Fundamentals *of*

Multiagent Systems

with NetLogo Examples

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Fundamentals of Multiagent Systems
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Contents

Preface

The goal of this book is to cover all the material that a competent multiagent practitioner or researcher should be familiar with. Of course, since this is a relatively new field the list of required material is still growing and there is some uncertainty as to what are the most important ideas. I have chosen to concentrate on the theoretical aspects of multiagent systems since these ideas have been around for a long time and are important for a wide variety of applications. I have stayed away from technological issues because these are evolving very fast. Also, it would require another textbook to describe all the distributed programming tools available. A reader interested in the latest multiagent technologies should visit the website `www.multiagent.com`.

The book is linked to a large number of sample NetLogo programs (Wilensky, 1999). These programs are meant to be used by the reader as an aid in understanding emergent decentralized behaviors. It is hard for people, especially those new to distributed systems, to fully grasp the order that can arise from seemingly chaotic simple interactions. As Resnick notices:

Resnick points to examples such as surveys of 8–15 year old kids, half of which believe that the government sets all prices and salaries.

> *"But even as the influence of decentralized ideas grows, there is a deep-seated resistance to such ideas. At some deep level, people seem to have strong attachments to centralized ways of thinking. When people see patterns in the world (like a flock of birds), they often assume that there is some type of centralized control (a leader of the flock). According to this way of thinking, a pattern can exist only if someone (or something) creates and orchestrates the pattern. Everything must have a single cause, and ultimate controlling factor. The continuing resistance to evolutionary theories is an example: many people still insist that someone or something must have explicitly designed the complex, orderly structures that we call Life."* (Resnick, 1994)

Resnick created StarLogo in order to teach the *decentralized* mindset. Wilensky, one of his students, later extended StarLogo and created NetLogo.

The reader is assumed to be familiar with basic Artificial Intelligence techniques (Russell and Norvig, 2003). The reader should also be comfortable with mathematical notation and basic computer science algorithms. The book is written for a graduate or advanced undergraduate audience. I also recommend (Mas-Colell et al., 1995; Osborne and Rubinstein, 1999) as reference books.

0.1 Usage

If you are using the PDF version of this document you can click on any of the citations and your PDF reader will take you to the appropriate place in the bibliography. From there you can click on the title of the paper and your web browser will take you to a page with a full description of the paper. You can also get the full paper if you have the user-name and password I provide in class. The password is only available to my students due to licensing restrictions on some of the papers.

ABTgc

Whenever you see an icon such as the one on this margin it means that we have NetLogo implementation of a relevant problem. If you are using the PDF version of this document you can just click on the name and your browser will take you to the appropriate applet. Otherwise, the URL is formed by pre-pending `http://jmvidal.cse.sc.edu/netlogomas/` to the name and appending `.html` at the end. So the URL for this icon is `http://jmvidal.cse.sc.edu/netlogomas/ABTgc.html`.

0.2 Acknowledgments

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Chapter 1

Multiagent Problem Formulation

The goal of multiagent systems' research is to find methods that allow us to build complex systems composed of autonomous agents who, while operating on local knowledge and possessing only limited abilities, are nonetheless capable of enacting the desired global behaviors. We want to know how to take a description of what a *system of agents* should do and break it down into individual agent behaviors. At its most ambitious, multiagent systems aims at reverse-engineering emergent phenomena as typified by ant colonies, the economy, and the immune system. Multiagent systems approaches the problem using the well proven tools from game theory, Economics, and Biology. It supplements these with ideas and algorithms from artificial intelligence research, namely planning, reasoning methods, search methods, and machine learning.

A more relaxed introduction to the topics in this chapter is given in (Russell and Norvig, 2003, Chapters 16–17).

These disparate influences have lead to the development of many different approaches, some of which end up being incompatible with each other. That is, it is sometimes not clear if two researchers are studying variations of the same problem or completely different problems. Still, the model that has thus far gained most attention, probably due to its flexibility as well as its well established roots in game theory and artificial intelligence, is that of modeling agents as utility maximizers who inhabit some kind of Markov decision process. This is the model we present in this chapter and the one we will use most often throughout the book. Note, however, that while this is a very general model it is not always the best way to describe the problem. We will also examine the traditional artificial intelligence planning model and, in later chapters, models inspired by Biology. Another popular model is that of agents as logical inference machines. This model is favored by those working on semantic or logical applications. We will sometimes make reference to **deductive** agents which can deduce facts based on the rules of logic, and **inductive** agents (see Chapter 5) which use machine learning techniques to extrapolate conclusions from the given evidence.

DEDUCTIVE
INDUCTIVE

1.1 Utility

A common simplifying assumption is that an agent's preferences are captured by a **utility function**. This function provides a map from the states of the world or outcome of game to a real number. The bigger the number the more the agent likes that particular state. Specifically, given that S is the set of states in the world the agent can perceive then agent i's utility function is of the form

UTILITY FUNCTION

In game theory it is known as the von Neumann-Morgenstern utility function.

$$u_i : S \to \Re. \tag{1.1}$$

Notice also that the states are defined as those states of the world that the agent can perceive. For example, if a robot has only one sensor that feeds him a binary input, say 1 if it is bright and 0 if its dark, then that robot has a utility function defined over only two states regardless of how complicated the real world might be, such as $u_i(0) = 5$, $u_i(1) = 10$. In practice, agents have sophisticated inputs and it is impractical to define a different output for each input. Thus, most agents also end up mapping their raw inputs to a smaller set of world states. Creating

this mapping function can be challenging as it requires a deep understanding of the problem setting.

PREFERENCE ORDERING

Given an agent's utility function we can define a **preference ordering** for the agent over the states of the world. By comparing the utility values of two states we can determine which one is preferred by the agent. This ordering has the following properties.

- *reflexive*: $u_i(s) \geq u_i(s)$
- *transitive*: If $u_i(a) \geq u_i(b)$ and $u_i(b) \geq u_i(c)$ then $u_i(a) \geq u_i(c)$.
- *comparable*: $\forall_{a,b}$ either $u_i(a) \geq u_i(b)$ or $u_i(b) \geq u_i(a)$.

We can use utility functions to describe the behavior of almost every agent. Utility functions are also useful for capturing the various tradeoffs that an agent must make, along with the value or expected value of its actions. For example, we can say that a robot receives a certain payment for delivering a package but also incurs a cost in terms of the electricity used as well as the opportunity cost—he could have been delivering other packages. If we translate all these payments and costs into utility numbers then we can easily study the tradeoffs among them.

Once we have defined a utility function for all the agents then all they have to do is take actions which maximize their utility. As in game theory, we use the word **selfish** to refer to a rational agent that wants to maximize its utility. Notice that this use is slightly different from the everyday usage of the word which often implies a desire to harm others, a true selfish agent cares exclusively about its utility. The use of selfish agents does not preclude the implementation of cooperative multiagent systems. We can view a cooperative multiagent system as one where the agents' utility functions have been defined in such a way so that the agents seem to cooperate. For example, if an agent receives a higher utility for helping other agents then the resulting behavior will seem cooperative to an observer even though the agent is acting selfishly.

SELFISH

selfish, adj. Devoid of consideration for the selfishness of others. —*The Devil's Dictionary*

We use utility functions as a succinct way of representing an agent's behavior. In the actual implementations these functions will sometimes be probabilistic, sometimes they will be learned as the agent acts in the world, sometimes they will be incomplete, sometimes they will be the result of some inferencing or induction. Still, by assuming that the function exists we can more easily design and study a multiagent system.

1.1.1 Utility is Not Money

Note that while utility represents an agent's preferences it is not necessarily equated with money. In fact, the utility of money has been found to be roughly logarithmic. For example, say Bill has \$100 million while Tim has \$0 in the bank. They both contemplate the possibility of winning one million dollars. Clearly, that extra million will make a significant difference in Tim's lifestyle while Bill's lifestyle will remain largely unchanged, thus Tim's utility for the same million dollars is much larger than Bill's. There is experimental evidence that shows most people have these type of conditional preferences. Very roughly, people's utility for smaller amounts of money is linear but for larger amounts it becomes logarithmic. Notice that we are considering the **marginal utility** of money, that is, it is the utility for the next million dollars. We assume that both Bill and Tim have the same utility for their first million dollars.

MARGINAL UTILITY

You can determine your own curve by repeatedly asking yourself for increasing values of x: which one would you rather have, x dollars, or a 50/50 chance at winning $2 \cdot x$ dollars? When you start preferring the first choice you are dropping below the $y = x$ line.

Recent results in behavioral economics have shown that the true utility function is more complicated than what can be captured by a mathematical function (Camerer et al., 2003). One experiment shows that people are less likely to take a gamble when the wording of the question is such that the person must give back some money, even if the gamble is mathematically identical to another one that uses different wording. The experiment in question is the following: which option would you prefer (a) I give you \$10,000 and a 50/50 chance at winning another \$10,000,

or (b) I give you $20,000 and then flip a coin and if it comes out heads you have to give me $10,000. Both of these are equivalent but a large majority of people prefer option (a).

The fact that people are not entirely rational when making choices about money becomes important to us when we start to think about building agents that will buy, sell, or negotiate for humans. It is reasonable to assume that in these systems people will demand agents that behave like people. Otherwise, the agent would be making decisions the user finds disagreeable. People's apparent irrationality also opens up the possibility that we could build agents that are better negotiators than us.

1.1.2 Expected Utility

Once we have utility functions we must then determine how the agents will use them. We assume each agent has some sensors which it can use to determine the state of the world and some effectors which it can use to take action. These actions can lead to new states of the world. For example, an agent senses its location and decides to move forward one foot. Of course, these sensors and effectors might not operate perfectly: the agent might not move exactly one foot or its sensors might be noisy. Let's assume, however, that the agent does know the probability of reaching state s' given that it is in state s and takes action a. This probability is given by $T(s, a, s')$ which we call the **transition function**. Since the transition function returns a probability then it must be true that the sum of $T(s, a, s')$ over all possible a and s' is equal to one.

TRANSITION FUNCTION

Using this transition function the agent can then calculate its **expected utility** for taking action a in state s as

EXPECTED UTILITY

$$E[u_i, s, a] = \sum_{s' \in S} T(s, a, s') u_i(s'), \tag{1.2}$$

where S is the set of all possible states.

An agent can use the expected utility function to determine the value of a piece of added information it might acquire such as an extra sensor reading or a message from another agent. For example, consider a piece of new information that leads the agent to determine that it is not really in state s but it is instead in state t. Perhaps a second agent tells him that he is near a precipice and not in the middle of a plateau as the agent originally thought. In this case the agent can compare the expected utility it would have received under the old state against the new one it will now receive. That is, the **value of information** for that piece of information is given by

VALUE OF INFORMATION

$$E[u_i, t, \pi_i(t)] - E[u_i, t, \pi_i(s)]. \tag{1.3}$$

The value of the piece of information is the expected utility the agent receives now that he knows he is in t and takes action accordingly minus the utility it would have received if he had instead mistakenly assumed it was in s and taken action accordingly. This equation provides a simple and robust way for agents to make meta-level decisions about what knowledge to seek: what messages to send, which sensors to turn on, how much deliberation to perform, etc.

Specifically, an agent can use it to play *"what if"* games to determine its best action in the same way a person might. For example, say a doctor has made a preliminary decision about which drug to prescribe for a patient but is uncertain about the patient's specific disease. She can calculate the value of information acquired from performing various tests which could identify the specific disease. If a test, regardless of its outcome, would still lead the doctor to prescribe the same drug then the value of the information from that test's result is zero: it does not lead to a change in action so (1.3) is zero. On the other hand, a test that is likely to change the doctor's action will likely have a high value of information.

s	$r(s)$
s_1	0
s_2	0
s_3	1
s_4	0

s_i	a	s_j	$T(s_i, a, s_j)$
s_1	a_1	s_1	0.2
s_1	a_1	s_2	0.8
s_1	a_2	s_1	0.2
s_1	a_2	s_4	0.8
s_2	a_2	s_2	0.2
s_2	a_2	s_3	0.8
s_2	a_3	s_2	0.2
s_2	a_3	s_1	0.8
s_3	a_4	s_2	1
s_3	a_3	s_4	1
s_4	a_1	s_4	0.1
s_4	a_1	s_3	0.9
s_4	a_4	s_4	0.2
s_4	a_4	s_1	0.8

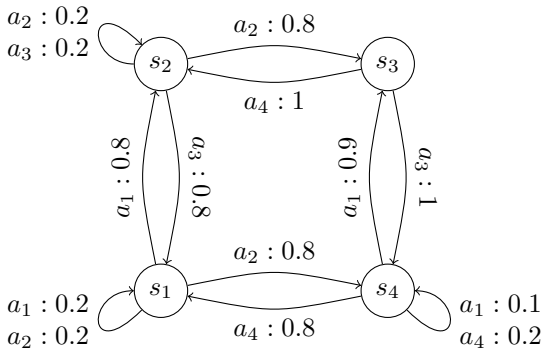


Figure 1.1: Graphical representation of a sample Markov decision process along with values for the transition and reward functions. We let the start state be s_1.

Andrey Markov. 1856–1922.

1.2 Markov Decision Processes

So far we have assumed that only the agent can change the state of the world. But, the reality in most cases is that agents inhabit an environment whose state changes either because of the agent's action or due to some external event. We can think of the agent sensing the state of the world then taking an action which leads it to a new state. We also make the further simplifying assumption that the choice of the new state therefore depends only on the agent's current state and the agent's action. This idea is formally captured by a **Markov decision process** or MDP.

MARKOV DECISION PROCESS

DETERMINISTIC

NON-DETERMINISTIC

Definition 1.1 (Markov Decision Processess)**.** *An* MDP *consists of an initial state s_1 taken from a set of states S, a transition function $T(s, a, s')$ and a reward function $r : S \to \Re$.*

The transition function $T(s, a, s')$ gives us the probability that an agent in state s who takes action a will end up in state s'. For a purely **deterministic** world we have that this function will be zero for all s' of each given s, a pair except one, for which it will be one. That is, in a deterministic world an agent's action has a completely predictable effect. This is not the case for a **non-deterministic** environment where an agent's action could lead to a number of different states, depending on the value of this fixed probability function.

Figure 1.1 shows a typical visualization of an MDP, this one with four states. Notice that in this example the agent's actions are limited based on the state, for example, in state s_1 the agent can only take actions a_1 and a_2. Also note that when the agent is in state s_1 and it takes action a_1 there is a 0.2 probability that it will stay in the same state. In this example the agent receives a reward of 1 only when it reaches s_3, at which point its only option is to leave s_3.

POLICY

OPTIMAL POLICY

MAXIMUM EXPECTED UTILITY

The agent's behavior is captured by a **policy** which is a mapping from states to actions. We will use π to denote the agent's policy. The agent's goal then becomes that of finding its **optimal policy** which is the one that maximizes its expected utility, as we might expect a selfish agent to do. This strategy is known as the principle of **maximum expected utility**. Agent i's optimal policy can thus be defined as

$$\pi_i^*(s) = \arg\max_{a \in A} E[u_i, s, a]. \tag{1.4}$$

But, in order to expand the expected value within that equation we must first determine how to handle future rewards. That is, is it better to take 100 actions with no reward just to get to a state which gives the agent a reward of 100, or is

it better to take 100 actions each of which gives the agent a reward of 1? Since no agent is likely to live forever we will not want to wait forever for that big reward. On the other hand, it seems reasonable to give up a small reward in the next couple of steps if we know there is a very big reward afterwards. As such, we generally prefer to use **discounted rewards** which allow us to smoothly reduce the impact of rewards that are farther off in the future. We do this by multiplying the agent's future rewards by an ever-decreasing **discount factor** represented by γ, which is a number between zero and one.

DISCOUNTED REWARDS

DISCOUNT FACTOR

For example, if an agent using policy π, starts out in state s_1 and visits states $s_1, s_2, s_3, \ldots$ then we say that its discounted reward is given by

$$\gamma^0 r(s_1) + \gamma^1 r(s_2) + \gamma^2 r(s_3) + \cdots \tag{1.5}$$

Note, however, that we only know s_1. The rest of the states, $s_2, s_3, \ldots$ depend on the transition function T. That is, from state s_1 we know that the probability of arriving at any other state s' given that we took action a is $T(s, a, s')$ but we do not know which specific state the agent will reach. If we assume that the agent is an utility maximizing agent then we know that it is going to take the action which maximizes its expected utility. So, when in state s the agent will take action given by (1.4), which when expanded using (1.2) gives us

$$\pi^*(s) = \arg\max_a \sum_{s'} T(s, a, s')u(s'), \tag{1.6}$$

where $u(s')$ is the utility the agent can expect from reaching s' and then continuing on to get more rewards for successive states while using π^*. We can now work backwards and determine what is the value of $u(s)$. We know that when the agent arrives in s it receives a reward of $r(s)$, but we now also know that because it is in s it can take its action based on $\pi^*(s)$ and will get a new reward at the next time. Of course, that reward will be discounted by γ. As such, we can define the real utility the agent receives for being in state s as

$$u(s) = r(s) + \gamma \max_a \sum_{s'} T(s, a, s')u(s'). \tag{1.7}$$

Richard Bellman. 1920–1984. Norbert Wiener prize, IEEE Medal of Honor.

This is known as the **Bellman equation** and it captures the fact that an agent's utility depends not only on its immediate rewards but also on its future discounted rewards. Notice that, once we have this function defined for all s then we also have the agent's true optimal policy $\pi^*(s)$, namely, the agent should take the action which maximizes its expected utility, as given in (1.6).

