-i}, \vec{a}'), \quad (5.12)$$

where  $\mathbf{Pr}[\vec{a}']$  is the probability that  $\vec{a}'$  happens. The aristocrat utility measures the difference in the global utility between the agent's action and its average or expected action. It has been shown that this reward function also performs well, sometimes better than the wonderful life utility (Wolpert and Tumer, 2001) .

ARISTOCRAT UTILITY



## 5.7 Summary

We have seen how the theory of learning in games provides us with various equilibrium solution concepts and often tells us when some of them will be reached by simple learning models. On the other hand, we have argued that the reason learning is used in a multiagent system is often because there is no known equilibrium or the equilibrium point keeps changing due to outside forces. We have also shown how the CLRI theory and n-level agents are attempts to characterize and predict, to a limited degree, the dynamics of a system given some basic learning parameters.

## 5.8 Recent Advances

The GAMUT (Nudelman et al., 2004) software can produce game matrices from thirty-five base classes of games. These matrices can be used to determine how well learning algorithms perform under all possible game types and sizes.



## Chapter 6

### Negotiation

In a negotiation problem multiple agents try to agree on a deal. Each agent is assumed to have a preference over all possible deals. The agents send messages to each other in the hope of finding a deal that all agents can agree on. The agents in a negotiation face an interesting problem. They want to maximize their own utility but they also face the risk of a break-down in negotiation, or expiration of a deadline for agreement. As such, each agent must decide whether the current deal is good enough or whether it should ask for more and risk agreement failure.

Automated negotiation can be very useful in multiagent systems as it provides a distributed method of aggregating distributed knowledge (Davis and Smith, 1983). That is, in a problem where each agent has different local knowledge negotiation can be an effective method for finding the one global course of action which maximizes utility without having to send all the local knowledge bases to a central location for consideration. For example, in a NASA rover mission to Mars the various engineering teams and science teams negotiate over what feature to include in the rover. The scientists are concerned with having the proper equipment in Mars so that they can do good science while the engineers try to ensure that everything will work as expected. Often it is the case that one side does not understand exactly why the other wants or rejects a particular feature but by negotiating with each other they arrive at a rover that is engineered solidly enough to survive the trip to Mars and has enough equipment to do useful science while there. Negotiation is a way of aggregating the knowledge of multiple individuals in order to make decisions which are better, for the whole, than if they were made by any one individual.

See (Squyres, 2005) for the full story on the MER mission to Mars. An exciting read.

#### 6.1 The Bargaining Problem

A specific version of the negotiation problem has been studied in game theory. It is known as the **bargaining problem** (Nash, 1950). Formally, we say that each agent  $i$  has a utility function  $u_i$  defined over the set of all possible deals  $\Delta$ . That is,  $u_i : \Delta \rightarrow \mathbb{R}$ . We also assume that there is a special deal  $\delta^-$  which is the no-deal deal. Without loss of generality we will assume that for all agents  $u_i(\delta^-) = 0$  so that the agents will prefer no-deal than accepting any deal with negative utility. The problem then is finding a protocol  $f$  which will lead the agents to the best deal. But, what is the best deal? several possible solutions have been proposed . There are multiple solutions because, as with standard-form games, in negotiation it is not clear which solution is the best one.

#### BARGAINING PROBLEM

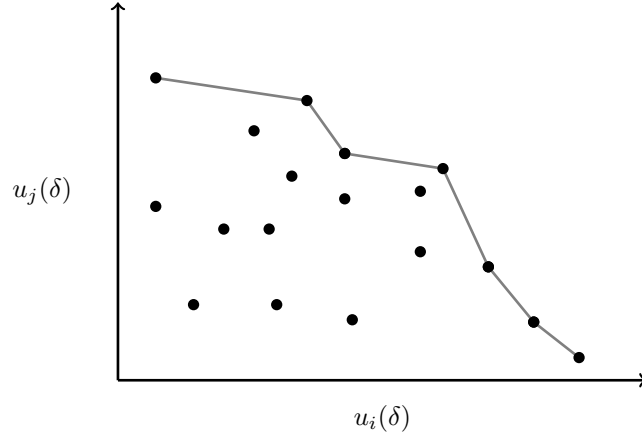
See (Osborne and Rubinstein, 1999, Chapter 7) or (Osborne and Rubinstein, 1990) for a more extended introduction to bargaining.

##### 6.1.1 Axiomatic Solution Concepts

There are many possible requirements we might want to impose on a possible solution deal.

**Definition 6.1** (Pareto optimal). *A deal  $\delta$  is Pareto optimal if there is no other deal such that everyone prefers it over  $\delta$ . That is, there is no  $\delta'$  such that*

$$\forall_i u_i(\delta') > u_i(\delta).$$



**Figure 6.1:** The dots represent possible deals. The deals on the gray line are Pareto optimal. The line is the Pareto frontier.

This is an obvious requirement because if a deal is not Pareto optimal then that means that there is some other deal which all of the agents like better. As such, it does not make sense to use the non-Pareto deal when we could find this other deal which everyone prefers.

With only two agents,  $i$  and  $j$ , we can visualize the set of Pareto-optimal deals by using an  $xy$ -plot where the  $x$  and  $y$  coordinates are the utilities each agent receives for a deal. Each deal then becomes a point in the graph, as seen in figure 6.1. The **pareto frontier** is represented by the line shown in the upper-right.

**Definition 6.2** (Independence of utility units). *A negotiation protocol is independent of utility units if when given  $U$  it chooses  $\delta$  and when given  $U' = \{(\beta_1 u_1, \dots, \beta_I u_I) : u \in U\}$  it chooses  $\delta'$  where*

$$\forall_i u_i(\delta') = \beta_i u_i(\delta).$$

That is, if an agent in a protocol that is independent of utility units used to get a utility of 10 from the result deal and now has multiplied its utility function by 5 then that agent will now get a utility of 50 from the new resulting deal. So, the utilities the agents receive remain proportional under multiplication.

**Definition 6.3** (Symmetry). *A negotiation protocol is symmetric if the solution remains the same as long as the set of utility functions  $U$  is the same, regardless of which agent has which utility.*

That is, if two agents swapped utility function then they also end up swapping the utilities they get from the resulting deal. In other words, the specific agents do not matter, all that matter is the utility functions.

**Definition 6.4** (Individual rationality). *A deal  $\delta$  is individually rational if*

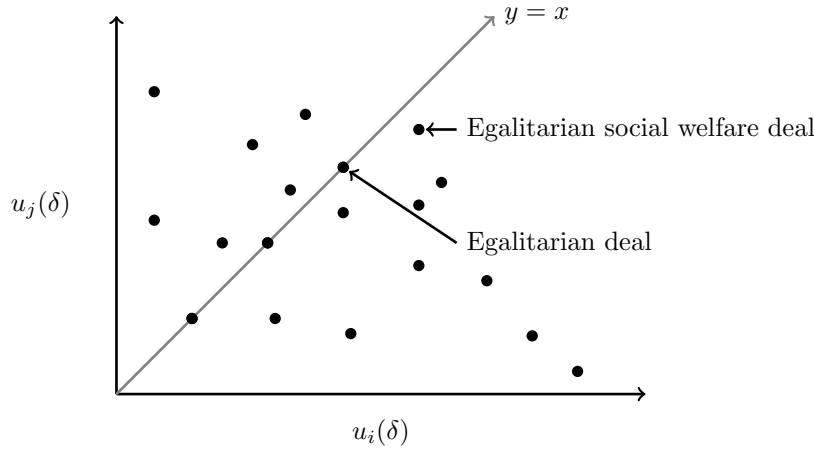
$$\forall_i u_i(\delta) \geq u_i(\delta^-).$$

Which means that  $u_i(\delta) \geq 0$  since we will be assuming that  $u_i(\delta^-) = 0$ . A deal is individually rational if all the agents prefer it over not reaching an agreement.

**Definition 6.5** (Independence of irrelevant alternatives). *A negotiation protocol is independent of irrelevant alternatives if it is true that when given  $\Delta$  it chooses  $\delta$  and when given  $\Delta' \subset \Delta$  where  $\delta \in \Delta'$  it again chooses  $\delta$ , assuming  $U$  stays constant.*

That is, a protocol is independent of irrelevant alternative is the deal it chooses does not change after we remove a deal that lost. Only removal of the winning deal changes the deal the protocol chooses.

Given these requirements we can now consider various possible solutions to a negotiation problem. In the **egalitarian solution** the gains from cooperation are



**Figure 6.2:** Egalitarian and egalitarian social welfare deals.

split equally among the agents. That is, the egalitarian deal is the one where all the agents receive the same utility and the sum of their utilities is maximal, that is

$$\delta = \arg \max_{\delta' \in E} \sum_i u_i(\delta') \quad (6.1)$$

where  $E$  is the set of all deals where all agents receive the same utility, namely

$$E = \{\delta \mid \forall_{i,j} u_i(\delta) = u_j(\delta)\}.$$

We can find the egalitarian solution visually for the two agent case by simply drawing the line  $y = x$  and finding the deal on this line that is farther away from the origin, as seen in figure 6.2. Note that the egalitarian deal in this case is not Pareto optimal. However, if we allowed all possible deals (the graph would be solid black as every pair of  $x, y$  coordinates would represent a possible deal) then the egalitarian deal would also be a Pareto deal. It would be the point on the  $y = x$  line which marks the intersection with the Pareto frontier. Also note that the egalitarian deal does not satisfy the independence of utility units requirement since it assumes the utility units for the agents are comparable, in fact, it assumes they are in the same units.

A variation on the pure egalitarian deal is the **egalitarian social welfare** solution which is the deal that maximizes the utility received by the agent with the lowest utility. That is, it is the deal  $\delta$  that satisfies

$$\delta = \arg \max_{\delta} \min_i u_i(\delta). \quad (6.2)$$

The egalitarian social welfare solution is especially useful in scenarios where no deal exists which provides all agents with the same utility since every problem is guaranteed to have an egalitarian social welfare solution. However, the solution itself can in some cases seem very un-egalitarian. For example, a deal where two agents receive utilities of 10 and 100 respectively is the egalitarian social welfare solution even when another deal exists which gives the agents 9 and 11 respectively.

The **utilitarian solution** is the deal that maximizes the sum of the utilities, that is

$$\delta = \arg \max_{\delta} \sum_i u_i(\delta). \quad (6.3)$$

The utilitarian deal is, by definition, a Pareto optimal deal. There might be more than one utilitarian deals in the case of a tie. The utilitarian deal violated the independence of utility units assumption as it also assumes utilities are comparable.

We can find the utilitarian deal visually for the two agent case by drawing a line with slope of  $-1$  and, starting at the top right moving it perpendicular to  $y = x$  until it intersects one or more deals. The first deals intersected by the line are utilitarian deals, as shown in figure 6.3.

**egalitarian n:** one who believes in the equality of all people.

EGALITARIAN SOCIAL  
WELFARE

UTILITARIAN SOLUTION

**utilitarian n :** someone who believes that the value of a thing depends on its utility.

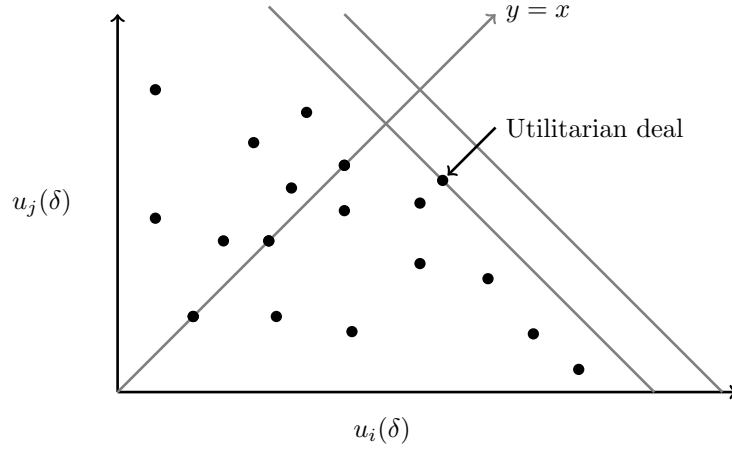


Figure 6.3: Utilitarian deal.

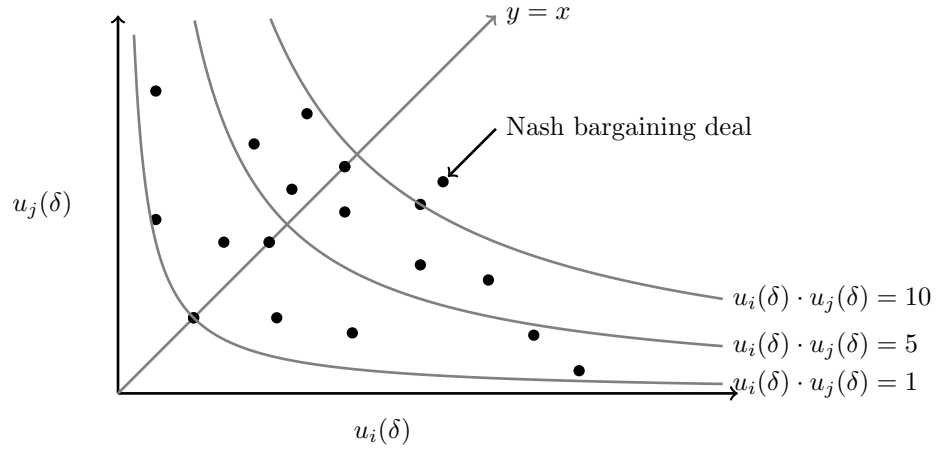


Figure 6.4: Nash bargaining deal.

The utilitarian and egalitarian solutions both seem like fairly reasonable solutions. Unfortunately, both violate the independence of utility units assumption. It was Nash who proposed a third solution which does not violate this assumption. The **Nash bargaining solution** states that the deal chosen is the one that maximizes the product of the utilities. That is,

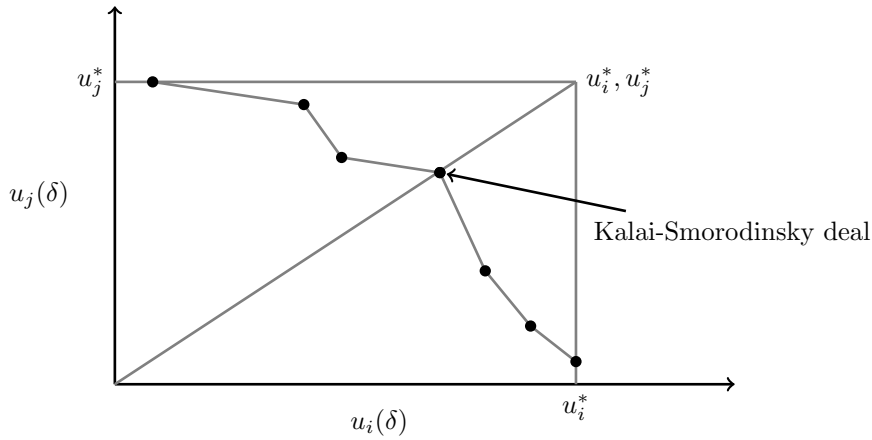
$$\delta = \arg \max_{\delta'} \prod u_i(\delta'). \quad (6.4)$$

The Nash solution is also Pareto efficient (Definition 6.1), independent of utility units (Definition 6.2), independent of irrelevant alternatives (Definition 6.5), and symmetric (Definition 6.3). In fact, it is the *only* solution that satisfies these four requirements (Nash, 1950). This means that if we want a solution that satisfies these four requirements then our only choice is the the Nash bargaining solution.

Figure 6.4 shows a visualization of the Nash bargaining deal. Each curve represents all pairs of utilities that have the same product. As we move northeast, following the line  $y = x$ , we cut across indifference curves of monotonically higher products. As such, the last deal to intersect a curve is the one which maximizes the product of the utilities, so it is the Nash bargaining solution.

Yet another possible solution is to find the deal that distributes the utility in proportion to the maximum that the agent could get (Kalai and Smorodinsky, 1975). Specifically, let  $u_i^*$  be the maximum utility that  $i$  could get from the set of all deals in the Pareto frontier. That is, the utility that  $i$  would receive if he got his best possible deal from the Pareto frontier. Then, find the deal that lies in the line between the point  $\delta^-$  and the point  $(u_i^*, u_j^*)$ . This solution is known as the **Kalai-Smorodinsky solution**. If all deals are possible then this solution is Pareto optimal. However, if there are only a finite set of possible deals then this solution





**Figure 6.5:** Kalai-Smorodinsky bargaining deal. We only show deals on the Pareto frontier.

might not exist. That is, there might not be a point in the specified line. This is a serious drawback to the use of the Kalai-Smorodinsky solution in discrete setting.

The Kalai-Smorodinsky solution, like the Nash bargaining solution, it is also independent of utility units requirement. On the other hand, it is not independent of irrelevant alternatives.

There is no general agreement as to which one of these axiomatic solutions is better. The social sciences try to develop experiments that will tell us which one of the solutions, if any, is arrived at by humans engaged in negotiation. Others try to justify some of these solutions as more fair than others. We, as multiagent designers, will probably find that all of them will be useful at some point, depending on the requirements of the system we are building.

### 6.1.2 Strategic Solution Concepts

Another way to think about what solution will be arrived at in a bargaining problem is to formalize the bargaining process, assume rational agents, and then determine their equilibrium strategies for their bargaining process. That is, define the solution concept as the solution that is reached by rational agents in a bargaining problem. This method does raise one large obstacle: we need to first formally define a bargaining process that allows all the same “moves” as real-life bargaining. If you have ever haggled over the price of an item then you know that it is impossible to formalize all the possible moves of market-vendor. We need a negotiation protocol that is simple enough to be formally analyzed but still allows the agents to use most of their moves.

One such model is the **Rubinstein’s alternating offers** model (Rubinstein, 1982). In this model two agents try to reach agreement on a deal. The agents can take actions only at integer time steps:  $t = 0, 1, 2, \dots$ . In each time step one of the agents, say  $i$ , proposes a deal  $\delta$  to the other who either accepts it or rejects it. If the offer is rejected then we move to the next time step where the other agent,  $j$ , gets to propose a deal. Once a deal has been rejected it is considered void and cannot be accepted at a later time. The agents, however, are always free to propose any deal and to accept or reject any deal as they wish. We further assume that the agents know each other’s utility functions.

The alternating offers models, as it stands, does not have a dominant strategy. Imagine that the two agents are bargaining over how to divide a dollar. Each agent wants to keep the whole dollar to itself and leave the other agent with nothing. Under the basic alternating offers protocol the agents’ best strategy is to keep proposing this deal to the other agent. That is, each agent keeps telling the other one “I propose that I keep the whole dollar” and the other agent keeps rejecting this proposal. This scenario is not very interesting.

In order to make it more interesting, we further assume that time is valuable

RUBINSTEIN’S  
ALTERNATING OFFERS

to the agents. That is, the agents' utility for all possible deals is reduced as time passes. For example, imagine that instead of haggling over a dollar the agents are haggling over how to split an ice cream sundae which is slowly melting (and, they don't like melted ice cream). Formally, we say that agent  $i$ 's utility at time  $t$  for deal  $\delta$  is given by  $\lambda_i^t u_i(\delta)$  where  $\lambda_i$  is  $i$ 's discount factor, similarly the utility for  $j$  is  $\lambda_j^t u_j(\delta)$ . Thus, the agents' utility for every possible deal decreases monotonically as a function of time with a discount factor given by  $\lambda$ . Note that if  $\lambda_i = 0$  then the agent must agree to a deal at time 0 since after that it will receive a utility of 0 for any deal. Conversely, if  $\lambda_i = 1$  then the agent can wait forever without any utility loss.

Furthermore, let's assume that the agents' utilities are linear and complimentary. Specifically, imagine that the deal  $\delta$  is simply a number between 0 and 1 and represents the amount of utility an agent receives so that  $u_i(\delta) = \delta$  and  $u_j(\delta) = 1 - \delta$ . In this scenario, it can be shown that a unique subgame perfect equilibrium strategy exists.

**Theorem 6.1** (Alternating Offers Bargaining Strategy). *The Rubinstein's alternating offers game where the agents have complimentary linear utilities has a unique subgame perfect equilibrium strategy where*

- agent  $i$  proposes a deal

$$\delta_i^* = \frac{1 - \lambda_j}{1 - \lambda_i \lambda_j}$$

and accepts the offer  $\delta_j$  from  $j$  only if  $u_i(\delta_j) \leq u_i(\delta_i^*)$ ,

- agent  $j$  proposes a deal

$$\delta_j^* = \frac{1 - \lambda_i}{1 - \lambda_i \lambda_j}$$

and accepts the offer  $\delta_i$  from  $i$  only if  $u_j(\delta_i) \leq u_j(\delta_j^*)$ .

(Rubinstein, 1982; Muthoo, 1999).

We can understand how these deals are derived by noting that since both agents have utilities that decrease with time the best deal will be reached in the first step. That means that each agent must propose a deal that the other will accept. Specifically, agent  $i$  must propose a deal  $\delta_i^*$  such that  $u_j(\delta_i^*) = \lambda_j u_j(\delta_j^*)$  because if it proposes a deal that gives  $j$  lower utility then  $j$  will reject it and if it proposes a deal that gives  $j$  higher utility then  $i$  is needlessly giving up some of its own utility. Conversely,  $j$  must propose a deal  $\delta_j^*$  such that  $u_i(\delta_j^*) = \lambda_i u_i(\delta_i^*)$ . We thus have two equations.

Since  $u_i(\delta) = \delta$  and  $u_j(\delta) = 1 - \delta$ , we can replace these definitions into the above equations to get

$$1 - \delta_i^* = \lambda_j(1 - \delta_j^*) \tag{6.5}$$

$$\delta_j^* = \lambda_i \delta_i^*. \tag{6.6}$$

Upon solving these two equations for  $\delta_i^*$  and  $\delta_j^*$  we get the equilibrium values from Theorem 6.1. More generally, it has been shown that a unique subgame perfect equilibrium exists even when the utility functions are not linear but are simply monotonic in  $\delta$ .

The theorem tells us that the best strategy for these agents is propose a bid on the first time step which will be accepted by the other agent. This action makes sense because we know that utilities only decrease with time so the best possible deal will be had on the first time step. The value of the proposed deal, on the other hand, is very interesting. Notice that the deal  $i$  proposes depends only on the agents' discount factor, not on their utility values. The important thing is how fast does each agent lose utility over time. For example, notice that if  $\lambda_j = 0$  then  $i$  will propose  $\delta_i^* = 1$ . That is,  $i$  will propose to keep all the utility to himself. Agent  $j$

## MONOTONIC-CONCESSION

```

1   $\delta_i \leftarrow \arg \max_{\delta} u_i(\delta)$ 
2  Propose  $\delta_i$ 
3  Receive  $\delta_j$  proposal
4  if  $u_i(\delta_j) \geq u_i(\delta_i)$ 
5      then Accept  $\delta_j$ 
6  else  $\delta_i \leftarrow \delta'_i$  such that  $u_j(\delta'_i) \geq \epsilon + u_j(\delta_i)$  and  $u_i(\delta'_i) \geq u_i(\delta^-)$ 
7  goto 2

```

**Figure 6.6:** The monotonic concession protocol.

will accept this proposal since he knows that given that his  $\lambda_j = 0$  if he waits for the next time step then he will receive utility of 0 in every possible deal.

Similar techniques have been used (Kraus, 2001) to show the existence of equilibria in other types of bargaining games. The games are all alternating offer games but assume different utility discounts such as a fixed loss utility function that is reduced by a constant amount each time and an interest rate utility which models the opportunity loss, among others. Depending on the type of utility discount and the type of game, some of these games can be proven to have a unique perfect equilibrium strategy that involves only one actions, others have multiple equilibria and thus would require some other coordination method, and yet others can be shown to be NP-complete in the time it takes to find the unique equilibrium. In general, however, the technique used to solve these bargaining games is the same.

1. Formalize bargaining as a Rubinstein's alternating offers games.
2. Generate extended-form game, or at least part of it.
3. Determine equilibrium strategy for this game.

In practice, step 3 is often computationally intractable, at least for the general case of the problem.

## 6.2 Monotonic Concession Protocol

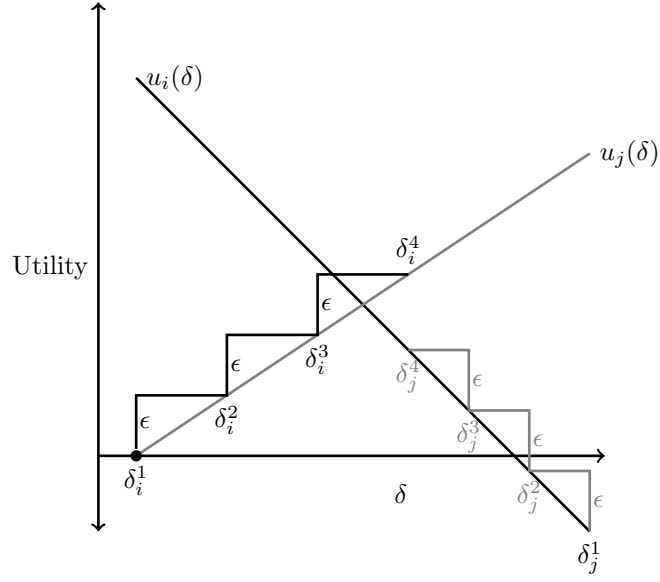
The simplest negotiation protocol following the Rubinstein's model of alternating offers is the **monotonic concession protocol** (Rosenschein and Zlotkin, 1994). In this protocol the agents agree to always make a counter offer that is slightly better for the other agent than its previous offer. Specifically, in monotonic concession the agents follow the algorithm shown in figure 6.6.

MONOTONIC CONCESSION  
PROTOCOL

Each agent starts by proposing the deal that is best for itself. The agent then received the proposal from the other agent. If the utility the agent receives from the other's proposal is bigger than the utility it gets from its own proposal, or vice-versa for the other agent, then it accepts it and negotiation ends. If no agreement was reached then the agent must propose a deal that is at least an increase of  $\epsilon$  in the other agent's utility. If neither agent makes a new proposal then there are no more messages sent and negotiations fail, so they agree on the no-deal deal  $\delta^-$ . The monotonic concession protocol makes it easy for the agents to verify that the other agent is also obeying the protocol. If  $i$  ever receives a proposal whose utility is less than a previous proposal then it knows that  $j$  is not following the protocol.

The protocol can be visualized as in figure 6.7. Here we see a simple example with linear utility functions where the agents reach an agreement ( $\delta^4$ ) after four time steps. Notice that the agreement reached is *not* the point at which the lines intersect. The monotonic concession protocol does is not guaranteed to arrive at any particular axiomatic solution concept. All that it guarantees is that it will stop.

Monotonic concession has several drawbacks. It can be very slow to converge. Convergence time is dictated by the number of possible deals, which is usually very



**Figure 6.7:** Monotonic concession protocol visualization. Both agents have linear utility functions over the set of deals. The superscripts indicate the time so  $\delta_i^1$  is  $i$ 's proposal at time 1. At time 4 we have that  $\delta_i^4 = \delta_j^4$  so  $u_i(\delta_j^4) = u_i(\delta_i^4)$  and the agents agree on this deal.

A common workaround is to assume its a zero-sum game and use the agent's own utility function.

large, and the value  $\epsilon$ . It is also impossible to implement this algorithm if the agent does not know the other agent's utility function. In practice, it is rare for an agent to know its opponent's utility function. Finally, the monotonic concession protocol has a tricky last step. Namely, the two agents could make simultaneous offers where each one ends up preferring the other agent's offer to the one it just sent. This is a problem as both of them now want to accept different offers. This can be solved by forcing the agents to take turns. But, if we did that then neither will want to go first as the agent that goes first will end up with a slightly worse deal.

### 6.2.1 The Zeuthen Strategy

The monotonic concession protocol we presented implements a simplistic strategy for the agent: always concede at least  $\epsilon$  to the other agent. But, if an agent knows that the other one will always concede then it might choose to not concede at all. Smarter agents will examine their opponent's behavior and concede in proportion to how much they are conceding.

This idea is formalized in the **Zeuthen strategy** for the monotonic concession protocol. We start by defining the willingness to risk a breakdown in negotiations to be

$$\text{risk}_i = \frac{u_i(\delta_i) - u_i(\delta_j)}{u_i(\delta_i)}. \quad (6.7)$$

The agent can then calculate the risks for both agents. The Zeuthen strategy tells us that the agent with the the smallest risk should concede just enough so that it does not have to concede again in the next time step. That is, the agent that has the least to lose by conceding should concede. More formally, we define the new protocol as shown in figure 6.8.