BELLMAN EQUATION

The problem we now face is how to calculate $u(s)$ given the MDP definition. One approach is to solve the set of equations formed. Given n states we have n Bellman equations each one with a different variable. We thus have a system of n equations with n variables so, theoretically, we can find values for all these variables. In practice, however, solving this set of equations is not easy because of the max operator in the Bellman equation which makes the equations non-linear.

Another approach for solving the problem is to use **value iteration**. In this method we start by setting the values of $u(s)$ to some arbitrary numbers and then iteratively improve these numbers using the **Bellman update** equation:

VALUE ITERATION

BELLMAN UPDATE

$$u^{t+1}(s) \leftarrow r(s) + \gamma \max_a \sum_{s'} T(s, a, s')u^t(s'). \tag{1.8}$$

The reader will notice that this equation is nearly the same as (1.7), except that we are now updating the u values over time. It has been shown that this process will eventually, and often rapidly, converge to the real values of $u(s)$. We also know that if the maximum change in utility for a particular time step is less than $\epsilon(1-\gamma)/\gamma$ then the error is less than ϵ. This fact can be used as a stopping condition.

valueiter

The VALUE-ITERATION algorithm is shown in figure 1.2. This algorithm is instance of **dynamic programming** in which the optimal solution to a problem is

DYNAMIC PROGRAMMING

```
VALUE-ITERATION(T, r, γ, ε)
1   do
2       u ← u'
3       δ ← 0
4       for s ∈ S
5           do u'(s) ← r(s) + γ max_a Σ_s' T(s, a, s')u(s')
6              if |u'(s) − u(s)| > δ
7                 then δ ← |u'(s) − u(s)|
8       until δ < ε(1 − γ)/γ
9   return u
```

Figure 1.2: The VALUE-ITERATION algorithm. It takes as input an MDP given by $T(\cdot)$, a discount value γ, and an error ϵ. It returns a $u(s)$ for that MDP that is guaranteed to have an error less than ϵ.

	Time (t)				
	0	1	2	3	4
$u(s_1)$	0	0	0	$.5(.8).45 = .18$	$.5(.036 + .376) = .206$
$u(s_2)$	0	0	$.5(.8)1 = .4$	$.5(.88)1 = .44$	$.5(.088 + .96) = .52$
$u(s_3)$	0	1	1	$1 + .5(1).45 = 1.2$	$1 + .5(.47) = 1.2$
$u(s_4)$	0	0	$.5(.9)1 = .45$	$.5(.9 + .045) = .47$	$.5(1.1 + .047) = .57$

s	$\pi^*(s)$
s_1	a_2
s_2	a_2
s_3	a_3
s_4	a_1

Figure 1.3: Example application of the VALUE-ITERATION algorithm to the MDP from figure 1.1, assuming $\gamma = .5$ and $\epsilon = .15$. The algorithm stops after $t = 4$. The bottom table shows the optimal policy given the utility values found by the algorithm.

found by first finding the optimal solutions to sub-problems. In our case, the sub-problems are the variables themselves. Finding a value for one variable helps us find values for other variables.

For example, figure 1.3 shows how the utility values change as the VALUE-ITERATION algorithm is applied to the example from figure 1.1. The utilities start at 0 but s_3 is quickly set to 1 because it receives a reward of 1. This reward then propagates back to its immediate neighbors s_2 and s_4 at time 2 and then at time 3 it reaches s_1. At time 4 the algorithm stops because the biggest change in utility from time 3 to time 4 was 0.13, for s_4, which is less than or equal to $\epsilon(1 - \gamma)/\gamma = .15$.

1.2.1 Multiagent Markov Decision Processes

The MDP model represents the problems of only one agent, not of a multiagent system. There are several ways of transforming an MDP into a multiagent MDP. The easiest way is to simply place all the other agents' effects into the transition function. That is, assume the other agents don't really exist as entities and are merely part of the environment. This technique can work for simple cases where the agents are not changing their behavior since the transition function in an MDP must be fixed. Unfortunately, agents that change their policies over time, either because of their own learning or because of input from the users, are very common.

A better method is to extend the definition of MDP to include multiple agents all of which can take an action at each time step. As such, instead of having a transition function $T(s, a, s')$ we have a transition function $T(s, \vec{a}, s')$, where $\vec{a}$ is a vector of size equal to the number of agents where each element is an agent's action, or a symbol representing non-action by that agent. We also need to determine how the reward $r(s)$ is to be doled out amongst the agents. One possibility is to divide it evenly among the agents. Unfortunately, such a simplistic division can mislead agents by rewarding them for bad actions. A better method is to give each agent a reward proportional to his contribution to the system's reward. We will see how this can be done in chapter 5.6.

1.2.2 Partially Observable MDPs

In many situations it is not possible for the agent to sense the full state of the world. Also, an agent's observations are often subject to noise. For example, a robot has only limited sensors and might not be able to see behind walls or hear soft sounds and its microphone might sometimes fail to pick up sounds. For these scenarios we would like to be able to describe the fact that the agent does not know in which state it is in but, instead, believes that it can be in any number of states with certain probability. For example, a robot might believe that it is in any one of a number of rooms, each with equal probability, but that it is definitely not outdoors. We can capture this problem by modeling the agent's **belief state** $\vec{b}$ instead of the world state. This belief state is merely a probability distribution over the set of possible states and it indicates the agent's belief that it is in that state. For the case with four states, the vector $\vec{b} = \langle \frac{1}{2}, \frac{1}{2}, 0, 0 \rangle$ indicates that the agent believes it is either in s_1 or s_2, with equal probability, and that it is definitely not in s_3 or s_4.

BELIEF STATE

We also need an **observation model** $O(s, o)$ which tells the agent the probability that it will perceive observation o when in state s. The agent can then use the observations it receives to update its current belief $\vec{b}$. Specifically, if the agent's current belief is $\vec{b}$ and it takes action a then its new belief vector $\vec{b}'$ can be determined using

OBSERVATION MODEL

$$\forall_{s'} \; \vec{b}'(s') = \alpha O(s', o) \sum_s T(s, a, s') \, \vec{b}(s), \tag{1.9}$$

where $\vec{b}(s)$ is the value of $\vec{b}$ for s and α is a normalizing constant that makes the belief state sum to 1. When we put all these requirements together we have a **partially observable Markov decision process** or POMDP which are a very natural way of describing the problems faced by an agent with limited sensors. Of course, since the agent does not know the state then it cannot use value iteration to solve this problem.

PARTIALLY OBSERVABLE MARKOV DECISION PROCESS

Luckily, it turns out that we can use (1.9) to define a new transition function

$$\tau(\vec{b}, a, \vec{b}') = \begin{cases} \sum_{s'} O(s', o) \sum_s T(s, a, s') \, \vec{b}(s) & \text{if (1.9) is true for } \vec{b}, a, \vec{b}' \\ 0 & \text{otherwise,} \end{cases} \tag{1.10}$$

and a new reward function

$$\rho(\vec{b}) = \sum_s \vec{b}(s) r(s). \tag{1.11}$$

Solving a POMDP on a physical state amounts to solving this MDP on the belief state. Unfortunately, this MDP can have an infinite number of states since the beliefs are continuous values. Luckily, there do exist algorithms for solving these type of MDPs. These algorithms work by grouping together beliefs into regions and associating actions with each region. In general, however, when faced with a POMDP problem it is usually easier to use a dynamic decision networks to represent and solve the problem.

See (Russell and Norvig, 2003, Chapter 17.5) for introduction to dynamic decision networks.

1.3 Planning

In the **artificial intelligence planning** problem an agent is also given a set of possible states and is asked for a sequence of actions that will lead it to a desirable world state. However, instead of a transition function the agent is given a set of operators. Each operator has pre-requisites that specify when it can be used—in which states—and effects which specify the changes the operator will cause in the state. The planning problem is to find a sequence of operators that take the agent from the start state to the goal state.

ARTIFICIAL INTELLIGENCE PLANNING

It should be clear that this problem is a special case of an MDP, one where only one state provides a reward and all the transitions have probability of 1 or 0. The transitions are generated by applying the operators to the current state. By

describing the problem using operators we achieve a much more succinct definition of the problem than by enumerating all states and transition probabilities as done in an MDP. Still, our goal is solving the problem, not defining it with as few bits as possible. The more detailed description of the states does, however, provide an advantage over the simple state listing used in MDP, namely, it provides added information about the composition of the state. For example, if we want to reach a state that has properties of *is-blue*? and *is-wet*? then we can use this knowledge to set about getting the state of the world blue and, once that is done, to get it wet without changing its color. This kind of information is opaque in a MDP description which simply describes a wet blue state as s_{11} and a dry blue state as s_{23}. Operators and their corresponding state descriptions thus provide the agent with more knowledge about the problem domain.

There exists many planning algorithms which solve the basic planning problem and variations of it, including cases where there is a possibility that the operators don't have the desired effect (making it even more like an MDP), cases where the agent must start taking actions before the plan can be finished, and cases where there is uncertainty as to what state the agent is in (like POMDP). Most modern planning algorithms use a graphical representation of the problem. That is, they end up turning the problem into an MDP-like problem and solving that. These algorithms are sophisticated and are available as libraries to use with your programs. If the need arises to solve a planning problem or an MDP it is advisable to use one of the many available libraries or, at worst, implement one of the published algorithms rather than trying to implement your own.

See (Russell and Norvig, 2003, Chapters 11–12) for pointers to these programs.

1.3.1 Hierarchical Planning

A very successful technique for handling complexity is to recursively divide a problem into smaller ones until we find problems that are small enough that they can be solved in a reasonable amount of time. This general idea is known as the **divide and conquer** approach in AI. Within planning, the idea can be applied by developing plans whose primitive operators are other plans. For example, in order to achieve your goal of a holiday at the beach you must first arrange transportation, arrange lodging, and arrange meals. Each one of these can be further decomposed into smaller actions, and there might be different ways of performing this decomposition. Within the planning model this amounts to building virtual operators which are planning problems themselves, instead of atomic actions. The problem then becomes one of deciding which virtual operators to build. In fact, the problem is very similar to that of deciding which functions or classes to write when implementing a large software project.

DIVIDE AND CONQUER

Within the MDP model we can imagine building a hierarchy of policies, each one taking of from states that fit a particular description (say, dry states) to other types of states (say, red and wet). We can then define new MDP problems which use sets of states as atomic states and the new policies as transition functions, similar to how we built POMDPs. These techniques are studied in **hierarchical learning**.

HIERARCHICAL LEARNING

Hierarchical planning and learning are usually studied as ways of making the planning and learning problems easier, that is, as ways to develop algorithms that will find solutions faster. However, once developed these hierarchies provide added benefits in a multiagent system. Specifically, they can be used to enable coordination among agents (Durfee, 1999). By exchanging top-level plan names (or policies) the agents have a general idea of what each other is doing without the need for exchanging detailed instructions of what they are doing at each step. For example, with two robots moving boxes in a room one might tell the other that its moving a box from the South corner to the East corner, the other agent then knows that if it stays in the Northwest part of the room it does not need to worry about the exact location of its partner—the robots stay out of each other's way without detailed coordination. This technique of using goal/plan/policy hierarchies for multiagent coordination has been very successful. We will examine it in more detail in

Chapter 9.

1.4 Summary

The view of an autonomous agent as an utility-maximizing agent that inhabits an MDP, or variation thereof, is most popular because of its flexibility, applicability to disparate domains, and amenability to formal analysis with mathematical or computational tools. As such, this book will largely adopt this model as the basis for formulating the various multiagent problems. Note, however, that the MDP is a tool for describing the problem faced by an agent. In practice, it is rare that a practical solution to a real world problem is also implemented as an algorithm to solve the raw MDP. More commonly we find algorithms which use a much more succinct, and therefore practical, method for representing the problem. Unfortunately, these representations tend to be very domain specific; they cannot be used in different domains.

Exercises

1.1 Marvin is a robot that inhabits a 3 by 3 grid and can only move North, South, East, and West. Marvin's wheels sometimes spin so that on each action there is a .2 probability that Marvin remains stationary. Marvin receives a reward of 1 whenever it visits the center square.

1. Draw an MDP which describes Marvin's problem.
2. What is the optimal policy?

1.2 The dynamic programming approach can be used to solve a wide variety of problems. For example, the Google search engine ranks results using the PageRank algorithm in which the pagerank of a webpage is proportional to the number of other pages that link to it weighted by their own pagerank. Thus, the pagerank can be calculated using a simple variation of the value iteration algorithm. However, one problem is that we do not know how long it will take for the algorithm to converge.

Implement the VALUE-ITERATION algorithm from figure 1.2 and run experiments to determine how much time it takes to converge as you increase the number of states given a fixed edge to node ratio. Try different ratios.

1.3 You have a table with three blocks: A, B, C. Assume the state of the world is completely defined by the position of the blocks relative to each other, where a block can only be on the table on on top of another block. At most one block can be on top of another block block. You are further given the following operators:

- MOVE-TO-TABLE(*block*): Requires that *block* has no other block on top of it. Results in the block now being on the table.
- MOVE-TO-BLOCK(*b1*, *b2*): Requires that block *b2* not have any other block on top of it. Results in block *b1* being on top of block *b2*.

Draw an MDP for this domain assuming operators always have their intended results.

1.4 Implement a NetLogo program where the patches are randomly set to one of three colors: black, red, and green.There is a turtle in this world who can only move North, South, East, and West by one patch at a time. Each time it lands in a patch it receives a reward determined by the color of the patch: black is 0, red is -1, and green in 1.

The turtle's movement is noisy, so when it decides to move North it ends up at the desired North tile with probability .5, and the tile NorthEast of its

current location (a diagonally adjacent tile) with probability .25, and at the tile NorthWest of its current location with a probability of .25.

Implement the VALUE-ITER algorithm for this domain and find the optimal policy.

Chapter 2

Distributed Constraints

Most multiagent systems are characterized by a set of autonomous agents each with local information and ability to perform an action when the set of actions of all must be coordinated so as to achieve a desired global behavior. In all these cases you own all the agents in question and can program them to do whatever you want. Thus, there is no need to properly incentivise them as there would be in an open system, which we study in later Chapters. However, there is still the problem of coordination. Since each agent only has local information it might be hard for it to decide what to do.

In this chapter we look at some algorithms for performing distributed search in cooperative multiagent systems where each agent has some local information and where the goal is to get all the agents to set themselves to a state such that the set of states in the system is optimal. For example, imagine a group of small sensors in a field. Each sensor can communicate only with those that are near him and has to decide which of its modalities to use (sense temperature, point radar North, point radar South, etc.) so that the group of sensors gets a complete view of the important events in the field. Or, imagine a group of kids who have been told to stand in a circle, each one must move find a spot to stand on but the position of that spot depends on everyone else's location. In these examples the agents have local information, can take any one of their available actions at any time, but the utility of their actions depends on the actions of others. We find that many multiagent problems can be reduced to a distributed constraints problem. Thus, the algorithms we present in this chapter have many different applications.

2.1 Distributed Constraint Satisfaction

We start by formally describing the problem. In a **constraint satisfaction problem** (CSP) we are given a set of variables, each with its own domain along with a set of constraints. The goal is to set every variable to a value from its domain such that no constraints are violated. Formally,

CONSTRAINT SATISFACTION PROBLEM

Definition 2.1 (Constraint Satisfaction Problem)**.** *Given a set of variables* x_1, x_2,...,x_n *with domains* $D_1, D_2, \ldots, D_n$ *and a set of boolean constraints* P *of the form* $p_k(x_{k1}, x_{k2}, \ldots, x_{kj}) \rightarrow \{0, 1\}$*, find assignments for all the variables such that no constraints are violated.*

The most widely studied instance of a constraint satisfaction problem is the graph coloring problem, shown in figure 2.1. In this problem we are given a graph and a set of colors. The problem is to find out if there is a way to color each node with one of the given colors such that no two nodes that are connected by an edge have the same color. We can easily map the graph coloring problem to the formal definition of a CSP by considering each node to be a variable, the domains to be the set of colors, and each edge becomes a constraint between its two nodes that is true only if their values are different. Notice that, in graph coloring constraints are only over two variables instead of over any set of variables as we find in the general constraint satisfaction problem.

The constraint satisfaction problem is NP-complete. As such, me use search algorithms to find a solution and hope that the actual running time will be less than

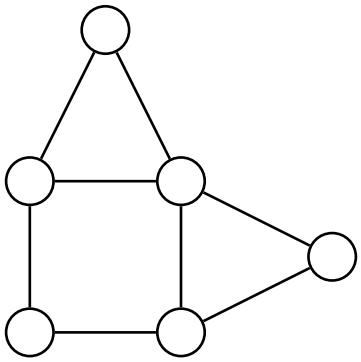


Figure 2.1: Sample graph coloring problem. You must color each node either black, white, or gray so that no two nodes connected by an edge have the same color. Can you find a coloring?

DEPTH-FIRST-SEARCH-CSP(i, g)

```
1  if i > n
2     then return g
3  for v ∈ D_i
4        do if setting x_i ← v does not violate any constraint in P given g
5              then g' ← DEPTH-FIRST-SEARCH-CSP(i + 1, g + {x_i ← v})
6                   if g' ≠ ∅
7                      then return g'
8
9  return ∅
```

Figure 2.2: A centralized depth first search algorithm for the CSP. Variable g holds the partial assignment of variable values. The algorithm is called with DEPTH-FIRST-SEARCH-CSP$(1, \emptyset)$.

the worst-case exponential time. In practice we find that there are many special cases where we can solve the problem really fast. For example, in the **n-queens problem** we are given an 8×8 chess board with 8 queens for which we must find places on the board such that no queen can take another queen. This problem is a special case of the CSP but it is one for which there are algorithms that can find a solution very quickly for large numbers of queens as the problem is under constrained.