Figure 6.9 shows a graphical representation of the first step in a Zeuthen negotiation. After the initial proposal the agents calculate their risks to be

$$\text{risk}_i^0 = \frac{5 - (-1)}{5} = \frac{6}{5}$$

and

$$\text{risk}_j^0 = \frac{4 - 0}{4} = 1.$$

Sine  $j$  has a lower risk it must concede. The new deal must be such that  $j$  will not be forced to concede again. That is, it must insure that

$$\text{risk}_i = \frac{5 - (5 - \delta_j)}{5} < \frac{\frac{2}{3}\delta_j - 0}{\frac{2}{3}\delta_j} = \text{risk}_j$$

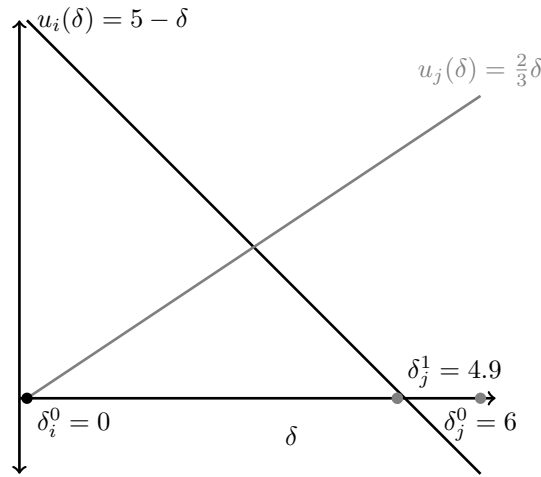
## ZEUTHEN-MONOTONIC-CONCESSION

```

1   $\delta_i \leftarrow \arg \max_{\delta} u_i(\delta)$ 
2  Propose  $\delta_i$ 
3  Receive  $\delta_j$  proposal
4  if  $u_i(\delta_j) \geq u_i(\delta_i)$ 
5      then Accept  $\delta_j$ 
6   $\text{risk}_i \leftarrow \frac{u_i(\delta_i) - u_i(\delta_j)}{u_i(\delta_i)}$ 
7   $\text{risk}_j \leftarrow \frac{u_j(\delta_j) - u_j(\delta_i)}{u_j(\delta_j)}$ 
8  if  $\text{risk}_i < \text{risk}_j$ 
9      then  $\delta_i \leftarrow \delta'_i$  such that  $\text{risk}_i(\delta'_i) > \text{risk}_j(\delta'_j)$ 
10     goto 2
11 goto 3

```

**Figure 6.8:** The monotonic concession protocol using the Zeuthen strategy.



**Figure 6.9:** Visualization of the Zeuthen strategy at work. The initial proposals are  $\delta_i^0 = 0$  and  $\delta_j^0 = 6$ .

which simplifies to  $\delta_j < 5$ . As such,  $j$  can pick any deal  $\delta$  less than 5. In the figure the agent chooses  $\delta_j^1 = 4.9$ .

The Zeuthen strategy is guaranteed to terminate and the agreement it reaches upon termination is guaranteed to be individually rational and Pareto optimal. Furthermore, it has been shown that two agents using the Zeuthen strategy will converge to a Nash bargaining solution.

**Theorem 6.2** (Zeuthen converges to Nash solution). *If both agents use the Zeuthen strategy they will converge to a Nash bargaining solution deal, that is, a deal that maximizes the product of the utilities (Harsanyi, 1965).*

The proof of the theorem shows how the maximum of both agents' utilities, that is  $\max u_i(\delta), u_j(\delta)$ , monotonically increases such that after the protocol reaches an agreement. As such, if there is any other agreement that has higher utility than the one they agreed upon then that agreement would have been proposed in the negotiation. Therefore, there can be no deal with higher utility for any agent than the one they agreed on.

A problem with the Zeuthen strategy might arise in the last step if the agents' risks are exactly the same. Specifically, if both agents realize that their risks are the same and that the next proposal that either of them makes will be accepted by their opponent then the agents will want to wait for the other agent to concede. This is an example of the game of chicken. As such, selfish agents should play the Nash equilibrium strategy which, in this case, is a mixed strategy. Since it is a mixed strategy it means that there remains a possibility for both of them to decide to wait for the other one, thereby breaking down the negotiations when a solution was possible.

## ONE-STEP-NEGOTIATION

- 1  $E \leftarrow \{\delta \mid \forall \delta', u_i(\delta)u_j(\delta) \geq u_i(\delta')u_j(\delta')\}$
- 2  $\delta_i \leftarrow \arg \max_{\delta \in E} u_i(\delta)$
- 3 Propose  $\delta_i$
- 4 Receive  $\delta_j$
- 5 **if**  $u_i(\delta_j)u_j(\delta_j) < u_i(\delta_i)u_j(\delta_i)$
- 6     **then** Report error,  $j$  is not following strategy.
- 7 Coordinate with  $j$  to choose randomly between  $\delta_i$  and  $\delta_j$ .

**Figure 6.10:** The one step negotiation protocol (Rosen-schein and Zlotkin, 1994).

The Zeuthen strategy is also attractive because, when combined with the mixed Nash strategy we described, it is in a Nash equilibrium. That is, if one agent declares that it will be using the Zeuthen strategy then the other agent has nothing to gain by using any other negotiation strategy. As such, an agent that publicly states that it will be using the Zeuthen strategy can expect that every agent it negotiates with will also use the Zeuthen strategy.

### 6.2.2 One-Step Protocol

Once we have a multi-step protocol that (usually) reaches a particular solution, we immediately think about the possibility of skipping all those intermediate steps and jumping right to the final agreement. Specifically, we could define a protocol that asks both agents to send a proposal. Each agent then has two proposals: the one it made and the one it received. The agents must then accept the proposal that maximizes the product of the agents' utilities. If there is a tie then they coordinate and choose one of them at random.

Given the rules established by this protocol, an agent's best strategy is to find of the deals that maximize the product of the utilities and then, from these deals, propose the one that is best for the agent. The agent's algorithm is shown in figure 6.10. The strategy implemented by this algorithm is a Nash equilibrium for the one-step protocol. That is, if one agent decides to implement this algorithm then the other agent's best strategy is to also implement the same algorithm.

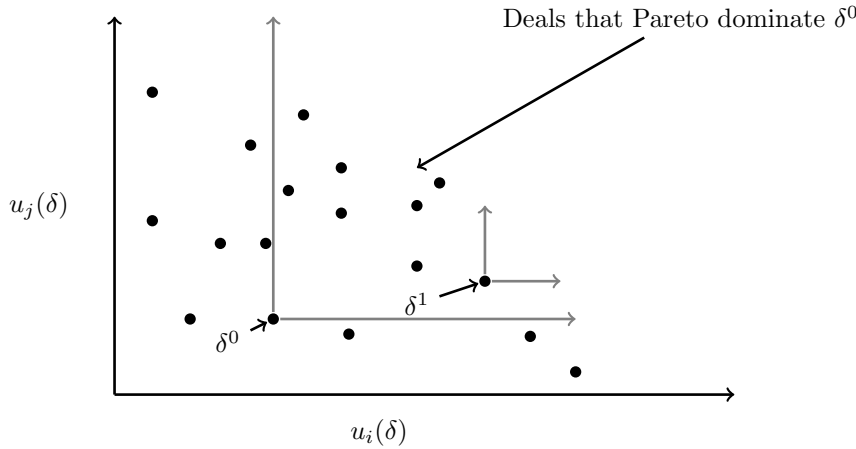
### 6.3 Negotiation as Distributed Search

If we take a step back and look at the bargaining problem, we realize that the problem of finding a chosen solution deal is a distributed search problem. That is, the system searches for the solution deal which you, as system designer, chose. A simple and natural way to carry out this search is by starting with some initial deal and at each time step moving to another deal. The process is repeated until there is no other deal which the agents can agree on.

For example, figure 6.11 shows an initial deal  $\delta^0$  as well as all the other possible deals. Any deal that is above and to the right of this deal, as delineated by the gray lines centered at  $\delta^0$ , dominates  $\delta^0$ . That means that any deal in this quadrant is preferred by all the agents. Thus, we can expect selfish agents to cheerfully accept any one of those deals. Our simple search process will continue moving in this up-and-right direction.

However, the search might get stuck in a local optima. For example, suppose that we go directly from  $\delta^0$  to  $\delta^1$  as shown in the figure. This new deal is not dominated by any other deal, thus our simple hill-climbing algorithm is now stuck at that deal. Notice also that this deal is not the social welfare maximizing deal nor is it the Nash bargaining deal. Thus, simple hill-climbing among selfish agents can lead us to sub-optimal solutions.

In real applications the problem is further compounded by the fact that it is often impossible to move directly from any one deal to any other deal because the number of possible deals is simply too large to consider or there are other external



**Figure 6.11:** Hill-climbing on the Pareto landscape. The quadrant to the top and right of  $\delta^0$  contains all the deals that dominate  $\delta^0$ , similarly for  $\delta^1$ . There are no deals that dominate  $\delta^1$  and yet it is not the social welfare or Nash bargaining solution.

limitations imposed on the agents. That is, most applications add an undirected graph to the bargaining problem. In this graph the nodes and edge between two nodes denotes the fact that it is possible to move between those two deals. Given such a graph, it is possible that the agents would find themselves in one deal that is pareto dominated by a lot of other deals but which are unreachable in one step because there is no edge between those deals and the current deal. Thus, we have even more local optima.

Researchers have tried to circumvent the problem of local optima in bargaining via a number of different methods, as we shall see in the next few sections.

#### 6.4 Ad-hoc Negotiation Strategies

Deployed multiagent negotiation systems, such as **ADEPT** (Faratin et al., 1998; Binmore and Vulkan, 1999), have implemented agents with ad-hoc negotiation strategies. In ADEPT, a handful of selected negotiation **tactics** were tested against each other to determine how they fared against each other. The basic negotiation model used was an alternating offers model with time discounts. The tactics the agents could use included: a *linear* tactic that concedes a fixed amount each time, a *conceder* tactic that concedes large amounts of utility on earlier time steps, an **impatient** tactic which conceded very little at first and requested a lot. We can think of each one of these tactics as stylized versions of the negotiation strategies people might use in a negotiation.

This ad-hoc approach is used by multiagent developers who need to build a negotiating multiagent system which achieves a good-enough solution and where all the agents are owned by the same party and, thus, do not need to behave truly selfishly. It allows us to use somewhat selfish that encapsulate some of the system's requirements and then use the negotiation protocol to aggregate these individual selfish interests into a global solution which, although not guaranteed to be the utilitarian solutions, can nonetheless be shown via experimentation to be a good-enough solutions. We must always remember that these systems are not to be opened up to outside competition. Given that we know the other agents' ad-hoc tactics, it is generally very easy to implement an agent that can defeat those tactics, usually at the cost of a much lower system utility. That is, one renegade agent can easily bring down the system.

#### 6.5 The Task Allocation Problem

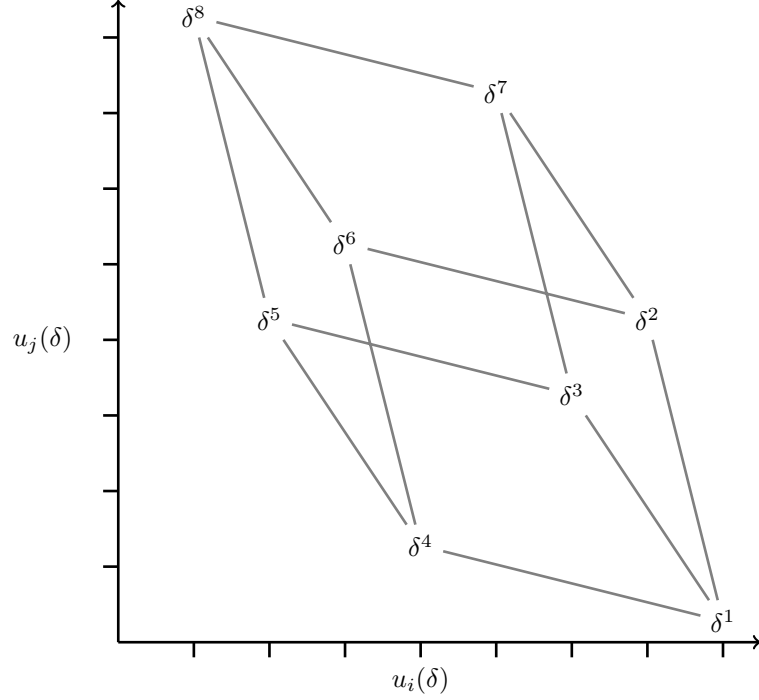
A common problem in multiagent systems is deciding how to re-allocate a set of tasks among a set of agents. This is known as the **task allocation problem**. In this problem there is a set of tasks  $T$ , a set of agents, and a cost function  $c_i : s \rightarrow \mathbb{R}$  which tells us the cost that agent  $i$  incurs in carrying out tasks  $s \subseteq T$ . In some

ADEPT

TACTICS

TASK ALLOCATION PROBLEM

| $\delta$   | $s_i(\delta)$       | $s_j(\delta)$       | $c_i(\delta)$ | $c_j(\delta)$ | $u_i(\delta) = 8 - c_i(\delta)$ | $u_j(\delta) = 8 - c_j(\delta)$ |
|------------|---------------------|---------------------|---------------|---------------|---------------------------------|---------------------------------|
| $\delta^1$ | $\emptyset$         | $\{t_1, t_2, t_3\}$ | 0             | 8             | 8                               | 0                               |
| $\delta^2$ | $\{t_1\}$           | $\{t_2, t_3\}$      | 1             | 4             | 7                               | 4                               |
| $\delta^3$ | $\{t_2\}$           | $\{t_1, t_3\}$      | 2             | 5             | 6                               | 3                               |
| $\delta^4$ | $\{t_3\}$           | $\{t_2, t_3\}$      | 4             | 7             | 4                               | 1                               |
| $\delta^5$ | $\{t_2, t_3\}$      | $\{t_1\}$           | 6             | 4             | 2                               | 4                               |
| $\delta^6$ | $\{t_1, t_3\}$      | $\{t_2\}$           | 5             | 3             | 3                               | 5                               |
| $\delta^7$ | $\{t_1, t_2\}$      | $\{t_3\}$           | 3             | 1             | 5                               | 7                               |
| $\delta^8$ | $\{t_1, t_2, t_3\}$ | $\emptyset$         | 7             | 0             | 1                               | 8                               |



**Figure 6.12:** An example task allocation problem is shown in the table and its graphical representation as a bargaining problem is shown in the graph. The edges on the graph connect deals that can be reached by moving a single task between the agents.  $s_i(\delta)$  is the set of tasks  $i$  has under deal  $\delta$ .

simplified versions of the problem we assume that all agents share the same cost function. The agents start out each one with a set of tasks such that all tasks are distributed. We can think of this initial allocation as  $\delta^-$  since, if negotiations break down then each agent is responsible for the tasks it was originally assigned. Similarly, every allocation of tasks to agents is a possible deal  $\delta$  where  $s_i(\delta)$  is the set of tasks allocated to  $i$  under deal  $\delta$ . The problem we then face is how to design a negotiation protocol such that the agents can negotiate task re-allocations and arrive at a final re-allocation of the tasks that is one of the axiomatic solution concepts, such as the utilitarian deal.

#### POSTMAN PROBLEM

An example of this type of problem is the **postman problem** where a set of postmen are placed around the city each with letters that must be delivered. Clearly, a postman prefers to have all his letter with delivery addresses that are very close to each other so as to minimize his delivery costs. The postmen can negotiate with each other in order to make this happen. In this example the tasks are the letters to be delivered and the cost function is the distance traveled to deliver a set of letters. Different cost functions arise when the postmen have different final destinations, for example, they go to their respective homes after delivering all letter, or if they prefer certain areas of town, etc.

Once we are given task allocation problem we must then decide how the agents will exchange tasks. The simplest method is for agents to pair up and exchange a single task. This means that not all deals are directly accessible from other deals. We can represent this constraint graphically by drawing an edge between every pair



of deals that are reachable from one another by the exchange of a single task. An example of such a graph is shown in figure 6.12. The table in this figure shows the costs that each one of the agents incurs for carrying out various subsets of tasks. We let the utility be 8 minus the cost in order to have positive numbers. Generally, utility is simply the negative of the cost. The graph shows all the possible deals and the edges connect deals that can be reached from one another by moving a single task between agents.

Note that the example in figure 6.12 has a Pareto frontier that consists of deals  $\delta^1$ ,  $\delta^2$ ,  $\delta^7$  and  $\delta^8$ . Thus, if we were to apply our simple hill-climbing search strategy where we always move to a neighboring dominant deal we are guaranteed to always end up stuck at one of these four deals. In terms of the task allocation problem this greedy search corresponds to the agents exchanging a task only when both of them are better off from the exchange. In other words, if your solution to a task allocation problem is to have the agents exchange a single task at a time when both agents have a utility gain from the transaction then, eventually, the system will converge to an allocation that is on the Pareto frontier. Of course, this means that, for example, you might end up at deal  $\delta^8$  from figure 6.12 which is perhaps not so desirable. A more desirable deal is  $\delta^7$  which is the utilitarian solution for this problem.

### 6.5.1 Payments

One possible way to allow the agents to find deals that are closer to the utilitarian deal is by allowing them to use **monetary payments**. For example, agent  $i$  might convince  $j$  to take a deal that gives  $j$  less utility if  $i$  can sweeten the deal by giving  $j$  some money to take that deal. This basic idea was implemented in the **contract net protocol** (Smith, 1981; Davis and Smith, 1983). In contract net each agent takes the roles of either a contractor or contractee. The contractor has a task that it wants someone else to perform. The contractor is also willing to pay to get that task done. In the protocol, the contractor first announces that it has a task available by broadcasting a call for bids. All agents receive this call for bids and, if they want, reply to the contractor telling him how much they charge for performing the task. The contractor then chooses one of these bids, assigns the task to the appropriate agent and pays him the requested amount.

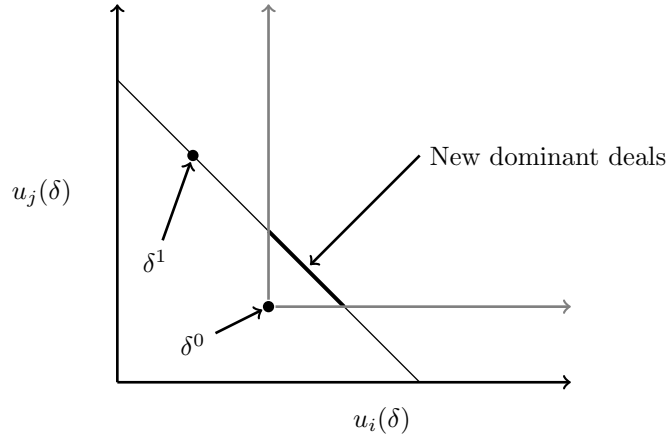
For example, given the current task allocation  $\delta$  agent  $i$  might find that one of his current tasks  $t \in s_i(\delta)$  costs him a lot. That is,  $c_i(s_i(\delta)) - c_i(s_i(\delta) - t)$  is a large number. In this case  $i$  will be willing to pay up to  $c_i(s_i(\delta)) - c_i(s_i(\delta) - t)$  in order to have some other agent perform  $t$ —the agent will pay up to the utility he will gain from losing the task, any more than that does not make sense as the agent can simply do the task himself. Similarly, any other agent  $j$  will be willing to perform  $t$  as long as it gets paid an amount that is at least equal to any cost increase he will endure by performing the task, that is, the payment must be at least  $c_j(s_j(\delta)) - c_j(s_j(\delta) + t)$ . Note that in the general case some of these numbers could be negative, for example, an agent could actually have lower costs from performing an extra task (however, these sub-additive cost functions are rare in practice).

The use of monetary payments has the effect of turning one deal into an infinite number of deals: the original deal plus the original deal and the infinite number of payments, from  $-\infty$  to  $\infty$  that can be made between the agents. Figure 6.13 shows a graphical representation of this process. Here we see deal  $\delta^1$  transformed into the set of deals represented by the line going thru  $\delta^1$ . The figure also shows us how this transformation creates new deals that dominate  $\delta^0$ . Thus, if the system was in  $\delta^0$  and we were doing hill-climbing then we would be stuck there as  $\delta^1$  does not dominate  $\delta^0$ . But, if we used the contract net then agent  $j$  could make a payment to  $i$ , along with the task transfer, which would give  $i$  a higher total utility. This new task allocation plus payment is represented by one of the deals in the thick area of the line. Thus, contract net allows us to reach a task allocation that is the utilitarian solution where. Unfortunately, this is not always the case, it depends on

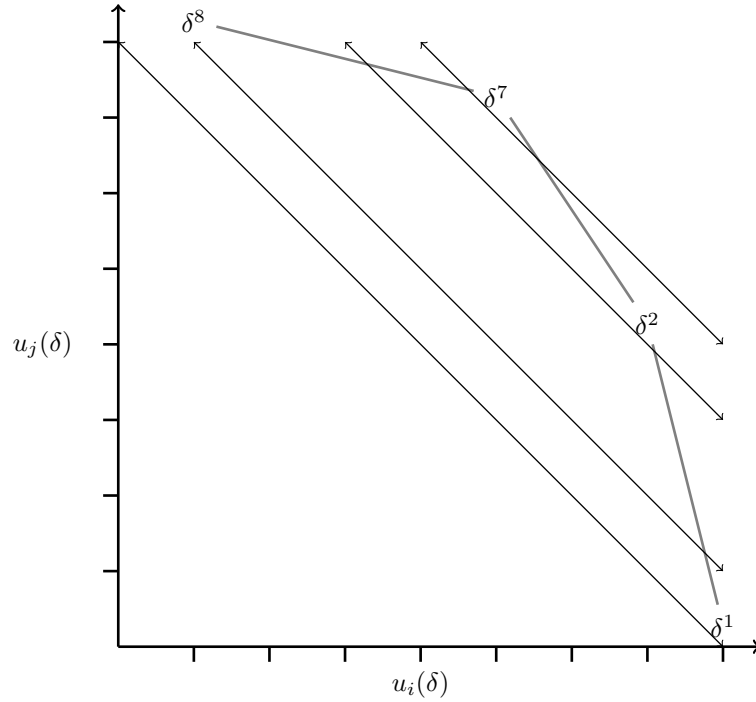
MONETARY PAYMENTS

CONTRACT NET PROTOCOL

**Figure 6.13:** Deal  $\delta^1$  is turned into an infinite number of deals, represented by the line going thru  $\delta^1$ , by the use of payments. Some of those new deals Pareto dominate  $\delta^0$ , as shown by the thicker part of the line.



**Figure 6.14:** Monetary payments in with an additive cost function. The deals on the Pareto frontier expand into an infinite set of deals represented by lines intersecting the original deals. The utilitarian deal  $\delta^7$  is the only one not dominated by any other deal.



the particular characteristics of the cost function.

One type of cost functions that have been found to be easy to search over are additive cost functions where the cost of doing a set of tasks is always equal to the sum of the costs of doing each task individually.

**Definition 6.6.** A function  $c(s)$  is an **additive cost function** if for all  $s \subseteq T$  it is true that

$$c(s) = \sum_{t \in s} c(t).$$

In these scenarios it has been shown that the contract net protocol, or any other protocol that uses payments and always moves to dominant deals, will eventually converge to the utilitarian social welfare solution.

**Theorem 6.3.** In a task allocation problem with an additive cost function where we only allow exchange of one task at a time, any protocol that allows payments and always moves to dominant deals will eventually converge to the utilitarian solution (Endriss et al., 2006).

For example, in figure 6.12 we saw how simple hill climbing would lead us to converge into one of the deals on the Pareto frontier but not necessarily the utilitar-

| $\delta$   | $s_i(\delta)$ | $s_j(\delta)$ | $u_i(\delta)$ | $u_j(\delta)$ |
|------------|---------------|---------------|---------------|---------------|
| $\delta^1$ | $\emptyset$   | $\{t_1\}$     | 1             | 3             |
| $\delta^2$ | $\{t_1\}$     | $\emptyset$   | 2             | 1             |

| $\delta$   | $s_i(\delta)$  | $s_j(\delta)$  | $u_i(\delta)$ | $u_j(\delta)$ |
|------------|----------------|----------------|---------------|---------------|
| $\delta^1$ | $\emptyset$    | $\{t_1, t_2\}$ | 1             | 5             |
| $\delta^2$ | $\{t_1\}$      | $\{t_2\}$      | 2             | 3             |
| $\delta^3$ | $\{t_2\}$      | $\{t_1\}$      | 2             | 3             |
| $\delta^4$ | $\{t_1, t_2\}$ | $\emptyset$    | 8             | 1             |

**Figure 6.15:** Task creation example. The top table shows the original utility values, where  $\delta^1$  is the Nash bargaining solution. The bottom table shows the values after agent  $i$  creates a phony task  $t_2$ . Here,  $\delta^4$  is the Nash bargaining solution.

ian solution ( $\delta^7$ ). Figure 6.14 reproduces this problem but this time we only show the deals on the Pareto frontier along with all the other deals those deals generate when using payments. The new deals are represented by the parallel lines which intersect each of the original deals. It is easy to confirm that every non-utilitarian dominant deal (that is,  $\delta^1$ ,  $\delta^2$  and  $\delta^8$ ) is dominated by a new deal. Thus, if we continue moving to successively dominant deals we are guaranteed to end up at  $\delta^7$  which is the utilitarian deal, and is the only deal not dominated by any other deals. The proof of Theorem 6.3 follows the same line of thought: it shows that any deal (with payments) that is not dominated by some other deal must be the utilitarian social welfare solution.

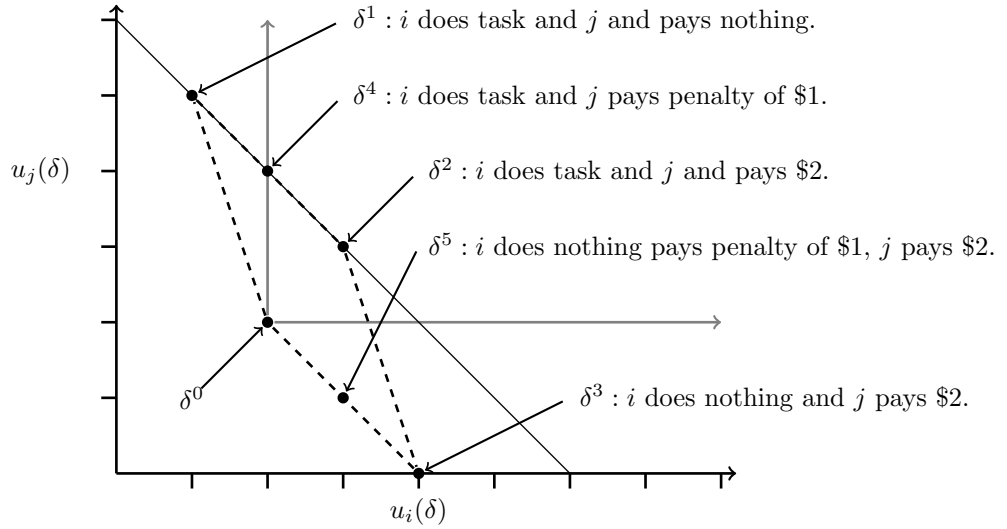
If we allow arbitrary cost functions then there is very little we can say about what solution a hill-climbing protocol will reach, except for one scenario. In the case where every pair of deals is connected by an edge—we can get to any deal from every other deal in one step, imagine the graph in figure 6.12 fully connected—then using payments guarantees we will converge to the utilitarian social welfare (Sandholm, 1997; Sandholm, 1998; Andersson and Sandholm, 1998). We can easily verify this by looking again at figure 6.14. Here we can see how each deal is converted into a line of slope  $-1$  and the utilitarian deal will always be the one with the line that is furthest to the northeast. Thus, any protocol that successively moves via dominant deals (hill-climbing towards the northeast) will always arrive at the utilitarian solution because, as we assumed, every deal is always reachable from every other deal. We can directly connect every deal to every other deal by using a very flexible contracting language, such as **OCSM contracts** which allows all possible task transfer combinations.

OCSM CONTRACTS

### 6.5.2 Lying About Tasks

In some cases it might be worth it for an agent to not tell others about some of the tasks it has to perform, or to make up tasks and tell others that it has perform them, or to make up these tasks and, if someone else offers to perform the phony tasks then actually create the new tasks. All these methods of cheating result in the modification of the set of existing deals. For example, if an agent creates a new task then we now have to consider all the possible ways in which the new bigger set of tasks can be distributed among the agents. If an agent hides a task from the agents then the system has fewer possible allocations to consider. We can then ask the question, when is it in an agent's best interest to lie?

Such is the approach taken in *Rules of Encounter* (Rosenschein and Zlotkin, 1994). In it, the authors assume that the agents will use some protocol which will arrive at the Nash bargaining solution, presumably by either using alternating offers with Zeuthen strategy or the ONE-STEP-NEGOTIATION protocol from figure 6.10. Once an agent knows that the final agreement deal will be the Nash bargaining solution then all it has to do is check each possible lie to see if the solution reached when it tells that lie gives him a higher utility. Fortunately for the agent, and unfortunately for us, it has been shown that such lies will do generally exist and will benefit the agent.



**Figure 6.16:** Levelled commitment contracts illustration. The agents start at  $\delta^0$  where  $j$  is performing the one available task and  $i$  is idle.  $\delta^1$  and  $\delta^3$  are the two de-commitment contracts. When we add a de-commitment penalty of 1 then these two become  $\delta^4$  and  $\delta^5$  respectively.