N-QUEENS PROBLEM

A simple but effective method for solving the centralized constraint satisfaction problem is with the use of a **depth first search** algorithm, as shown in figure 2.2. The algorithm simply chooses an arbitrary ordering for the variables. On each call it tries to set the current variable x_i to all its possible values to see which ones do not violate any of the constraints in P given the partial variable assignment g. Whenever a value v works, the algorithm adds that assignment to g and recursively tries to assign the next variable in the list. This algorithm performs a complete search and is thus guaranteed to find a solution if one exists. Depth first search is also practical because it only requires an amount of memory that is linear on the number of variables, that is, it has $O(n)$ memory usage. As we shall see later in this chapter, some of the distributed algorithms try to parallelize this basic algorithm.

DEPTH FIRST SEARCH

A common modification is to improve the variables' ordering.

In the **distributed constraint satisfaction** (DCSP) problem we give each agent one variable. The agent is responsible for setting the value of its own variable. The agents do not know the values of any other variable but can communicate with other agents. Formally, we define:

DISTRIBUTED CONSTRAINT SATISFACTION

Definition 2.2 (Distributed Constraint Satisfaction Problem)**.** *Give each agent one of the variables in a* CSP*. Agents are responsible for finding a value for their variable and can find out the values of the other agents' variables via communication*

Usually the agents are limited so that they can only communicate with their neighbors—the agents with whom they share a constraint. Some algorithms, however, allow agents to communicate with any other agent. Also, most algorithms assume that the agents know about all the constraints that involve their variable.

The DCSP is pervasive in multiagent research and many real-world problems can be reduced to a DCSP, or a variation thereof. As such, many DCSP algorithms have

```
FILTERING()
1  for j ∈ {neighbors of i}          ▷ i is this agent.
2        do REVISE(x_i, x_j)

HANDLE-NEW-DOMAIN(j, D')
1  D_j ← D'
2  REVISE(x_i, x_j)

REVISE(x_i, x_j)
1  old-domain ← D_i
2  for v_i ∈ D_i
3        do if there is no v_j ∈ D_j consistent with v_i
4              then D_i ← D_i − v_i
5  if old-domain ≠ D_i
6     then ∀_{k∈{neighbors of i}} k.HANDLE-NEW-DOMAIN(i, D_i)
```

Figure 2.3: The filtering algorithm. Each agent i executes FILTERING().

been developed, each one with its own set of strengths and weaknesses. In the next sections we examine some of the most popular algorithms.

2.1.1 Filtering Algorithm

Picture yourself as one of the agents. You know your constraints, the domain of your variable, and the domain of the variables of your neighbors. You can use this information to determine if any of the values from your variable's domain is impossible. That is, if there is a value which violates some constraint no matter what value one of your neighbors uses then that value will clearly not be part of the solution so you can get rid of it. Similarly, all other agents might do the same.

This idea is implemented by the **filtering algorithm** (Waltz, 1975) shown in figure 2.3. Every agent i executes the FILTERING procedure which leads it to do a REVISE with every one of its neighbors' variables x_j. If the agent does succeed in eliminating one or more values from its domain it then tells its neighbors its new domain. When an agent receives new domains from its neighbors it again executes REVISE for those neighbors. This process repeats itself until all agents finish their computations and no more messages are sent.

FILTERING ALGORITHM

The filtering algorithm is guaranteed to stop since there are only a finite number of values in all the domains and the algorithm only sends a message when a value is eliminated from a domain. Once it stops, the filtering algorithm might have reduced the size of some domains. Unfortunately, it might stop at a point where one or more of the agents still has many values in its domain. Thus, it can stop before a solution is found.

Figure 2.4 shows an example of the filtering algorithm as applied to the graph coloring problem. In this problem the agents have three possible colors but start out with already limited domains as indicated by the black crosses. For example, agent x_1 can only be either black or gray. At the first step x_1 executes REVISE(x_1, x_3) and realizes that since x_3 must be gray that it cannot be gray. As such, it eliminates gray from its domain and tells all the other agents about this. The other agents incorporate this knowledge. Then, agent x_2 does a REVISE(x_2, x_3) and realizes that it also can't be gray and thus eliminates gray from its domain. At this point x_2 knows the final configuration. When it tells the other agents about its new domain then all the agents will also know the final solution.

In this example the filtering algorithm found a specific solution. Unfortunately, this is a rare occurrence. More often we find that the filtering algorithm only slightly reduces the size of the domains. For example, if we run the example from figure 2.4 but start with all possible colors in all domains then the filtering algorithm will not be able to do anything. That is, none of the agents are able to eliminate any color

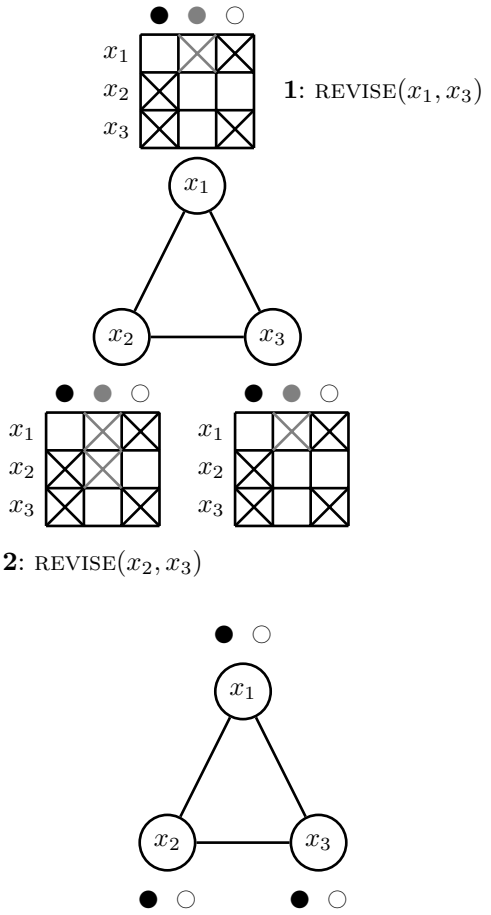


Figure 2.4: Filtering example. The agents start out with some prohibited colors as indicated by the black crosses. On the first step x_1 does his REVISE and eliminates the color gray from consideration. It then tells everyone else about this. Then x_2 does its revise and eliminates the color gray from its domain.

Figure 2.5: Example of a problem that does not have a solution and the filtering algorithm cannot that fact.

from their domain because many colorings are possible.

The filtering algorithm might also fail to eliminate values from the domains even when no coloring exists. That is, the filtering algorithm cannot reliably detect problems that do not have a solution. This problem can be illustrated with the example in figure 2.5. Here the nodes have to choose between either white or black. We know that there is no possible way to color these nodes without violating a constraint. However, the filtering algorithm will get stuck at the first step in this example and will not be able to reduce the size of the domains. Namely, each agent considers that if he is black then the other two could be white and if he is white then the other two could be black. As such, no agent eliminates colors from its domain.

2.1.2 Hyper-Resolution Based Consistency Algorithm

The reason the filtering algorithm fails for this example is because the REVISE function considers only the agent's own value and one other agent's value at a time. More generally, we need to consider the agent's value against all possible combinations of the values of two of its neighbors, of three of its neighbors, and so on all the way to considering the values of all of its neighbors at the same time.

More formally, we say that if an agent can find a suitable value regardless of the values of k of its neighbors then it can be consistent with anything they might do. If we can say this for all agents then we have achieved **k-consistency**.

k-CONSISTENCY

Definition 2.3 (k-consistency)**.** *Given any instantiation of any $k-1$ variables that satisfy all constraints it is possible to find an instantiation of any k^{th} variable such that all k variable values satisfy all constraints.*

If we can prove that problem is j-consistent for $j = 2, 3, \ldots$ up to some number k then we could say that the problem is strongly k-consistent.

	$\mathbf{x_1}$	$\mathbf{x_2}$	$\mathbf{x_3}$
	$x_1 = ○ \vee x_1 = ●$	$x_2 = ○ \vee x_2 = ●$	$x_3 = ○ \vee x_3 = ●$
	$\neg(x_1 = ○ \wedge x_2 = ○)$	$\neg(x_1 = ○ \wedge x_2 = ○)$	$\neg(x_1 = ○ \wedge x_2 = ○)$
Time	$\neg(x_1 = ● \wedge x_2 = ●)$	$\neg(x_1 = ● \wedge x_2 = ●)$	$\neg(x_1 = ● \wedge x_2 = ●)$
1	$\neg(x_2 = ○ \wedge x_3 = ●)$	$\neg(x_2 = ○ \wedge x_3 = ●)$	$\neg(x_2 = ○ \wedge x_3 = ●)$
2	$\neg(x_2 = ● \wedge x_3 = ○)$	$\neg(x_3 = ●)$	$\neg(x_2 = ● \wedge x_3 = ○)$
3		$\neg(x_2 = ● \wedge x_3 = ○)$	$\neg(x_3 = ○)$
4		$\neg(x_3 = ○)$	$\neg(x_3 = ●)$

Table 2.1: Example databases for a sample run of the hyper-resolution based consistency algorithm as applied to the graph of figure 2.5. Only a few of the nogoods produced by the graph's constraints are shown due to space constraints. You can infer the rest. New statements are added starting at time 1.

Definition 2.4 (Strongly k-consistent)**.** *A problem is* strongly k-consistent *if it is j-consistent for all $j \leq k$.*

We now face the problem of proving that a particular DCSP is strongly k-consistent. Notice that the filtering algorithm guarantees 2-consistency since it ensures that all agents can find a value regardless of the value any one of their neighbors might set. We need to extend this procedure to more than one neighbor. This can be done using the **hyper-resolution rule**.

HYPER-RESOLUTION RULE

Definition 2.5 (Hyper-Resolution Rule)**.**

$$\frac{\begin{array}{c} A_1 \vee A_2 \vee \cdots \vee A_m \\ \neg(A_1 \wedge A_{11} \wedge \cdots) \\ \neg(A_2 \wedge A_{21} \wedge \cdots) \\ \vdots \\ \neg(A_m \wedge A_{m1} \wedge \cdots) \end{array}}{\neg(A_{11} \wedge \cdots \wedge A_{21} \wedge \cdots \wedge A_{m1} \wedge \cdots)}.$$

The hyper-resolution rule is a simple deduction rule from formal logic whose form just so happens to correspond exactly to what we need. The idea is that the agents' domains can be represented as an OR statement like the first one in the rule. For example, if agent x_1 can be either gray, black, or white then we write $x_1 = \text{gray} \vee x_1 = \text{black} \vee x_1 = \text{white}$. The constraints are represented as negations like the one in the second line of the hyperesolution rule. For example, if there is a constraint that says that x_1 and x_2 cannot both be white we would write that as $\neg(x_1 = \text{white} \wedge x_2 = \text{white}$. Once we have represented both the domain and the constraints in this logical form then we can use the hyper-resolution rule to generate new constraints which are added to the agent's knowledge and communicated to the other agents. Each one of these new constraints is called a **nogood**.

NOGOOD

We can use the hyper-resolution rule to implement a distributed constraint satisfaction algorithm that, unlike the filtering algorithm, will detect all cases where there is no solution and will find a solution when there is one. The algorithm is basically the same as the filtering algorithm but we modify the REVISE function to instead use the hyper-resolution to generate new nogoods. These nogoods are then sent to the other agents. When an agent receives nogoods from other agents it adds them to its set of nogoods. The process continues until no new nogoods can be derived by any agent or until a contradiction is reached. When a contradiction is found it means that no solution exists to this problem.

We now show how the hyper-resolution algorithm would solve the problem from figure 2.5. Table 2.1 shows a few of the initial statements in the agents' databases. Note that many of the nogoods are missing due to space limitations but you can easily infer them as they correspond to the constraints in the problem. One possible run of the algorithm is as follows:

Let x_1 start out by doing

$$\frac{\begin{array}{c} x_1 = \bigcirc \vee x_1 = \bullet \\ \neg(x_1 = \bigcirc \wedge x_2 = \bigcirc) \\ \neg(x_1 = \bullet \wedge x_3 = \bullet) \end{array}}{\neg(x_2 = \bigcirc \wedge x_3 = \bullet)}$$

at time 1. It then sends $\neg(x_2 = \bigcirc \wedge x_3 = \bullet)$ to x_2 and x_3. x_2 then can do

$$\frac{\begin{array}{c} x_2 = \bigcirc \vee x_2 = \bullet \\ \neg(x_2 = \bigcirc \wedge x_3 = \bullet) \\ \neg(x_2 = \bullet \wedge x_3 = \bullet) \end{array}}{\neg(x_3 = \bullet)}$$

Meanwhile, x_1 can do

$$\frac{\begin{array}{c} x_1 = \bigcirc \vee x_1 = \bullet \\ \neg(x_1 = \bullet \wedge x_2 = \bullet) \\ \neg(x_1 = \bigcirc \wedge x_3 = \bigcirc) \end{array}}{\neg(x_2 = \bullet \wedge x_3 = \bigcirc)}$$

x_1 then sends the nogood $\neg(x_2 = \bullet \wedge x_3 = \bigcirc)$ to x_2 and x_3. Using this message, x_2 can now do

$$\frac{\begin{array}{c} x_2 = \bigcirc \vee x_2 = \bullet \\ \neg(x_2 = \bigcirc \wedge x_3 = \bigcirc) \\ \neg(x_2 = \bullet \wedge x_3 = \bigcirc) \end{array}}{\neg(x_3 = \bigcirc)}$$

x_2 then sends $\neg(x_3 = \bigcirc)$ and $\neg(x_3 = \bullet)$ to x_3. Using this message, x_3 can then do

$$\frac{\begin{array}{c} x_3 = \bigcirc \vee x_3 = \bullet \\ \neg(x_3 = \bigcirc) \\ \neg(x_3 = \bullet) \end{array}}{\text{Contradiction}}$$

This last step derives a contradiction which means that the algorithm has been able to prove that no solution exists.

In practice, the hyper-resolution algorithm is very slow because there is often little guidance as to which nogoods should be derived first. That is, at any one time each agent can derive a large number of different nogoods. Most of these are useless to the other agents and there is no easy way for the agent to determine which nogoods might be most useful since that would require that it also know about the other agents' databases. As such, we find that in practice the hyper-resolution algorithm spends a lot of time deriving useless nogoods. This is especially true as the size of the nogoods—their number of terms—increases. Also, if the problem does have a solution then the algorithm only stops when all the agents prove to themselves that they cannot generate any new nogood, which can take a very long time. In summary, hyper-resolution in its raw form is impractical for most cases with the possible exception of highly over-constrained problems.

2.1.3 Asynchronous Backtracking

Earlier, in figure 2.2, we saw a depth first search algorithm for solving a constraint satisfaction problem. The algorithm sets a variable to a specific value and, recursively, tries to set a new variable to some value which would not violate any of the constraints with existing values. If no such value exists the algorithm backs up and re-sets one of the variables it has already set. This type of depth first search is also known as a **backtracking** algorithm because we set the variables to some values and then if there is a conflict we backtrack, unset variables, and try again with different values.

BACKTRACKING

ASYNCHRONOUS BACKTRACKING

The **asynchronous backtracking** algorithm (ABT), is a distributed asynchronous version of the centralized depth first search algorithm for constraint satisfaction (Yokoo and Hirayama, 2000). ABT performs the same kind of search but it does so via the use of messages between agents and under the understanding that only the owner of the variable can change its value.

The ABT algorithm also implements some performance improvements on the basic depth first search. As we mentioned earlier, there is often much to be gained by properly ordering the variables. In general, we want to place variables that are involved in more constraints towards the top of the tree and search them first so as to avoid having to do some backtracking later on. This heuristic has been shown to work well and is employed by ABT.

Each agent i in ABT is responsible for variable x_i. All agents are given a fixed *priority* ordering, from 1 to n. Each agent i can change the value of its variable x_i. The agents keep track of the others' variables in *local-view*. The list of *neighbors* is initially set to be just the other agents with whom the agent shares a constraint but, as we will see, this list can grow. The agents find out other agents' variable values by sending messages. There are only three types of messages that ABT sends. Thus, only three remote procedures need to be implemented.

1. HANDLE-OK?(j, x_j) where j is the name of another agent and x_j is its current variable value. This message asks the receiver if that assignment does not violate any of his constraints.

2. HANDLE-NOGOOD$(j, nogood)$ which means that j is reporting that it can't find a value for his variable because of *nogood*.

3. HANDLE-ADD-NEIGHBOR(j) which requests the agent to add some other agent j as its neighbor. This message is handled by simply adding that agent to *neighbors*.

The full algorithm can be seen in figure 2.6. All agents start out by setting their x_i variable to some value and then asking all their neighbors to HANDLE-OK? After that they become event-driven, handling messages as they arrive. ABT assumes that messages are never lost and arrive and in the same order they were sent.

The procedure CHECK-LOCAL-VIEW checks that the agent's current value x_i is consistent with the values it thinks its neighbors have. If not then it checks to see if it can find some other value that will work. If it can't find one then this means it must BACKTRACK and find a new value. If it can then it sets x_i to that value and informs its neighbors.

Nogoods are handled by HANDLE-NOGOOD. The first thing this procedure does is to see if any of the agents in the nogood is not in *neighbors* so it can add it. This step is needed to ensure the algorithm will arrive at a correct answer. The nogood is added to the set of constraints and CHECK-LOCAL-VIEW is called to ensure there are no conflicts.

Finally, the BACKTRACK procedure is responsible for generating and sending new nogoods. As we saw, it is only called when there is no way for the agent to set is value so that no constraints are violated. This means that someone else must change their value. That someone else is the agent with the lowest priority in the nogood. BACKTRACK first uses the hyper-resolution rule (or similar) to generate one or more nogoods. Each one of these nogoods is then sent to the agent in that nogood that has the lowest priority. Also, if the hyper-resolution generated a contradiction, represented as an empty set then this means that no solution exists and thus the algorithm is terminated.