Figure 6.15 shows an example where an agent has an incentive to create a phony tasks. The table on the left shows the initial utility values for a simple game with only one task. In this game the Nash bargaining solution is  $\delta^1$  which leaves agent  $i$  with a utility of 1. Noticing this,  $i$  decides to create a phony task  $t_2$  whose utility values are given on the table to the right. In this new game the Nash bargaining solution is  $\delta^4$  which gives both tasks to agent  $i$ . Thus, by creating this task  $i$  was able to get a utility of 2 instead of the original 1. Since  $t_2$  was assigned to  $i$ , the agent does not have to worry about perform this task nor does he have to worry about other agents trying to perform a task he made up.

### 6.5.3 Contracts

In our discussion of payments we have thus far assumed that if one agent says that he will pay another agent to perform some tasks then both of them will abide by that contract. The tasks will be performed and the money will be paid. Of course, things are not so simple for real applications. Specifically, in a dynamic environment an agent that is given a set of tasks to perform might find that right after he agrees to perform the tasks a new deal appears which will give him much higher utility. In this case the agent will be tempted to de-commit on his current contract and establish a new one. Similarly, it is possible that right after an agent agrees to pay a certain amount to get some tasks done that a new agent appears which is willing to perform those tasks for less. Again, the agent will be tempted to de-commit on his payment.

For example, in figure 6.16 the agents start out with  $\delta^0$ . Lets say that in this scenario there is only one task to perform and, under  $\delta^0$ , agent  $j$  is going to perform it. Both agents get a utility of 2 under this initial deal. However,  $j$  notices that if he can give it to  $i$  then they will be at  $\delta^1$  which gives  $j$  a utility of 4 and  $i$  a utility of 1. Thus,  $j$  reasons that he could pay  $i$  the amount of \$2 to perform the task. This new deal is  $\delta^2$  which gives both agents a utility of 3.  $\delta^2$  dominates  $\delta^0$  and will thus be accepted by both. The problem is the risks involved with this deal. If  $j$  fails to pay  $i$  then we are at  $\delta^1$  where  $i$  loses a utility of 1. Similarly, if  $j$  pays but  $i$  fails to perform the task then  $j$  has to do them and we end up at  $\delta^3$  where  $j$  has lost 2 in utility.

The simple way to fix this problem is to enforce all contracts. If the agents agree to a deal then each one must deliver on that promise. However, there are cases when this might be impossible or undesirable. Specifically, in dynamic environments where new opportunities can appear at any time allowing agents to de-commit might increase social welfare. In this cases we might wish to use more sophisticated contracts.

One solution is to use **leveled commitment contracts** (Sandholm and Lesser, 2002; Sandholm and Zhou, 2002) which have the added stipulation that if an agent breaks the contract then it must pay a penalty. For example, we could add a penalty of \$1 to the contract from figure 6.16. In this case, instead of the de-commitments leading us to  $\delta^1$  and  $\delta^3$ , they will lead us to  $\delta^4$  and  $\delta^5$ . Note how these new deals are closer to  $\delta^0$  thus reducing the risks of loss for both agents. Specifically, if  $j$  were to de-commit and fail to pay the \$2, he must still pay the \$1 penalty to  $i$  so  $i$ 's final utility is 2 even if  $j$  de-commits, the same as  $u_i(\delta^0)$ . This means that there is no risk for  $i$  to take this contract. Similarly, if  $i$  de-commits and fails to do the task then we end up at  $\delta^4$  because  $i$  pays a penalty of \$1 to  $j$  who must perform the task. In this case,  $j$ 's utility is 1 which is greater than  $u_j(\delta^3) = 0$  so  $j$  has reduced his risk. Such risk reduction means that in dynamic situations where the agents are constantly trying to tradeoff current income against future possibilities—deciding whether or not to wait for a better deal to come along—the agents will be able to accept more contracts and, hopefully, the system will perform better.

Note that the de-commitment decision itself is not obvious, even when the agent receives a better offer from somewhere else. For example, say that after agreeing to  $\delta^2$  agent  $i$  receives another offer that will give him a utility of 10 but to do that task he must de-commit on the task from  $j$ . Clearly  $i$  wants to de-commit but, a strategic agent  $i$  would also realize that there is a chance that  $j$  will de-commit first. If  $j$  de-commits then  $i$  gets the penalty payment from  $j$  and still gets to perform the other task. The agents thus face a version of the game of chicken (figure 3.5) with uncertainty about the other agent's payoff. Namely,  $i$  does not know if  $j$  wants to de-commit or not. If  $i$  has some probabilistic data on  $j$ 's offers then these could be used to generate an extended-form game. Agent  $i$  can then find the Nash equilibrium of this game and play that strategy.

Another solution is to extend the negotiation protocol to include tentative contracts. For example, (Aknine et al., 2004) extend the contract net protocol to include pre-bid and pre-assignment steps. This allows agents to make tentative commitments which they can make binding at a later time when they are more certain that no new offers will arrive from other agents.

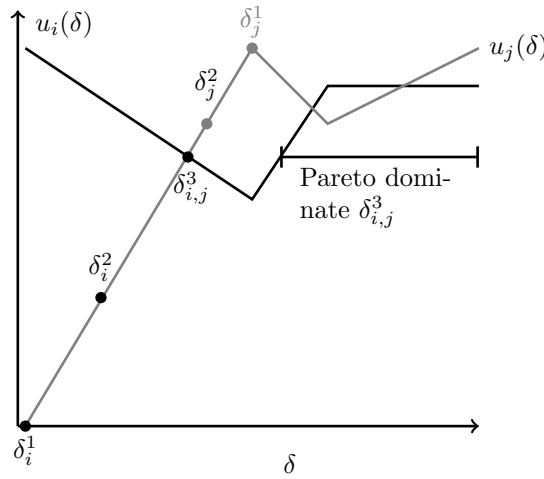
## 6.6 Complex Deals

Thus far we have considered deals to be atomic units that cannot be broken down into smaller pieces. In contrast, real world deals are known to be composed of many different items such as price, warranty, delivery time, color, etc. For example, two agents negotiating over the purchase of a car will need to agree on the price to be paid, the color of the car, the number of years in the warranty, the value of the trade-in car, the type of financing and interest rate provided, and many other possible parameters. These dimensions could be used explicitly during the negotiation when, for example, an agent claims that it cannot pay more than \$5,000 for the car or that it really likes red cars. We describe these complex preference languages in Section 6.7. These dimensions inevitably lead to an explosion in the space of possible deals.

More formally we define a **multi-dimensional deal** as composed of a set of variables  $x_1, x_2, \dots, x_n$  with domains  $D_1, D_2, \dots, D_n$ , respectively. For example, one variable could correspond to price and its domain could be all integer numbers in the range 0 to 100. We can also re-define the agent's utility functions so that they explicitly deal with each variable. For example, instead of an opaque  $u_i(\delta)$ , we could have a more expressive  $u_i(\delta) = c_1 u_i^1(x_1) + c_2 u_i^2(x_2) + \dots + c_n u_i^n(x_n)$  or some other combination of the individual variables. The negotiation problem remains that of finding a deal that corresponds to a chosen solution concept, such as the utilitarian deal or the Nash bargaining deal.

The astute reader will notice that this is nearly the exact same problem as the distributed constraint optimization problem from Chapter 2. In fact, they are the same problem but negotiation and constraint optimization assume different

**Figure 6.17:** Example of convergence to non-Pareto deal. The agents converge to deal  $\delta_{i,j}^3$  which is Pareto-dominated by all the deals indicated on the right.



data distribution requirements. In distributed constraint optimization it is assumed that each agent owns one or more variables whose value it can change and the agents generally only care about the value of their variable and a few other ones. In negotiation the variables are now owned by any agent and the agents generally have preferences over all the variables. Finally, in constraint optimization there is a clear solution concept—minimize constraint valuations—while in negotiation each agent is assumed to have a utility function and there is no one obviously better solution concept.

When the problem uses multi-dimensional deals we often find that the total number of deals is extremely large—the number of possible deals grows exponentially with the number of variables. Thus, it becomes even more likely that a hill-climbing negotiation algorithm will get stuck in a local optima. For example, figure 6.17 shows how we could converge to a non-Pareto deal via protocol similar to monotonic concession. The agents start out by proposing deals that are the best possible for them and then concede to the other agent. However, the agents only consider deals that differ from the last proposed deal by changing only a few variables—they are “near” the last proposed deal if we define the distance to be the number of variables that have changed and by how much. Since they only consider a subset of the deals they might end up ignoring better deals. Specifically, the agents in the figure ignore the deals on the right part of the chart. As such, they end up converging on a deal which is Pareto-dominated by many other possible deals.

The reason for ignoring the deals is often the simple fact that the space of possible deals is so large that there is not enough time for the agent to sample it well enough to find other deals which are better than the ones it has proposed. In these cases agents choose a few attributes and change their values monotonically. For example, when negotiating a complex car deal you might wish to keep all other attributes at fixed values and negotiate only over price. In that case you might end up at a non-Pareto deal if your chosen values for the other attributes are not the right ones, such as the case where the dealer would be willing to accept a much lower price for a car with a one-year warranty rather than the three-year warranty you are asking for.

We can formalize this strategy by having the agents agree on one dimension at a time. For example, we could declare that the agents first agree on the price at which the car will be sold, then go on to haggle on the warranty, then on the color, and so on until all the dimensions are covered. The order in which these negotiations are carried out is called an **agenda**. As you might expect, ordering the negotiation in this way can sometimes lead to sub-optimal outcomes but does make the negotiation problem tractable for a lot more cases. For example, (Fatima et al., 2004) compare the results of doing agenda-based negotiations versus full negotiation, using specific negotiation strategies, and show which combination of strategies leads to a social

The Hamming distance, a similar concept from information theory, between two binary strings is the number of positions which are occupied by different values. Thus the distance between 101 and 110 is 2.

ANNEALING-MEDIATOR

```

1  Generate random deal  $\delta$ .
2   $\delta_{\text{accepted}} \leftarrow \delta$ 
3  Present  $\delta$  to agents.
4  if both accept
5      then  $\delta_{\text{accepted}} \leftarrow \delta$ 
6           $\delta \leftarrow \text{mutate}(\delta)$ 
7          Goto 2
8  if one or more reject
9      then  $\delta \leftarrow \text{mutate}(\delta_{\text{accepted}})$ 
10 goto 2

```

**Figure 6.18:** Procedure used by annealing mediator.

welfare solution.

### 6.6.1 Annealing Over Complex Deals

A common solution to the problem of searching for an optimal solution over a very large space of possible answers is to use an **annealing** method. Simulated annealing algorithms start out with a randomly chosen deal which becomes the best deal. New possible deals are generated by mutating this deal and are accepted as the new best deal if they are better than the current best deal or, with a certain probability, they are accepted even if they are worse than the current best. The probability of accepting a worse deal and the severity of the mutations both decrease with time. In this way the algorithm is guaranteed to converge to a locally optimal solution. In practice it has been found that this solution is often also the global optimum.

ANNEALING

Simulated annealing was used in (Klein et al., 2003) to implement several simple negotiation protocols. The protocol they present uses a mediator agent instead of having agents negotiate with each other. At each step the mediator presents a deal to both agents. The agents can either accept the deal or reject it. If both of them accept the deal the mediator mutates the deal slightly and offers the new deal to both agents, who can once again either accept it or reject it. If one or more of the agents rejected the proposed deal then a mutation of the most recently accepted deal is used instead. The protocol implemented by the mediator is shown in figure 6.18.

They then implemented two types of agents.

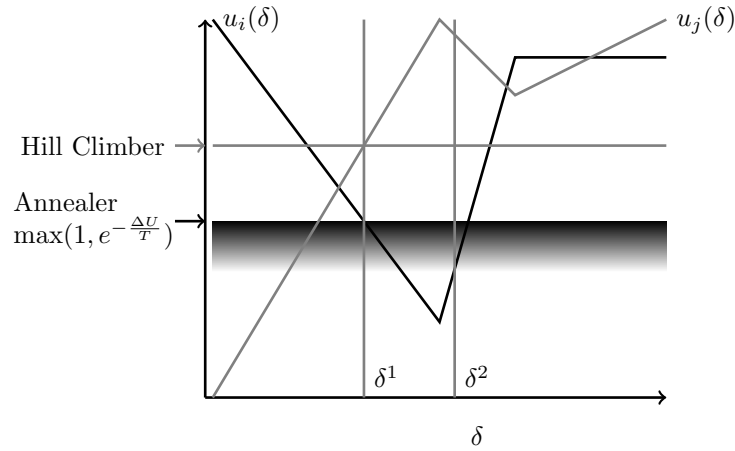
**Hill Climber** Accepts a deal only if it has utility higher than its reservation price  $u_i(\delta^-)$  and higher than that of the last deal it accepted. That is, it monotonically increases its reservation price as it accepts deals with higher utility.

**Annealer** Use a simulated annealing algorithm. That is, they maintain a temperature  $T$  and accept deals worse than the last accepted deal with probability  $\max(1, e^{-\frac{\Delta U}{T}})$ , where  $\Delta U$  is the utility change between the contracts.

Annealer agents, along with the annealing mediator, effectively implement a simulated annealing search over the set of possible deals. The hill climbing agents implement a hill climbing search over the set of possible deals.

For example, in figure 6.19 agent  $i$  is a hill climber and  $j$  is an annealer. After the mediator presents  $\delta_1$  both of them accept since its utility is higher than their reservation prices. The hill climber sets a new reservation price below which he will not accept any deal. The annealer sets up a probability of accepting deals that are below his new reservation price. When  $\delta_2$  appears,  $i$  will accept it because it is above its reservation price and  $j$  might also accept it, but with a small probability.

This type of annealing algorithm is a good example of how we can take a standard search technique like annealing and apply to a distributed negotiation problem. Annealing works because there is some randomness in the sequence of deals that



**Figure 6.19:** Two steps in the annealing algorithm with one hill climber  $j$  and one annealing agent  $i$ . After the mediator presents  $\delta^1$  both agents set up their new reservation prices shown by the line, for  $j$ , and the fuzzy bar for  $i$ .

are explored. The annealing negotiation protocol places this randomness within a negotiation. Unfortunately, generating proposal deals purely at random means ignoring any knowledge that the agents have about the shape of their utility function. Also, an annealing agent is acting irrationally when it accepts a deal that is worse for it than a previous deal. We would need some way to justify why this agent occasionally acts irrationally.

### 6.7 Argumentation-Based Negotiation

We can further relax the idea of an agenda, where agents negotiate sequentially over each dimension of the deal, and instead define a more complex negotiation language which agents can use for negotiation. That is, thus far we have only considered **1-sided proposal** where a deal is proposed and it is either accepted or rejected. In **argument-based protocols** the agents use a more sophisticated language for communications. There are currently no standard languages for argumentation although there is a lot of work being done on preference languages, as we will see in Chapter 7. Still, we can categorize the various types of utterances that an argumentation language might support (Jennings et al., 2001).

Specifically, an agent might be able to **critique** the proposal submitted by the other agent. Critiques provide the agents with information about others' utility functions, specifically with respect to the previous proposal. This information can be used to rule out some deals from consideration. For example,

$i$ : I propose that you provide me with  $x_1 = 10$  under conditions  $x_2 < 20$  and delivery  $x_3 < 20061025$ .

$j$ : I am happy with the price of  $x_2 < 20$  but the delivery date  $x_3$  is too late.

$i$ : I propose that I will provide you with service  $x_1 = 9$  if you provide me with  $x_1 = 10$ .

$j$ : I don't want  $x_1 = 9$ .

An agent might come back with a **counter-proposal** which are new deals, just like in the alternating offers models, but which are generally assumed to be related to the last offer. For example,

$i$ : I propose that you provide me with service  $x_1 = 10$  under conditions  $x_2 < 20$  and delivery  $x_3 < 20061025$ .

$j$ : I propose that you provide me with service  $x_1 = 10$  under conditions  $x_2 < 30$  and delivery  $x_3 \geq 20061025$ .

1-SIDED PROPOSAL  
ARGUMENT-BASED  
PROTOCOLS

CRITIQUE

COUNTER-PROPOSAL



An agent might be able to **justify** his negotiation stance with other statements. Statements it hopes will convince the other agent to accept his proposal. These statements are just ways of giving more knowledge to the other agent in the hopes that it will use that knowledge to eliminate certain deals from consideration. For example,

JUSTIFY

*i*: My warehouses is being renovated and it will be impossible to deliver anything before the end of the month, that is  $x_3 > 20061031$  .

An agent might try to **persuade** the other agent to change its negotiation stance. Persuasion is just another way of giving more knowledge tot the other agent. For example,

PERSUADE

*i*: Service  $x_1 = 9$  is much better than you think, look at this report.

Finally, an agent might also employ **threats**, **rewards**, and **appeals** in order to convince the others to accept his proposal. These techniques help agents build better models of the others' utility functions, eliminate sets of deals from consideration, and change the agents utility functions in order to make them better reflect the reality of the situation.

THREATS  
REWARDS  
APPEALS

Argument-based negotiation closely matches human negotiation. Unfortunately, that also means that it is very hard to build agents that can understand these complex negotiation languages. A common approach is to build the agent using Prolog or some other logic-based language. The agent can then keep a database of the messages (facts) sent by the opponent and try to infer the set of possible contracts. Even when exploiting the inference powers of a logic-based programming language the problem of implementing correct argument-based negotiators is still very hard. No one has been able to implement a general argument-based negotiator. Even when limiting the problem to a specific domain, there are very few examples of successful programs that do argument-based negotiation. Still, there is a growing research community developing new preference description languages to be used by auctions, as we will see in Chapter 7. These languages could just as easily be used by an argumentation-based agent as by a centralized auctioneer.

## 6.8 Negotiation Networks

It is possible that an agent might be involved in **concurrent negotiations** with several other agents where the resulting deals might not be compatible with each other. For example, you might be negotiating with several car dealers at the same time, as well as negotiating a car loan with several lenders and several insurance agencies for your car insurance. Once you make a purchasing deal with a specific dealer then you can stop negotiating with the other dealers. Furthermore, if the deal you made includes a loan agreement you can also stop negotiating with the lenders. Thus, as you negotiate with the various parties their last offers will tend to have an effect on how you negotiate with the others, for example, if a lender offers you a very low interest rate then you might be able to afford a more expensive car and you will not be swayed by the dealer's offer that includes a higher interest rate loan.

CONCURRENT  
NEGOTIATIONS

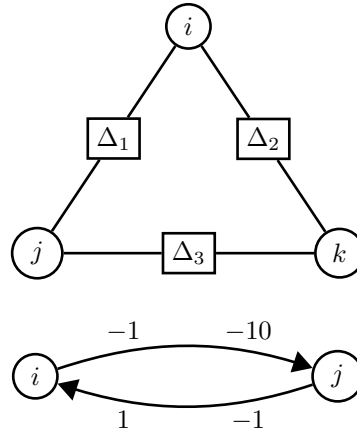
More formally, we can define a negotiation network problem as one were a set of agents negotiate over a set of deals such that all agents end up agreeing to a set of deals that are compatible with each other.

**Definition 6.7.** A *negotiation network* problem involves a set of agents  $A$  and set of sets of deals. Each set of deals  $\Delta_i$  involves only a subset of agents  $\Delta_i^a \subseteq A$  and always includes the no-deal deal  $\delta^-$ . A solution  $\vec{\delta}$  to the problem is a set of deals, one from each  $\Delta_i$  set, such that all the deals that each agent is involved in are compatible with each other. We thus define

NEGOTIATION NETWORK

$$c_i(\delta, \delta') = \begin{cases} 1 & \text{if } \delta \text{ and } \delta' \text{ are compatible} \\ 0 & \text{otherwise} \end{cases}$$

**Figure 6.20:** Graphical representation of a negotiation network with agents  $i$ ,  $j$ , and  $k$ , where  $\Delta_1$  is a set of deals that  $i$  and  $j$  can agree upon.



**Figure 6.21:** The coercion network.

Figure 6.20 shows a graphical representation of a negotiation network problem with three agents:  $i$ ,  $j$  and  $k$ , and where  $i$  and  $j$  can enter into any of the deals in  $\Delta_1$ , similarly  $i$  and  $j$  can enter into a deal from  $\Delta_2$  and  $j$  and  $k$  can enter into a deal from  $\Delta_3$ . We also need to define the boolean functions  $c_i$ ,  $c_j$ , and  $c_k$  which tell us which pair of deals are compatible.

In the negotiation networks the standard negotiation problem is further exacerbated by the fact that each agent must maintain simultaneous negotiations with several other agents and these negotiations impact each other. The approaches at solving these type of problem has thus far consisted of using ad-hoc negotiation strategies and running tests to determine how these fare against each other (Nguyen and Jennings, 2004; Zhang et al., 2005a; Zhang et al., 2005b).

### 6.8.1 Network Exchange Theory

Another way to approach the problem of negotiation networks is to look at what humans do and try to imitate that, the **descriptive approach**. Luckily for us, researchers in Sociology have developed and studied a model which they call Network Exchange Theory (NET) (Willer, 1999) that is very similar to the negotiation networks problem. In this model humans are represented by nodes in a graph. The annotated edges between nodes represent the possibility of a negotiation between those two humans. Sociology researchers have placed actual humans in these networks and tested them to see what deals they arrive at. They then found equations that can be used to predict these deals from the initial structure of the problem.

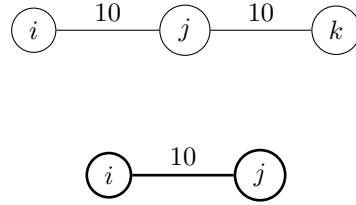
Figure 6.21 shows a simple NET network. The nodes represent agents  $i$  and  $j$ . The edges are directional and represent interaction possibilities from the source agent to the destination agent where only the source agent can decide whether to enact the interaction. For example, in the graph above  $i$  could decide to enact its edge which would give  $i$  a utility of  $-1$  and would give  $j$  a utility of  $-10$ . Similarly  $j$  could decide to enact its edge which would give it a utility of  $-1$  and give  $i$  a utility of  $1$ . This particular network is known as a coercion scenario because  $i$  can threaten  $j$  to give him the  $1$  otherwise  $i$  will punish  $j$  by giving it  $-10$ . Even though it is not rational for  $i$  to want go punish  $j$  (for it is punishing itself at the same time) the threat works with people. Another type of edge used is the resource pool edge where an amount of utility is to be divided among the agents but the agents must decide how much each one gets. A sample is shown in figure 6.23. This network shows two agents,  $i$  and  $j$ , who are negotiating over how to divide 10 dollars.

NET can also represent various deal compatibility requirements. One such example is exclusive connections where one agent can exchange with any one of a number of other agents but exchange with one of them precludes exchange with any of the other ones. This is typically simply represented by just adding more nodes and edges to the graph, as shown in figure 6.22.

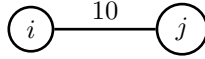
NET tells us that each agent has a **resistance** to each particular payment  $p$

DESCRIPTIVE APPROACH

RESISTANCE



**Figure 6.22:** A network with three agents. Agent  $j$  can only split 10 dollars with either  $i$  or  $k$ , but not both.



**Figure 6.23:** A sample exchange network with 10 units to be distributed among two agents.

given by the agent's resistance equation.  $i$ 's resistance to payment  $p$  is given by

$$r_i = \frac{p_i^{\max} - p_i}{p_i - p_i^{\text{con}}} \quad (6.8)$$

where

$$p_i^{\max} = \text{Maximum } i \text{ could get (10)}$$

and

$$p_i^{\text{con}} = \text{Conflict deal, 0.}$$

$j$ 's resistance is similarly defined. Furthermore, since in this case we know there are 10 dollars to be split then we know that  $p_i + p_j = 10$ .

The resistance equation is meant to capture the person's resistance to agreeing to a deal at any particular price. The higher the resistance the less willing the person is to agree to the deal. Note how this equation is not linear, as might be expected if people were rational, but has an exponential shape. This shape tells us a lot about our irrational behavior.

NET tells us that exchange happens at the **equi-resistance point**

$$r_i = \frac{p_i^{\max} - p_i}{p_i - p_i^{\text{con}}} = \frac{p_j^{\max} - p_j}{p_j - p_j^{\text{con}}} = r_j. \quad (6.9)$$

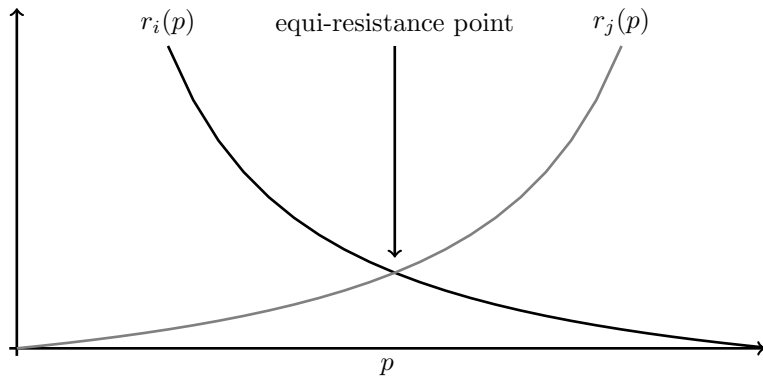
EQUI-RESISTANCE POINT

Notice that the equi-resistance equation tells us the agreement that people will eventually reach via negotiation, but it does not tell us how the negotiation took place, that is, it does not tell us their negotiation tactics.

We can represent the equi-resistance point graphically by simply replacing  $p_j$  with  $10 - p_i$  in  $j$ 's resistance equation  $r_j$  and plotting the two curves  $r_i$  and  $r_j$ . The point at which the curves cross is the point of exchange, as shown in figure 6.24. Of course, the exchange does not always happen at the midpoint. For example, if  $i$  had an offer from some other agent for 6 dollars if it refused to negotiate with  $j$ , this would mean that  $p_i^{\text{con}} = 6$  which would change the equi-resistance point.

We can solve complex NETs using the **iterated equi-resistance algorithm**. The algorithm simply uses the equi-resistance equation repeatedly on each edge of the graph in order to calculate the payments that these agents can expect to receive. This is repeated for all edges until payments stop changing. For example, for the graph in figure 6.22 we would follow these steps.

ITERATED EQUI-RESISTANCE  
ALGORITHM



**Figure 6.24:** Resistance of two agents to each possible deal  $p$ .

1. Apply Equi-resistance to  $\textcircled{i} \xrightarrow{10} \textcircled{j}$ . Gives us  $p_j = 5$ .
2. Apply Equi-resistance to  $\textcircled{j} \xrightarrow{10} \textcircled{k}$ . Let  $p_j^{con} = 5$  and apply equi-resistance again.
3. Repeat until quiescence.

The iterated equi-resistance algorithm is not guaranteed to converge and it might converge to different solutions when the edges are sorted in differently. Even when it does converge the deal it reaches is not guaranteed to be the one that humans would reach, that would mean we can predict human behavior. However, many experiments have been performed with humans in *small* networks (less than 12 nodes) which have shown that the iterated equi-resistance algorithm correctly predicts the resulting deal. This algorithm gives a new solution for the negotiation problem which, unlike the solutions in Section 6.1.1, is not based on some desirable properties of the solution but is based on evidence from human negotiation. If you want to implement agents that reach the same solution as humans then you probably want to use the iterated equi-resistance solution.

### Exercises

- 6.1 Given the following utility values for agents  $i$  and  $j$  over a set of possible deals  $\delta$ :

| $\delta$   | $u_i(\delta)$ | $u_j(\delta)$ |
|------------|---------------|---------------|
| $\delta^1$ | 1             | 0             |
| $\delta^2$ | 0             | 1             |
| $\delta^3$ | 1             | 2             |
| $\delta^4$ | 3             | 1             |
| $\delta^5$ | 2             | 2             |
| $\delta^6$ | 1             | 1             |
| $\delta^7$ | 8             | 1             |

1. Which deals are on the Pareto frontier?
  2. Which one is the egalitarian social welfare deal?
  3. Which one is the utilitarian deal?
  4. Which one is the Nash bargaining deal?
  5. Which one is the Kalai-Smorodinsky deal?
- 6.2 In a distributed workflow enactment application, a workflow is defined as a set of tasks all of which must be performed in order to complete the workflow and collect the payment. The following table lists the available workflow instances, their tasks and their payments:

| Workflow | Tasks                | Payment |
|----------|----------------------|---------|
| $w_1$    | $t_1, t_1, t_1$      | 6       |
| $w_2$    | $t_1, t_1, t_2$      | 5       |
| $w_3$    | $t_1, t_1, t_3$      | 3       |
| $w_4$    | $t_1, t_2, t_2$      | 1       |
| $w_5$    | $t_2, t_2, t_3, t_3$ | 8       |
| $w_6$    | $t_2, t_2$           | 1       |

There are three agents. An agent can perform at most two tasks at a time, as long as those tasks are **different**. Also, a task is performed for only a single workflow. That, if an agent performs  $t_1$  for  $w_1$  then this  $t_1$  cannot be used for any other workflow. The goal is for the agents to find the set of workflows that maximizes the total payment.