Some versions of ABT use one nogood in line 1 of BACKTRACK, others use more.

Notice that, since the nogoods are always sent to the lowest priority agent and the agents have linear priorities, the nogoods always flow from the high priority agents to the low priority agents. Namely, the highest priority agent will never receive a nogood, the second highest priority agent will only receive nogoods from the highest priority agent, and so on. This suggests to us that the lower priority agents will end up doing more work.

ABTgc

```
HANDLE-OK?(j, x_j)
1  local-view ← local-view +(j, x_j)
2  CHECK-LOCAL-VIEW()

CHECK-LOCAL-VIEW()
1  if local-view and x_i are not consistent
2     then if no value in D_i is consistent with local-view
3             then BACKTRACK()
4             else select d ∈ D_i consistent with local-view
5                  x_i ← d
6                  ∀_{k∈neighbors} k.HANDLE-OK?(i, x_i)

HANDLE-ADD-NEIGHBOR(j)
1  neighbors ← neighbors +j

HANDLE-NOGOOD(j, nogood)
1  record nogood as a new constraint
2  for (k, x_k) ∈ nogood where k ∉ neighbors
3        do k.HANDLE-ADD-NEIGHBOR(i)
4           neighbors ← neighbors +k
5           local-view ← local-view +(k, x_k)
6  old-value ← x_i
7  CHECK-LOCAL-VIEW()
8  if old-value ≠ x_i
9     then j.HANDLE-OK?(i, x_i)

BACKTRACK()
1  nogoods ← {V | V = inconsistent subset of local-view using hyper-resolution rule}
2  if an empty set is an element of nogoods
3     then broadcast that there is no solution
4          terminate this algorithm
5  for V ∈ nogoods
6        do select (j, x_j) where j has lowest priority in V
7           j.HANDLE-NOGOOD(i, V)
8           local-view ← local-view −(j, x_j)
9  CHECK-LOCAL-VIEW()
```

Figure 2.6: ABT algorithm. All agents start by setting themselves to some color and invoking a HANDLE-OK? on all their neighbors. After this they become event driven.

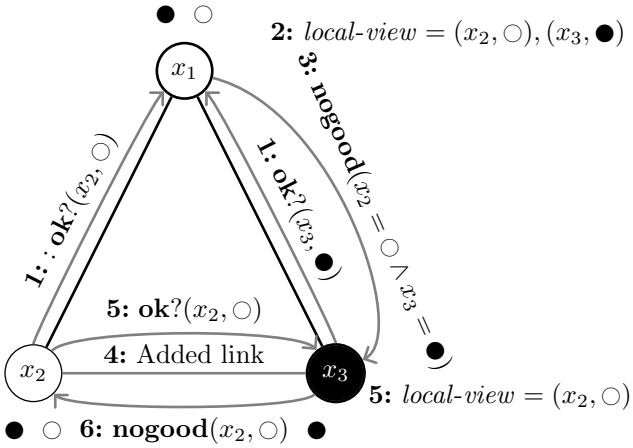


Figure 2.7: ABT example. The numbers preceding each message indicate the time at which the particular action took place. Only the first six steps are shown.

An example of ABT at work can be seen in figure 2.7. Here agents x_2 and x_3 start by setting their colors to be gray and black, respectively, and asking x_1 to HANDLE-OK?. Agent x_1 then generates a new nogood and sends a *nogood* to x_3. x_3 notices that x_2 is in the nogood but not in his *neighbors* and so adds it as his neighbor and asks x_2 to add him as a neighbor. In response to this x_2 asks x_3 to HANDLE-OK? with his new color. Agent x_3 uses x_2's new color to generate a new nogood that it sends to x_2. The figure ends at this point but the algorithm would continue with x_2 realizing that it must set itself to black and sending the appropriate HANDLE-OK? messages, and so on.

The ABT algorithm has been shown to be complete.

Theorem 2.1 (ABT is Complete)**.** *The* ABT *algorithm always finds a solution if one exists and terminates with the appropriate message if there is no solution (Yokoo et al., 1998).*

Proof. The proof works by induction. In order to prove it we must first show that the agent with the highest priority never enters an infinite loop. Then show that given that all the agents with lower priority that k never fall into an infinite loop then k will not fall into an infinite loop. □

Unfortunately, after running a few examples of this algorithm one quickly notices some problems. The most important is the uneven division of labor. In ABT the lowest priority agent ends up doing many times more work than the highest priority agents, which is reflected by the number of messages each agent receives. The difference only gets worse as the number of agents increases. Another drawback is the fact that in practice ABT can take very long as it also suffers from the same problem as regular backtracking. That is, if the high priority agents make bad decisions (choose colors that will not work) then it takes a long time to fix these as all the other agents have to prove that those colors will not work by trying all possible combinations of their colors. In the graph coloring problem we notice that the time to solve a problem increases exponentially as the ratio of edges to nodes increases. This means that as the problem becomes more constrained the time to find a solution explodes.


ABTmm

Another possible improvement to ABT can be found in line 1 of BACKTRACKING which tells us to find new nogoods. As you can see, the line does not specify which subset of nogoods to generate. One way to improve ABT is by managing which nogoods get generated such that only those that are likely to lead to a solution get generated (Jiang and Vidal, 2005; Jiang and Vidal, 2008). This strategy has the effect of greatly reducing the number of cycles needed to find a solution.


ABTkgc

We can also extend ABT by allowing agents to solve the problem without the need to add new links, thus limiting the agent to agent communication. This enhancement has been implemented by the kernel ABT algorithm (Bessière et al., 2005). This algorithm could be used in applications where it is impossible to change the original communication network.

2.1.4 Asynchronous Weak-Commitment Search

As we mentioned, ABT suffers from the fact that the agents have a fixed priority so that if the highest priority agent makes a bad decision then all the other agents must perform an exhaustive search to prove him wrong. The **Asynchronous Weak-Commitment** (AWC) algorithm tries to avoid this problem by giving the agents dynamic priorities. The agents all start out with the same priority but whenever an agent is forced to backtrack and resolve a new nogood then it raises its priority to be higher than anyone else's. The agents inform each other about their priorities by including them in the HANDLE-OK? and HANDLE-NOGOOD remote procedure calls.

ASYNCHRONOUS WEAK-COMMITMENT

AWC also uses the min-conflict heuristic (Minton et al., 1992) in order to reduce the risk of making a bad decision. The min-conflict heuristic tells the agents to choose the value for their variable which minimizes the number of constraint violations with other agents given the current values.


AWCgc

CHECK-LOCAL-VIEW

1 **if** x_i is consistent with *local-view*
2 **then return**
3 **if** no value in D_i is consistent with *local-view*
4 **then** BACKTRACK()
5 **else** select $d \in D_i$ consistent with *local-view* and which minimizes constraint violations with lower priority agents.
6 $x_i \leftarrow d$
7 $\forall_{k \in neighbors}$ k.HANDLE-OK?$(i, x_i, priority)$

BACKTRACK

1 generate a nogood V
2 **if** V is empty nogood
3 **then** broadcast that there is no solution
4 terminate this algorithm
5 **if** V is a new nogood
6 **then** $\forall_{(k,x_k) \in V}$ k.HANDLE-NOGOOD$(i, j, priority)$
7 $priority \leftarrow 1 + \max\{neighbors$'priorities$\}$
8 select $d \in D_i$ consistent with *local-view* and which minimizes constraint violations with lower priority agents.
9 $x_i \leftarrow d$
10 $\forall_{k \in neighbors}$ k.HANDLE-OK?$(i, x_i, priority)$

Figure 2.8: Asynchronous weak-commitment search algorithm. The rest of the procedures are the same as in ABT, from figure 2.6 except that they now also record the calling agent's priority.

The AWC algorithm is the same as ABT except that it re-defines both CHECK-LOCAL-VIEW and BACKTRACK. The re-defined functions from the AWC algorithm can be seen in figure 2.8. As you can see, it amounts to only a small modification to ABT. The agents now must keep track of all their neighbors' priorities. Whenever an agent detects a nogood it sets its priority to be higher than all its neighbors. This technique results in having the most constrained agent pick its value first.

Note that the algorithm, as presented in figure 2.8, glosses over some important implementation details. Line 5 of BACKTRACK asks us to check if V is a new nogood. This means that the nogoods need to be stored in a data structure with a fast search function, perhaps a hash table or a database. Line 5 in CHECK-LOCAL-VIEW, as well as line 8 in BACKTRACK, ask us to solve a minimization problem which could also take a long time for agents with high priority. Finally, line 1 of BACKTRACK does not specify which nogood we should generate and, as we mentioned in the previous section, is thus amenable to improvements.

In practice, a problem that plagues both AWC and ABT is the fact that the implementation of a good hyper-resolution function is not easy. Even if implemented correctly, finding a new nogood could take a very long time. In fact, one expects that the time required to generate new nogoods would increase linearly, perhaps exponentially, as the number of known nogoods increases.

Just like its predecessor, AWC has been shown to be complete. The only tricky part to proving its completeness lies in those changing priorities. Luckily, we note that these priorities cannot be forever changing as there are only a finite number of impossible value assignments. Thus, at some point we will have tried all possible value combinations and the priorities will stop increasing.

Theorem 2.2 (AWC is complete)**.** *The* AWC *algorithm always finds a solution if one exists and terminates with the appropriate message if there is no solution (Yokoo and Hirayama, 2000).*

Proof. The priority values are changed if and only if a new nogood is found. Since the number of possible nogoods is finite the priority values cannot be changed indefinitely. When the priority values are stable AWC becomes ABT, which is complete. □

It has been shown that AWC outperforms ABT in that it finds the solution much

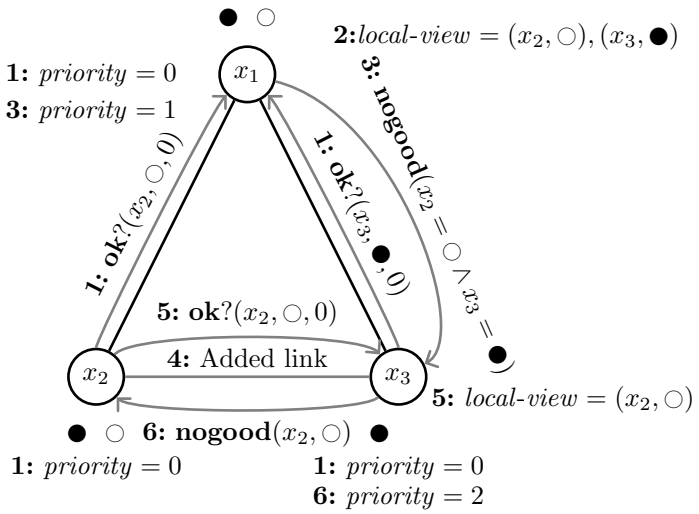


Figure 2.9: AWC example. The numbers indicate the time at which the particular action took place. Only the first six steps are shown. Note how the priorities of the agents change.

faster in randomly generated graphs (Yokoo and Hirayama, 2000). Neither ABT nor AWC provide us with guidance as to which nogood should be generated. That is, at any step there are many different nogoods that can be generated and these nogoods get bigger as the number of neighbors increases.

2.1.5 Distributed Breakout

Another way to approach the problem of distributed constraint satisfaction, different from the backtracking idea, is to use a hill-climbing method. In hill-climbing we start by assigning all variables randomly chosen values, then we change the values so as to minimize the constraint violations. We repeat the process until we cannot improve on the current solution. In a distributed version of hill-climbing each agent is a variable and all agents change their color in parallel, sending messages to their neighbors which indicate their new color.

All hill-climbing algorithms suffer from the problem of local minima. That is, it is possible for the system to reach a state were no one agent can find a better value for its variable and yet that is not the best possible value assignment for the system. That is, a local minimum can be recognized by the fact that no one agent can reduce the total number of constraint violations by changing its color but that number could be reduced by having two or more agents change their color simultaneously. As such, identifying the fact that the system is in a local minimum requires all agents to broadcast the fact that they are in a local minimum and these broadcasts must be further annotated to ensure that the agents' local views are coherent with each other—they were all in the same global state when they sent out their broadcasts. Implementing all these techniques would lead to a complex algorithm with large amounts of communications and would result in long run times.

A local minimum is akin to a Nash equilibrium while the optimal solution is akin to the utilitarian solution in game theory.

The **distributed breakout** algorithm tries to bypass this problem by recognizing a **quasi-local minimum** instead of a the full local minimum (Yokoo and Hirayama, 1996; Hirayama and Yokoo, 2005; Yokoo and Hirayama, 2000). The quasi-local minimum has the advantage that it can be recognized by an individual agent.

DISTRIBUTED BREAKOUT
QUASI-LOCAL MINIMUM

Definition 2.6 (Quasi-local minimum)**.** *An agent x_i is in a* quasi-local minimum *if it is violating some constraint and neither it nor any of its neighbors can make a change that results in lower total cost for the system.*

Unfortunately, the fact that an agent is in a quasi-local minimum does not necessarily imply that it is in a local minimum since it is possible that some non-neighboring agents could make changes that would lower the total cost. On the other hand, if the system is in a local minimum then at least one agent will find itself in a quasi-local minimum. Thus, to escape local minima the algorithm identifies quasi-local minima and increases the cost of the constraint violations.

```
HANDLE-OK?(j, x_j)
1  received-ok[j] ← TRUE
2  agent-view ← agent-view +(j, x_j)
3  if ∀_{k∈neighbors} received-ok[k] = TRUE
4     then SEND-IMPROVE()
5          ∀_{k∈neighbors} received-ok[k] ← FALSE

SEND-IMPROVE()
1  new-value ← value that gives maximal improvement
2  my-improve ← possible maximal improvement
3  ∀_{k∈neighbors} k.HANDLE-IMPROVE(i, my-improve, cost)

HANDLE-IMPROVE(j, improve)
1  received-improve[j] ← improve
2  if ∀_{k∈neighbors} received-improve[k] ≠ NONE
3     then SEND-OK
4          agent-view ← ∅
5          ∀_{k∈neighbors} received-improve[k] ← NONE

SEND-OK()
1  if ∀_{k∈neighbors} my-improve ≥ received-improve[k]
2     then x_i ← new-value
3  if cost > 0 ∧ ∀_{k∈neighbors} received-improve[k] ≤ 0 ▷ quasi-local optimum
4     then increase weight of constraint violations
5  ∀_{k∈neighbors} k.HANDLE-OK?(i, x_i)
```

Figure 2.10: The distributed breakout algorithm. Agents start by setting themselves to some color and then calling HANDLE-OK? on all their neighbors.

As such, the breakout algorithm maintains a weight associated with each constraint. These weights are initially set to one. Whenever an agent finds itself in a quasi-local-minimum it increases the weights of the constraints that are being violated. The weights in turn are used to calculate the cost of a constraint violation. That is, rather than simply counting the number of violated constraints, distributed breakout uses the weighted sum of the violated constraints. In this way, a constraint that was violated increases in weight so it becomes more important to satisfy that constraint in the next time step.

Distributed breakout implements two remote procedures.

- HANDLE-OK?(i, x_i) where i is the agent and x_i is its current value,
- HANDLE-IMPROVE$(i, improve)$ where *improve* is the maximum i could gain by changing to some other color.

The full algorithm can be seen in figure 2.10. In it we can see that the agents spend their time either waiting to HANDLE-OK? or HANDLE-IMPROVE from all their neighbors. Once an agent gets all the HANDLE-OK? invocations it calculates its new cost and possible improvement and sends a HANDLE-IMPROVE to all its neighbors. Once an agent gets all the HANDLE-IMPROVE messages then the agent determines if it is in a quasi-local-minimum. If so then it increases the weights of the constraint violations and sends a HANDLE-OK? to its neighbors.

Figure 2.11 shows an example application of the distributed breakout algorithm. We start out with a random coloring for all the nodes as seen on the first picture on the left. Once the agents receive all the HANDLE-OK? from their neighbors they check to see how much they could improve by changing and send the HANDLE-IMPROVE messages. In this case, it turns out that for all agents *my-improve* ≤ 0. Therefore, the agents increase weights to that shown on the second picture. At this point nodes 1, 2, 4 and 6 have a *my-improve* $= 1$ while 2, 5 are at -3. We break

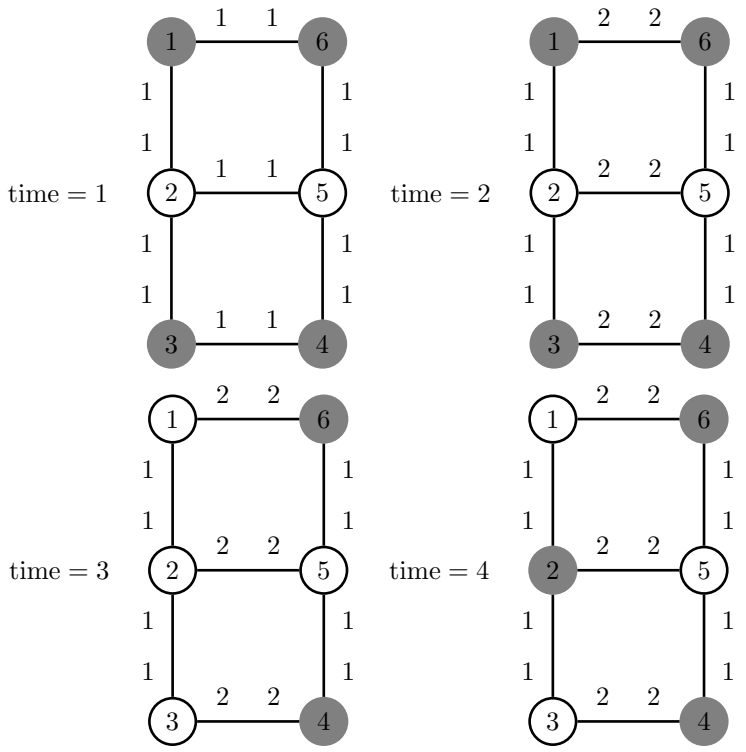


Figure 2.11: Example of distributed breakout. The only colors allowed are white and gray. The numbers represent the weights that the agents give to each of their constraints. We show four steps, starting at the top left.

ties using the agent number where smaller number wins. As such, agents 1 and 3 are the ones that get to change their color. They do so in the third picture. They then tell their neighbors their new color by calling HANDLE-OK? on them. At this point agent 2 has a *my-improve* $= 4$ (that is $1+1+2$) which is higher than anyone else's so it gets to change its color which moves us to the last picture.