1. Re-state this problem as a negotiation problem, show the set of possible deals. (Hint: note that the agents have identical capabilities, so you do not need to worry about which one does which task).
2. We further constrain this negotiation by only allowing agents to either drop one workflow, add one workflow, or exchange one workflow for another. But, they can only do so if this increases the total payment (thus, you quickly conclude that they will never drop a workflow). Which deals become local optima under these constraints?



## Chapter 7

### Auctions

Auctions are a common and simple way of implementing resource allocation in a multiagent system. Agents can express how much they want a particular task via their bid and a central auctioneer can make the allocation based on these bids. Of course, this requires the use of a centralized auctioneer. Work is being done on reducing this bottleneck. Still, even with centralized auctions there are many details which you need to understand.

#### 7.1 Valuations

Before we begin to talk about the various types of auctions we must first clarify how people value the goods being sold. We have used the notation  $u_i(s)$  to refer to the utility that agent  $i$  derives from state  $s$ . Similarly, if  $S$  is an item, or set of items, for sale we can say that  $v_i(S)$  is the **valuation** that  $i$  assigns to  $S$ . We furthermore assume that this valuation is expressed in a common currency, thus  $v_i(S)$  then becomes the maximum amount of money that agent  $i$  is willing to pay for  $S$ . When studying auctions we generally assume that all agents have a valuation function over all the items being sold.

VALUATION

In the simplest case this valuation function reflects the agent's utility of owning the given items. For example, if you plan to eat a meal then the amount you are willing to pay for each item in the menu depends solely on how hungry you are and how much you like each item. In these cases we say that the agent has a **private value** function.

PRIVATE VALUE

On the other hand, there are items which you cannot consume and gain no direct utility from but might still have a resale value. The classic example are stocks. When you buy a share in some company you cannot do anything with that share, except sell it. As such, your valuation on that share depends completely on the value that other attribute, and will attribute, to that share. These are known as **common value** functions.

COMMON VALUE

Most cases, however, lie somewhere in the middle. When you buy a house you take into consideration the value that you will derive from living in that house as well as its appreciation prospects: the price you think others will pay when you finally sell it. This is an example of a **correlated value** function and is very common in the real world with durable high priced goods.

CORRELATED VALUE

The type of valuation function that the agents use changes their optimal behavior in an auction. For example, if an agent has a common value function then it will likely pay more attention to what the other agents are bidding. Most multiagent implementations use agents with private value functions as they don't take the time to implement secondary markets for the items being sold.

#### 7.2 Simple Auctions

There are times when there are many agents and the only thing they need to negotiate over is price. In these occasions it makes sense to use an auction since they are fast and require little agent communication. However, auctions are not as

ENGLISH AUCTION  
FIRST-PRICE OPEN-CRY  
ASCENDING

RESERVATION PRICE

WINNER'S CURSE

FIRST-PRICE SEALED-BID



Ontario Flower Growers Co-op, an example of a Dutch auction at work. The two large circles in the back are used to show the descending price.

DUTCH  
OPEN-CRY DESCENDING  
PRICE

VICKREY  
SECOND-PRICE SEALED-BID



William Vickrey. 1914–1996.  
Nobel Prize in Economics.

simple as they might appear at first and there are many ways in which things can go wrong when using them.

The actual mechanisms used for carrying out an auction are varied. The most common of all is the **English auction**. This is a **first-price open-cry ascending** auction. It is the standard one used in most auction houses. In it, the auctioneer raises the price as people yell higher bids. Once no one is willing to bid higher, the person with the highest bid gets the item and pays his bid price. These auctions sometimes have an initial or **reservation price** below which the seller is not willing to sell. The dominant strategy in an English auction, with private value, is to bid the current price plus some small amount until either the auction is won or one's reservation price is reached.

If an English auction is common or correlated value then it suffers from the **winner's curse**. For example, when you buy a stock in an English auction it means that you paid more than anyone else was willing to pay. As such, your valuation of that share must now be less than what you paid for it. In the real world we gamble that at some point in the future the others will realize that this stock really is worth that much more.

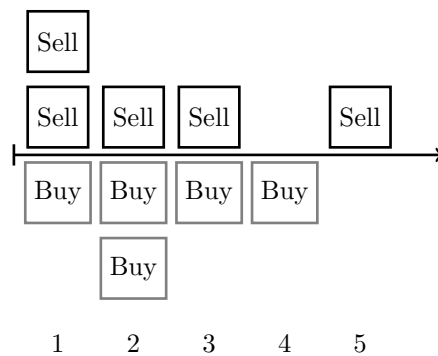
A similar auction type is the **First-price sealed-bid** auction. In this auction each person places his bid in a sealed envelope. These are given to the auctioneer who then picks the highest bid. The winner must pay his bid amount. These auctions have no dominant strategy. The buyer's best strategy is to spy on the other bidders in order to determine what they are going to bid and then bid slightly higher than that, as long as that is less than one's reservation price. As such, these auctions lead to a lot of inefficiencies when paired with intelligent agents.

The **Dutch** auction is an **open-cry descending price**. In this auction the seller continuously lowers price until a buyer hits the buzzer, agreeing to buy at the current price. The name comes from its use by the Dutch to sell flowers. The Dutch flower markets have been around for centuries and are still thriving. Every morning carts of flowers are paraded before eager flower shop owners who are equipped with an buzzer. Each cart stops before the buyers and a clock starts ticking backwards from a high number. When the first buyer hits his buzzer the flowers are sold to him at the current price. Analysis of it shows that it is equivalent to a first-price sealed-bid auction in terms of strategy. That is, there is no dominant strategy. However, it has the nice property of being real-time efficient. That is, the auction closes really fast and the auctioneer can make it move even faster by lowering the price faster. This real-time efficiency makes it a very attractive auction for selling cut flowers as these lose their value quickly after being harvested.

The **Vickrey** auction has some very interesting properties. It is a **second-price sealed-bid** auction. All agents place their bids and the one with the highest bid wins the auction but the price he pays is the price of the second highest bid. Analysis of this auction has shown that bidding one's true valuation, in a private value auction, is the dominant strategy. For example, let your valuation for the good being sold be  $v$ . If you bid less than  $v$  then you are risking the possibility that some other agent will  $w < v$  and get the item even though you could have won it. Moreover, since  $w$  is less than  $v$  you could have bid  $v$  and paid only  $w$ . As such, you have nothing to gain by bidding less than  $v$  but risk the possibility of losing an action that you could have won at an acceptable price. If you bid  $v'$ , which is bigger than  $v$ , then you are risking that some other agent will bid  $w$ , where  $v' > w > v$ , and thereby cause you to pay more than your reservation price  $v$ . At the same time, you do not gain anything by bidding higher than  $v$  because the only auction that you might win by bidding  $v'$  instead of  $v$  are those where you have to pay more than  $v$ .

As such, the Vickrey auction eliminates the need for strategizing. Since there is an easy dominant strategy the agents do not have to think about what they should do. They just take their dominant strategy and bid their true valuation. It is for this reason that most people don't like Vickrey auctions. People are often hesitant about revealing their true valuations for an item because we know that the world





**Figure 7.1:** Graphical representation of a double auction

is an iterated game and this information could be used against us in some future auction. As such, Vickrey auctions are seldom used in the real world.

Finally, the **double auction** is a way of selling multiple units of the same item. It is the auction used in most stock markets. Each buyer places either a buy or a sell order at a particular price for a number of instances of the good (number of shares in the stock-market). The buy and sell bids can be visualized in a simple graph such as the one shown in Figure 7.1.

DOUBLE AUCTION

Once we have all the bids then it is time to clear the auction. There are many different ways to clear a double auction. For example, on the example above we could try matching pairs of buy/sell bids. But, there are many different ways to match up these pairs and it is not clear which one is better.

One metric we might wish to maximize (or minimize) is the amount of surplus (known as the spread by traders). That is, the sum of the differences between the buy bids and the sell bids. In some auctions this surplus is kept by the auctioneer who then has an incentive to maximize it. Another option is to use it to enable more bids to clear. In the example above the total supply and demands are 12, therefore all bids should clear. One way to do this is to pair up all the small sell bids and give these sellers exactly what they asked for then give the surplus to the sell bid of 5 in order to clear it with the remaining buy bid.

Another metric we could use is a uniform price. That is, rather than giving each seller and buyer the exact price they asked for, we give them all one uniform clearing price. Clearly, the only buy bids that will clear are those that are above the clearing price and the only sell bids that clear are those below the clearing price.

### 7.2.1 Analysis

Now that we know the various auction types, there is an obvious question that we must ask ourselves. On which auction do sellers make more money? This question is answered by the following theorem.

**Theorem 7.1** (Revenue Equivalence). *All four auctions produce the same expected revenue in private value auctions with bidders that are risk-neutral.*

We also know that if the bidders are risk-averse then the Dutch and First-price are better. A risk-averse bidder is willing to pay a bit more than their private valuation in order to get the good. In a Dutch or First-price auction a risk-averse agent can insure himself by bidding more than would be required.

In common or correlated value cases the English auction gives a higher revenue to the seller. The increasing price causes others to increase valuation. That is, once the agent sees others bidding very high for the item the agent realizes that the item is really worth more to the other agents so it also raises its valuation of the item.<sup>1</sup>

<sup>1</sup>An interesting example of this was a British auction for 3G bandwidth licenses. The standard English auction was modified so that everyone must agree to buy at the current price or leave the room. This led to the licenses selling for 1000 times the expected amount (Harford, 2005).

**Table 7.1:** Example of inefficient allocation.

|                      | tasks      | Agent 1 | Agent 2 |
|----------------------|------------|---------|---------|
| Costs of Doing Tasks | $t_1$      | 2       | 1.5     |
|                      | $t_2$      | 1       | 1.5     |
|                      | $t_1, t_2$ | 2       | 2.5     |

BIDDER COLLUSION

SELF-ENFORCE COLLUSION

LYING AUCTIONEER

shill: a decoy who acts as an enthusiastic customer in order to stimulate the participation of others.

As it is often the case when money is involved, we have to be on the look out for ways in which the agents might cheat. The problem of **bidder collusion** affects all 4 auctions. In bidder collusion the bidders come to an a-priori agreement about what they will bid. They determine which one of them has the higher valuation and then all the other bidders refrain from bidding their true valuation so that the one agent can get it for a much lower price. The winner will then need to payback the others. The English and Vickrey auctions are especially vulnerable to bidder collusion as they **self-enforce collusion** agreements. For example, say there are 10 buyers and they realize that one of them has a valuation of 100 while the others have a valuation of 50 for the item. They agree to let him buy it for 1. In an English auction one of the 99 agents could defect and bid 2. However, this would only make the high-valuation agent bid 3, and so on until they get to 51. The high-valuation agent will get the item for 51 so the other agent gets nothing by defecting. The same logic applies in a Vickrey auction.

Another problem might be that a **lying auctioneer** can make money from a Vickrey auction. All he has to do is to report a higher second-price than the one that was announced. The winner then pays this higher second price to the auctioneer who gives the buyer the real second price and pockets the difference. This requires that the bids are not revealed and that the buyer does not pay the seller directly. If the buyer paid the seller directly then a lying auctioneer and the seller could collude to charge the buyer a higher price. A lying auctioneer can also place a shill in an English auction. That is, assuming that the auctioneer gets paid a percentage of the sales price. If the auctioneer gets paid a fixed amount then there is no incentive for him to increase the sales price.

When auctioning items in a series when their valuations are interrelated, such as chairs in a dining set or bandwidth rights in adjacent counties, it is possible to arrive at inefficient allocations. For example, the problem in Table 7.1 leads to an inefficient allocation if we first auction  $t_1$  and then  $t_2$ . Specifically, if we auction  $t_1$  first then Agent 2 will get it as it has the lower cost. When we then auction  $t_2$  both agents have the same cost (1) but, no matter who gets it the total cost is 2.5. If, on the other hand agent 1 had won both tasks then the total cost would be 2. matter who gets it the total cost is 2.5. This is the problem of **inefficient allocation**.

INEFFICIENT ALLOCATION

We could solve this problem if we made the agents use full lookahead. With full lookahead the agents can build a tree where each level corresponds to a task being auctioned. In this way agent 1 can see that it will only cost him 2 to do  $t_1$  and  $t_2$  so it can reduce its cost for  $t_1$  from 2 to 1. Of course, this puts agent 1 at risk of not getting  $t_2$  since agent 1 generally will not know agent 2’s cost for  $t_2$  so it does not know if it will win that auction. Another much better way of solving it is to use a combinatorial auction, which we will learn about in Section 7.3.

7.2.2 Auction Design

When designing an auction for a multiagent system you must make many decisions. You must first determine what kind of control you have over the system. It is possible that you control only the bidding agent and the auction is already implemented, as when building agents to bid on Ebay. It is possible that you control only the auction mechanism, as when building an auction website. Finally, it is possible that you might control both agents and mechanism, as when building a closed multiagent system.

The case where you control the mechanism is especially interesting. You must



| Price | Bid items         |
|-------|-------------------|
| \$1   | Beast Boy         |
| \$3   | Robin             |
| \$5   | Raven, Starfire   |
| \$6   | Cyborg, Robin     |
| \$7   | Cyborg, Beast Boy |
| \$8   | Raven, Beast Boy  |

**Figure 7.2:** Teen Titans figurines: (from top left) Beast Boy, Cyborg, Robin, Raven, and Starfire. The set of combinatorial bids received for them in on the table at the right.

then decide what bidding rules you will use: when bids are to be placed, when they can no longer be placed, what form can these bids takes, and what rules they must follow. For example, you might set a rule that a new bid has to always be for a higher value. You also set up clearing rules which determine when the goods are sold. We explained some of the problems with various clearing rules in the double auction. The four standard auction types already have clearing rules but you might want to modify these. Finally, you must decide on information rules: how much information the agents are to know about what the other agents bid, whether to reveal information during the bidding process itself or after clearing (Wurman et al., 2002).

7.3 Combinatorial Auctions

Arguably, the most useful auction for multiagent systems has been the **combinatorial auction** in which agents can place bids for sets of goods. In many systems we have the problem that there is a set of tasks or jobs that needs to be distributed among the agents but the agents have complex preferences over the set of tasks. For example, a workflow application we have a set of workflows, each composed of a set of web services, which must be performed by certain deadlines. Each agent can perform some of the services but each one has different costs which might depend on the agent's type, its current load, the services it has done before, etc. Our problem as system designers is to allocate the workflows to agents so that we maximize the total number of workflows completed. Other examples of combinatorial auctions include the selling of broadcasting rights by the federal government where cellular companies prefer to own the same frequencies in nearby locations, or at least to own some bandwidth in all the neighborhoods of a city. As well as the buying parts to put together a PC. One needs a motherboard, CPU, ram, etc. Each part can be bought independently but only some bundles work together. These problems, an all problems of this type, can be solved by a combinatorial auction.

COMBINATORIAL AUCTION

Formally, we define a combinatorial auction over a set of items  $M$  as being composed of a set of bids, where each agent can supply many different bids for, presumably, different subset of items. Each bid  $b \in B$  be composed of  $b^{items}$  which is the set of items the bid is over,  $b^{value}$  the value or price of the bid, and  $b^{agent}$  the agent that placed the bid.

For example, say you had a set of 5 figurines, one each of a different Teen Titan and you received 6 different combinatorial bids, as shown in Figure 7.2. The question you then face is how do your determine which are the winning bids so as to maximize the amount of revenue you get? Note that you can sell each item only once since you only have one of each. This is the problem of winner determination.

7.3.1 Centralized Winner Determination

The **winner determination** problem is finding the set of bids that maximizes the

WINNER DETERMINATION

seller's revenue. Or, more formally, find

$$X^* = \arg \max_{X \subseteq C} \sum_{b \in X} b^{\text{value}} \quad (7.1)$$

where  $C$  is a set of all bid sets where none of the bids share an item, that is

$$C = \{Y \subseteq B \mid \forall b, b' \in Y, b^{\text{items}} \cap b'^{\text{items}} = \emptyset\}. \quad (7.2)$$

Unfortunately, this is not a simple problem as there are, in the worst case, many  $X$ s. Specifically, if bids exists for all subsets of items then  $X$  is a way of partitioning the set of goods  $S$ . That is, take the set of goods  $S$  and figure out how many ways it can be split into smaller sets. We can calculate this number by remembering that the Stirling number of the second kind give us the number of ways to partition a set of  $n$  elements into  $k$  non-empty sets. Namely,

$$S(n, k) = \frac{1}{k!} \sum_{i=0}^{k-1} (-1)^i \binom{k}{i} (k-i)^n.$$

Using this great formula we can easily determine that the total number of allocations of  $m$  goods is

$$\sum_{i=1}^m S(m, i), \quad (7.3)$$

which is

$$O(m^m) \text{ and } \omega(m^{m/2}).$$

This means that a brute force search of all possible allocations of goods to agents is computationally intractable. In fact, no approach will work in polynomial time.

**Theorem 7.2.** *Winner Determination in Combinatorial Auction is NP-hard. That is, finding the  $X^*$  that satisfies (7.1) is NP-hard (Rothkopf et al., 1998).*

Even simplifying the problem does not make it easier to solve. For example, say that instead of trying to find the best allocation we simply want to check if there exists an allocation with total revenue of at least  $w$ . We call this the **decision version** of the winner determination problem. And, we further restrict the types of bids the agents can submit. Even under these circumstances the problem remains hard.

**Theorem 7.3.** *The decision version of the winner determination problem in combinatorial auctions is NP-complete, even if we restrict it to instances where every bid has a value equal to 1, every bidder submits only one bid, and every item is contained in exactly two bids (Cramton et al., 2006, Chapter 12).*

So, the problem is very hard, even when we try to limit its complexity. But, there is some hope. The winner determination problem in combinatorial auctions can be reduced to a linear programming (LP) problem and, therefore, solved in polynomial time with well-known algorithms but only if prices can be attached to single items in the auction (Nisan, 2000). That is, there needs to be a singleton bid for every item. In many cases we can satisfy this by simply adding the missing singleton bids, each with a value of 0. Specifically, the linear program which models the winner determination problem is:

**Maximize:**

$$\sum_{b \in B} x[b] b^{\text{value}}$$

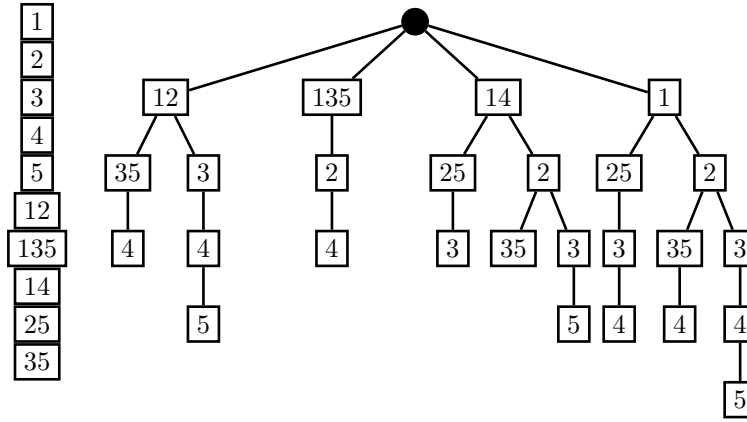
**Subject to:**

$$\begin{aligned} \sum_{b \mid j \in b^{\text{items}}} x[b] &\leq 1, \forall j \in M \\ x[b] &\in \{0, 1\}, \forall b \in B, \end{aligned}$$

A variation on this problem is when agents can submit XOR bids. That is, when an agent can say that it wants only one of his bids to win. Computationally, both problems are similar.

DECISION VERSION

Simplex is the most widely used LP algorithm but it does have worst-case exponential time, but in practice it is much faster. Other algorithms exist that are guaranteed polynomial.



**Figure 7.3:** Branch on items search tree for winner determination in combinatorial auctions. Note that this tree has 9 leafs (9 possible ways of selling all goods given the bids) while the total number of dividing 5 items into subsets is 52.

where  $x[b]$  is a bit which denotes whether bid  $b$  is a winning bid. That is, maximize the sum of the bid values given that each item can be in, at most, one winning bid. (Nisan, 2000) also shows that the LP problem will solve a combinatorial auction when the bids satisfy any one of the following criteria.

1. All bids are for consecutive sub-ranges of the goods.
2. The bids are hierarchical.
3. The bids are only OR-of-XORs of singleton bids.
4. The bids are all singleton bids.
5. The bids are downward sloping symmetric.

Another approach is to conduct one of the standard AI-searches over all possible allocations, given the bids submitted. The advantage over using an LP solver is that we can tweak our AI search algorithms and optimize them to solve our specific problem. That is, we can put some of our domain knowledge into the algorithm to make it run faster, as we shall see.

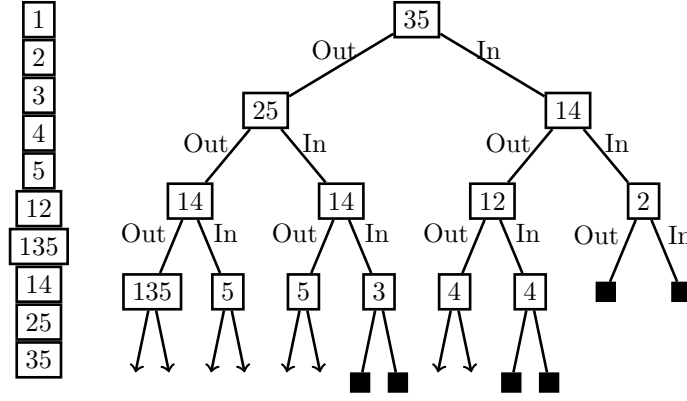
Before we can do search we need to define our search tree. One way we can build a search tree is by having each node be a bid and each path from the root to a leaf correspond to a set of bids where no two bids share a good. Figure 7.3 shows an example tree built in this fashion. In this case we have five goods for sale, numbered 1–5. The column on the left lists all the bids received. We omit the bid amount and only show the set of goods for each bid. The search algorithm uses these bids to build the tree shown on the right of the figure. We start at the first level from the top. All the children of the root are bids that have a 1 in them. Then, we proceed to add children to each node. The children of every node will be all the bids that contain the smallest number that is **not** on the path from the root to the node. Since the algorithm has the provision of adding a singleton bid, with value 0, for every good, we are guaranteed to find a suitable bid as a children of every node. The only time we cannot find such a bid is when the path from the root to the node contains all goods. In this case the node is a leaf and the set of bids from root to leaf constitutes one possible a working bid set.

The speedup can be confirmed by the fact that this tree has 9 leafs, therefore only 9 working bid sets exists, while application of the Stirling formula gives us

$$\sum_{i=1 \dots 5} S(5, i) = 52.$$

Clearly, fewer bids means faster run time which is the central idea of the algorithm. In general, we know that the number of leafs in the tree is bounded.

**Figure 7.4:** Branch on bids partial tree. The black boxes indicate search paths that have terminated because they denote a complete set of bids, that is, no more bids can be added because they contain items already sold.



**Theorem 7.4.** *The number of leaves in the tree produced by BUILD-BRANCH-ON-ITEMS-SEARCH-TREE is no greater than  $(|B|/|M|)^{|M|}$ . The number of nodes is no greater than  $|M|$  times the number of leaves plus 1 (Sandholm, 2002).*

We can also build a binary tree where each node is a bid and reach edge represents whether or not that particular bid is in the solution. We refer to this tree as a **branch on bids** search tree, an example is shown in Figure 7.4. Each edge on the tree indicates whether the parent node (bid) is to be considered as part of the final bidset. For example, the rightmost branch of the tree consists of all “In” branches so the rightmost leaf node corresponds to the bidset (35)(14)(2) which forms a complete allocation. In practice, the branch on bids search tree is often faster than the previous tree because it gives us more control over the order of bids so we can better optimize the bid order. Also, the branch on bids search does not require us to add dummy singleton bids.

We now have to decide how to search our chosen tree. Since both trees have a number of nodes that is exponential on the number of bids a breadth first search would require too much memory. However, a depth first search should be possible, but time consuming. A branch and bound algorithm like the one we used for DCOP in Chapter 2.2 further helps reduce the search space and speed up computation. In order to implement it we first need a function  $h$  which gives us an upper bound on the value of allocating all the items that have yet to be allocated. One such function is  $h$

$$h(g) = \sum_{j \in M - \bigcup_{b \in g} b^{\text{items}}} \max_{b|j \in b^{\text{items}}} \frac{b^{\text{value}}}{|b^{\text{items}}|}, \quad (7.4)$$

where  $g$  is the set of bids that have been cleared. The function  $h$  simply adds up the maximum possible revenue that each item not in  $g$  could contribute by using the bid that pays the most for each item, divided by the number of items on the bid. This function provides an upper bound since it is likely that the best bids for each individual item will conflict with each other by having other items in common.

Given the upper bound  $h(g)$  we can then implement the branch and bound algorithm shown in Figure 7.5. This algorithm searches the branch on bids tree. It maintains a partial solution  $g$  to which it adds one bid on each recursive call. Whenever it realizes that that partial solution will never be able to achieve revenue that is higher than the best revenue it has already found then it gives up on that subtree, see line 8 of BRANCH-ON-BIDS-CA-HELPER. The algorithm is complete.

We can also use the same heuristic function to do an  $A^*$  search. Unfortunately, since  $A^*$  acts much like a breadth first search it generally consumes too much memory. A viable solution is to use iterative deepening  $A^*$ .  $IDA^*$  guesses how much revenue we can expect and runs a depth-first search that prunes nodes that have used more than that. If a solution is not found then the guess is reduced and we try again.  $IDA^*$ , with some optimizations, was implemented by the **Bidtree** algorithm (Sandholm, 1999) on the branch on items search tree. In practice, this approach was found to often be slower than branch and bound search.

BRANCH ON BIDS

BIDTREE

BRANCH-ON-BIDS-CA()

```

1   $r^* \leftarrow 0$        $\triangleright$  Max revenue found. Global variable.
2   $g^* \leftarrow \emptyset$    $\triangleright$  Best solution found. Global variable.
3  BRANCH-ON-BIDS-CA-HELPER( $\emptyset$ )
4  return  $g^*$ 

```

BRANCH-ON-BIDS-CA-HELPER( $g$ )

```

1  if  $\bigcup_{b \in g} b^{\text{items}} = M$                                  $\triangleright g$  covers all items
2      then if  $\sum_{b \in g} b^{\text{value}} > r^*$                          $\triangleright g$  has higher revenue than  $r^*$ 
3          then  $g^* \leftarrow g$ 
4               $r^* \leftarrow \sum_{b \in g} b^{\text{value}}$ 
5          return
6  for  $b \in \{b \in B \mid b^{\text{items}} \cap \bigcup_{b_1 \in g} b_1^{\text{items}} = \emptyset\}$    $\triangleright b$ 's items do not overlap  $g$ 
7      do  $g' \leftarrow g + b$ 
8          if  $\sum_{b_1 \in g'} b_1^{\text{value}} + h(g') > r^*$ 
9              then BRANCH-ON-BIDS-CA-HELPER( $g'$ )

```

**Figure 7.5:** A centralized branch and bound algorithm that searches a branch on bids tree and finds the revenue maximizing solution given a set  $B$  of combinatorial bids over items  $M$ .

BRANCH-ON-ITEMS-CA()

```

1   $r^* \leftarrow 0$        $\triangleright$  Max revenue found. Global variable.
2   $g^* \leftarrow \emptyset$    $\triangleright$  Best solution found. Global variable.
3  BRANCH-ON-ITEMS-CA-HELPER(1,  $\emptyset$ )
4  return  $g^*$ 

```

BRANCH-ON-ITEMS-CA-HELPER( $i, g$ )

```

1  if  $i = m$                                  $\triangleright g$  covers all items
2      then if  $\sum_{b \in g} b^{\text{value}} > r^*$                          $\triangleright g$  has higher revenue than  $r^*$ 
3          then  $g^* \leftarrow g$ 
4               $r^* \leftarrow \sum_{b \in g} b^{\text{value}}$ 
5          return
6  for  $b \in \{b \in B \mid i \in b^{\text{items}} \wedge b^{\text{items}} \cap \bigcup_{b_1 \in g} b_1^{\text{items}} = \emptyset\}$   $\triangleright b$ 's items do not overlap  $g$ 
7      do  $g' \leftarrow g + b$ 
8          if  $\sum_{b_1 \in g'} b_1^{\text{value}} + h(g') > r^*$ 
9              then BRANCH-ON-BIDS-CA-HELPER( $g'$ )

```

**Figure 7.6:** A centralized branch and bound algorithm that searches a branch on items tree and finds the revenue maximizing solution given a set  $B$  of combinatorial bids over items  $M$ .

The BRANCH-ON-BIDS-CA algorithm is the basic framework for the **CABOB** algorithm (Combinatorial Auction Branch on Bids). CABOB improves the performance of the basic algorithm in several ways, one of which is by improving the search for new bids to add to the partial solution. Specifically, we note that a naive implementation of line 6 of BRANCH-ON-BIDS-CA-HELPER would mean that we would build this set on each recursive call to the function. That would be very time consuming as there are an exponential number of bids in  $B$ . CABOB handles this problem by maintaining graph data structure which has all the bids that can still be used given  $g$ . The nodes in the graph are the bids that are still available and the edges connect every pair of bids that share an item. In this way when a new bid is added to  $g$  it is removed from the graph as well as all the other bids that are connected to it (Sandholm et al., 2005).