The performance of distributed breakout is, on average, superior to AWC which is in turn superior to ABT. However, we should always remember that distributed breakout is not complete.

Theorem 2.3 (Distributed Breakout is **not** Complete)**.** *Distributed breakout can get stuck in local minimum. Therefore, there are cases where a solution exists and it cannot find it.*

Distributed breakout will also not be able to identify that a particular problem lacks a solution. Specifically, the algorithm will either continue running or get stuck in a local minimum when given a problem that lacks a solution. As such, breakout cannot be used when there is any chance that the problem lacks a solution and you want to determine if the problem has or does not have a solution.

Tests have also shown that the load distribution of distributed breakout is uneven. That is, some agents end up handling several times more messages than other agents. There is likely a correlation between the number of edges an agent has and the number of messages it must process. Still, distributed breakout has been shown to, in practice, find the local optima a very large percentage of the time, and variations of the basic algorithm perform even better. Specifically, **Multi-DB**$^{++}$ has been shown to, in practice, always find the solution for 3SAT problems (Hirayama and Yokoo, 2005).

MULTI-DB^{++}

2.2 Distributed Constraint Optimization

The **distributed constraint optimization** problem is similar to the constraint satisfaction problem except that the constraints return a real number instead of a boolean value and the goal is to minimize the value of these constraint violations. The problem arises when multiple, perhaps all, solutions are valid but some are

DISTRIBUTED CONSTRAINT OPTIMIZATION

```
BRANCH-AND-BOUND-COP()
1  c* ← ∞      ▷ Minimum cost found. Global variable.
2  g* ← ∅      ▷ Best solution found. Global variable.
3  BRANCH-AND-BOUND-COP-HELPER(1, ∅)
4  return g*

BRANCH-AND-BOUND-COP-HELPER(i, g)
1  if i = n
2     then if P(g) < c* ▷ Cost of violations in g is less than current bound.
3             then g* ← g
4                  c* ← P(g)
5          return
6  for v ∈ D_i
7        do g' ← g + {x_i ← v}
8           if P(g') < c*
9              then BRANCH-AND-BOUND-COP-HELPER(i + 1, g')
```

Figure 2.12: A centralized branch and bound algorithm for constraint optimization. It assumes there are n variables, $x_1, \ldots, x_n$, and that $P(g)$ returns the total of any constraint violations given partial assignment g and constraints P.

preferred to others. For example, say you are trying to schedule all your classes or weekly meetings and any schedule where no class overlaps another one is a valid schedule. However, you might prefer schedules with the least number of early morning classes.

We formally define a constraint optimization problem as follows:

Definition 2.7 (Constraint Optimization Problem (COP))**.** *Given a set of variables $x_1, x_2, \ldots x_n$ with domains $D_1, D_2, \ldots D_n$ and a set of constraints P of the form $pk(x_{k1}, x_{k2}, \ldots, x_{kj}) \rightarrow \Re$, find assignments for all the variables such that the sum of the constraint values is minimized.*

COP is an NP-Complete problem. That means that, at worse, all algorithms will take an exponential time (in the number of variables) to finish. Even parallel algorithms will, at worst, need exponential time. However, those are worst case scenarios. In practice we find some algorithms that do very well on most problems.

BRANCH AND BOUND

The most common approach to solving COPs is to use a **branch and bound** algorithm such as the one shown in figure 2.12. Branch and bound algorithms perform a depth first search of the variable assignments but also maintain a bound, c^*, which is the cost of the best solution found thus far. If a partial variable assignment already has a cost that is equal or higher than c^* then we know that there is no need to find values for the rest of the variables as the cost will only increase. The algorithm is complete so it will find the best possible solution to the problem, but doing so might require a lot of time. Since it is based on a depth first search its memory requirements are linear on the number of variables.

The distributed constraint optimization problem distributes the variables to the agents.

Definition 2.8 (Distributed Constraint Optimization Problem (DCOP))**.** *Give each agent one of the variables in a* COP*. Agents are responsible for finding a value for their variable and can find out the values of their neighbors' via communication*

We now look at some algorithms for solving the DCOP problem.

2.2.1 Adopt

ADOPT

The **Adopt** algorithm (Modi et al., 2005) is a recent addition to the family of DCOP algorithms. It is roughly a depth-first search on the set of possible value assignments, but with a lot of improvements on the basic search strategy. Namely, each agent in the depth-first tree keeps a lower and upper bound on the cost for the sub-problem below him (given assignments from above) and on the sub-problems

```
RESET-VARIABLES(d, c)
1  lower-bound[d, c] ← 0
2  t[d, c] ← 0
3  upper-bound[d, c] ← ∞
4  context[d, c] ← {}

INITIALIZE()
1  threshold ← 0
2  received-terminate ← FALSE
3  current-context ← {}
4  ∀_{d∈D_i, c∈children} RESET-VARIABLES(d, c)
5  x_i ← d ∈ D_i which minimizes: my cost plus Σ_{c∈children} lower-bound[d, c]
6  BACKTRACK()

HANDLE-THRESHOLD(t, context)
1  if context is compatible with current-context
2     then threshold ← t
3          MAINTAIN-THRESHOLD-INVARIANT()
4          BACKTRACK()

HANDLE-TERMINATE(context)
1  received-terminate ← TRUE
2  current-context ← context
3  BACKTRACK()

HANDLE-VALUE(j, x_j)
1  if ¬ received-terminate
2     then current-context[j] ← x_j
3          for d ∈ D_i, c ∈ children such that context[d, c] is
                                        incompatible with current-context
4               do RESET-VARIABLES(d, c)
5          MAINTAIN-THRESHOLD-INVARIANT()
6          BACKTRACK()

HANDLE-COST(k, context, lb, ub)
 1  d ← context[i]
 2  delete context[i]
 3  if ¬ received-terminate
 4     then for (j, x_j) ∈ context and j is not my neighbor
 5               do current-context[j] ← x_j
 6          for d' ∈ D_i, c ∈ children such that context[d, c] is
                                        incompatible with current-context
 7               do RESET-VARIABLES(d', c)
 8          if context compatible with current-context
 9             then lower-bound[d, k] ← lb
10                  upper-bound[d, k] ← ub
11                  context[d, k] ← context
12                  MAINTAIN-CHILD-THRESHOLD-INVARIANT()
13                  MAINTAIN-THRESHOLD-INVARIANT()
14          BACKTRACK()
```

Figure 2.13: The Adopt algorithm. Before the algorithm is even called the agents must form a depth first search tree. Each agent has a *parent* variable which points to its parent. The root's parent is set as null.

```
BACKTRACK()
 1  if threshold = min_{d∈D_i} cost(d) + Σ_{c∈children} upper-bound[d, c]
 2     then x_i ← arg min_{d∈D_i} cost(d) + Σ_{c∈children} upper-bound[d, c]
 3  elseif threshold < cost(x_i) + Σ_{c∈children} lower-bound[x_i, c]
 4     then x_i ← arg min_{d∈D_i} cost(d) + Σ_{c∈children} lower-bound[d, c]
 5  ∀_{k∈neighbors ∧ k has lower priority} k.HANDLE-VALUE(i, x_i)
 6  MAINTAIN-ALLOCATION-INVARIANT()
 7  if threshold = min_{d∈D_i} cost(d) + Σ_{c∈children} upper-bound[d, c] and
                (received-terminate or I am root)
 8     then current-context[i] ← x_i
 9          ∀_{c∈children} c.HANDLE-TERMINATE(current-context)
10          exit
11  parent.HANDLE-COST(current-context,
            min_{d∈D_i} cost(d) + Σ_{c∈children} lower-bound[d, c],
            min_{d∈D_i} cost(d) + Σ_{c∈children} upper-bound[d, c])

MAINTAIN-THRESHOLD-INVARIANT()
1  if threshold < min_{d∈D_i} cost(d) + Σ_{c∈children} lower-bound[d, c]
2     then threshold ← min_{d∈D_i} cost(d) + Σ_{c∈children} lower-bound[d, c]
3  if threshold > min_{d∈D_i} cost(d) + Σ_{c∈children} upper-bound[d, c]
4     then threshold ← min_{d∈D_i} cost(d) + Σ_{c∈children} upper-bound[d, c]

MAINTAIN-ALLOCATION-INVARIANT()
1  while threshold > cost(x_i) + Σ_{c∈children} t[x_i, c]
2        do chosen ← c' ∈ children such that upper-bound[x_i, c'] > t[x_i, c']
3           t[x_i, chosen] ← t[x_i, chosen] + 1
4  while threshold < cost(x_i) + Σ_{c∈children} t[x_i, c]
5        do chosen ← c' ∈ children such that lower-bound[x_i, c'] < t[x_i, c']
6           t[x_i, chosen] ← t[x_i, chosen] − 1
7  ∀_{c∈children} c.HANDLE-THRESHOLD(t[x_i, chosen], current-context)

MAINTAIN-CHILD-THRESHOLD-INVARIANT()
1  for d ∈ D_i, c ∈ children
2        do if lower-bound[d, c] > t[d, c]
3              then t[d, c] ← lower-bound[d, c]
4  for d ∈ D_i, c ∈ children
5        do if upper-bound[d, c] < t[d, c]
6              then t[d, c] ← upper-bound[d, c]
```

Figure 2.14: The Adopt algorithm, continued.

for each one of his children. It then tells the children to look for a solution but ignore any partial solution whose cost is above the lower bound because it already knows that it can get that lower cost.

The general idea in Adopt is that each node in the tree calculates upper and lower bounds on the cost for all the constraints that involve all the nodes in the subtree rooted at that node, given an assignment for the nodes on the path to the root. Each node changes to the value (color) whose lower bound is smallest at each time and then asks its children to calculate new bounds given the new value. Upon receiving this message each node again changes its value to the one with the lowest lower bound and the process is repeated. Figures 2.13 and 2.14 show the full algorithm.

The basic workings of Adopt are best shown via an example. Table 2.2 shows the cost function we will be using in our example. Figure 2.15 shows a trace of an application of Adopt to a simple problem. As you can see, all the constraints are binary, as Adopt can only handle binary constraints. All the agents keep track of the

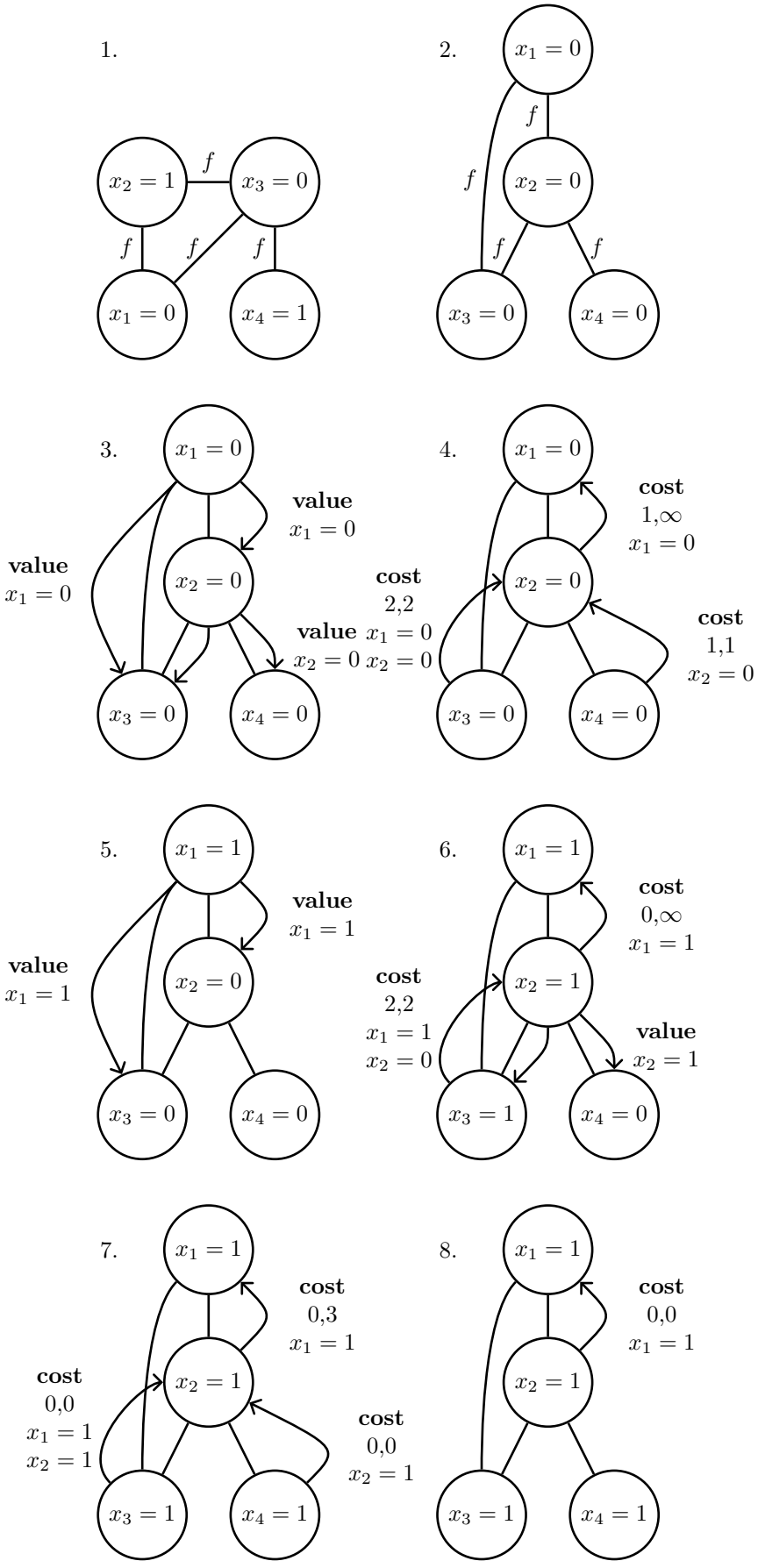


Figure 2.15: Adopt example. Start at the top left and read left to right. The domain for all nodes is the set $\{0, 1\}$.

Table 2.2: Cost function for Adopt example in figure 2.15.

d_i	d_j	$p(d_i, d_j)$
0	0	1
0	1	2
1	0	2
1	1	0

upper and lower bounds sent by their children along with the associated context. They use these bounds to calculate bounds on their own costs. Initially, all lower bounds are set to 0 and all upper bounds to infinity.

The first diagram is the constraints graph itself. The first thing we must do before even starting Adopt is to form a depth-first search graph. That is, a graph such that all constraints emanating from a node go to either one of its descendants—the tree below it—or one of its ancestors—agents on the path from the node to the root. There exists several algorithms for finding such a tree and generally multiple trees can be formed from one constraints graph. The second diagram shows a possible depth-first search tree for our example.

The third diagram shows the first step in the algorithm. All the nodes invoke HANDLE-VALUE on all the descendants with whom they share a constraint. That is why x_1 sends a message to x_3 but not x_4. These HANDLE-VALUE are similar to the HANDLE-OK? functions we saw before. They simply inform the other agents of the new value, in this case the original values.

ADOPTgc

The agents use these values to calculate both upper and lower bounds on their cost and report these back up to their parents, as shown in the fourth diagram. For example, x_3 calculated that, given that $x_1 = 0$ and $x_2 = 0$ if it set itself to 0 the cost would be 2 and if set itself to 1 the cost would be 4, remember that we are using the cost function from Table 2.2, thus it stays at 0 and sends a HANDLE-COST to its parent saying that its lower and upper bounds, given $x_1 = 0$ and $x_2 = 0$, are 2. This message means that the cost of the constraints for the subtree rooted at x_3 (since x_3 is a leaf this then means just all the constraints in which x_3 is involved) given $x_1 = 0$ and $x_2 = 0$, are no lower or no higher than 2. x_3 can set both its upper and lower bounds because it has values for all its constraints. On the other hand, x_2 sends an upper bound of ∞ because it does not know the upper bounds of its children, so it does not know how much those costs could add up to.

In the fifth diagram the agents calculate new best values and so x_1 sends HANDLE-VALUE messages. In the sixth diagram x_2 receives the cost messages from before and sets himself to 1 because the value 1 has a lower lower bound (because all lower bounds are initially set to 0). x_2 then sends the appropriate HANDLE-VALUE messages.

In the seventh diagram both x_3 and x_4 calculate that both their upper and lower bounds are 0 given that $x_1 = 0$ and $x_2 = 0$ and send this message up to x_2. In the eighth diagram x_2 uses this message to calculate its new bounds, also 0 and 0, and sends these up to the root. Upon receiving this message the root knows it can stop the algorithm because it has identical upper and lower bounds. We note that the Adopt algorithm is actually a bit more complicated than this example shows as we ignored any HANDLE-THRESHOLD invocations.

Test results on Adopt show that there is a very wide discrepancy in the number of messages each agent must handle and, therefore, the individual workload. Namely, agents at the leafs of the tree end up doing all of the work while the root sits idle. This gets worse as the tree becomes thinner (like a line). Since the tree must be a depth-first search graph, this means that problems with a lot of constraints will be doubly bad because they will need a thinner tree and the extra constraints means more work for the agents.

Also, it must be noted that if each agent handles each message as it comes (and calls the BACKTRACK routine) then Adopt will never finish. For Adopt to work the agents need to accumulate a number of HANDLE-VALUE and HANDLE-COST

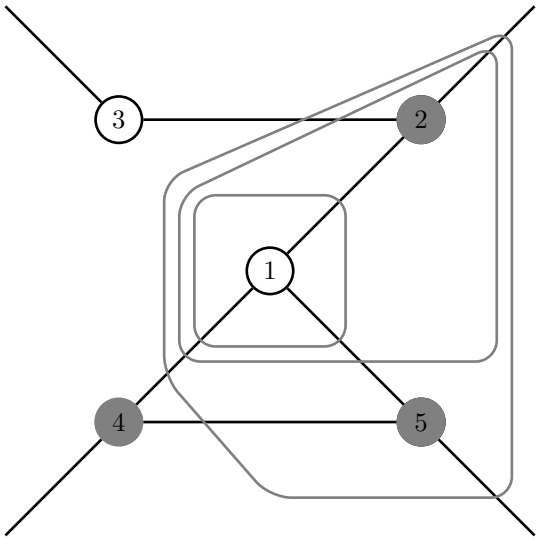


Figure 2.16: Trace of APO algorithm showing the growing size of node 1's *good-list*. Initially only 1 is in the *good-list*, then 2 joins, finally 5 joins.

invocations before running their BACKTRACK procedure.

2.2.2 OptAPO

A different approach to solving the distributed constraint optimization problem was taken by the Optimizing Asynchronous Partial Overlay (**optapo**) algorithm (Mailler and Lesser, 2004a), which is the cousin of the **apo** algorithm (Mailler and Lesser, 2004b) for distributed constraint satisfaction.

OPTAPO
APO

Both of these algorithms use the same basic idea. The agents start out doing individual hill-climbing, that is, each one changes its color so as to minimize constraints violations with its neighbors. When an agent realizes that it cannot find a value with no violations it tries to find a coloring for a (growing) set of agents which includes itself. More specifically, each agent maintains a variable called its *good-list* which initially contains only the agent. As the agent finds conflicts with other agents that it cannot resolve it adds these agents to its *good-list*. Then, the conflict is resolved by electing one of the agents in the conflict to be the *mediator* of the conflict. The mediator agent then finds a coloring for all the agents in its *good-list* and which minimizes violations with agents outside the *good-list* using any centralized algorithm, such as the branch and bound algorithm from figure 2.12. The agent then informs the other agents of their new color. The agents change to their new color as instructed by the mediator.

Figure 2.16 gives an idea of how APO works. Initially, node 1 has only itself in its *good-list*. If it finds that it cannot resolve a conflict with agent 2 then they enter into a negotiation phase to elect a mediator. In this case 1 becomes the mediator and then finds colors for both 1 and 2. Later on another conflict between 1 and 3 might arise which would lead to 1 becoming mediator for 1,2,3. Notice that if we give optAPO a graph coloring problem that has no solution, that is, there is always at least one conflict then it will eventually end up performing a complete depth first search of the complete problem in one node. This will be extremely time consuming.

In practice, APO and optAPO can be very fast, but their theoretical worst-case bound is still exponential.

Theorem 2.4 (APO worst case is centralized search)**.** *In the worst case* APO *(opt*APO*) will make one agent do a completely centralized search of the complete problem space.*

optAPO requires fewer number of synchronous cycles (that is, steps) than Adopt but performs a lot more computations as measured by number of constraints checks (Davin and Modi, 2005). That is, optAPO centralizes the problem much more than Adopt. When an agent becomes a mediator it centrally solves a large part of the

problem and tells the other agents what to do. As such, that one agent ends up doing a lot of work while everyone else does nothing (generally there can be more than one mediator, each mediating over non-overlapping sets of agents, but this is rare). In this way optAPO can finish in a much smaller number of steps simply because some of those steps take a long time.

In general we find that Adopt is better when agent to agent communications are fast and optAPOis better when communications are slow in comparison to the agent's processing speed. Both of them have exponential worst-case running times but perform reasonable well for small problems.

Exercises

2.1 In both ABT and AWC the load distribution among the agents is very uneven, as demonstrated in our NetLogo implementations. Can you come up with an equation that predicts how many messages, proportionally, each agent receives given the initial problem structure and priorities?


DBsodoku

2.2 The Sodoku puzzle is an instance of a constraint satisfaction problem. We can view it as a distributed constraint satisfaction problem by simply assuming that each empty square on the board corresponds to an agent. Implement any one of the distributed constraint satisfaction algorithms in this chapter to solve Sodoku puzzles.

2.3 Provide an example which proves Theorem 2.3.

2.4 Provide an example which proves Theorem 2.4.

2.5 All these algorithms assume a static constraint graph. Unfortunately, many real world applications are dynamic and can best be represented by a series of constraint satisfaction problems each of which is a little bit different from the previous one. In these applications it seems wasteful to re-start the algorithm from the beginning when the new problem is only slightly different from the old one.

Implement one of the constraint satisfactions algorithms for graph coloring and run tests to determine how it behaves as the graph changes. For example,

1. Start with a randomly generated graph and an initial coloring.
2. Run the algorithm starting with he current coloring.
3. Change the graph by adding or deleting a random node.
4. Goto step 2.

Does the algorithm take the same time in the first step as in all the other steps? What type of changes to the graph make the most difference in the run time?

Chapter 3

Standard and Extended Form Games

John Von Neumann. 1903–1957. Pioneer of the digital computer, game theory and cellular automata.

GAME THEORY

In all multiagent systems we have a set of autonomous agents each performing its own actions using whatever information is has available. Since the other agents are also taking actions, each agent must also take these into account when deciding what to do. Thus, what one does depends on what the other one does and vice-versa. The agent must decide what to do when their choice of action depends on the others' choices. These types of problems are very common in different fields, from Economics to Biology, and their solution is sometimes of immense importance. As such, a set of mathematical tools has been developed over the years to model and solve these problems. It is known as **game theory** and is the subject of this and several other chapters in this book.

Game theory was first formally introduced in the book *"The Theory of Games and Economic Behavior"* (Neumann and Morgenstern, 1944). The book introduced a mathematical way of analyzing certain types of decisions where two or more parties take actions that impact all. In this chapter we present the two most basic forms of games: normal and extended form games. These games are known as **non-cooperative games** because the agents' preferred sets of actions can be in conflict with each other. That is, what is good for one agent might be bad for the others. Of course, the fact that they can be in conflict does not mean that they have to be, thus, non-cooperative games can lead to cooperation.

NON-COOPERATIVE GAMES

3.1 Games in Normal Form

In the simplest type of game we have two agents each of which must take one of two possible actions. The agents take their actions at the same time. They will then each receive a utility value, or payoff, based on their joint actions. Games such as this one can be represented using a **payoff matrix** which shows the utility the agents will receive given their actions. Figure 3.1 shows a sample game matrix in **normal form**, also known as **strategic form**, a phrase introduced by Shapley in 1965. In this game if Bob takes action a and Alice takes action c then Bob will receive a utility of 1 and Alice a utility of 2. We can extend the payoff matrix to any number of players and actions. In these games we always assume that the players take their actions simultaneously.

PAYOFF MATRIX

NORMAL FORM
STRATEGIC FORM

Normal form games also assume that the players have **common knowledge** of the utilities that all players can receive. That is, everybody knows that everybody knows that everybody knows, and so on, the values in the payoff matrix. This situation is different from having the agents know the values in the matrix but not know that the others know those values. For example, if you know that I am giving you a surprise party then you might go along and act surprised when we all jump

COMMON KNOWLEDGE

(Fagin et al., 1995) describes a logic for representing agents' knowledge about others' knowledge.

		Alice	
		c	d
Bob	a	1,2	4,3
	b	3,2	2,4

Figure 3.1: Sample game matrix in normal form.

and yell "surprise!". However, if you know that I know that you know that I will be giving you a surprise party then the deception will no longer work.

It is also interesting to note that in message-passing multiagent systems where messages can be lost it is impossible for agents to ever achieve common knowledge about anything (Fagin et al., 1995). The problem is historically described as the **Byzantine generals problem** where two generals from the same army are poised on opposite sides of a valley which is occupied by the enemy. The generals must both attack at the same time in order to defeat the enemy. However, their only method of communication is by sending a messenger who could be captured by the enemy. We can see that, if one general sends a message to the other saying "We attack at dawn" it has no way of confirming whether the other general received this message. Similarly, if a general does receive the message and sends another messenger back acknowledging receipt then it has no way of confirming whether the other general received the acknowledgment. Thus, it is impossible to agree on a time. In practice, however, multiagent systems that need common knowledge either give their agents this common knowledge from the beginning or assume that communications are reliable enough that it is safe to assume that all messages are delivered.

BYZANTINE GENERALS PROBLEM

Getting back to the normal form game, we define a **strategy** s to be the set of actions all players take. In this case a strategy of $s = (a, c)$ would give Bob a utility of 1 and Alice a utility of 2, in other words, $u_{\text{Bob}}(s) = 1$ and $u_{\text{Alice}}(s) = 2$. We also refer to Bob's strategy in s as s_{Bob}, which is a in this case. This strategy is also an example of a **pure strategy**: one where the agents take a specific action. In contrast, a **mixed strategy** is one where the agents take different actions, each with some fixed probabilities. For example, a mixed strategy for Bob is to take action a with probability of .3 and action b with a probability of .7. Note that in a mixed strategy the probabilities for all actions of each agent have to add up to 1. Typically, game theory further assumes that players are **rational**, which we use as a synonym for selfish. That is, a rational player always acts so as to maximize its utility.

STRATEGY

PURE STRATEGY
MIXED STRATEGY

RATIONAL

A special type of game are those in which the values in every box of the matrix add up to zero. These games are known as **zero-sum** games and represent scenarios where one agent's gain must come at a loss to he other agents. For example, in a zero-sum game with two players if for a particular strategy s one player gets a utility of 5 then the other player must receive a utility of -5. In these games cooperation is unlikely. Note that every competitive sport is a zero-sum game as the fact that one team wins means the other must lose. Luckily, real-world problems are rarely zero-sum, even if our tendency is often to perceive them as such.

ZERO-SUM

3.1.1 Solution Concepts

Given a game matrix we can't help but ask: what strategy should they use? Which is best the best strategy in any given game? The problem, of course, is that there is no simple way to define what's best since what is best for one agent might not be good for another. As such, different solution concepts have been proposed.

The earliest solution concepts were proposed by Von Neumann. He realized that in any given game an agent could always take the action which maximized the worst possible utility it could get. This is known as the **maxmin strategy**, or minmax, if we are talking about losses instead of gains. Specifically, in a game with two agents, i and j, agent i's maxmin strategy is given by

MAXMIN STRATEGY

$$s_i^* = \max_{s_i} \min_{s_j} u_i(s_i, s_j). \tag{3.1}$$

That is, i assumes that no matter what it does j will take the action that is worst for i. Thus, i takes its best possible action given this assumption. Unfortunately, the strategy where both players play their maxmin strategy might not be stable in the general case. In some payoff matrices it can happen that if i knows that j will play its maxmin strategy then i will prefer a strategy different from its maxmin strategy. For example, in figure 3.1 the maxmin strategy is (b, d) but if Alice plays d then

Bob should play a. Thus, that strategy is not stable. This problem has prevented the maxmin strategy from becoming a popular solution concept.

The existence of a solution concept is also important to us since, if a solution concept does not exist for the game we are interested in then it is not of any use to us. For the maxmin strategy we have the **minimax theorem** which states that a strategy that minimizes the maximum loss, a minmax strategy, can always be found for all two-person zero-sum games (Neumann, 1928). Thus, we know it will work at least for this subset of games.

MINIMAX THEOREM

Another approach is to look for strategies that are clearly better. We say that s is the **dominant** strategy for agent i if the agent is better off doing s regardless of which strategies the others use. Formally, we say that a pure strategy s_i is dominant for agent i if

DOMINANT

$$\forall_{s_{-i}} \forall_{r_i \neq s_i} u_i(s_{-i}, s_i) \geq u_i(s_{-i}, r_i), \tag{3.2}$$

where s_{-i} represents the strategies of all agents except i. This idea can be expanded into the **iterated dominance** solution in which dominated strategies are eliminated in succession. First we eliminate strategies from one agent, then from another, and so on in a round-robin manner (and repeating agents) until we check all agents in successions and none has a dominated action. Unfortunately, the iterated dominance algorithm almost always ends before a solution is found. That is, in most games none of the players has a dominant strategy.

ITERATED DOMINANCE

We can also take a step back and look at the problem from a system designer's perspective. With our new enlightened outlook we might think that the right solution is to maximize overall welfare for all players. The **social welfare** strategy is the one that maximizes the sum of everyone's payoffs. That is

SOCIAL WELFARE

$$s^* = \arg\max_s \sum_i u_i(s). \tag{3.3}$$

However, once again we have the problem that a social welfare strategy might not be stable. Each selfish agent cares only about his own utility and will thus play a different strategy if it can get a higher utility, even if it means everyone else is much worse. Also, the social welfare strategy might not seem fair as it could be one where one agent gets an extremely high utility and everyone else gets almost nothing.

Vilfredo Pareto. 1848–1923.

We can solve the unfairness problem by defining a solution concept that does not take into account the agents' absolute utility values. A strategy s is said to be **Pareto optimal** if there is no other strategy s' such that at least one agent is better off in s' and no agent is worse off in s' than in s. There can be more than one Pareto optimal solutions for a given problem. The set of all Pareto strategies for a given problem is formally defined to be the set

PARETO OPTIMAL

$$\{s \mid \neg\exists_{s' \neq s}(\exists_i u_i(s') > u_i(s) \wedge \neg\exists_{j \in -i} u_j(s) > u_j(s'))\} \tag{3.4}$$

where $-i$ represents the set of all agents except i. Pareto solutions are highly desirable from a social welfare perspective as they ensure that no one agent can complain that it could get more utility without hurting someone else in the process. In Economics the Pareto solution of often referred to as Pareto efficient or, simply, the **efficient** solution. Unfortunately, Pareto solutions might also be unstable in that one player might have an incentive to play a different action because he gets higher utility, which means others get lower utility. In a dynamic multiagent system stability can be very important as designers often want the system to converge.

EFFICIENT

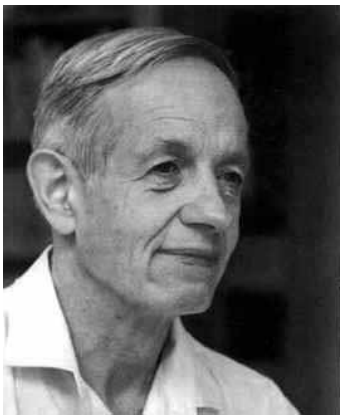
The problem of lack of stability was solved by John F. Nash. We say that a strategy s is a **Nash equilibrium** if for all agents i, s_i is i's best strategy given that all the other players will play the strategies in s. That is, if everyone else is playing the Nash equilibrium then the best thing for everyone to do is to play the Nash equilibrium. As with the Pareto solution, a game can have more than one Nash equilibrium. Formally, the set of all Nash equilibrium strategies for a given game is given by

NASH EQUILIBRIUM

$$\{s \mid \forall_i \forall_{a_i \neq s_i} u_i(s_{-i}, s_i) \geq u_i(s_{-i}, a_i)\}. \tag{3.5}$$

		A	
		Stays Silent	Betrays
B	Stays Silent	Both serve six months.	B serves 10 years; A goes free.
	Betrays	A serves 10 years; B goes free.	Both serve two years.

Figure 3.2: Payoff matrix for original prisoner's dilemma problem.



John F. Nash. 1928–. Winner of 1994 Nobel prize on Economics.

Nash showed that all game matrices have at least one equilibrium strategy, but this strategy might be mixed. The only problem with the Nash equilibrium is the fact that in many games there are many such equilibria. Also, some of those Nash equilibria might be better for some players than other. Thus, the players might end up arguing about which specific one to adopt. Still, once the agents have agreed on one it is a very stable solution. Note that there is no general relation between the Nash equilibrium and the Pareto solution. That is, a strategy can be a Nash equilibrium but not be a Pareto solution. Similarly, a strategy can be Pareto optimal but not be a Nash equilibrium. On the other hand, the social welfare strategy must be a Pareto optimal.

All these solution concepts make it clear that it is not a simple matter to define what is the best answer. With multiple players, what is best depends on how much we are willing to respect the agents' selfishness, how we trade-off the various agents utilities, and how fair we wish to be.

The most common problem we face when designing multiagent systems is the existence of multiple equilibria. That is, we can generally design a system so that selfish agents will converge to an equilibrium solution if there is one. But, if there are more than one solution then we need to add extra coordination mechanisms to ensure that the agents converge to the same strategy.

3.1.2 Famous Games

Prisoner's Dilemma

The most famous game of all is the **Prisoner's Dilemma**. Its story typically goes something like the following.

> Two suspects A, B are arrested by the police. The police have insufficient evidence for a conviction, and having separated both prisoners, visit each of them and offer the same deal: if one testifies for the prosecution against the other and the other remains silent, the silent accomplice receives the full 10-year sentence and the betrayer goes free. If both stay silent, the police can only give both prisoners 6 months for a minor charge. If both betray each other, they receive a 2-year sentence each.