We can also perform the branch and bound search on the branch on items search tree, as shown in Figure 7.6. This algorithm is the basis for the **CASS** (Combinatorial Auction Structured Search) algorithm which also implements further refine-

CABOB

CASS

ments (Fujishima et al., 1999).

Most algorithms for centralized winner determination in combinatorial auction expand on the basic branch and bound search by using specialized data structures to speed up access to the information need—the viable bids given the current partial solution—and implement heuristics which have been shown to reduce the size of the search space, especially for certain popular bid distributions. Some heuristics that have been found useful include the following:

- Keep only the highest bid for any set. That is, if there is a bid of \$10 for items 1,2 and another bid of \$20 for 1,2 then we get rid of the \$10 bid.
- Remove provably noncompetitive bids (i.e. they are dominated by another bid or sets of bids). For example, if there is a bid for \$10 for item 1 and another bid for \$5 for items 1,2 then the \$10 bid dominates the \$5 bid—any situation in which we choose the \$5 bid would be made better if we changed that bid for the \$10 bid.
- Decompose bids into connected sets, each solved independently. If we can separate the set of bids into two or more sets of bids where all bids for any item are to be found in only one of the sets then this set of bids becomes one smaller, and independent, winner determination problem.
- Mark noncompetitive tuple of bids. For example, if there are bids \$1:(1,2), \$1:(3,4), \$10:(1,3), \$10:(2,4) then the pair of \$10 bids dominates the pair of \$1 bids, so we can get rid of them.
- In the branch-on-items tree place the items with the most bids first on the tree. This way the most constrained items are tried first thereby creating fewer leafs.
- If the remaining search subtree is the same for different nodes in the search tree, as can happen when different items are cleared but by different bids, then these subtrees can be cached. The subtree is solved once and the answer, that is, the best set of bids found in it, is saved for possible future use.

In general the best speed attainable by the best algorithms varies greatly depending on the type of bids submitted. For example, if the number of items in each bid is chosen from a flat probability distribution then we can solve problems with thousands of items and tens of thousands of bids in seconds. On the other hand, if each bid contains exactly five randomly chosen items and a price of 1 then we can only solve problems with tens of items and hundreds of bids in a minute. The Combinatorial Auction Test Suite (**CATS**) can generate realistic types of bid distributions so new algorithms can be compared to existing ones using realistic bid sets (Leyton-Brown et al., 2000). It generates these bids by using several sample scenarios. In one scenario there is a graph where the nodes represent cities and the edges are railroad tracks that connect these cities. The items for sale are the tracks between cities, that is, the edges. The agents are given pairs of host/destination cities and are told that they need to establish a train route between their city pairs. Thus, each agent determines all the possible sets of edges which connect his city pairs and submits XOR combinatorial bids on them. The value of each path depends on the total distance; shorter routes are preferred.

### 7.3.2 Distributed Winner Determination

One problem with the centralized winner determination algorithms, aside from the bottleneck, is that they require the use of a trusted auctioneer who will perform the needed computations. Another option is to build a peer-to-peer combinatorial auction protocol which lets the sellers themselves determine the winning set of bids and discourages them from cheating.



### Incremental Auctions: Distribute over Bidders

One way to distribute the winner determination calculation is by offloading it on the bidding agents. We can do this by using an increasing price auction and making the bidders figure out which bids would be better than the current standing bid. This is the approach taken by the Progressive Adaptive User Selection Environment or **PAUSE** combinatorial auction (Kelly and Stenberg, 2000) (Cramton et al., 2006, Chapter 6).

PAUSE

A PAUSE auction for  $m$  items has  $m$  stages. Stage 1 consists of having simultaneous ascending price open-cry auctions for each individual item. During this stage the bidders can only place individual bids on items. At the end of this state we will know what is the highest bid for each individual good and who placed that bid. In each successive stage  $k = 2, 3, \dots, m$  we hold an ascending price auction where the bidders must submit sets of bids that cover all goods but each one of the bids must be for  $k$  goods or less. The bidders are allowed to use bids that other agents have placed in previous rounds. Also, any new bid set has to have a sum of bid prices which is bigger than the currently winning bid set.

At the end of each stage  $k$  all agents know the best bid for every subset of size  $k$  or less. Also, at any point in time after stage 1 has ended there is a standing bid set whose value increases monotonically as new bid sets are submitted. Since in the final round all agents consider all possible bid sets, we know that the final winning bid set will be one such that no agent can propose a better bidset. Note, however, that this bid set will generally not be  $X^*$  since we are using ascending price auction so the winning bid will be only slightly bigger than the second highest bid for the particular set of goods.

In the general case, the PAUSE auction has been shown to be **envy-free** in that at the conclusion of the auction no bidder would prefer to exchange his allocation with that of any other bidder. However, it is not guaranteed to find the utilitarian solution (7.1).

ENVY-FREE

The PAUSE auction makes the job of the auctioneer very easy. All it has to do is make sure each new bidset adds up to a number that is bigger than the current best as well as make sure that any bids an agent places that are not his do indeed correspond to other agents' bids. The computational problem shifts from one of winner determination to one of bid generation. Each agent must search over the space of all bid sets which contain at least one of its bids. The search is made easier by the fact that the agent need only consider the current best bids and that in stage  $k$  all bid sets must contain at least one bid of size  $k$  since they would have otherwise been bid in a previous stage. The **pausebid** algorithm uses the same branch and bound techniques used in centralized winner determination but expands them to include the added constraints an agent faces. As such, the pausebid algorithm can be used to find the myopically optimal bid for agents in the PAUSE auction (Vidal, 2006). The research also reveals that agents using pausebid reach the same allocation as the utilitarian solution (where the agents bid their true valuations) at least 95% of the time.

PAUSEBID

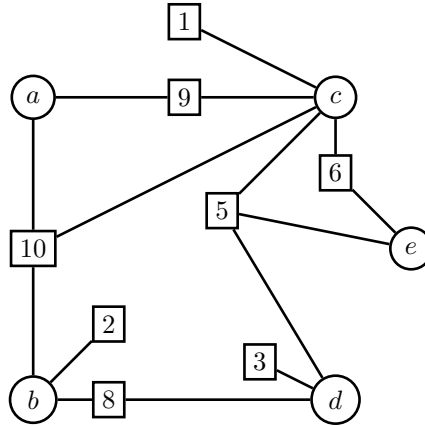
Another way of distributing the winner determination problem among the bidders is provided by the Virtual Simultaneous Auction (**VSA**) (Fujishima et al., 1999) which is based on market-oriented programming ideas (Wellman, 1996). The VSA is an iterative algorithm where successive auctions for the goods are held and the bidders change their bids based on the last auction's outcome. The auction is guaranteed to find the optimal solution when the bidding terminates. Unfortunately, there is no guarantee that bidding will terminate and experimental results show that in most cases bidding appears to go on forever.

VSA

### Distribute over Sellers

Another way to distribute the problem of winner determination is to distribute the actual search for the winning bid set among the sellers. We envision a distributed

**Figure 7.7:** Graphical representation of a distributed winner determination problem. The circles represent agents/goods while the squares represent combinatorial bids.



system where each person that wants to sell an item runs an agent. The agent advertises that the item is for sale. Every person who wants to place a, possibly combinatorial, bid does so by telling all the agents present in the bid about it. After some time the agents have gathered some bids and begin the clearing process. The set of agents and bids can be visualized in a graph such as Figure 7.7.

Here we see that agent  $b$  has received 3 three bids. One of them is a singleton bid worth \$2, the other two are combinatorial bids one of them for \$8 and including agent  $d$  and the other for \$10 and including agents  $a$  and  $c$ . The problem we face is how can the agents  $a$ – $e$  determine the set of winning bids. comparing some simple approaches.

The simplest solution is to do a **complete search via sequentialized ordering**. In this method we use the same search tree as before but instead of implementing it centrally we pass messages among the agents. Agent 1 handles the top level of the tree. It tentatively sets one of its bids as cleared and sends a message to agent 2. In this way, each agent (roughly) handles one level of the tree. Note the agents are sequentialized so there is no parallelism, even though it is distributed.

Another option is to **partition** the problem into smaller pieces then have sets of agents to a complete search via sequentialized ordering on each of the parts. That is, we first partition the set of agents then do a complete search on each subset. This method means that each subset works in parallel. However, if there is any bid that contains agents from more than one subset then the solution found is no longer guaranteed to be optimal.

We can maximize the available parallelism by having the agents do **individual hill-climbing**. Each agent starts by ordering all its bids based on their price divided by the number of agents in the bid under the assumption that the agent gets  $1/n$  of a bid that includes  $n$  agents. The agent picks the first bid in the list (the highest valued) and tells all the other agents in the bid that it wants to clear it. If the agent receives similar messages from all the agents in the bid this means that they all wanted to clear it so the bid is considered cleared and all the agents in it are removed from the protocol. The remaining agents repeat the same process with their next highest bid and so on until they run out of bids. This algorithm is guaranteed to terminate but will only find, at best, a local optima.

### Winner Determination as Constraint Optimization

We can reduce the winner determination problem to a constraint optimization problem as described in Chapter 2.2 in two different ways. One way is to let the variables  $x_1, \dots, x_m$  be the items to be sold and their domains be the set of bids which include the particular good. That is, each item  $k$  is represented by a variable  $x_k$  with domain  $D_k$  which contains all the bids that involve  $k$ . The constraints are given by the bids. Every bid is replaced with a constraint which returns the value of the bid if all the items in the bid have a value equal to that bid. That is, if all the items

are cleared for that bid/constraint then that constraint returns its value, otherwise the constraints returns a value of zero. In this problem we now want to maximize the value returned by the constraints.

We can also reduce the winner determination problem by letting the variables be the bids themselves with binary domains which indicate whether the bid has been cleared or not. We then need two sets of constraints. One set of constraints has to be generated such that they give a very large penalty if any two bids with goods in common have been cleared, so as to make sure that never happens. The other set simply gives a value for every cleared bid equal to the value of that bid.

Since both of these are standard constraint optimization problems we can use the algorithms from Chapter 2.2 to solve them in a distributed manner. However, as those algorithms are generic, it seems likely that they will not perform as well as the specialized algorithms.

Can we reduce winner determination to a constraint satisfaction problem? and use distributed breakout?

### 7.3.3 Bidding Languages

We have thus far assumed that each buyer can submit a set of bids, where each bid  $b$  specifies that he is willing to pay  $b^{\text{value}}$  for the set of items  $b^{\text{items}}$ . Implicit in the set of bids is the fact that the agent is also willing to accept winning two or more of his bids. That is, if  $b$  and  $b'$  are two bids for non-overlapping sets of items then any agent that places them should also be happy to win both bids. This bidding language is known as **OR bids**, because agents can place multiple **atomic bids** and they can win any feasible combination of the bids they submit. That is, the agent expresses his valuation as  $b_1 \text{ OR } b_2 \text{ OR } \dots \text{ OR } b_k$ .

OR BIDS  
ATOMIC BIDS

A limitation of OR bids is that they cannot represent sub-additive valuations. For example, a sub-additive valuation arises if you have a value of 10 for a red hat and 10 for a blue hat but together value them at 11 because you really only need one hat. In this scenario if you placed the individual bids it could be that you end up paying 10 for both hats. We thus say that OR bids are not a complete bidding language since they cannot represent all possible valuations.

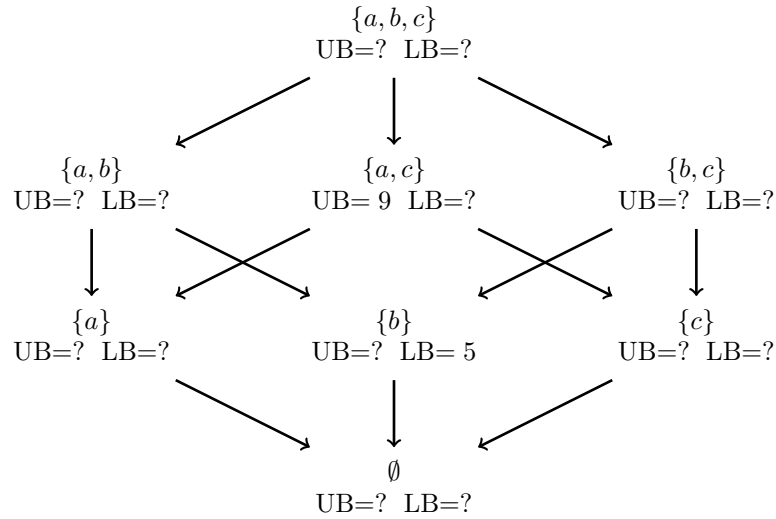
**XOR bids**, on the other hand, can represent all possible valuations. An XOR bid takes the form of a series of atomic bids joined together by exclusive-or operations:  $b_1 \text{ XOR } b_2 \text{ XOR } \dots \text{ XOR } b_k$ . This bid represents the fact that the agent is willing to win any *one* of the bids, but not more than one. Thus, you can use it to place a bid that says you are willing to pay 10 for a red hat or 10 for a blue hat, but only want to buy one hat.

XOR BIDS

One problem with XOR bids is that they can get very long for seemingly common valuations that can be more succinctly expressed using the OR bids. For example, say an agent has a purely additive valuation function over a set of items, that is if  $s = s' \cup s''$  then  $v(s) = v(s') + v(s'')$ . This agent could have expressed this valuation by placing an OR bid where each atomic bid was just for one item. Implicit in this bid is the fact that the agent would be willing to accept any subset of the items as long as he gets paid the sum of the individual valuation. If the same agent had to place an XOR bid it would have to place an atomic bid for every subset of items, and there are  $2^{|s|}$  such subsets.

Another practical problem with XOR bids is that most of the winner determination algorithms are designed to work with OR bids. The problem can be solved by adding dummy items to OR bids, these bids are known as **OR\* bids**. For example, if an agent wants to place a bid for item  $a$  and  $b$ , but not both, it could generate a dummy item  $d$  and place an OR bid for items  $\{a, d\}$  and  $\{b, d\}$ . This way the agent guarantees that it will not win both  $a$  and  $b$ . OR\* combines the completeness of XOR bids with the succinctness of OR bids without adding too many new dummy items. In fact, any bid that can be expressed by OR or XOR using  $x$  atomic bids can be represent using an OR\* bids with at most  $x^2$  dummy items (Nisan, 2000).

OR\* BIDS



**Figure 7.8:** Constraint network for determining an agent's valuation. Directed edges indicate preferred sets.

### 7.3.4 Preference Elicitation

We can also try to reduce the amount of information the bidders must supply by trying to only ask them about those valuations that are important in finding the utilitarian solution (Hudson and Sandholm, 2004). This can best be done in the common case of **free disposal** where there is no cost associated with the disposal of an item, thus, if  $S \subseteq T$  then  $v(S) \leq v(T)$ . For example, if we know that an agent values item  $a$  at 10 then we know that it must also value the set  $(a, b)$  at *least* at 10. Similarly, if the agent values items  $(a, b)$  at 5 then we know that its value for item  $a$  is at *most* 5.

Given free disposal, an auctioneer can incrementally explore a network like the one in Figure 7.8 which shows all the possible subsets of items and associates with each one a upper bound (UB) and a lower bound (LB) on the valuation that the agent has for that particular set of items. The directed edges indicate which sets are preferred over other ones. For example, the agent will always prefer the set  $\{a, b, c\}$  over the set  $\{a, b\}$  and even, transitively, over the set  $\{a\}$ . The graph also makes it easy to see how the auctioneer can propagate the bounds he learns on one set to the others. Namely, the lower bounds can be propagated upstream and the upper bounds can be propagated downstream. For example, in the figure there is a lower bound of 5 for the set  $\{b\}$ , knowing this the auctioneer can immediately set the lower bounds of  $\{b, c\}$  and  $\{a, b, c\}$  to 5. Similarly, the upper bound of 9 for the set  $\{a, c\}$  can be propagated down to  $\{a\}$ ,  $\{c\}$ , and  $\emptyset$ . Note also that if the agent tells the auctioneer its exact valuation for a particular set then the auctioneer will set both the upper and lower bounds of that set to the given value.

Of course, the goal of an elicitation auctioneer is to minimize the amount of questions that it asks the bidders while still finding the best allocation. If we limit the auctioneer to only asking questions of the type “What is your  $n^{\text{th}}$  most preferred set?” then we will be unable to find the revenue maximizing allocation. However, we can still find the Pareto optimal allocations.

The **PAR algorithm**, shown in Figure 7.9, allows an elicitation auctioneer to find a Pareto optimal solution by only using rank questions (Conen and Sandholm, 2002). It does this by incrementally building a solution lattice for the bidders. Figure 7.10 shows an example of a complete lattice for two bidders. The PAR algorithm maintains a set, called the *fringe*, of possible Pareto optimal allocations. At the first time step the auctioneer adds the solution  $\{1, 1\}$  to the *fringe*, where  $\{1, 1\}$  represents the solution formed by using both agents' most preferred solution. In every succeeding step the auctioneer picks some solution from the *fringe* and asks the agents for those sets, if it does not already know them. This communication with the bidders occurs within the FEASIBLE procedure. In the example figure both

FREE DISPOSAL

PAR ALGORITHM

```

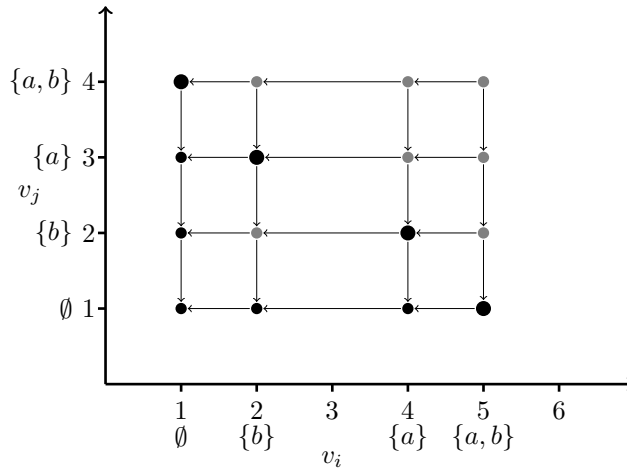
PAR()
1  fringe ← {{1, ..., 1}}
2  while fringe ≠ ∅
3      do c ← first(fringe)
4      fringe ← rest(fringe)
5      successors ← CHILDREN(c)
6      if FEASIBLE(c)
7          then pareto-solutions ← pareto-solutions ∪ c
8          fringe ← fringe successors
9      else for n ∈ successors
10         do if n ∉ fringe ∧ UN-DOMINATED(n, pareto-solutions)
11            then fringe ← fringe ∪ n

```

```

CHILDREN({k1, ..., kn})
1  for i ∈ 1 ... n
2      do c ← {k1, ..., kn}
3      c[i] ← c[i] + 1
4      result ← result ∪ c
5  return result

```



**Figure 7.9:** The PAR algorithm. The procedure  $\text{FEASIBLE}(\{k_1, \dots, k_n\})$  asks each bidder  $i$  for its  $k_i$  most valued set, if we haven't asked before. It uses these sets to determine if, together, they form a feasible allocation. The  $\text{CHILDREN}$  procedure returns a set of possible solutions, by adding 1 to each position in  $\{k_1, \dots, k_n\}$ .

**Figure 7.10:** Rank lattice. The dots represent all possible allocations. Directed edges represent Pareto dominance. Grey dots are infeasible while black dots are feasible allocations. The PAR search starts at the top rightmost point and stops when it has identified the complete Pareto frontier of feasible allocations—the larger black dots.

agents prefer the set  $\{a, b\}$  so they will respond with those sets. Since the set of bids  $(\{a, b\}, \{a, b\})$  is not feasible the algorithm checks to make sure that each one of the  $\text{CHILDREN}$  of  $\{1, 1\}$ , in this case  $\{1, 2\}$  and  $\{2, 1\}$ , is not dominated by any other allocation in the set  $\text{pareto-solutions}$  and adds it to the  $\text{fringe}$  if its not already there. The algorithm continues in this way until the  $\text{fringe}$  is empty, at that point the variable  $\text{pareto-solutions}$  contains all the Pareto allocations for the problem.

Since the PAR algorithm does not ask the agents for their valuation values it cannot determine which one of the Pareto allocations is the utilitarian allocation. Of course, once we start asking for valuations we have a better idea of which bids will likely be part of the utilitarian allocation, namely those sets that have the highest value per item.

The **ebf algorithm** (efficient best first) performs a search similar to the one that PAR implements but it also asks for the values of the sets and always expands the allocation in the  $\text{fringe}$  which has the highest value (Conen and Sandholm, 2002). Figure 7.11 shows the algorithm. This algorithm will find the utilitarian allocation.

Unfortunately, both PAR and EBF have worst case running times that are exponential in the number of items. They also perform rather poorly in practice. PAR

```

EBF()
1  fringe  $\leftarrow \{\{1, \dots, 1\}\}$ 
2  if  $|fringe| = 1$ 
3      then  $c \leftarrow \text{first}(fringe)$ 
4      else  $M \leftarrow \{k \in fringe \mid v(k) = \max_{d \in fringe} v(d)\}$ 
5          if  $|M| \geq 1 \wedge \exists_{d \in M} \text{FEASIBLE}(d)$ 
6              then return  $d$ 
7          else  $c \leftarrow \text{PARETO-SOLUTION}(M)$ 
8  if  $\text{FEASIBLE}(c)$ 
9      then return  $c$ 
10 successors  $\leftarrow \text{CHILDREN}(c)$ 
11 for  $n \in successors$ 
12     do if  $n \notin successors$ 
13         then  $fringe \leftarrow fringe \cup \{n\}$ 
14 goto 2

```

**Figure 7.11:** The EBF algorithm.  $\text{FEASIBLE}(d)$  returns true if  $d$  is a feasible allocation.  $\text{PARETO-SOLUTION}(M)$  returns one allocation from  $M$  which is not Pareto-dominated by any other allocation in  $M$ .

does not make any effort at trying to pick a good set to ask about, it simply chooses randomly, so its bad performance is not surprising. EBF's elicitation strategy also has been shown to perform poorly in experiments—it asks too many questions. Its performance also degrades as more agents are added.

A more general framework for elicitation is to maintain a set of allocations which are potentially optimal. Initially, this set would contain all allocations in the general case. In cases where we can assume some structure for the value function, such as free disposal, then it contains all those allocations that are not dominated by another. The elicitation algorithm can then choose one allocation from this set and ask the agents their values for the sets they receive in that allocation. These values can then be propagated to other sets and a new allocation chosen (Conen and Sandholm, 2001b; Conen and Sandholm, 2001a).

Within this general framework there are various elicitation strategies we could try. The simplest one is to randomly choose an allocation from the set. This technique has been shown to require a number of elicitations that is proportional to  $n2^m$  where  $n$  is the number of agents and  $m$  is the number of items. Another strategy is to choose the candidate with the highest lower bound value on the expectation that a candidate with a higher value is more likely to be the optimal choice, or at least will need to be eliminated from competition. Experiments have shown that this strategy performs better than random elicitations.

### 7.3.5 VCG Payments

VCG payments (see Chapter 8.2) can be applied to combinatorial auctions (MacKie-Mason and Varian, 1994). In a VCG combinatorial auction the bid set with maximum payment is chosen as the winner but the bidders do not have to pay the amounts they bid. Instead, each bidder pays the difference in the total value that the *other* bidders would have received if he had not bid (and the best set of bids was chosen from their bids) minus the total value the other bidders receive given his bid. Each bidder thus get a payment that is proportional to his contribution to everyone else's valuation. This has the desirable effect of aligning the bidders' incentives with the utilitarian allocation and thus eliminating the incentive to lie about their true valuation. However, it increases the computational requirements as we now have to also solve a winner determination problem for every subset of  $n - 1$  agents in order to calculate the payments. That is, VCG payments increase the work by a factor of  $n$ , where  $n$  is the number of agents.

**Exercises**

- 7.1 The branch and bound algorithm for the branch on bids search tree, seen in figure 7.5, does not specify in which order the bids should be searched. Give a good heuristic for this ordering and explain why it should, on average, help find a solution quicker than expanding bids in a random order.





Chapter 8

Voting and Mechanism Design

A common problem when designing multiagent systems with selfish agents is finding a way to poll them so that the final results represents the knowledge of the group. One issue is that simple voting mechanism do not always lead to the desired or expected outcome. The other issue is that if we tell the agents how their vote will in turn dictate a future event then the agents might lie in order to get a preferred event. These are the problems of voting and of mechanism design.

8.1 The Voting Problem

Voting is easy. Each agent proclaims its preferences, these are tallied up and the result is revealed. The problem comes if we want this result to match, in some way, the agents' preferences. It turns out that the commons voting mechanisms which you are familiar with all have some serious problems once we analyze them closely.

Figure 8.1 shows 15 mathematicians getting ready to go to party. They must first decide which beverage the math department will serve at this party. There are three choices: beer, wine, and milk. As the figure shows, 6 mathematicians prefer milk over wine over beer, 5 prefer beer over wine over milk and 4 prefer wine over beer over milk. Which drink will they choose?

One option is to have a **plurality vote** where each one votes for their favorite drink, the votes are tallied and the drink with the most votes is the winner. Under this voting scheme beer would get 5 votes, wine 4, and milk 6. Therefore, the mathematicians should clearly choose milk as the drink for the party.

Another option is to have a **runoff** election (primaries) then pick the two winners and have another election with only those two (these technique can be easily extended to any number of runoff elections). Under this scheme the first election would lead to the same votes as before so the second election would consist of beer

PLURALITY VOTE

RUNOFF

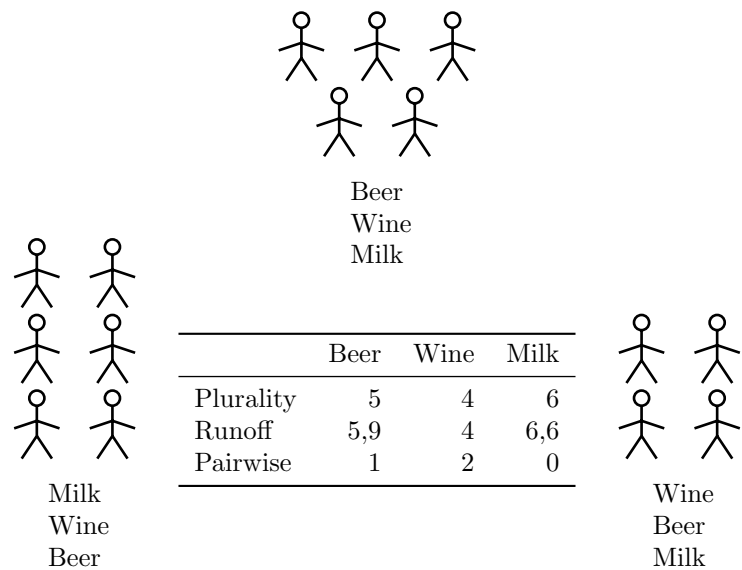


Figure 8.1: Fifteen mathematicians trying to decide whether they should buy beer, wine, or milk.

PAIRWISE

and milk. With only these two candidates beer would get 9 votes while milk would get 6 votes. Therefore, the mathematicians should clearly choose beer as the drink for the party.

Another option is to hold three elections each one with only two candidates (that is, implement all **pairwise** elections) and the candidate that wins the most elections is the chosen one. Under this scheme we would see that if beer and wine are paired then wine wins, if beer and milk are paired then beer wins, and if wine and milk are paired then wine wins. Wine wins two elections so it is the chosen one. Therefore, the mathematicians should clearly choose wine as the drink for the party.

The mathematicians wisely decide to make it “bring your own drink”.

### 8.1.1 Possible Solutions

As always, we want fairness. What does fairness look like in this voting problem? A possible solution is to look a symmetry. There are two different types of symmetry that we can identify in this problem.

- *Reflectional symmetry*: If one agent prefers A to B and another one prefers B to A then their votes should cancel each other out.
- *Rotational symmetry*: If one agent prefers A,B,C and another one prefers B,C,A and another one prefers C,A,B then their votes should cancel out.

If we look back at the three types of schemes presented we notice that the plurality votes violates reflectional symmetry for there are 8 agents that prefer beer over milk and wine over milk, but only 6 that have the opposite preferences, and yet milk wins. Similarly, since the runoff election is just a series of plurality votes it also violates reflectional symmetry.

It has also been shown that pairwise comparison violates rotational symmetry.

With all these negative results one begins to wonder if a symmetric voting mechanism actually exists. It does. It is called the Borda count after its inventor and it works as follows:

1. With  $x$  candidates, each agent awards  $x$  to points to his first choice,  $x - 1$  points to his second choice, and so on.
2. The candidate with the most points wins.

Borda satisfies both reflectional and rotational symmetry. It is most useful when there are multiple candidates and we want to choose one by taking into account all agents' knowledge, equally.

We can now formally define the voting problem.

**Definition 8.1** (Voting Problem). *We are given set of agents  $A$  and a set of outcomes  $O$ . Each agent  $i$  has a preference function  $>_i$  over the set of outcomes. Let  $>^*$  be the global set of social preferences. That is, what we want the outcome to be.*

Given this notation we can now try to dream up of the kind of  $>^*$  that we would like. Namely, we are probably interested in a  $>^*$  that is fair to everyone. After thinking about it for some time, you would probably come up with a set of voting conditions similar to the following.

**Definition 8.2** (Desirable Voting Outcome Conditions). *A voting protocol is desirable if it obeys the following conditions:*

1.  $>^*$  exists for all possible inputs  $>_i$
2.  $>^*$  exists for every pair of outcomes



Jean-Charles de Borda.  
1733–1799.

3.  $>^*$  is asymmetric and transitive over the set of outcomes
4.  $>^*$  should be Pareto efficient. That is, if all agents prefer Beer over Milk then  $>^*$  should also prefer Beer over Milk.
5. The scheme used to arrive at  $>^*$  should be independent of irrelevant alternatives. That is, if in one world all agents prefer Beer to Milk and in another world all agents again prefer Beer to Milk then in both cases the rankings of Beer and Milk should be the same, regardless of how they feel about Wine.
6. No agent should be a dictator in the sense that  $>^*$  is always the same as  $>_i$ , no matter what the other  $>_j$  are.

Unfortunately, Arrow showed that no voting mechanism exists which satisfies all these conditions.

**Theorem 8.1** (Arrow's Impossibility). *There is no social choice rule that satisfies the six conditions.*

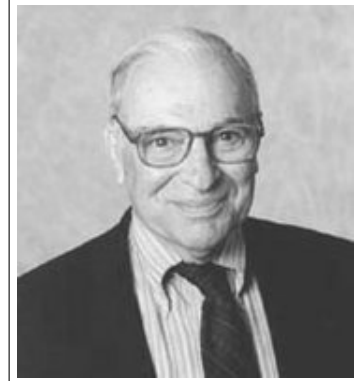
Specifically, we can show that plurality voting violates conditions 3 and 5 when there are three or more candidates. Similarly, since runoff elections are just several plurality votes they also violate conditions 3 and 5. Pairwise voting can violate condition 5 as does the Borda count. We can show that the Borda count violates condition 5 with the following example:

1.  $a > b > c > d$
2.  $b > c > d > a$
3.  $c > d > a > b$
4.  $a > b > c > d$
5.  $b > c > d > a$
6.  $c > d > a > b$
7.  $a > b > c > d$

These list the preferences of 7 agents over 4 choices ( $a, b, c, d$ ). If we applied the Borda count to these agents we would find that  $c$  gets 20 points,  $b$  gets 19,  $a$  gets 18, and  $d$  13. As such,  $c$  wins and  $d$  comes out last. If we then eliminate  $d$  we would then have the following preferences.

1.  $a > b > c$
2.  $b > c > a$
3.  $c > a > b$
4.  $a > b > c$
5.  $b > c > a$
6.  $c > a > b$
7.  $a > b > c$

If we ran Borda on this scenario we would find that  $a$  gets 15 votes,  $b$  gets 14, and  $c$  gets 13, so  $a$  wins. So, originally  $c$  wins and  $d$  comes out last but by eliminating  $d$  we then get that  $a$  wins! This is a violation of condition 5.



Kenneth Joseph Arrow. 1921–. Nobel prize in Economics.

**Table 8.1:** List of individual desires for painting the house.

| Name     | Wants house painted? |
|----------|----------------------|
| Alice    | Yes                  |
| Bob      | No                   |
| Caroline | Yes                  |
| Donald   | Yes                  |
| Emily    | Yes                  |

8.1.2 Voting Summary

In practice we find voting mechanism often used in multiagent systems but without much thought given to which voting mechanism is used. Thus, plurality vote is often used. In general, the Borda count should be the preferred voting mechanism but it does have some drawbacks. Namely, the Borda count requires the agents to generate a complete preference ordering over all items, which could be computationally expensive. For example, if each choice is an allocation of packages that the agent must deliver then the agent must solve a traveling salesman problem for each choice in order to determine how much it would cost him to deliver all those packages.

8.2 Mechanism Design

The mechanism design problem is a well-known problem that has been studied by Economists for some time. The problem is interesting to us because it maps very well to open multiagent systems with selfish agents. Mechanism design asks how we can provide the proper incentives so the agents will want to tell the truth about their preferences. In this chapter we present the standard mechanism design problem as studied in Economics as well as the distributed mechanism design extension which is an even better model for many multiagent system design problems.

8.2.1 Problem Description

Alice lives in a house with four other housemates. They are thinking about paying someone to paint the outside of their house and have decided to hold a vote where everyone will vote either Yes, if they want the house painted, or No if they don't. The votes will be public and the set of people who vote for painting will share equally in the cost of the painters, as long as two or more people vote for painting. The people who voted against painting will pay nothing. We note that, since the paint covers the whole outside of the house everyone will be able to enjoy the new cleaner house. Each person knows whether or not they want the house to be painted. Their desires are shown in Table 8.1.

Alice wants the house painted but does not want to pay for it. She realizes that if only two people voted yes that the house will be painted. As such, she has an incentive to vote against painting (that is, lie about her true preferences) in the hope that some other two agents will vote for it and the house will get painted anyway. This means that Alice's strategy will be to try and determine what the others are likely to vote and see if there are enough Yes votes so that she can safely lie. Unfortunately, all that scheming is very inefficient from a system's perspective. It would be much easier if everyone wanted to tell the truth.

We would like to create a protocol where these types of incentives do not exist. We would like for all agents to want to vote truthfully rather than lie or try to find out how the other agents are going to vote. That way the agents do not waste their resources trying to beat the system and instead use them (hopefully) to work for the system.

Mechanism design (Mas-Colell et al., 1995, Chapter 23) studies how private information can be elicited from individuals. It tells us how to build the proper incentives into our protocols such that the agents will want to tell the truth about their preferences. It also tells us about some circumstances when this is impossible.

| $i$      | $\theta_i$    | $v_i(\text{Paint}, \theta_i)$ | $v_i(\text{NoPaint}, \theta_i)$ |
|----------|---------------|-------------------------------|---------------------------------|
| Alice    | WantPaint     | 10                            | 0                               |
| Bob      | DontWantPaint | 0                             | 0                               |
| Caroline | WantPaint     | 10                            | 0                               |
| Donald   | WantPaint     | 10                            | 0                               |
| Emily    | WantPaint     | 10                            | 0                               |

**Table 8.2:** Values for the house painting problem where  $O = \{\text{Paint}, \text{NoPaint}\}$  and the agents are either of type WantPaint or type DontWantPaint.

More formally, we define a **mechanism design** problem as consisting of a set of agents with the following properties.

MECHANISM DESIGN

- Each agent  $i$  has a **type**  $\theta_i \in \Theta_i$  which is private. That is, only the agent knows its type, no one else does.
- We let  $\theta = \{\theta_1, \theta_2, \dots, \theta_A\}$  be the set of types.
- The **mechanism**  $g$  we are to implement will map from the set of agents' actions to a particular **outcome**  $o \in O$ .
- Each agent  $i$  gets a value  $v_i(o, \theta_i)$  for outcome  $o$ .
- The **social choice function**  $f : \theta \rightarrow O$  tells us the outcome we want to achieve.

TYPE

MECHANISM  
OUTCOME

SOCIAL CHOICE FUNCTION

For example, the social choice function

$$f(\theta) = \arg \max_{o \in O} \sum_{i=1}^n v_i(o, \theta_i) \quad (8.1)$$

is the social welfare solution. It tries to maximize the sum of everyone's utility. You can, however, choose to implement a different social choice function. Other popular choices include minimizing the difference in the agents' utility, maximizing the utility of the agent that receives the highest utility, and the **paretian** social choice function  $f$  such that for all  $\theta$  there is no  $o' \neq o = f(\theta)$  such that some agent  $i$  gets a higher utility from  $o'$  than it would have received under  $f(\theta)$ . That is, there is some  $i$  for which  $v_i(o', \theta_i) > v_i(f(\theta), \theta_i)$  and for all  $i$   $v_i(o', \theta_i) \geq v_i(f(\theta), \theta_i)$ .

PARETIAN

Note also that since the agent's type are usually fixed (an agent cannot change its true type, only lie about it) then the  $v_i$  usually only needs to be defined for the agent's particular  $\theta_i$ .

If we apply this notation to the example from Table 8.1 we get the values shown in Table 8.2. We have that  $\Theta = \{\text{WantPaint}, \text{DontNeedPaint}\}$  since there are only two types of agents: those that want the house painted and those that think it does not need paint. Also,  $O = \{\text{Paint}, \text{NoPaint}\}$  since either the house gets painted or it doesn't. Notice that we had to add some arbitrary number for the agents' utilities for all possible actions. We decided that the agents that want the house painted would get a value of 10 from seeing it painted and 0 if it does not get painted while those who think the house is fine as it is get a value of 0 either way. Lets further assume that we want to maximize social welfare. That is, our social choice function is (8.1). Finally, we assume that the cost of painting the house is 20. We now face the problem of designing a protocol that will decide whether or not to paint the house and how to pay for it.

One possible way to solve this problem is to let all the agents vote Yes or No. We then count the votes and if a majority voted for painting the house then the house will be painted and the cost (20) will be divided evenly among the 5 agents. That is, each agent will have to pay 4 no matter what. This scheme works fine for all agents except for Bob who did not want the house painted and must now pay 4. We are imposing a tax on Bob for something he does not want.

Another way is to let everyone vote Yes or No and then split the cost of painting the house among those who voted Yes. This seems fairer but it has the problem

that it gives all the agents, except Bob, and incentive to lie, as we explained before. They would want to lie in the hopes that someone else would vote Yes and spare them having to pay for it. In general, we are looking for a protocol (mechanism) which implements the social choice function. This idea can be formalized as follows:

**Definition 8.3** (*g Implements f*). A mechanism  $g : S_1 \times \dots \times S_A \rightarrow O$  **implements** social choice function  $f(\cdot)$  if there is an equilibrium strategy profile  $(s_1^*(\cdot), \dots, s_A^*(\cdot))$  of the game induced by  $g$  such that

$$\forall_{\theta} g(s_1^*(\theta_1), \dots, s_A^*(\theta_A)) = f(\theta_1, \dots, \theta_A)$$

where  $s_i(\theta_i)$  is agent  $i$ 's strategy given that it is of type  $\theta_i$ .

The definition might sound a little bit circular but it isn't. Say you start out with a set of agents each one with a type (which you don't know about) then you tell them that you are going to use  $g(\cdot)$  to calculate the final outcome (you tell them how the function does its work). The agents will use their knowledge of  $g$  to figure out their best action and will take that action. You then plug in this set of actions into  $g$  to come up with the outcome. If the outcome is the same as  $f(\theta_1, \dots, \theta_A)$  then you just implemented  $f$ . As you can see, the tricky part is that you have to pick  $g$  such that it will encourage the agents to take the appropriate actions.

Another point of confusion might be that we have changed from types to actions. In the previous examples the agents actions (votes) were merely the revelation of their types (that is, there was a one-to-one mapping from types to actions). However, in general this need not be the case. We could, for example, have a system with 20 different types but only 2 possible actions.

We have also not defined what we mean by an "equilibrium strategy" as used in Definition 8.3. As you will remember from Chapter 3, there are many equilibrium concepts that can be applied to a game. The equilibrium we will concern ourselves with is the dominant strategy equilibrium.<sup>1</sup> A player has a dominant strategy (action) if the agent prefers to use this strategy regardless of what anyone else will do. In our mechanism design problem we formally define a dominant strategy as follows:

**Definition 8.4** (Dominant Strategy Equilibrium). We say that a strategy profile  $(s_1^*(\cdot), \dots, s_A^*(\cdot))$  of the game induced by  $g$  is a **dominant strategy equilibrium** if for all  $i$  and all  $\theta_i$ ,

$$v_i(g(s_i^*(\theta_i), s_{-i}), \theta_i) \geq v_i(g(s'_i, s_{-i}), \theta_i)$$

for all  $s'_i \in S_i$  and all  $s_{-i} \in S_{-i}$ .

We can now specialize Definition 8.3 for dominant equilibria.

**Definition 8.5** (*g Implements f in Dominant Strategies*). A mechanism  $g : S_1 \times \dots \times S_A \rightarrow O$  **implements** social choice function  $f(\cdot)$  **in dominant strategies** if there is a dominant strategy equilibrium strategy profile  $(s_1^*(\cdot), \dots, s_A^*(\cdot))$  of the game induced by  $g$  such that  $g(s_1^*(\theta_1), \dots, s_A^*(\theta_A)) = f(\theta_1, \dots, \theta_A)$  for all  $\theta \in \Theta$ .

Before we go into how to find this magical  $g$  let's explore some simplifications of the problem. The first simplification is one that our first example made. Namely, that the agents' strategies correspond to the revelation of their types. That is, let's assume that the agents' actions are simply type revelations. The only thing an agent can do is say "I am of type  $\theta_x$ " (of course, he could be lying). We call this a **direct revelation mechanism** because the agents directly reveal their types rather than taking an action that might or might not be correlated to their type.

In these cases we would want to design a mechanism  $g$  which implements a social choice function  $f$  and encourages all agents to tell their true type. This might or might not be possible for a particular  $f$ . If it is possible then we say that  $f$  is strategy-proof.

<sup>1</sup>The other popular equilibrium is the Bayesian Nash equilibrium, see (Mas-Colell et al., 1995, Chapter 23.D) for more information.

IMPLEMENTS

DOMINANT STRATEGY  
EQUILIBRIUM

IMPLEMENTS SOCIAL  
CHOICE FUNCTION  $f(\cdot)$  IN  
DOMINANT STRATEGIES

DIRECT REVELATION  
MECHANISM

**Definition 8.6** (Strategy-Proof). *The social choice function  $f(\cdot)$  is **truthfully implementable in dominant strategies** (or **strategy-proof**) if for all  $i$  and  $\theta_i$  we have that  $s_i^*(\theta_i) = \theta_i$  is a dominant strategy equilibrium of the direct revelation mechanism  $f(\cdot)$ . That is, if for all  $i$  and all  $\theta_i \in \Theta_i$ ,*

STRATEGY-PROOF

$$v_i(f(\theta_i, \theta_{-i}), \theta_i) \geq v_i(f(\hat{\theta}_i, \theta_{-i}), \theta_i)$$

for all  $\hat{\theta}_i \in \Theta_i$  and all  $\theta_{-i} \in \Theta_{-i}$ .

That is, the value that each agent gets under the outcome prescribed by the social choice function when all tell the truth is bigger than or equal to the value it gets if it lied about its type. Notice how we have plugged in  $f$  directly as the mechanism (instead of using  $g$ , in other words  $g = f$ ). As you might guess, this would make  $g$  trivial to implement because we are given  $f$ . For example, if I ask you to find a mechanism that implements social function  $f$  and you look at  $f$  and realize that it is strategy-proof then all you have to do is directly use  $f$ . That is, you would ask the agents for their types, they would all tell the truth because telling the truth is their dominant strategy, you would then plug these values into  $f$  and out would come the desired outcome.

These strategy-proof social choice functions make it trivial to find a  $g$  that implements them, namely  $g = f$ . But, we might worry that the particular  $f$  we have been given to implement is not strategy-proof but there might exist some mechanism  $g$  which implements  $f$  in dominant strategies. That is,  $g$  lets the agents take some action (which is different from revealing their type) and uses these actions to come up with the same outcome that  $f$  would have resolved using the agents true types. Furthermore, the actions the agents take are dominant given that they know about  $g$ .

Well we can now stop worrying about finding such a  $g$  as it has been proven that no such  $g$  exists. This is known as the **revelation principle**.

REVELATION PRINCIPLE

**Theorem 8.2** (Revelation Principle). *If there exists a mechanism  $g$  that implements the social choice function  $f$  in dominant strategies then  $f$  is truthfully implementable in dominant strategies.*

That is, for any mechanism that we might have found that uses complex actions we could have found a mechanism that just asks the agents to reveal their types.

### An Example Problem and Solution

Let's now use all this notation in an example. Imagine that you want to sell an item. There are a bunch of prospective buyer agents. You want to sell it to the agent that wants it the most, but you can't trust them to tell you the truth. More formally, we can describe this problem as consisting of the following variables.

- $\theta_i \in \mathbb{R}$ : types are the valuations.
- $o \in \{1, \dots, n\}$ : index of agent who gets the item.
- $v_i(o, \theta_i) = \theta_i$  if  $o = i$ , and 0 otherwise.
- $f(\theta) = \arg \max_i(\theta_i)$
- Each agent gets a  $p_i(o)$  so that  $u_i(o, \theta_i) = v_i(o, \theta_i) + p_i(o)$ .

Given this problem we must now try to figure out how to implement the payments  $p$  as well as how to determine the outcome given the agents' reported types, both of these together constitute the desired mechanism  $g$ . That is, as with most research in mechanism design, we are only interested in mechanism that involve paying or taxing the agents some amount of money in order to change their utility valuation.

After thinking about this problem for a while, you suddenly realize that this is a problem you have already seen. The solution is a Vickrey auction. Set  $p(o)$  such that the agent who wins must pay a tax equal to the second highest valuation. No one else pays/gets anything. This results in the agents having final utilities as follows.

$$u_i(o, \theta_i) = \begin{cases} \theta_i - \max_{j \neq i} \theta_j & \text{if } o = i \\ 0 & \text{otherwise.} \end{cases}$$

That is, you define  $g$  such that it returns an outcome  $o$  which contains the index of the agent that sent you the highest bid (type). Your  $g$  also charges this winning agent an amount equal to the second highest bid and charges everyone else 0.

In fact, we can use the notation we have set up to prove that telling the truth is the dominant strategy in this scenario which, along with the fact that we implement the social choice function, makes the Vickrey auction strategy-proof for this social choice function.

*Truth-Telling is Dominant in Vickrey Payments Example.* We can prove that telling the truth is the dominant strategy by following these steps.

1. Let  $b_i(\theta_i)$  be  $i$ 's bid given that his true valuation is  $\theta_i$ .
2. Let  $b' = \max_{j \neq i} b_j(\theta_j)$  be the highest bid amongst the rest.
3. If  $b' < \theta_i$  then any bid  $b_i(\theta_i) > b'$  is optimal since

$$u_i(i, \theta_i) = \theta_i - b' > 0$$

4. If  $b' > \theta_i$  then any bid  $b_i(\theta_i) < b'$  is optimal since

$$u_i(i, \theta_i) = 0$$

5. Since we have that if  $b' < \theta_i$  then  $i$  should bid  $> b'$  and if  $b' > \theta_i$  then  $i$  should bid  $< b'$ , and we don't know  $b'$  then  $i$  should bid  $\theta_i$ .

□

### The Groves-Clarke Mechanism

We just showed how to check that a mechanism implements a particular social choice function can be truthfully implemented in dominant strategies. However, we did not show how we came up with the mechanism itself (the Vickrey payments). We would like a general formula that can be used to calculate the agents' payments no matter what social choice function is given to us. We do not know of such a formula.

However, if we instead assume that the social function is the social welfare solution and further assume that the agents have quasilinear preferences. That is, their utilities are of the form  $u_i(o, \theta_i) = v_i(o, \theta_i) + p_i(o)$ . In this case the Groves-Clarke mechanism gives us the desired payments (Groves, 1973; Clarke, 1971).

**Theorem 8.3** (Groves-Clarke Mechanism). *If we have a social choice function*

$$f(\theta) = \arg \max_{o \in O} \sum_{i=1}^n v_i(o, \theta_i)$$

*then calculating the outcome using*

$$f(\tilde{\theta}) = \arg \max_{o \in O} \sum_{i=1}^n v_i(o, \tilde{\theta}_i),$$

*where  $\tilde{\theta}$  are reported types, and giving the agents payments of*

$$p_i(\tilde{\theta}) = \sum_{j \neq i} v_j(f(\tilde{\theta}), \tilde{\theta}_j) - h_i(\tilde{\theta}_{-i}), \quad (8.2)$$

*where  $h_i(\theta_{-i})$  is an arbitrary function, results in a strategy-proof mechanism.*



Theodore Groves.



Edward H. Clarke. 1939–.



| Name     | $v_i(o, \tilde{\theta})$ | $v_i(o, \theta) + \sum_{j \neq i} v_j(\tilde{\theta})$ |
|----------|--------------------------|--|
| Alice    | $10 - \frac{20}{4} = 5$  | $5 + 15 = 20$  |
| Bob      | $0 - 0 = 0$              | $0 + 20 = 20$  |
| Caroline | $10 - \frac{20}{4} = 5$  | $5 + 15 = 20$  |
| Donald   | $10 - \frac{20}{4} = 5$  | $5 + 15 = 20$  |
| Emily    | $10 - \frac{20}{4} = 5$  | $5 + 15 = 20$  |

**Table 8.3:** Groves-Clarke payments for house painting assuming that all agents tell the truth.

| Name     | $v_i(o, \tilde{\theta})$           | $v_i(o, \theta) + \sum_{j \neq i} v_j(\tilde{\theta})$ |
|----------|------------------------------------|--|
| Alice    | $0 - 0 = 0$                        | $10 + (\frac{10}{3} \cdot 3) = 20$                     |
| Bob      | $0 - 0 = 0$                        | $0 + (\frac{10}{3} \cdot 3) = 10$                      |
| Caroline | $10 - \frac{20}{3} = \frac{10}{3}$ | $\frac{10}{3} + (\frac{10}{3} \cdot 2) = 10$           |
| Donald   | $10 - \frac{20}{3} = \frac{10}{3}$ | $\frac{10}{3} + (\frac{10}{3} \cdot 2) = 10$           |
| Emily    | $10 - \frac{20}{3} = \frac{10}{3}$ | $\frac{10}{3} + (\frac{10}{3} \cdot 2) = 10$           |

**Table 8.4:** Groves-Clarke payments for house painting assuming that Alice lies and all others tell the truth.

Notice that the payments that  $i$  receives are directly proportional to the sum of everybody else's value. This is the key insight of the Groves-Clarke mechanism. In order to get the agents to tell the truth so that we may improve the social welfare (sum of all utilities) we must pay the agents in proportion to this social welfare. Another way to look at it (perhaps a bit cynically) is to say that the way to get individuals to care about how everyone else is doing is to pay them in proportion to how everyone else is doing. For example, companies give shares of their company to employees in the hope that this will make them want the company as a whole to increase its profits (even if it means they have to work longer or take a pay-cut). In effect, the Groves-Clarke mechanism places the social welfare directly into the agent's utility function.

Lets apply the Groves-Clarke Mechanism to the house painting example from Table 8.2. Remember that the second solution we tried, where the cost of painting was divided among those who voted to paint, was not strategy-proof. Perhaps we can add Groves-Clarke payments to make it strategy-proof. To do this we must first re-evaluate the agents' value which will be decreased from 10 since they might have to pay for part of the painting (if they voted yes) and then calculate the agents' payments using Theorem 8.3. The set of payments the agents would receive if they all told the truth is shown in Table 8.3. As you can see, all the agents get the same utility (20) from telling the truth.

Now, what if Alice lied? Would she get a higher utility? Table 8.4 shows the payments and utility values for the case where Alice lies and the rest tell the truth. As you can see, Alice still gets the same 20 of utility. As such, she has nothing to gain by lying (you should repeat these calculations for Bob to make sure you understand how the equations are used). It is interesting to note how everyone else's utilities have dropped due to Alice's lie. Of course Alice, being purely selfish, does not care about this.

Finally, notice how the payments add up to 80 on the first example and 40 in the second example. Where does this money come from? We have to give the agents real money otherwise it does not work. We cannot simply tell them to imagine I'm giving you \$20. What would be really nice is if some of the agents payed us money and we payed some back to the other agents such that the total amount we pay equals the total amount we receive. This is known as the **revenue equivalence** problem.

**Table 8.5:** VCG payments for house painting assuming Alice tells the truth.

| Name     | $v_i(o, \tilde{\theta})$ | $\sum_{j \neq i} v_j(f(\tilde{\theta}_{-i}), \tilde{\theta}_j)$ | $\sum_{j \neq i} v_j(f(\tilde{\theta}), \tilde{\theta}_j) - \sum_{j \neq i} v_j(f(\tilde{\theta}_{-i}), \tilde{\theta}_j)$ |
|----------|--------------------------|---|--|
| Alice    | $10 - \frac{20}{4} = 5$  | $(10 - \frac{20}{3}) \cdot 3 = 10$                              | $15 - 10 = 5$  |
| Bob      | $0 - 0 = 0$              | $(10 - \frac{20}{4}) \cdot 4 = 20$                              | $20 - 20 = 0$  |
| Caroline | $10 - \frac{20}{4} = 5$  | $(10 - \frac{20}{3}) \cdot 3 = 10$                              | $15 - 10 = 5$  |
| Donald   | $10 - \frac{20}{4} = 5$  | $(10 - \frac{20}{3}) \cdot 3 = 10$                              | $15 - 10 = 5$  |
| Emily    | $10 - \frac{20}{4} = 5$  | $(10 - \frac{20}{3}) \cdot 3 = 10$                              | $15 - 10 = 5$  |

### The Vickrey-Clarke-Groves Mechanism

Another well-known payment mechanism is **Vickrey-Clarke-Groves**, which is just a small variation on Groves-Clarke but is closer to achieving revenue equivalence.

**Theorem 8.4** (Vickrey-Clarke-Groves (VCG) Mechanism). *If*

$$f(\theta) = \arg \max_{o \in O} \sum_{i=1}^n v_i(o, \theta_i)$$

*then calculating the outcome using*

$$f(\tilde{\theta}) = \arg \max_{o \in O} \sum_{i=1}^n v_i(o, \tilde{\theta}_i)$$

*(where  $\tilde{\theta}$  are reported types) and giving the agents payments of*

$$p_i(\tilde{\theta}) = \sum_{j \neq i} v_j(f(\tilde{\theta}), \tilde{\theta}_j) - \sum_{j \neq i} v_j(f(\tilde{\theta}_{-i}), \tilde{\theta}_j) \quad (8.3)$$

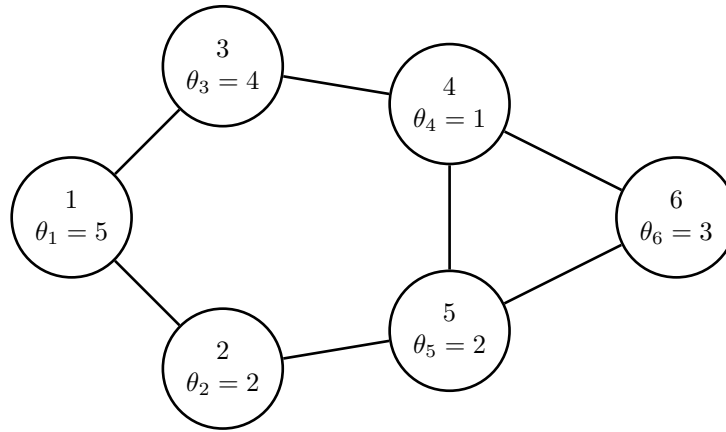
*results in a strategy-proof mechanism.*

VCG is almost identical to Groves-Clarke except that the payments are slightly different. The payments in VCG measure the agent's contribution to the whole, what some call the “wonderful life” utility (Wolpert and Tumer, 2001). You might have seen that old Frank Capra film “It’s a Wonderful life”. In it, George Bailey gets a chance to see how the world would have been had he never existed. He notices that the social welfare in the world without him is lower than in the world with him in it. As such, he calculates that his existence has had a positive effect on social welfare and therefore decides not to end his life, which makes us conclude that he wanted to increase social welfare. Nice guy that George.

That is exactly what the VCG payments calculate. In the first term the  $f(\tilde{\theta})$  captures the outcome had agent  $i$  not existed. So the first term captures the social welfare had  $i$  not existed while the second term captures the social welfare with  $i$  in the picture. I should point out that my experience has been that  $f(\tilde{\theta})$  is, in practice, very hard to compute. For many multiagent systems it is not clear what would happen if we took out one of the agents (imagine a soccer team without a player, or an assembly line without a worker, or workflow without an agent). George had supernatural help. We unfortunately don’t.

The advantage of VCG over Groves-Clarke is that it results in a lower net revenue gain/loss. This can be seen if we re-calculate the payments for the house painting example, as seen in Table 8.5. The payments in this case add up to 20, as compared with 40 and 80 for the Groves-Clarke payments. Of course, 20 is not 0 so we have not yet achieve total revenue equivalence. Economists have many theorems that say that revenue equivalence for this and that case is impossible to achieve. So, it is best to consult their work before trying to come up with a revenue equivalent mechanism.

The big problem with both Groves-Clarke and VCG payments is that calculating these payments takes exponential time on the number of agents. This means that they can only be used when we have a small number of agents.



**Figure 8.2:** Example Inter-domain routing problem. The nodes represent networks. Each one shows the cost it would incur in transporting a packet through its network. The agents' types are their costs.