From this story we can generate a payoff matrix as shown in figure 3.2. We can replace the prison sentences with utility values and arrive at the standard prisoner's dilemma payoff matrix shown in figure 3.3, note that longer prison terms translate into lower utility. Thus, a 10 year sentence gets a utility of 0, a 2 year sentence has utility of 1, 6 months is 3, and no time served has utility of 5. The actions are labeled cooperate and defect because the suspects can either cooperate with each other and maintain their silence or defect from their coalition and tell on the other one.

Analysis of this matrix reveals that the social welfare solution is (C,C), the set of Pareto optimal strategies is {(C,C) (D,C) (C,D)}, the dominant strategy for both players is Defection, and the Nash equilibrium strategy is (D,D).

The Prisoner's dilemma is interesting because the best choice is not the rational choice. That is, even thought the (C,C) solution Pareto dominates the (D,D)

		A	
		Cooperate	Defect
B	Cooperate	3,3	0,5
	Defect	5,0	1,1

Figure 3.3: Prisoner's dilemma with standard payoff values.

		Alice	
		Ice Hockey	Football
Bob	Ice Hockey	4,7	0,0
	Football	3,3	7,4

Figure 3.4: Battle of the sexes game.

solution, a rational player will only play D as that strategy is dominant. The game is widely studied because it applies to many real-world situations like nuclear disarmament and getting kids to clean up their room. There are some that object to the fact that defection is the equilibrium and claim that real people do not act like that.

BATTLE OF THE SEXES

The **battle of the sexes** is another popular game. It is shown in figure 3.4. In this game Alice and Bob like each other and would like to spend time together but must decide, without communicating with each other, where to go. Alice likes ice hockey while Bob likes football. As such, they have a coordination problem where each prefers to go to a different place but they would like to be together. This type of problem arises frequently in multiagent systems where we often have agents that want to cooperate with each other to achieve some larger goal but have conflicting priorities about how to achieve that goal. For example, the agents in a company want to get a new contract so that they can all get paid but each one wants to do as little work as possible while getting as much money as possible. Another example is agents that want to deliver a package but have different preferences on where the hand-off from one agent to the other should occur.

After some analysis of this game we can determine that the social welfare solutions are (I,I) (F,F), the Pareto optimal solutions are also (I,I) (F,F), there is no dominant strategy, and the Nash equilibrium solutions are (I,I) (F,F). As you can see, the problem here is that there are two strategies both of which are equally attractive. This is the type of problem that we could fix easily if we just had a little communication.

GAME OF CHICKEN

The **game of chicken**, shown in figure 3.5, is also common. In this story, two maladjusted teenagers drive their cars towards each other at high speed. The one who swerves first is a chicken and thus loses the game. But, if neither of them swerves then they both die in a horrible crash. After some analysis we can see that this game is very similar to the battle of the sexes. Its social welfare, Pareto optimal, and Nash strategies are all the same, namely (C,S) (S,C), and there is no dominant strategy. Once again there is a coordination problem: who will be the chicken?

One way out of the problem of multiple Nash equilibria is for one of the players to eliminate one of his choices, say by soldering the steering wheel of the car so that it does not turn. This would make it so that the other player's rational choice is to swerve. It might seem counter intuitive but in cases like this an agent can increase

		Alice	
		Continue	Swerve
Bob	Continue	-1,-1	5,1
	Swerve	1,5	1,1

Figure 3.5: The game of chicken.

		Pig	
		Nothing	Press Lever
Piglet	Nothing	0,0	5,1
	Press Lever	-1,6	1,5

Figure 3.6: The pig and the piglet.

its payoff by limiting its set of possible actions.

The final game is taken from the world of zoology. Imagine there is a big pen with a lever which, when pressed, delivers a small amount of food at the other end of the pen. The pen is occupied by one big, but slow, pig and one small and fast piglet. We then try to determine which one of them will press the lever and which one will eat. The situation can be summarized by the matrix in figure 3.6. Namely, if the big pig presses the lever then the piglet can stay standing by where the food comes out and eat some of it before the pig comes and shoves him out. If the piglet presses the lever then it will not get any food because the big pig will be standing by the food dispenser and will not let the piglet get close to the food. If both of them press it then the piglet will be able to eat a little bit since it can run faster than the pig.

An analysis of this game shows that the social welfare solutions are (N,P) (P,P), the Pareto optimal solutions are (N,P) (P,P) (P,N), the piglet has a dominant strategy of N, and the Nash equilibrium is (N,P). Since the piglet has a dominant strategy, it is clear that it will chose that, as such the big pig will have to press the lever. The end result has the piglet siting next to where the food comes out and the big pig running back and forth between the lever and the food.

Animal behaviorists have actually conducted experiments similar to this one and indeed found that the big pig usually ends up pushing the lever. Of course, the pigs do not actually think about their options. They simply try the different actions until they find the one that works best. This tells us that agents can arrive at an equilibrium solution without having much, if any, intelligence and use adaptation instead. We will learn about learning and how we can use it to solve coordination problems in a later chapter.

3.1.3 Repeated Games

We saw how in the prisoner's dilemma the dominant strategy was to defect but both players could have received a higher utility by cooperating. One way to try to get out of this conundrum, and to better simulate real-world interactions, is to let two players play the same game some number of times. This new game is known as the **iterated prisoner's dilemma** and, in general, we refer to these type of games as repeated games. Repeated games with a finite horizon are a special case of extended form games, which we will cover in the next section. They have, however, sometimes been studied independently.

ITERATED PRISONER'S DILEMMA

One way to analyze repeated games that last for a finite number of periods is to backtrack from the end. Let's say you are playing an iterated prisoner's dilemma which will last for 50 rounds. You know that at the last round you will defect because that is the dominant strategy and there is no need to be nice to the other player as that is the last game. Of course, since you realized this then you also know that the other player realized the same thing and so he will also defect at the last round. As such, you know that you have nothing to gain by cooperating at round 49 so you will defect at 49, and so will he. This backward induction can be repeated any number of times leading us to the conclusion that for any finite number of games the rational strategy is to always defect. However, people don't act like this.

We can also formally prove a cooperative equilibrium for the iterated prisoner's dilemma if instead of a fixed known number of interactions there is always a small probability that every interaction will be the last interaction. That is, the players

never know if this will be their last interaction or not. In this scenario we can show that the dominant strategy for the iterated prisoner's dilemma is to cooperate, with a certain probability.

More generally, however, the **folk theorem** tells us that in a repeated game any strategy that is not Pareto-dominated by another and where every agent gets a utility that is higher than his maxmin utility (3.1) is a feasible equilibrium strategy. In these cases, each player knows that he can get his maxmin utility by playing the appropriate action, regardless of what the others do. Thus, no player will be satisfied with less than his maxmin utility. On the other hand, if the agents agree to a strategy that gives everyone at least their maxmin and is not Pareto-dominated then any agent that diverges from it could be penalized by the other agents so that any gains he incurred from the deviation would be erased. In this way the agents can police any chosen strategy and thus ensure that it is a stable strategy.

FOLK THEOREM

Such analysis still leaves open the question of which, in practice, is the optimal strategy for the iterated prisoner's dilemma. In the early 1980's Robert Axelrod performed some experiments on the iterated prisoner's dilemma (Axelrod, 1984). He sent out an email asking people to submit fortran programs that played the prisoner's dilemma against each other for 200 rounds. The winner was the one that accumulated the most points. Many entries were submitted. They included the following strategies.

- *ALL-D*: always play defect.
- *RANDOM*: pick action randomly.
- *TIT-FOR-TAT*: cooperate in the first round, then do whatever the other player did last time.
- *TESTER*: defect on the first round. If other player defects then play tit-for-tat. If he cooperated then cooperate for two rounds then defect.
- *JOSS*: play tit-for-tat but 10% of the time defect instead of cooperating.

The **tit-for-tat** strategy won the tournament. It still made less than ALL-D when playing against it but, overall, it won more than any other strategy. It was successful because it had the opportunity to play against other programs that were inclined to cooperate. It cooperated with those that could cooperate and defected against the rest. This result, while intriguing, is not theoretically robust. For example a tit-for-tat strategy would lose against a strategy that plays tit-for-tat but defects on the last round. Still, the tit-for-tat strategy has been widely used and is considered to be a simple yet robust strategy.

TIT-FOR-TAT

Another method for analyzing repeated games is by assuming that the players use some learning algorithm and then trying to determine which strategy their learning will converge to. This research area is known as learning in games, which we present in section 5.3.

3.2 Games in Extended Form

In **extended form** games the players take sequential actions. These games are represented using a tree where the branches at each level correspond to a different player's actions and the payoffs to all agents are given at the leafs. Extended form games can also represent simultaneous actions by using dotted ellipses to group together nodes which are equivalent to the rest of the agents because, for example, they could not see the actions taken by the agent.

EXTENDED FORM

Figure 3.7 shows the extended form version of the payoff matrix from figure 3.1. In it, the first number inside the parentheses is the utility that Bob receives and the second number is Alice's utility. The dotted line groups together two states which are invisible to Bob: it shows that Bob does not know the action Alice took. If we

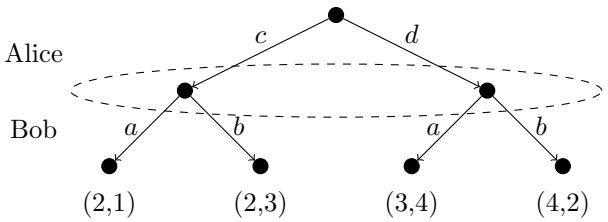


Figure 3.7: Game in extended form that corresponds to the normal form game from Figure 3.1.

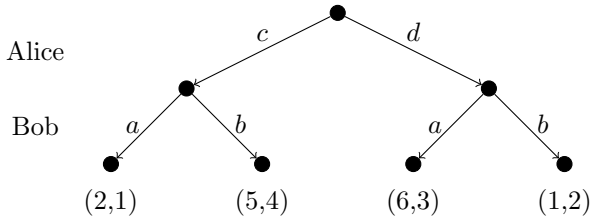


Figure 3.8: Game in extended form.

Extended form games, without the dotted ellipses, are nearly identical those built by the minimax algorithm (Russell and Norvig, 2003, Chapter 6).

eliminated the dotted line then in that new game Alice takes an action, which Bob can see, and then Bob takes his action.

In an extended game a player's strategy s_i is no longer just an action but can now be a series of actions if the player gets to take more than one action in the tree, or if the player can have different actions depending on which node in the tree it is in. That is, if a player can see others' actions that come before him then his action can vary. For example, in figure 3.8 a rational Bob would choose b if Alice has chosen c but would choose a if Alice has chosen d. As before, agent i's utility from strategy s is given by $u_i(s)$ and corresponds to the values on the leaf node reached when all players play s.

3.2.1 Solution Concepts

We can apply the Nash equilibrium idea from normal form games to extended games. Namely, we say that an extended game has a Nash equilibrium strategy s^* if for all agents i it is true that they can't gain any more utility by playing a strategy different from s_i^* given that everyone else is playing s_{-i}^*. For example, for the game in figure 3.8 the Nash equilibrium is (c, b)—Alice plays c and Bob plays b. While this strategy is in equilibrium notice how, if there was some problem or noise in the system and Alice ends up playing d then Bob's strategy of playing b is no longer his best response. That is, the basic Nash equilibrium solution might not be stable under noisy conditions.

SUBGAME PERFECT EQUILIBRIUM

A stronger solution concept is the **subgame perfect equilibrium** strategy s^* which is defined as one where for all agents i and all subgames it is true that i can't gain any more utility by playing a strategy different from s_i^*. We define a subgame to be any subtree of the extended game. That is, a subgame perfect equilibrium gives all the agents their best response strategies for every node in the tree. For example, the subgame perfect equilibrium for figure 3.8 is for Alice to play c and Bob to play b if Alice plays c and a if Alice plays d.

Extended form games are a useful way to represent more complex multiagent interactions. Notice, however, that they are a special case of a multiagent Markov decision process. Most researchers have opted to use the more complete Markov model over the simpler extended form game for modeling multiagent systems.

These solution concepts provide us with solutions to shoot for when building multiagent systems, even when these solutions are impossible to find. For example, say you want to build a team of robot soccer players. Each robot will have a specific behavior that depends on its current perception of the world and its internal state. Each robot can also be assumed to receive a payoff whenever it, or someone in its team, scores a goal. If you consider the set of all possible agent behaviors for all agents in the team as the set of actions in a game then we can talk about the players

being at a Nash equilibrium, or about the players in one team being in the Pareto optimum given the other team's behaviors. Of course, such equilibria are impossible to calculate but they do give us a precise goal, of the intellectual kind, to shoot for. Also, as many researchers have done, we can try to break up that immense problem into smaller problems for which the agents can find equilibrium strategies.

3.3 Finding a Solution

There is an extensive literature on centralized algorithms for finding the various equilibrium strategies for a given game they are, however, generally considered to be outside the purview of multiagent research. Generally, the algorithms involve a complete search using heuristics for pruning. The **Gambit** software program (McKelvey et al., 2006) is an open source implementation of several of these algorithms.

GAMBIT

However, note that some of these solutions can be found by more multiagent-friendly distributed algorithms. In chapter 5 we show multiagent learning algorithms which allow learning agents to converge to Nash and other equilibria.

Exercises

3.1 Prove that a social welfare solution must be Pareto optimal.

3.2 Show a 2-person standard form game with 3 actions for each player in which iterated dominance leads to a unique equilibrium strategy.

3.3 Assume a 2-person standard form game with x actions for each player.

1. What is the maximum number of Pareto optimal pure strategies that such a game could have?
2. Assuming that all payoff values are different, what is the maximum number of Pareto optimal pure strategies that such a game could have?

3.4 Find the pure Nash equilibria, and the Pareto optimal solutions of the following game:

		Alice		
		d	e	f
Bob	a	1,2	2,3	2,3
	b	4,5	6,7	3,4
	c	5,4	6,5	5,6

3.5 The battle of the sexes game, seen in figure 3.4, is a classic coordination game in which the players must somehow coordinate to agree upon one of the Nash equilibria. These problems are solved by populations who adopt a social law after some trial and error adaptation phase. For example, in the US people drive on the right side of the rode while in England they drive on the left. Both solutions are equally valid.

coordination

Implement a NetLogo program where each patch repeatedly engages in a battle of the sexes game with one of its neighbors, chosen at random. Then try to come up with some simple adaptation strategy which the agents could use so that the population will quickly converge to an equilibrium.

Simple adaptation strategies for solving this problem exist (Shoham and Tennenholtz, 1997), as well as for more complex neighborhood definitions (Delgado, 2002).

3.6 Lets say that in a repeated version of the game of chicken, figure 3.5, Alice and Bob decided to converge to the strategy (Swerve, Swerve). Since this strategy satisfies the maxmin criteria (check this) the Folk theorem tells us that is will be stable since both players could play an iterated strategy that penalizes the other so that any gains from defection are erased after some rounds. Write the short algorithm which describes the player's iterated strategy for this situation, that is, the iterated strategy they must use to guarantee that (Swerve, Swerve) is the equilibrium strategy on every step of the iterated game.

3.7 A generalization of the Tit-for-Tat strategy is to, instead of simply doing exactly what the other agent did last time, make a stochastic decision based on the other agent's action. That is, if the other agent defected last time then you will be very likely, with a given probability, to defect and vice versa. Write a NetLogo program where you add these type of agents to a tournament similar to Axelrod's. Which strategy triumphs in the end?


packages

This general strategy is known as reciprocity and often occurs in human interaction: you are more likely, but not entirely sure, to be nice to those that were nice to you in the past. Simulations of populations with various numbers of reciprocating agents have shown that populations of reciprocating agents tend to do better as a whole (since they help each other) but can be exploited by selfish agents. This exploitation can be curbed by having the reciprocating agents share their opinions of other (Sen, 2002).

Chapter 4

Characteristic Form Games and Coalition Formation

There is another type of game studied in game theory: the **characteristic form** game or **coalition** game (Osborne and Rubinstein, 1999). In these games the agents decide how to form coalitions among themselves and each coalition receives some utility. For example, a group of people, each with different skills, all want to start new companies. The problem they face is deciding how to divide themselves into subgroups such that each subgroup has the needed set of skills to succeed in the marketplace. Similarly, a group of agents with different skills must decide how to divide itself into subgroups so as handle as many tasks as possible in the most efficient manner. Because the agents must cooperate with each other in order to form coalitions and an agent cannot unilaterally decide that it will form a coalition with a second agent, these games are known as **cooperative games**. Multiagent researchers have also extended the basic characteristic form into to the more general coalition formation, which we also present in this chapter.

CHARACTERISTIC FORM
COALITION

COOPERATIVE GAMES

It is interesting to note that most game theory textbooks focus exclusively on non-cooperative games as these have found many applications in Economics and Business and have been the focus of most of the research. However, when building multiagent systems we find that cooperative games are much more useful since they clearly and immediately model the problem of which agents should perform which tasks.

4.1 Characteristic Form Games

Formally, a game in characteristic form includes a set $A = \{1, \ldots, |A|\}$ of agents. The agents are assumed to deliberate and the final result of the deliberation is an **outcome** $\vec{u} = (u_1, \ldots, u_{|A|}) \in \Re^{|A|}$ which is just a vector of utilities, one for each agent. There is also a rule $V(\cdot)$ that maps every coalition $S \subset A$ to a utility possibility set, that is $V(S) \subset \Re^{|S|}$. Notice that $V(S)$ returns a set of utility vectors, not a single utility vector. As such, $V(\cdot)$ provides us the set of payoffs that players in S can achieve it they form a coalition. For example, for the players $\{1, 2, 3\}$ we might have that $V(\{1, 2\}) = \{(5, 4), (3, 6)\}$ meaning that if agents 1 and 2 formed a coalition they could either get 5 for agent 1 and 4 for agent 2, or they could get 3 for agent 1 and 6 for agent 2. The function V must be defined for all subsets of A.