### 8.2.2 Distributed Mechanism Design

Computer scientists have taken the results from mechanism design and tried to apply them to computer problems. The first extension of mechanism design they came up with is **algorithmic mechanism design** which proposes that the mechanism should run in polynomial time (Nisan and Ronen, 2001). These types of mechanism are sorely needed because the calculation of VCG and Groves-Clarke payments takes exponential time. The second extension was **distributed algorithmic mechanism design** (DAMD) (Feigenbaum et al., 2001) which proposes that the computations carried out by the mechanism should be done by the agents themselves (and in polynomial time, with added limitation to the number of messages that can be sent). In general, distributed mechanism design (Dash et al., 2003) refers to the idea that the mechanism should be carried out by the agents. Note that if the agents themselves are doing the mechanism we must also guard against their interference with the calculation of the outcome. This can be done either by design (for example, encrypting partial results) or by providing the agents with the proper incentives so they will not want to cheat.

Thus far, DAMD algorithms have been almost exclusively developed for computer network applications. Unlike traditional network research, these algorithms treat the individual routers as selfish agents that try to maximize their given utility function. That is, the Internet is no longer viewed as composed of dumb and obedient routers but is instead viewed as composed of utility-maximizing autonomous entities, which is a much better model of reality.

Figure 8.2 shows a sample inter-domain routing problem. In this problem each node represents a computer network. The edges represent how these networks are connected to each other. Each network is willing to handle packets whose source or destination address is within the network. However, they are reticent to carry packets that are just passing thru. Each network in the figure also shows the cost it incurs in routing one of these passing-thru packets. These costs correspond to the agents' types. That is, only the agent knows its cost. We further assume that we know how much traffic will travel from every start node to every destination node. The problem then is to find the routes for each source-destination pair such that the costs are minimized. This is a mechanism design problem which can be solved in a distributed manner (Feigenbaum et al., 2005), as we now show.

We can formally define the problem as consisting of  $n$  agents. Each agent  $i$  knows its cost to be  $\theta_i$ . An outcome  $o$  of our mechanism design problem corresponds to routing rules for all source-destination pairs. That is, the outcome  $o$  should tell us, for all  $i, j$ , which path a packet traveling from  $i$  to  $j$  should follow. The value  $v_i(o)$  that each agent receives for a particular outcome  $o$  is the negative of the total amount of traffic that  $i$  must endure given the paths specified in  $o$  and the traffic amounts between every pair.

ALGORITHMIC MECHANISM  
DESIGN

DISTRIBUTED ALGORITHMIC  
MECHANISM DESIGN

The solution we want is then simply

$$f(\theta) = \arg \max_o \sum_i v_i(o). \quad (8.4)$$

Since this is the social-welfare maximizing solution we are free to use VCG payments. Specifically, we can ask all agents to report their costs  $\tilde{\theta}$  and then use (8.3) to calculate everyone's payments thus ensuring that telling the truth is the dominant strategy for all agents. Note that, in this case, calculating the outcome given  $\tilde{\theta}$  amounts to calculating a minimum spanning tree for each destination node  $j$ . This problem, finding the routes that minimize the sum of all costs, is already solved, in a distributed manner, by the standard Internet Border Gateway Protocol (**BGP**). The protocol works by propagating back the minimum costs to a given destination. All agents tell each other their costs to destination. Each agent then updates its cost to a destination by adding its own transportation costs. The process then repeats until quiescence. It is a variation on Dijkstra's algorithm for finding shortest paths in a graph.

Note, however, that the payment calculation (8.3) requires us to find the sum of everyone else's valuation for the case where  $i$  does not exist (second term in (8.3)). For this problem, eliminating  $i$  simply means eliminating all links to  $i$ . However, it could be that doing this partitions the graph into two or more pieces thereby making it impossible for traffic from some nodes to reach other nodes. In such a case then it would be impossible to calculate the VCG payments. Thus, we must make the further assumption that the network is bi-connected: any one node can be eliminated and still every node can be reached from every other node.

We also note that the VCG payments can be broken down into a sum

$$p_i(\tilde{\theta}) = \sum_{a,b} p_i(a,b,\tilde{\theta}), \quad (8.5)$$

where  $p_i(a,b,\tilde{\theta})$  is the payment that  $i$  receives for the traffic that goes from node  $a$  to node  $b$ . This payment can be further broken down into its constituent VCG parts: the value everyone else gets and the value everyone else would get if  $i$  did not exist. We can find these by first defining the total cost incurred in sending a packet from  $a$  to  $b$ . We let  $lowest-cost-path(a,b,\tilde{\theta})$  be the lowest cost path, a list of nodes, for sending a packet from  $a$  to  $b$  given  $\tilde{\theta}$ . The total cost associated with that path is given by

$$cost(a,b,\tilde{\theta}) = \sum_{i \in lowest-cost-path(a,b,\tilde{\theta})} \tilde{\theta}_i, \quad (8.6)$$

which is simply the sum of the costs in all the intervening nodes. We can then determine that the payment that  $i$  gets is given by

$$p_i(a,b,\tilde{\theta}) = \tilde{\theta}_i - cost(a,b,\tilde{\theta}_i) + cost_{-i}(a,b,\tilde{\theta}_i), \quad (8.7)$$

where  $cost_{-i}(a,b,\tilde{\theta}_i)$  is defined in the same as  $cost$  but for a graph without agent  $i$ . Note how this payment equation is really just the VCG payments again. The first two terms capture the value that everyone else received while the third term capture the value that everyone else receives if  $i$  did not exist (since  $i$  does not exist he does not contribute cost). The main difference is that the signs are reversed since we are dealing with costs and not value.

We now note that since the payments depend solely on the costs of the lowest cost path from  $i$  to  $j$  then some of these costs could be found by adding other costs. For example, if the lowest cost path from  $i$  to  $j$  involves first going from  $i$  to  $a$  directly and then taking the lowest cost path from  $a$  to  $j$  then the  $cost(i,j,\tilde{\theta}) = \tilde{\theta}_i + cost(a,j,\tilde{\theta})$ . A careful analysis of these type of constraints leads us to deduce that for all  $k \in lowest-cost-path(i,j,\tilde{\theta})$  where  $a$  is a neighbor of  $i$ , we have that:

1. If  $a$  is  $i$ 's parent in  $lowest-cost-path(i,j,\tilde{\theta})$  then  $p_k(i,j) = p_k(a,j)$ .

```

INITIALIZE()
1  for  $j \leftarrow 1 \dots n$   $\triangleright$  for each destination  $j$ 
2      do calculate  $\text{lowest-cost-path}(i, j)$  and  $\text{cost}(i, j)$ 
3          for  $k \in \text{lowest-cost-path}(i, j)$ 
4              do  $p_k[i, j] \leftarrow \infty$ 

HANDLE-UPDATE( $a, j, \text{cost-aj}, \text{path}, \text{payments}$ )
     $\triangleright a$  is the agent sending the message (invoking this procedure).
     $\triangleright j$  is the destination node.
     $\triangleright \text{cost-aj}$  is the reported cost of sending a packet from  $a$  to  $j$ .
     $\triangleright \text{path}$  is a list of nodes on the lowest cost path from  $a$  to  $j$ .
     $\triangleright \text{payments}$  are the payments for each node on  $\text{path}$ .
1   $\text{modified} \leftarrow \text{FALSE}$ 
2  if  $a \in \text{lowest-cost-path}(i, j) \triangleright \text{Parent}$ 
3      then for  $k \in \text{path}$ 
4          do if  $p_k[i, j] > p_k[a, j]$ 
5              then  $p_k[i, j] \leftarrow p_k[a, j]$ 
6                   $\text{modified} \leftarrow \text{TRUE}$ 
7  elseif  $i \in \text{lowest-cost-path}(a, j) \triangleright \text{Child}$ 
8      then for  $k \in \{\text{All but last item in path}\}$ 
9          do if  $p_k[i, j] > p_k[a, j] + \tilde{\theta}_a + \tilde{\theta}_i$ 
10             then  $p_k[i, j] \leftarrow p_k[a, j] + \tilde{\theta}_a + \tilde{\theta}_i$ 
11                  $\text{modified} \leftarrow \text{TRUE}$ 
12  else
13       $t \leftarrow \text{position of last node for which path equals lowest-cost-path}(i, j)$ 
14      for  $k \in \text{lowest-cost-path}(i, j)[1..t]$ 
15          do if  $p_k[i, j] > p_k[a, j] + \tilde{\theta}_a + \text{cost-aj} - \text{cost}(i, j)$ 
16              then  $p_k[i, j] \leftarrow p_k[a, j] + \tilde{\theta}_a + \text{cost-aj} - \text{cost}(i, j)$ 
17                   $\text{modified} \leftarrow \text{TRUE}$ 
18      for  $k \in \text{lowest-cost-path}(i, j)[t+1..]$ 
19          do if  $p_k[i, j] > \tilde{\theta}_k + \tilde{\theta}_a + \text{cost-aj} - \text{cost}(i, j)$ 
20              then  $p_k[i, j] \leftarrow \tilde{\theta}_k + \tilde{\theta}_a + \text{cost-aj} - \text{cost}(i, j)$ 
21                   $\text{modified} \leftarrow \text{TRUE}$ 
22  if  $\text{modified}$ 
23      then  $\text{my-payments} \leftarrow p_k[i, j]$  for  $k \in \text{lowest-cost-path}(i, j)$ 
24      for  $b$  is my neighbor
25          do  $b.\text{HANDLE-UPDATE}(i, j, \text{cost}(i, j),$ 
               $\text{lowest-cost-path}[i, j], \text{my-payments})$ 

```

**Figure 8.3:** Algorithm for calculating VCG payments for the inter-domain routing problem.  $i$  refers to the agent running the algorithm. All agents start by executing INITIALIZE and then send a HANDLE-UPDATE to all their neighbors.

2. If  $a$  is  $i$ 's child in  $\text{lowest-cost-path}(i, j, \tilde{\theta})$  then  $p_k(i, j) = p_k(a, j) + \tilde{\theta}_i + \tilde{\theta}_a$ .
3. If  $i$  is neither a parent or a child of  $i$  and  $k \in \text{lowest-cost-path}(a, j, \tilde{\theta})$  then  $p_i(i, j) = \tilde{\theta}_a + \text{cost}(a, j) = \text{cost}(i, j)$ .
4. If  $i$  is neither a parent or a child of  $i$  and  $k \notin \text{lowest-cost-path}(a, j, \tilde{\theta})$  then  $p_k(i, j) = \tilde{\theta}_k + \tilde{\theta}_a + \text{cost}(a, j) - \text{cost}(i, j)$ .

We can then build an algorithm where the agents send each other payment tables and then update their own payment tables using the ones they received along with the above equations. Figure 8.3 shows such an algorithm. All agents start by executing the procedure INITIALIZE and then send HANDLE-UPDATE to their neighbors in order to get the process going. The algorithm has been shown to converge to the true prices for agents.

Note that the calculation of *lowest-cost-path* is itself a distributed calculation that is being carried out simultaneously by the BGP algorithm. This means that the values of *lowest-cost-path* are also changing and, thus, might be incorrect. However, it has been shown that the payment calculations will converge even in the presence of such changes.

### 8.2.3 Mechanism Design Summary

Mechanism design provides us with a solid mathematical framework for expressing many multiagent design problems. Specifically, we can represent problems involving selfish agents who are intelligent enough to lie if it will increase their utility. Furthermore, the Groves-Clarke and VCG payment equations give us a ready-made solution for these problems. Unfortunately, this solution only works if we can implement payments and, even then, the payments are not revenue-neutral so we need an infinite amount of cash to give the agents. Also, these solutions are centralized in that all types are reported to a central agent who is then in charge of calculating and distributing the payments.

Distributed mechanism design is a new research area and very few effective algorithms exist. Still, it holds great promise as it would solve many multiagent design problems.

### 8.2.4 Recent Advances in Mechanism Design

(Larson and Sandholm, 2005) have looked at mechanism design using deliberative agents. They define deliberative agents as those that do not have a priori knowledge of their  $v_i$  function and must instead compute them as needed or gather information needed to compute them. In their words “A deliberative agent is an agent who must compute or gather information in order to determine its preferences, has restrictions on its computing or information gathering capabilities, and who carefully considers how to use its available resources given its restrictions.” They give new definitions for desirable mechanism and then show how these are impossible—all negative results thus far.

(Dash et al., 2004) extend the mechanism design problem by defining trust-based mechanism in which agents take into account the degree of trust they have on each other when determining their allocations.

(Parkes and Shneidman, 2004) take a first look at a distributed implementation of VCG payment computation. They propose to have redundancy. That is, have at least two agents compute every payment and if there is a discrepancy then that payment needs to be computed again. Not very original but at least they provide a clean definition of the problem.

(Monderer and Tennenholtz, 1999) also look at the problem of distributing mechanism design in networks (not fully connected) of agents. They use redundancy to discourage agents from lying and simple encryption (XOR) to discourage agents from modifying others’ messages.

## 8.3 History

(Varian, 1995) was one of the first to propose the use of mechanism design for building multiagent systems.

(Feigenbaum and Shenker, 2002) is a good summary paper listing the most important results in DAMD in general and, specifically, as they pertain to network routing, overlay networks, and P2P file sharing problems. Finally, (Dash et al., 2003) is another introduction to mechanism design and distributed mechanism design.

## Chapter 9

### Coordination Using Goal and Plan Hierarchies

Goal, plan, and policy hierarchies have proven to be very successful methods for coordinating agents. In these approaches we assume the existence of one of these hierarchies and the problem then becomes that of determining which parts of the hierarchy are to be done by which agents. In this setting we assume that the agents are cooperative, that is, they will do exactly what we tell them to. The problem is one of finding a good-enough answer.

Hierarchies like the ones we show here are a way to solve the problem of finding an optimal policy in multiagent MDPs. That is, since the traditional mathematical tools for solving those problems are computationally intractable for real-world problems, researchers hoping to build real-world systems have had to find ways to add more of their domain knowledge into the problem description so as to make it easier to find a solution. In general, the more we tell the agent about the world the easier it is to solve the problems it faces. Unfortunately, if we want to tell the agent more information about its domain we need more sophisticated data structures. In this chapter we describe one of the most popular such data structures: TÆMS. Once we have these new data structures we will generally need new algorithms which take advantage of the new information. The algorithms used for TÆMS are captured in the GPGP framework which we also discuss.

#### 9.1 tæms

**tæms** is a language for representing large task hierarchies that contain complex constraints among tasks. You can think of it as a data structure for representing very complicated constraint optimization problems (Lesser et al., 2004). At its simplest, TÆMS represents a goal hierarchy. As such, TÆMS structures are roughly tree-shaped. The root is the top-level goals that we want the system to achieve. The children are the sub-goals that need to be achieved in order for the top goal to be achieved—a form of divide and conquer. The leaves of the tree are either goals that can be achieved by a single agent or tasks that can be done by an agent. These goals and tasks, however, might require the use of some resources or data. TÆMS represents these requirements with an arrow from the data or resource to the goal.

Figure 9.1 shows a simple example TÆMS structure. It tells us that in order to achieve goal  $G_0$  we must do  $G_1$  and  $G_2$  and  $G_3$ . But, in order to achieve  $G_3$  we must achieve either  $G_{23}$  or  $G_{31}$ . Note also how  $G_{23}$  is a subgoal of both  $G_2$  and  $G_3$ , which makes this a graph instead of a tree. The bottom row shows two data elements and two resources. Data elements are pieces of data that are needed in order to achieve the goals to which they are connected. Resources are consumable resources that are needed by their particular goals. The difference between data and resource is that a piece of data can be re-used as many times as needed while resources are consumed every time they are used and must therefore be replenished at some point. TÆMS also allows one to annotate links to resources to show how much of the resource is needed in order to achieve a particular goal. We can see that  $G_1$  requires  $data_1$  while  $G_{22}$  requires  $data_1$  and  $resource_1$ .

Figure 9.1 also shows **non local effects**, namely the *enables* constraint. The directed link (in red) from  $G_1$  to  $G_{22}$  indicates that  $G_1$  enables  $G_{22}$  which means

TÆMS

NON LOCAL EFFECTS

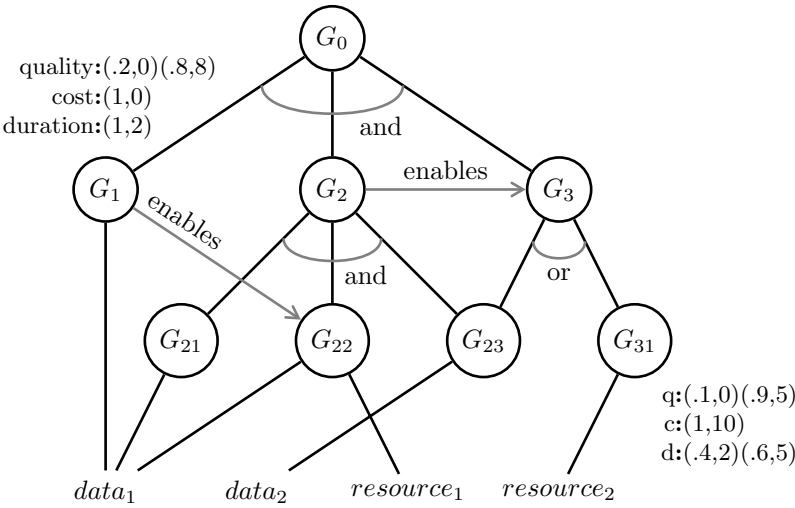


Figure 9.1: An example of a simple TÆMS structure.

Figure 9.2: Some of the quality accumulation functions available for a TÆMS structure.

| Function       | Description   |
|----------------|---|
| $q_{min}$      | minimum quality of all subtasks                               |
| $q_{max}$      | maximum quality of all subtasks                               |
| $q_{sum}$      | aggregate quality of all subtasks                             |
| $q_{last}$     | quality of most recently completed subtask                    |
| $q_{sum\_all}$ | as with $q_{sum}$ but all subtasks must be completed          |
| $q_{seq\_min}$ | as with $q_{min}$ but all subtasks must be completed in order |
| $q_{seq\_max}$ | as with $q_{max}$ but all subtasks must be completed in order |

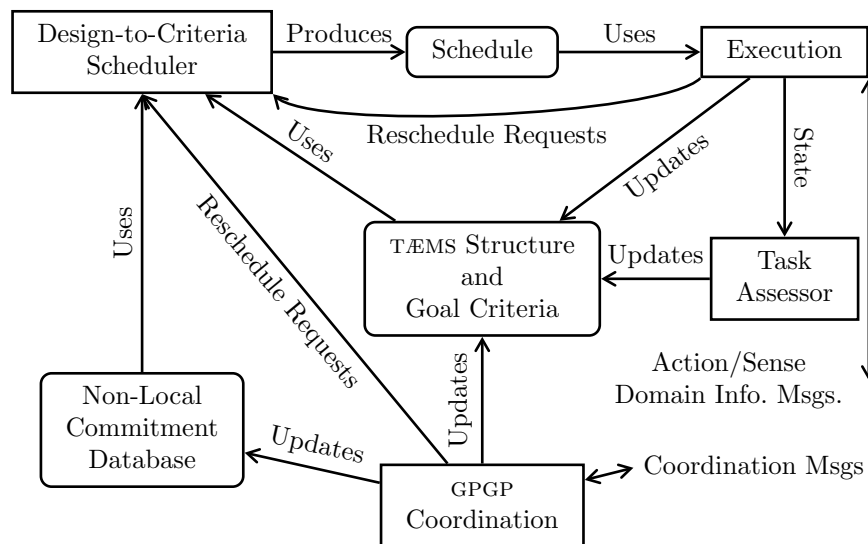
that  $G_1$  must be achieved before we can start to work on  $G_{22}$  if we want  $G_{22}$  to be achieved with quality.

The TÆMS formal model calls for each leaf goal or task to be described by three parameters: quality, cost, and duration. Figure 9.1 shows examples of these parameter values for only two nodes ( $G_1$  and  $G_{31}$ ) due to space constraints. In practice all leaf nodes should have similar values. These parameters represent the quality we expect to get when the goal is achieved (that is, how well it will get done), the cost the system will incur in achieving that goal, and the duration of time required to achieve the goal (or perform the task). The values are probabilistic. For example,  $G_{31}$  will with .1 probability be achieved with a quality of 0 (that is, it will fail to get achieved) while with a probability of .9 it will be achieved with quality of 5.  $G_{31}$  also has a fixed cost of 10 and it will take either 2 time units to finish, with probability .4, or 5 units to finish, with probability .6.

Figure 9.1 shows two ways of aggregating the quality of children nodes: the *and* and *or* functions. These are boolean functions and require us to establish a quality threshold below which we say that a goal has not been achieved. TÆMS usually assumes that a quality of 0 means that the task was not achieved and anything higher means that the task was achieved. For example,  $G_0$  will only be achieved if (because of the *and*)  $G_1$  is achieved, which only happens with probability .8 each time an agent tries.

TÆMS allows us to use many different functions besides simply *and* and *or*. Table 9.2 shows some of the **quality accumulation functions** (qafs) that are defined in TÆMS. Of course, the more different qafs we use the harder it will be, in general, to find a good schedule for the agents. Scheduling algorithms must always limit themselves to a subset of qafs in order to achieve realistic running times.





**Figure 9.3:** GPGP agent architecture. Components are in squares and data structures are in rounded squares.

## 9.2 GPGP

Generalized Partial Global Planning (GPGP) is the complete framework for multiagent coordination which includes, as one of its parts the TÆMS data structure. Over the years there have been many GPGP agent implementations each one slightly different from the others but all sharing some basic features. They have been applied to a wide variety of applications, from airplane repair scheduling, to supply chain management, to distributed sensor networks. The TÆMS data structure and the GPGP approach to coordination via hierarchical task decomposition are among the most successful and influential ideas in multiagent systems.

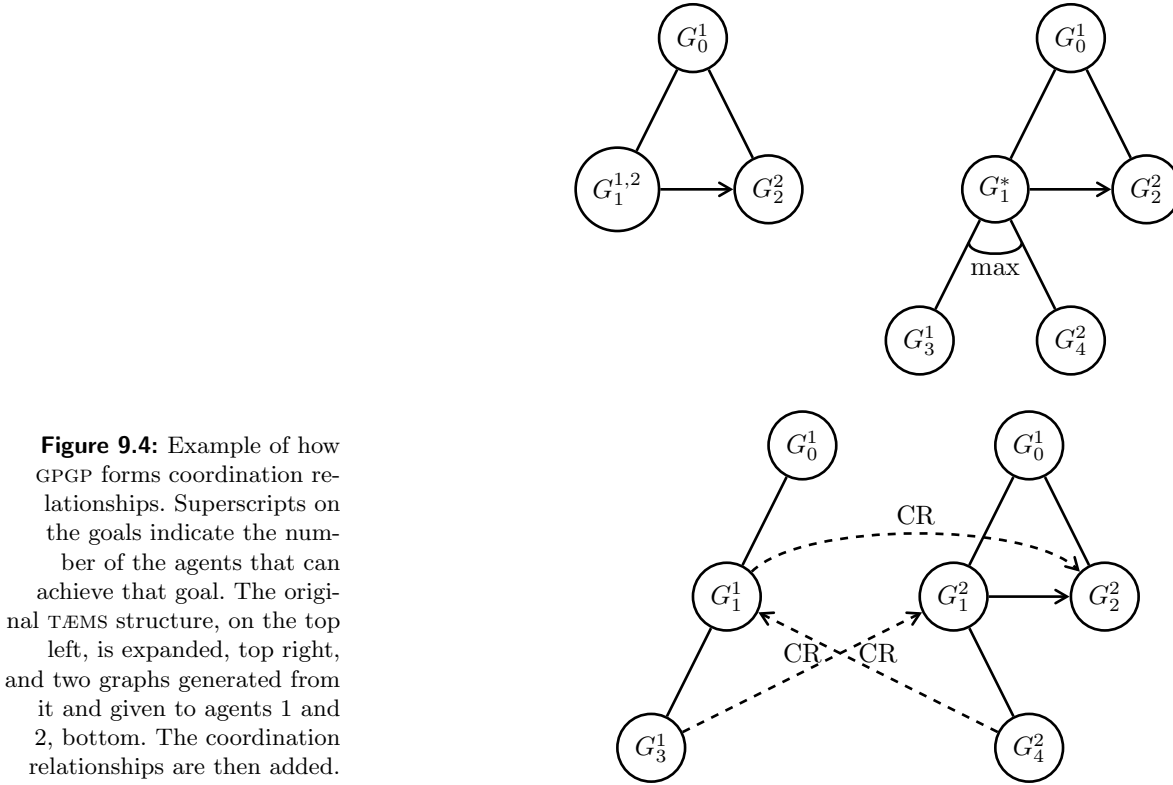
### 9.2.1 Agent Architecture

Figure 9.3 shows a typical agent architecture using GPGP. The red squares are software components. These typically operate in parallel within the agent. The black rounded squares are some of the most important data structures used by a GPGP agent.

The TÆMS data structure lies at the heart of a GPGP agent. This TÆMS structure is also annotated with the goal criteria. That is, it specifies which goals the agent should try to achieve. Remember that the basic TÆMS structure only lists all possible goals, it does not say which agent should try to achieve which goal. The TÆMS structure used by an agent does contain information that tells the agent who is trying to achieve which task. The agent gets this information from the GPGP coordination module which we will explain later.

The design to criteria scheduler is a component (program) which takes as input a TÆMS data structure, along with some commitments the agent has made, and generates a schedule. We will discuss the commitments in more detail later but, basically, the agent has the ability to make commitments to other agents that say that it will achieve a certain goal by a certain time. Once an agent has made a commitment the design to criteria scheduler does everything possible to ensure that the schedule it produces satisfies that commitment. The schedules produced by the scheduler are simply lists of tasks or goals the agent must execute and the times at which the agent will execute them.

The execution module handles plan execution as well as monitoring. That is, it uses the schedule generated by the scheduler and instructs the agent to perform the next task according to the schedule. It then uses the agent's sensors to make sure that the task was accomplished. In the case of a failure, or if it notices that the next task/goal is impossible to achieve in the current world state, it will ask the design to criteria scheduler to generate a new schedule. The execution module also sends



**Figure 9.4:** Example of how GPGP forms coordination relationships. Superscripts on the goals indicate the number of the agents that can achieve that goal. The original TÆMS structure, on the top left, is expanded, top right, and two graphs generated from it and given to agents 1 and 2, bottom. The coordination relationships are then added.

all the information it has about the world and the schedule's execution state to the task assessor module. The task assessor module has domain-specific knowledge and can determine if there are changes that need to be made to the TÆMS structure. For example, it might determine that one of the subgoals is no longer needed because of some specific change in the world. The execution module can also, in a more generalized manner, make changes to the TÆMS structure. These changes are all incorporated into the current TÆMS structure.

### 9.2.2 Coordination

In parallel with all these modules, the GPGP coordination module is always in communication with other agents and tries to keep the agent's commitments and TÆMS structure updated. The GPGP coordination module implements (mostly) domain independent coordination techniques and is the module that tells the agent which of the goals and tasks in its TÆMS structure it should achieve.

Figure 9.4 gives a pictorial view of the general steps the GPGP module follows in order to achieve coordination among agents. The coordination module starts with the TÆMS structure shown on the left. The structure is then modified so that every goal that can be achieved by more than one agent is replaced by a new dummy goal which has one child for every agent that could perform the goal. Each one of the children is a new goal that can only be performed by one child. The children are joined by a max qaf. In Figure 9.4 we see show  $G_1^{1,1}$ , which could be achieved by either 1 or 2 (as denoted by the superscript) is replaced by a new  $G_1$  with two children  $G_3^1$  and  $G_4^2$ .

The resulting graph is then divided among the agents such that each agent has the subgraph with all the goals that the agent can achieve plus all the ancestors of these goals (all the way to the root). This division can be seen in the last two graphs of Figure 9.4.

The GPGP coordination module identifies **coordination relationships (CR)**. These come from two different situations: when a non-local effect in the original graph now starts in one graph and ends in another, or when a non-local effect

or a subtask relationship has one end in one subgraph but the other end in both subgraphs.

Notice that these CRs denote situations where one of the agents needs to either wait for the other to finish or to determine whether the other one has failed. The GPGP module uses some mechanism to remove the uncertainty that these CRs cause. Specifically, it makes the agent commit to do those actions that are at the initiating end of the CR to the other agent. For example, in Figure 9.4 the GPGP module would make agent 1 send to agent 2 the messages: “Commit ( $\mathbf{Do}(G_1)$ )” and “Commit ( $\mathbf{Do}(G_3)$ )”.

### 9.2.3 Design-to-Criteria Scheduler

The design-to-criteria scheduler is a complex piece of software which uses search along with a bunch of heuristics to find a good schedule given the current TÆMS structure and the set of commitments. The scheduler only tries to schedule the first few tasks since it knows that it is likely that the problem will change and it will need to re-schedule. The scheduler also sometimes needs to create temporary commitments and generate schedules using those commitments for the GPGP coordination module. That is, when the GPGP coordination module wants to know how a certain commitment might affect the agent’s other commitments it will ask the design-to-criteria scheduler to create a new schedule using those commitments.

The job of the scheduler is very complicated. It must decide how to satisfy goals in the TÆMS() structure and which ones are likely to generate the maximum utility. As such, the scheduler is a large piece of code which is generally considered a black box by GPGP users. They just feed it the TÆMS structure and out comes a schedule. The scheduler solves this problem by using a number of search heuristics and other specialized techniques.

### 9.2.4 GPGP/tæms Summary

The GPGP/TÆMS framework utilizes some key concepts. It views the problem of coordination as distributed optimization, in fact, it reduces complex problems to something similar to the distributed constraint optimization problem of Chapter 2. It provides a family of coordination mechanism for situation-specific control. It uses TÆMS to provide proven domain-independent representation of agent tasks, with support for multiple goals of varying worth and different deadline. It also uses quantitative coordination relationships among tasks.

The general approach of GPGP has been widely successful and has proven itself in many domains. The TÆMS structure has been adopted not just as a programming tools but also as a design tool to be used to better understand the problem. However, one problem with GPGP has been that it has required modifications for each one of its applications. As such, GPGP is best thought of as a general approach to designing multiagent systems rather than as a specific tool/architecture.



## **Chapter 10**

### **Nature-Inspired Approaches**

(Abelson et al., 2000), (Nagpal, 2002)

#### **10.1 Ants and Termites**

(Kube and Bonabeau, 2000), (Bonabeau et al., 1999), (Parunak et al., 2002), (Sauter et al., 2002), (Parunak, 1997)

#### **10.2 Immune System**

(Kephart and White, 1991), (Kephart, 1994)

#### **10.3 Physics**

(Cheng et al., 2005)



## Bibliography

- Abelson, H., Allen, D., Coore, D., Hanson, C., Homsy, G., Knight, T. F., Nagpal, R., Rauch, E., Sussman, G. J., and Weiss, R. (2000). Amorphous computing. *Communications of the ACM*, 43(5):74–82.
- Aknine, S., Pinson, S., and Shakun, M. F. (2004). An extended multi-agent negotiation protocol. *Autonomous Agents and Multi-Agent Systems*, 8(1):5–45.
- Andersson, M. R. and Sandholm, T. (1998). Contract types for satisficing task allocation: Ii experimental results. In *AAAI Spring Symposium: Satisficing Models*.
- Axelrod, R. M. (1984). *The Evolution of Cooperation*. Basic Books.
- Bessière, C., Maestre, A., Brito, I., and Meseguer, P. (2005). Asynchronous backtracking without adding links: a new member in the ABT family. *Artificial Intelligence*, 161:7–24.
- Binmore, K. and Vulkan, N. (1999). Applying game theory to automated negotiation. *Netnomics*, 1(1):1–9.
- Bomze, I. M. (1986). Noncooperative two-person games in biology: A classification. *International Journal of Game Theory*, 15(1):31–57.
- Bonabeau, E., Dorigo, M., and Theraulaz, G. (1999). *Swarm Intelligence: From Natural to Artificial Systems*. Oxford.
- Brooks, C. H. and Durfee, E. H. (2003). Congregation formation in multiagent systems. *Journal of Autonomous Agents and Multi-agent Systems*, 7(1–2):145–170.
- Cheng, J., Cheng, W., and Nagpal, R. (2005). Robust and self-repairing formation control for swarms of mobile agents. In *Proceedings of the Twentieth National Conference on Artificial Intelligence*, pages 59–64, Menlo Park, California. AAAI Press.
- Clarke, E. H. (1971). Multipart pricing of public goods. *Public Choice*, 11(1).
- Conen, W. and Sandholm, T. (2001a). Minimal preference elicitation in combinatorial auctions. In *Proceedings of the International Joint Conference of Artificial Intelligence Workshop on Economic Agents, Models, and Mechanisms*.
- Conen, W. and Sandholm, T. (2001b). Preference elicitation in combinatorial auctions. In *Proceedings of the Third ACM conference on Electronic Commerce*, pages 256–259, New York, NY, USA. ACM Press.
- Conen, W. and Sandholm, T. (2002). Partial-revelation VCG mechanism for combinatorial auctions. In *Proceedings of the National Conference on Artificial Intelligence*, pages 367–372.
- Conitzer, V. and Sandholm, T. (2003). AWESOME: A general multiagent learning algorithm that converges in self-play and learns a best response against stationary opponents. In *Proceedings of the Twentieth International Conference on Machine Learning*.

- Conitzer, V. and Sandholm, T. (2006). AWESOME: A general multiagent learning algorithm that converges in self-play and learns a best response against stationary opponents. *Machine Learning*.
- Cramton, P., Shoham, Y., and Steinberg, R., editors (2006). *Combinatorial Auctions*. MIT Press.
- Dang, V. D. and Jennings, N. (2004). Generating coalition structures with finite bound from the optimal guarantees. In *Proceedings of the Third International Joint Conference on Autonomous Agents and MultiAgent Systems*, pages 564–571. ACM.
- Dash, R., Ramchurn, S., and Jennings, N. (2004). Trust-based mechanism design. In *Proceedings of the Third International Joint Conference on Autonomous Agents and MultiAgent Systems*, pages 748–755. ACM.
- Dash, R. K., Jennings, N. R., and Parkes, D. C. (2003). Computational mechanism design: A call to arms. *IEEE Intelligent Systems*, 18(6):40–47.
- Davin, J. and Modi, P. J. (2005). Impact of problem centralization in distributed constraint optimization algorithms. In (Dignum et al., 2005), pages 1057–1066.
- Davis, R. and Smith, R. G. (1983). Negotiation as a metaphor for distributed problem solving. *Artificial Intelligence*, 20:63–109.
- Delgado, J. (2002). Emergence of social conventions in complex networks. *Artificial Intelligence*, 141:171–185.
- Dignum, F., Dignum, V., Koenig, S., Kraus, S., Pechoucek, M., Singh, M., Steiner, D., Thompson, S., and Wooldridge, M., editors (2005). *Proceedings of the Fourth International Joint Conference on Autonomous Agents and Multi Agent Systems*, New York, NY. ACM Press.
- Durfee, E. H. (1999). Practically coordinating. *AI Magazine*, 20(1):99–116.
- Endriss, U., Maudet, N., Sadri, F., and Toni, F. (2006). Negotiating socially optimal allocations of resources. *Journal of Artificial Intelligence Research*, 25:315–348.
- Fagin, R., Halpern, J. Y., Moses, Y., and Vardi, M. Y. (1995). *Reasoning About Knowledge*. The MIT Press, Cambridge, MA.
- Faratin, P., Sierra, C., and Jennings, N. R. (1998). Negotiation decision functions for autonomous agents. *Robotics and Autonomous Systems*, 24(3–4):159–182.
- Fatima, S. S., Wooldridge, M., and Jennings, N. (2004). Optimal negotiation of multiple issues in incomplete information settings. In *Proceedings of the Third International Joint Conference on Autonomous Agents and MultiAgent Systems*, pages 1080–1089. ACM.
- Feigenbaum, J., Papadimitriou, C., Sami, R., and Shenker, S. (2005). A BGP-based mechanism for lowest-cost routing. *Distributed Computing*.
- Feigenbaum, J., Papadimitriou, C. H., and Shenker, S. (2001). Sharing the cost of multicast transmissions. *Journal of Computer and System Sciences*, 63(1):21–41.
- Feigenbaum, J. and Shenker, S. (2002). Distributed algorithmic mechanism design: Recent results and future directions. In *Proceedings of the 6th International Workshop on Discrete Algorithms and Methods for Mobile Computing and Communications*, pages 1–13. ACM Press, New York.
- Fudenberg, D. and Kreps, D. (1990). Lectures on learning and equilibrium in strategic-form games. Technical report, CORE Lecture Series.



- Fudenberg, D. and Levine, D. K. (1998). *The Theory of Learning in Games*. MIT Press.
- Fujishima, Y., Leyton-Brown, K., and Shoham, Y. (1999). Taming the computational complexity of combinatorial auctions: Optimal and approximate approaches. In *Proceedings of the Sixteenth International Joint Conference on Artificial Intelligence*, pages 548–553. Morgan Kaufmann Publishers Inc.
- Gmytrasiewicz, P. J. and Durfee, E. H. (1995). A rigorous, operational formalization of recursive modeling. In *Proceedings of the First International Conference on Multi-Agent Systems*, pages 125–132. AAAI/MIT press.
- Gmytrasiewicz, P. J. and Durfee, E. H. (2001). Rational communication in multi-agent systems. *Autonomous Agents and Multi-Agent Systems Journal*, 4(3):233–272.
- Groves, T. (1973). Incentives in teams. *Econometrica*, 41(4):617–631.
- Harford, T. (2005). *The Undercover Economist: Exposing Why the Rich Are Rich, the Poor Are Poor—and Why You Can Never Buy a Decent Used Car!* Oxford University Press.
- Harsanyi, J. C. (1965). Approaches to the bargaining problem before and after the theory of games: A critical discussion of zeuthen’s, hicks’, and nash’s theories. *Econometrica*, 24(2):144–157.
- Hirayama, K. and Yokoo, M. (2005). The distributed breakout algorithms. *Artificial Intelligence*, 161(1–2):89–115.
- Hu, J. and Wellman, M. P. (2003). Nash q-learning for general-sum stochastic games. *Journal of Machine Learning Research*, 4:1039–1069.
- Hudson, B. and Sandholm, T. (2004). Effectiveness of query types and policies for preference elicitation in combinatorial auctions. In (Jennings et al., 2004).
- Jennings, N. R., Faratin, P., Lomuscio, A. R., Parsons, S., Wooldridge, M., and Sierra, C. (2001). Automated negotiation: Prospects methods and challenges. *Group Decision and Negotiation*, 10(2):199–215.
- Jennings, N. R., Sierra, C., Sonenberg, L., and Tambe, M., editors (2004). *Proceedings of the Third International Joint Conference on Autonomous Agents and Multi Agent Systems*, New York, NY. ACM Press.
- Jiang, H. and Vidal, J. M. (2005). Reducing redundant messages in the asynchronous backtracking algorithm. In *Proceedings of the Sixth International Workshop on Distributed Constraint Reasoning*.
- Jiang, H. and Vidal, J. M. (2006). The message management asynchronous backtracking algorithm. *Journal of Experimental and Theoretical Artificial Intelligence*. To appear.
- Kalai, E. and Smorodinsky, M. (1975). Other solutions to nash’s bargaining problem. *Econometrica*, 43:513–518.
- Kelly, F. and Stenberg, R. (2000). A combinatorial auction with multiple winners for universal service. *Management Science*, 46(4):586–596.
- Kephart, J. O. (1994). A biologically inspired immune system for computers. In Brooks, R. A. and Maes, P., editors, *Proceedings of the Fourth International Workshop on Synthesis and Simulation of Living Systems*, pages 130–139. MIT Press, Cambridge, MA.

- Kephart, J. O. and White, S. R. (1991). Directed-graph epidemiological models of computer viruses. In *Proceedings of the IEEE Computer Society Symposium on Research in Security and Privacy*, pages 343–359.
- Klein, M., Faratin, P., Sayama, H., and Bar-Yam, Y. (2003). Negotiating complex contracts. *Group Decision and Negotiation*, 12:111–125.
- Kraus, S. (2001). *Strategic Negotiation in Multiagent Environments*. MIT Press.
- Kube, C. R. and Bonabeau, E. (2000). Cooperative transport by ants and robots. *Robotics and Autonomous Systems*, pages 85–101.
- Larson, K. and Sandholm, T. (2005). Mechanism design and deliberative agents. In (Dignum et al., 2005), pages 650–656.
- Lesser, V., Decker, K., Wagner, T., Carver, N., Garvey, A., Horling, B., Neiman, D., Podorozhny, R., Prasad, M. N., Raja, A., Vincent, R., Xuan, P., and Zhang, X. Q. (2004). Evolution of the GPGP/TAEMS domain-independent coordination framework. *Autonomous Agents and Multi-Agent Systems*, 9:87–143.
- Leyton-Brown, K., Pearson, M., and Shoham, Y. (2000). Towards a universal test suite for combinatorial auction algorithms. In *Proceedings of the 2nd ACM conference on Electronic commerce*, pages 66–76. ACM Press. <http://cats.stanford.edu>.
- Littman, M. L. (2001). Friend-or-foe q-learning in general-sum games. In *Proceedings of the Eighteenth International Conference on Machine Learning*, pages 322–328. Morgan Kaufmann.
- MacKie-Mason, J. K. and Varian, H. R. (1994). Generalized vickrey auctions. Technical report, University of Michigan.
- Mailler, R. T. and Lesser, V. (2004a). Solving distributed constraint optimization problems using cooperative mediation. In (Jennings et al., 2004), pages 438–445.
- Mailler, R. T. and Lesser, V. (2004b). Using cooperative mediation to solve distributed constraint satisfaction problems. In (Jennings et al., 2004), pages 446–453.
- Mas-Colell, A., Whinston, M. D., and Green, J. R. (1995). *Microeconomic Theory*. Oxford.
- McKelvey, R. D., McLennan, A. M., and Turocy, T. L. (2006). Gambit: Software tools for game theory. Technical report, Version 0.2006.01.20.
- Minton, S., Johnston, M. D., Philips, A. B., and Laird, P. (1992). Minimizing conflicts: a heuristic repair method for constraint satisfaction and scheduling problems. *Artificial Intelligence*, 58:161–205.
- Mitchell, T. M. (1997). *Machine Learning*. McGraw Hill.
- Modi, P. J., Shen, W.-M., Tambe, M., and Yokoo, M. (2005). Adopt: Asynchronous distributed constraint optimization with quality guarantees. *Artificial Intelligence*, 16(1–2):149–180.
- Monderer, D. and Tennenholtz, M. (1999). Distributed games: From mechanisms to protocols. In *Proceedings of the Sixteenth National Conference on Artificial Intelligence*, pages 32–37. AAAI Press. Menlo Park, Calif.
- Muthoo, A. (1999). *Bargaining Theory with Applications*. Cambridge University Press.

- Nagpal, R. (2002). Programmable self-assembly using biologically-inspired multi-agent control. In *Proceedings of the 1st International Joint Conference on Autonomous Agents and Multi-Agent Systems*.
- Nash, J. F. (1950). The bargaining problem. *Econometrica*, 18:155–162.
- Neumann, J. V. (1928). Zur theorie der gesellschaftsspiele. *Mathematische Annalen*, 100(1):295–320.
- Neumann, J. V. and Morgenstern, O. (1944). *Theory of Games and Economic Behavior*. Princeton University Press.
- Nguyen, T. D. and Jennings, N. (2004). Coordinating multiple concurrent negotiations. In *Proceedings of the Third International Joint Conference on Autonomous Agents and MultiAgent Systems*, pages 1064–1071. ACM.
- Nisan, N. (2000). Bidding and allocation in combinatorial auctions. In *Proceedings of the ACM Conference on Electronic Commerce*, pages 1–12.
- Nisan, N. and Ronen, A. (2001). Algorithmic mechanism design. *Games and Economic Behavior*, 35:166–196.
- Nudelman, E., Wortman, J., Leyton-Brown, K., and Shoham, Y. (2004). Run the GAMUT: A comprehensive approach to evaluating game-theoretic algorithms. In (Jennings et al., 2004).
- Osborne, M. J. and Rubinstein, A. (1990). *Bargaining and Markets*. Academic Press.
- Osborne, M. J. and Rubinstein, A. (1999). *A Course in Game Theory*. MIT Press.
- Parkes, D. C. and Shneidman, J. (2004). Distributed implementations of vickrey-clarke-groves auctions. In *Proceedings of the Third International Joint Conference on Autonomous Agents and MultiAgent Systems*, pages 261–268. ACM.
- Parunak, H. V. D. (1997). “go to the ant”: Engineering principles from natural agent systems. *Annals of Operation Research*, 75:69–101.
- Parunak, H. V. D., Brueckner, S., and Sauter, J. (2002). Synthetic pheromone mechanisms for coordination of unmanned vehicles. In *Proceedings of the First International Joint Conference on Autonomous Agents and Multiagent Systems*, pages 448–450, Bologna, Italy. ACM Press, New York, NY.
- Petcu, A. and Faltings, B. (2005). A scalable method for multiagent constraint optimization. In *Proceedings of the International Joint Conference on Artificial Intelligence*, pages 266–271, Edinburgh, Scotland.
- Petcu, A. and Faltings, B. (2006). ODPOP: An algorithm for open/distributed constraint optimization. In *Proceedings of the National Conference on Artificial Intelligence, AAAI-06*, pages 703–708, Boston, USA.
- Resnick, M. (1994). *Turtles, Termites, and Traffic Jams: Explorations in Massively Parallel Microworlds*. MIT Press.
- Rosenschein, J. S. and Zlotkin, G. (1994). *Rules of Encounter*. The MIT Press, Cambridge, MA.
- Rothkopf, M. H., Pekec, A., and Harstad, R. M. (1998). Computationally manageable combinational auctions. *Management Science*, 44(8):1131–1147.
- Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. *Econometrica*, 50(1):97–110.

- Russell, S. and Norvig, P. (2003). *Artificial Intelligence: A Modern Approach*. Prentice Hall, second edition.
- Sandholm, T. (1997). Necessary and sufficient contract types for optimal task allocation. In *Proceedings of the Fourteenth International Joint Conference on Artificial Intelligence*.
- Sandholm, T. (1998). Contract types for satisficing task allocation: I theoretical results. In *AAAI Spring Symposium: Satisficing Models*.
- Sandholm, T. (1999). An algorithm for winner determination in combinatorial auctions. In *Proceedings of the Sixteenth International Joint Conference on Artificial Intelligence*, pages 542–547.
- Sandholm, T. (2002). An algorithm for winner determination in combinatorial auctions. *Artificial Intelligence*, 135(1-2):1–54.
- Sandholm, T., Larson, K., Anderson, M., Shehory, O., and Tohmé, F. (1999). Coalition structure generation with worst case guarantees. *Artificial Intelligence*, 111(1-2):209–238.
- Sandholm, T. and Lesser, V. (2002). Leveled-commitment contracting: A backtracking instrument for multiagent systems. *AI Magazine*, 23(3):89–100.
- Sandholm, T., Suri, S., Gilpin, A., and Levine, D. (2005). CABOB: a fast optimal algorithm for winner determination in combinatorial auctions. *Management Science*, 51(3):374–391.
- Sandholm, T. and Zhou, Y. (2002). Surplus equivalence of leveled commitment contracts. *Artificial Intelligence*, 142(2):239–264.
- Sauter, J. A., Matthews, R., Parunak, H. V. D., and Brueckner, S. (2002). Evolving adaptive pheromone path planning mechanisms. In *Proceedings of the First International Joint Conference on Autonomous Agents and Multiagent Systems*, pages 434–440, Bologna, Italy. ACM Press, New York, NY.
- Sen, S. (2002). Believing others: Pros and cons. *Artificial Intelligence*, 142(2):179–203.
- Shehory, O. and Kraus, S. (1998). Methods for task allocation via agent coalition formation. *Artificial Intelligence*, 101(1-2):165–200.
- Shoham, Y. and Tennenholtz, M. (1997). On the emergence of social conventions: modeling, analysis, and simulations. *Artificial Intelligence*, 94:139–166.
- Smith, R. G. (1981). The contract net protocol: High-level communication and control in a distributed problem solver. *IEEE Transactions on Computers*, C-29(12):1104–1113.
- Squires, S. (2005). *Roving Mars: Spirit, Opportunity, and the Exploration of the Red Planet*. Hyperion.
- Stone, P. (2000). *Layered Learning in Multiagent Systems*. MIT Press.
- Stone, P. and Weiss, G., editors (2006). *Proceedings of the Fifth International Joint Conference on Autonomous Agents and Multi Agent Systems*, New York, NY. ACM Press.
- Sutton, R. S. and Barto, A. G. (1998). *Reinforcement Learning: An Introduction*. MIT Press.
- Taylor, P. and Jonker, L. (1978). Evolutionary stable strategies and game dynamics. *Mathematical Biosciences*, 16:76–83.

- Tumer, K. and Wolpert, D., editors (2004). *Collectives and the Design of Complex Systems*. Springer.
- Varian, H. R. (1995). Economic mechanism design for computerized agents. In *Proceedings of the First USENIX Workshop on Electronic Commerce*.
- Vidal, J. M. (2006). Multiagent coordination using a distributed combinatorial auction. In *AAAI Workshop on Auction Mechanism for Robot Coordination*.
- Vidal, J. M. and Durfee, E. H. (1998a). Learning nested models in an information economy. *Journal of Experimental and Theoretical Artificial Intelligence*, 10(3):291–308.
- Vidal, J. M. and Durfee, E. H. (1998b). The moving target function problem in multi-agent learning. In *Proceedings of the Third International Conference on Multi-Agent Systems*, pages 317–324. AAAI/MIT press.
- Vidal, J. M. and Durfee, E. H. (2003). Predicting the expected behavior of agents that learn about agents: the CLRI framework. *Autonomous Agents and Multi-Agent Systems*, 6(1):77–107.
- Waltz, D. (1975). Understanding line drawings of scenes with shadows. In Winston, P., editor, *The Psychology of Computer Vision*, pages 19–91. McGraw-Hill.
- Watkins, C. J. C. H. and Dayan, P. (1992). Q-learning. *Machine Learning*, 8(3-4):279–292.
- Wellman, M. P. (1996). Market-oriented programming: Some early lessons. In Clearwater, S., editor, *Market-Based Control: A Paradigm for Distributed Resource Allocation*. World Scientific.
- Wilensky, U. (1999). NetLogo: Center for connected learning and computer-based modeling, Northwestern University. Evanston, IL. <http://ccl.northwestern.edu/netlogo/>.
- Willer, D., editor (1999). *Network Exchange Theory*. Praeger Publishers, Westport CT.
- Wolpert, D. and Tumer, K. (2001). Optimal payoff functions for members of collectives. *Advances in Complex Systems*, 4(2–3):265–279.
- Wolpert, D., Wheeler, K., and Tumer, K. (1999). General principles of learning-based multi-agent systems. In *Proceedings of the Third International Conference on Autonomous Agents*, pages 77–83, Seattle, WA.
- Wolpert, D. H. and Macready, W. G. (1995). No free lunch theorems for search. Technical report, Santa Fe Institute.
- Wolpert, D. H. and Tumer, K. (1999). An introduction to collective intelligence. Technical report, NASA. NASA-ARC-IC-99-63.
- Wurman, P. R., Wellman, M. P., and Walsh, W. E. (2002). Specifying rules for electronic auctions. *AI Magazine*, 23(3):15–23.
- Yokoo, M., Conitzer, V., Sandholm, T., Ohta, N., and Iwasaki, A. (2005). Coalition games in open anonymous environments. In *Proceedings of the Twentieth National Conference on Artificial Intelligence*, pages 509–514, Menlo Park, California. AAAI Press.
- Yokoo, M., Durfee, E. H., Ishida, T., and Kuwabara, K. (1998). The distributed constraint satisfaction problem: Formalization and algorithms. *IEEE Transactions on Knowledge and Data Engineering*, 10(5):673–685.

- Yokoo, M. and Hirayama, K. (1996). Distributed breakout algorithm for solving distributed constraint satisfaction problems. In *Proceedings of the Second International Conference on Multiagent Systems*, pages 401–408.
- Yokoo, M. and Hirayama, K. (2000). Algorithms for distributed constraint satisfaction: A review. *Autonomous Agents and Multi-Agent Systems*, 3(2):185–207.
- Zhang, X., Lesser, V., and Abdallah, S. (2005a). Efficient management of multi-linked negotiation based on a formalized model. *Autonomous Agents and Multi-Agent Systems*, 10(2).
- Zhang, X., Lesser, V., and Podorozhny, R. (2005b). Multi-dimensional, multistep negotiation for task allocation in a cooperative system. *Autonomous Agents and Multi-Agent Systems*, 10(1):5–40.

## Index

- NASHQ-LEARNING, 68
- TÆMS, 135
- 1-sided proposal, 96
  
- ABT, 24
- additive cost function, 90
- additive valuation, 115
- ADEPT, 87
- Adopt, 32
- agenda, 94
- AI planning, 15
- algorithmic mechanism design, 131
- annealing, 95
- APO, 37
- appeals, 97
- argument-based protocols, 96
- aristocrat utility, 75
- asynchronous backtracking, 24
- Asynchronous Weak-Commitment, 27
- atomic bids, 115
- AWESOME, 65
  
- backtracking, 24
- bargaining problem, 77
- battle of the sexes, 42
- belief state, 15
- Bellman equation, 13
- Bellman update, 13
- BGP, 132
- bidder collusion, 106
- Bidtree, 110
- branch and bound, 32
- branch on bids, 110
- Byzantine generals problem, 39
  
- CABOB, 111
- CASS, 111
- CATS, 112
- characteristic form game, 49
- characteristic function, 49
- CLRI model, 70
- coalition formation, 56
- coalition game, 49
- coalition structure, 50
- COIN, 73
- combinatorial auction, 107
- common knowledge, 39
- common value, 103
- concurrent negotiations, 97
  
- constraint satisfaction problem, 19
- contract net protocol, 89
- cooperative games, 49
- coordination problem, 42
- coordination relationships, 138
- core, 50
- correlated value, 103
- counter-proposal, 96
- critique, 96
- CSP, 19
  
- DCSP, 20
- de-commit, 92
- decision version, 108
- depth first search, 20
- descriptive approach, 98
- deterministic, 12
- direct revelation mechanism, 126
- discount factor, 12
- discounted rewards, 12
- distributed algorithmic mechanism design,  
131
- distributed breakout, 29
- distributed constraint optimization, 31
- distributed constraint satisfaction, 20
- divide and conquer, 16
- dominant, 40
- dominant strategy equilibrium, 126
- double auction, 105
- Dutch, 104
- dynamic programming, 14
  
- EBF algorithm, 117
- egalitarian social welfare, 79
- egalitarian solution, 78
- English auction, 104
- envy-free, 113
- equal excess, 54
- equi-resistance point, 99
- excess, 53
- expected utility, 11
- exploration rate, 67
- extended form games, 44
  
- factored, 74
- feasible, 50
- fictitious play, 61
- filtering algorithm, 21
- first-price open-cry ascending, 104

- First-price sealed-bid, 104
- folk theorem, 44
- free disposal, 116
- Gambit, 46
- game of chicken, 42
- hierarchical learning, 16
- hill climbing, 90
- hyper-resolution rule, 23
- implements, 126
- implements social choice function  $f(\cdot)$  in
  - dominant strategies, 126
- individual hill-climbing, 114
- induction, 60
- induction bias, 60
- inefficient allocation, 106
- iterated dominance, 40
- iterated equi-resistance algorithm, 99
- iterated prisoner's dilemma, 43
- justify, 97
- $k$ -consistency, 22
- Kalai-Smorodinsky solution, 80
- kernel ABT, 27
- learning rate, 67
- leveled commitment contracts, 93
- lying auctioneer, 106
- machine learning, 59
- marginal utility, 10
- Markov decision process, 11
- maximum expected utility, 12
- maxmin strategy, 40
- mechanism, 125
- mechanism design, 125
- minimax theorem, 40
- mixed strategy, 40
- monetary payments, 89
- monotonic concession protocol, 83
- moving target function, 60
- Multi-DB<sup>++</sup>, 31
- multi-dimensional deal, 93
- n-level, 72
- n-queens problem, 20
- Nash bargaining solution, 80
- Nash equilibrium, 41
- Nash equilibrium point, 68
- negotiation network, 97
- no free lunch theorem, 60
- non local effects, 135
- non-cooperative games, 39
- non-deterministic, 12
- normal form, 39
- nucleolus, 53
- observation model, 15
- OCSM contracts, 91
- opacity, 74
- open-cry descending price, 104
- optAPO, 37
- optimal policy, 12
- OR bids, 115
- OR\* bids, 115
- outcome, 49, 125
- pairwise, 122
- PAR algorithm, 116
- paretian, 125
- Pareto frontier, 117
- pareto frontier, 78
- Pareto optimal, 41
- partially observable MDP, 15
- PAUSE, 113
- pausebid, 113
- payoff matrix, 39
- persuade, 97
- plurality vote, 121
- policy, 12
- postman problem, 88
- preference ordering, 9
- Prisoner's Dilemma, 41
- private value, 103
- pure strategy, 40
- Q-learning, 67
- quality accumulation functions, 136
- quasi-local minimum, 29
- rational, 40
- reciprocity, 47
- reinforcement learning, 67
- replicator dynamics, 63
- reservation price, 104
- resistance, 98
- revelation principle, 127
- revenue equivalence, 129
- rewards, 97
- Rubinstein's alternating offers, 81
- runoff, 121
- second-price sealed-bid, 104
- self-enforce collusion, 106
- selfish, 10
- Shapley value, 52
- social choice function, 125
- social law, 46
- social welfare, 40
- strategic form, 39
- strategy, 40
- strategy-proof, 127
- subgame perfect equilibrium, 45
- super-additive, 50



tactics, 87  
task allocation problem, 87  
testing set, 59  
threats, 97  
tit-for-tat, 44  
training set, 59  
transferable utility, 49  
transition function, 11  
type, 125  
  
utilitarian solution, 79  
utility function, 9  
  
valuation, 103  
value function, 49  
value iteration, 13  
value of information, 11  
Vickrey, 104  
Vickrey-Clarke-Groves, 130  
VSA, 113  
  
winner determination, 107  
winner's curse, 104  
wonderful life, 74  
  
XOR bids, 115  
  
zero-sum game, 40  
Zeuthen strategy, 84