OUTCOME

A special case of the characteristic form game–the one nearly all multiagent research focuses on—is the **transferable utility** game in characteristic form. This game assumes that the players can exchange utilities among themselves as they see fit. For example, if the utility payments are in the form of money then we only need to specify the total amount of money the coalition will receive and decide later how this money will be distributed among the agents in the coalition. More formally, we define a transferable utility game

TRANSFERABLE UTILITY

Definition 4.1 (Tranferable utility characteristic form game)**.** *These games consist of a set of agents* $A = \{1, \ldots, A\}$ *and a* characteristic function $v(S) \to \Re$ *defined for every* $S \subseteq A$*.*

The **characteristic function** $v(S)$ is also sometimes simply referred to as the **value function** for the coalitions. Characteristic form games with transferable

CHARACTERISTIC FUNCTION
VALUE FUNCTION

S	$v(S)$
(1)	2
(2)	2
(3)	4
(12)	5
(13)	7
(23)	8
(123)	9

(1)(2)(3)
$2 + 2 + 4 = 8$

(1)(23)
$2 + 8 = 10$

(2)(13)
$2 + 7 = 9$

(3)(12)
$4 + 5 = 9$

(123)
9

Figure 4.1: Sample characteristic form game with transferable utility for three agents: 1, 2 and 3. The table on the left shows the values of each coalition. On the right are the coalition structures. Below each one we calculate its value.

utility can represent many multiagent scenarios. For example, they can represent a task allocation problem where a set of tasks has to be performed by a set of agents, subsets of whom can sometimes improve their performance by joining together to perform a task. They can represent a sensor network problems where the sensors must join together in subgroups to further refine their readings or relay important information, or they can represent workflow scheduling systems where agents must form groups to handle incoming workflows.

4.1.1 Solution Concepts

As is often the case in game theory, there is no clear best solution to all characteristic form games. Instead, various solutions concepts have been proposed each one having its own advantages and disadvantages.

Before defining the solution concepts, we must first notice that the outcome as we have defined it allows for impossible utility values in the transferable utility game. Specifically, there might not be a set of coalitions such that, given v, the agents can all get their utility as promised by $\vec{u}$. In order to rectify this problem we first specify that we are only interested in **feasible** outcomes, that is, those that can be implemented given v.

FEASIBLE

Definition 4.2 (Feasible)**.** *An outcome $\vec{u}$ is* feasible *if there exists a set of coalitions $T = S_1, \ldots, S_k$ where $\bigcup_{S \in T} S = A$ such that $\sum_{S \in T} v(S) \geq \sum_{i \in A} \vec{u}_i$.*

That is, an outcome $\vec{u}$ is feasible if we can find a disjoint set of coalitions whose values are as much as that in $\vec{u}$, so we can payoff $\vec{u}$ with v. The set of disjoint coalitions T defined above is also often referred to as a **coalition structure**, also sometimes represented with the symbol CS.

COALITION STRUCTURE

Notice that, if the characteristic function is super-additive then we can check if an outcome $\vec{u}$ is feasible by simply ensuring that

$$\sum_{i \in A} \vec{u}_i = v(A). \tag{4.1}$$

We define a **super-additive** domain as one where, for all pairs of disjoint coalitions $S, T \subset A$, we have that $v(S \cup T) \geq v(S) + v(T)$. That is, there is nothing to be lost by merging into a bigger coalition. Unfortunately, multiagent systems are rarely supper-additive since agents have a habit of getting into each others' way, so that a team is not always better than letting each agent work on a task separately.

SUPER-ADDITIVE

The problem of finding feasible solutions can best be illustrated with an example. Figure 4.1 shows a sample transferable utility game for three agents along with the definition of the v function and all possible coalition formations. In this game the outcome $\vec{u} = \{5, 5, 5\}$ is not feasible since there is no way to divide the agents into subsets such that they can all get their utility. If we tried the coalition (123) then we only have a value of 9 to distribute and we need a total of 15. On the other hand, the outcome $\vec{u} = \{2, 4, 3\}$, is feasible because the coalition structures (1)(23), (2)(13), and (123) can satisfy it (but not the other ones). However, $\vec{u} = \{2, 4, 3\}$ does have a problem in that in it agent 3 is getting an utility of 3 while we have that $v((3)) = 4$. That is, agent 3 could defect any one of the three coalition structures

S	$v(S)$
(1)	1
(2)	2
(3)	2
(12)	4
(13)	3
(23)	4
(123)	6

(1)(2)(3)
$1+2+2=5$

(1)(23)
$1+4=5$

(2)(13)
$2+3=5$

(3)(12)
$2+4=6$

$\vec{u}$	in Core?
$\{2,2,2\}$	*yes*
$\{2,2,3\}$	*no*
$\{1,2,2\}$	*no*

(123)
6

Figure 4.2: Sample game showing some outcomes that are in the core and some that are not. The characteristic function v is super-additive.

we found, join the coalition (3), and get a higher utility than he currently has. This outcome thus seems unstable.

The Core

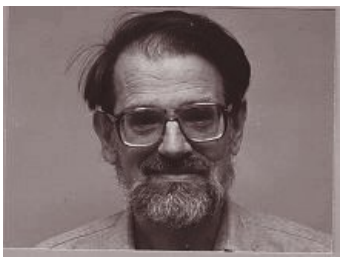
In general, we say that an outcome $\vec{u}$ is stable if no subset of agents gets paid more, as a whole, than what they get paid in $\vec{u}$. Stability is a nice property because it means that the agents do not have an incentive to go off into their own coalition. Our first solution concept, the **core**, refers to all the outcomes that are stable.

CORE

Definition 4.3 (Core)**.** *An outcome $\vec{u}$ is in the* core *if*

1. $$\forall_{S \subset A} : \sum_{i \in S} \vec{u}_i \geq v(S)$$

2. *it is feasible.*



Lloyd F. Shapley. 1923–. Responsible for the core and Shapley value solution concepts.

The first condition in this definition tells us that the utility the agents receive in outcome $\vec{u}$ is bigger than those of any coalition, for the agents in the coalition. In other words, that there is no coalition S whose $v(S)$ is bigger than the sum of payments the agents in S get under $\vec{u}$. The second condition merely checks that the total utility we are giving out is not more than what is coming in via $v(\cdot)$.

Figure 4.2 shows a new game with different payments and a list of outcomes some of which are in the core and some which are not. $\{2,2,2\}$ is in the core because it is feasible and there is no subset of agents S with a $v(S)$ that is bigger than what they could get in this outcome. $\{2,2,3\}$ is not in the core because it is not feasible. This outcome adds up to 7 and there is no coalition structure that adds up to 7. Note that, since the v is super-additive, all we need to check is the grand coalition. Finally, $\{1,2,2\}$ is not in the core because agents 1 and 2 are getting a total of 3 while if they formed the coalition (12) they would get a utility of 4.

We like the core because we know that any solution that is in the core cannot be improved by having any of the 2^A subsets of agents form a coalition of higher value than they are getting now. It is a very stable solution. Unfortunately, there are many games with empty cores. Figure 4.3 shows one such example. Try to find an outcome in the core for this example. You will see that every outcome is blocked by some other outcome.

Another problem with the core solutions is that when it is not empty then there are often many outcomes in the core. For example, in figure 4.2 any outcome $\vec{u} = \{x, y, z\}$ where $x + y + z \leq 6$ and $x + y \geq 4$ and $x + z \geq 3$ and $y + z \geq 4$ is in the core. Thus, we still have a coordination or negotiation problem where we must choose between these coalitions. Finally, there is the fact that the outcome does not tell us which coalitions are formed. For example, if we choose the outcome $\vec{u} = \{2, 2, 2\}$ then we must still also choose a coalition structure as there are two which would work: (123) and (12)(3).

S	$v(S)$
(1)	0
(2)	0
(3)	0
(12)	10
(13)	10
(23)	10
(123)	10

Figure 4.3: Set of payments for a game with an empty core

S	$v(S)$
()	0
(1)	1
(2)	3
(12)	6

Figure 4.4: Example game. If the agents form coalition (12) then how much utility should each one get?

The Shapley Value

While the core gives us one possible solution, it suffers from the fact that many games don't have any solutions in the core and from its lack of guidance in fairly distributing the payments from a coalition to its members. The Shapley value solves these problems by giving us one specific set of payments for coalition members, which are deemed fair.

The problem with identifying fairness in characteristic form games is best illustrated by an example. Figure 4.4 shows a game for two players. Clearly, we should choose the coalition (12) as it has the highest value. Now we must decide how much each agent should get. The simplest solution is to divide the total of 6 evenly amongst the coalition members, so that each agent gets 3. This seems unfair to agent 2 because agent 2 could have gotten 3 by simply staying on its own coalition (2). It seems like the fair thing to do is to give each agent a payment that is proportional to the value it contributes to the coalition, that is, the amount that value increases by having the agent in the coalition. But, how do we extend this idea to cases with more than 2 agents?

Shapley was able to extend this idea by realizing that each agent should get a payment that corresponds to its marginal contribution to the final value. An agent's marginal contribution to a coalition is the difference between the value before the agent joins the coalition and after he joined. For example, if before you join Initech their annual profits are \$10M but after you are there for a year they increase to \$11M then you can claim that your marginal contribution is to Initech is \$1M assuming, of course, that everything else stays the same during that year.

The one remaining problem is that there are many different orderings in which n agents could have joined the coalition, namely, there are $n!$ orderings of n elements. The **Shapley value** simply averages over all possible orderings. That is, the Shapley value gives each agent a utility proportional to its average marginal contribution to every possible coalition, in every possible order it could have been formed. More formally, we define the Shapley value as:

Shapley value

Definition 4.4 (Shapley Value)**.** *Let $B(\pi, i)$ be the set of agents in the agent ordering π which appear before agent i. The* Shapley value *for agent i given A agents is given by*

$$\phi(A, i) = \frac{1}{A!} \sum_{\pi \in \Pi_A} v(B(\pi, i) \cup i) - v(B(\pi, i)),$$

where Π_A is the set of all possible orderings of the set A. Another way to express

the same formula is

$$\phi(A,i) = \sum_{S \subseteq A} \frac{(|A| - |S|)!\,(|S| - i)!}{|A|!}[v(S) - v(S - \{i\})].$$

Notice that the Shapley values are calculated for a particular coalition A in the definition above. They are not meant as a way of determining which is the best coalition structure. They can only be used to distribute the payments of a coalition once it is formed.

Lets calculate the Shapley values for the game in figure 4.4 and the grand coalition (12). Since there are only two agents it means that there are only two possible orderings: (12) and (21). As such we have that

$$\begin{aligned}
\phi(\{1,2\},1) &= \frac{1}{2} \cdot (v(1) - v() + v(21) - v(2)) \\
&= \frac{1}{2} \cdot (1 - 0 + 6 - 3) = 2 \\
\phi(\{1,2\},2) &= \frac{1}{2} \cdot (v(12) - v(1) + v(2) - v()) \\
&= \frac{1}{2} \cdot (6 - 1 + 3 - 0) = 4
\end{aligned}$$

A somewhat surprising and extremely useful characteristic of the Shapley value is that it is always feasible. In our example the payments of 4 and 2 add up to 6 which is the same value we get in the grand coalition (12). Another nice feature of the Shapley value is that it always exists and is unique. Thus, we do not have to worry about coordination mechanism to choose among different payments. A final interesting result is that the Shapley value might not be in the core, even for cases where the core exists. This is a potential problem as it means that the resulting payments might not be stable and some agents might choose to leave the coalition in order to receive a higher payment on a different coalition.

Unfortunately, while the Shapley value has some very attractive theoretical properties, it does have some serious drawbacks when we try to use it for building multiagent systems. The biggest problem is computational. The Shapley value requires us to calculate at least $2^{|A|}$ orderings, this is only possible for very small sets A. It also requires that we know the value of v for every single subset S. In many real-world applications the calculation of v is complex. For example, it might require simulating how a particular coalition of agents would work together. These complex calculations could dramatically increase the total time. Finally, the Shapley value does not give us the actual coalition structure. Thus, it only solves the second part of the coalition formation problem. We must still determine which coalition the agents will form and how they will do it.

The Nucleolus

Since the core is often empty, researchers started looking for ways of relaxing it and find a new solution concept that would exist for every game. The problem with the core is that it says that there is no subset of agents that could get paid more than what they are currently getting paid in $\vec{u}$, because then they would be tempted to defect and form a new coalition. If it is impossible to find such an $\vec{u}$ then the next best thing would be to find the $\vec{u}$ that minimizes the total temptation felt by the agents. That is what the **nucleolus** aims to do.

NUCLEOLUS

We start by clarifying what we mean by temptation. Specifically, a coalition S is more tempting the higher its value is over what the agents get in $\vec{u}$. This is known as the **excess**.

EXCESS

Definition 4.5 (excess)**.** *The* excess *of coalition S given outcome $\vec{u}$ is given by*

$$e(S,\vec{u}) = v(S) - \vec{u}(S),$$

where

$$\vec{u}(S) = \sum_{i \in S} \vec{u}_i.$$

That is, a coalition S has a positive excess, given $\vec{u}$, if the agents in S can get more from $v(S)$ than they can from $\vec{u}$. The more they can get from S the higher the excess. Note that, by definition, if an outcome $\vec{u}$ is in the core then all coalitions have a excess that is less than or equal to 0 with respect to that outcome. But, since we are now concerned with outcomes that are not in the core we will instead look for those with minimal excess. Since excess is defined for all possible subsets S we first need a way to compare the excesses of two outcomes. We do this by putting them in a sorted list and comparing the list. The one with the higher excess first is declared more excessive. More formally, for each $\vec{u}$ we find its excess for all subsets S and order these in a list; the higher excesses come first, as such

$$\theta(\vec{u}) = \langle e(S_1^{\vec{u}}, \vec{u}), e(S_2^{\vec{u}}, \vec{u}), \ldots, e(S_{2^{|A|}}^{\vec{u}}, \vec{u}) \rangle, \tag{4.2}$$

where $e(S_i^{\vec{u}}, \vec{u}) \geq e(S_j^{\vec{u}}, \vec{u})$ for all $i < j$. We then define a lexicographical ordering $\succ$ over these lists where $\theta(\vec{u}) \succ \theta(\vec{v})$ is true when there is some number $q \in 1 \ldots 2^{|A|}$ such for all $p < q$ we have that $e(S_p^{\vec{u}}, \vec{u}) = e(S_p^{\vec{v}}, \vec{v})$ and $e(S_q^{\vec{u}}, \vec{u}) > e(S_q^{\vec{v}}, \vec{v})$ where the S_i have been sorted as per θ. That is, if $\theta(\vec{u}) \succ \theta(\vec{v})$ then that means that when we sort their excesses for all subsets their first, and greatest, excesses are all the same and on the first set for which they have a discrepancy $\vec{u}$ has the highest excess. For example, if we had the lists $\{(2,2,2),(2,1,0),(3,2,2),(2,1,1)\}$ they would be ordered as $\{(3,2,2),(2,2,2),(2,1,1),(2,1,0)\}$.

NUCLEOLUS

We can now define the **nucleolus** as the $\vec{u}$ which is not lexicographically bigger than anyone else.

Definition 4.6 (nucleolus)**.** *The* nucleolus *is the set*

$$\{\vec{u} \,|\, \theta(\vec{u}) \not\succ \theta(\vec{v}) \text{ for all } \vec{v}, \text{ given that } \vec{u} \text{ and } \vec{v} \text{ are feasible.}\}$$

In other words, the outcomes in the nucleolus are those where the excesses for all possible sets are lexicographically not greater than those of any other outcome. A nice feature of the nucleolus is that it is always unique for each coalition structure. That is, given a coalition structure there is only one nucleolus.

The nucleolus captures, to some degree, the idea of an outcome that minimizes the temptation the agents face. However, notice that the lexicographic order it defines only cares about the first coalition that has a higher excess, it does not care about the ones after that. This could lead to a situation where the sum of the excesses from the nucleolus is actually larger than that of some other outcome. For example, $(5.0, 0, 0)$ comes before $(4, 3, 3, 3)$. As such, the nucleolus does not seem to minimize the sum of temptations.

Equal Excess

EQUAL EXCESS

Another technique for calculating the agents' payoff, besides the Shapley value, is called **equal excess**. It is an iterative algorithm where we adjust the payments that the agents expected they will receive from each coalition that includes them. At each time step t we let $E^t(i, S)$ be agent i's expected payoff for each coalition S which includes him. Initially these are set to 0. We thus let

$$A^t(i, S) = \max_{T \neq S} E^t(i, T) \tag{4.3}$$

be agent i's expected payment from not choosing S and instead choosing the best alternative coalition. Then, at each time step we update the players' expected payments using

$$E^{t+1}(i, S) = A^t(i, S) + \frac{v(S) - \sum_{j \in S} A^t(j, S)}{|S|}. \tag{4.4}$$

(1)(2)(3)(4)

(12)(3)(4) (13)(2)(4) (14)(2)(3) (23)(1)(4) (24)(1)(3) (34)(1)(2)

(1)(234) (2)(134) (3)(124) (4)(123) (12)(34) (14)(23) (13)(24)

(1234)

Figure 4.5: Coalition structure formation possibilities for four agents, organized by the number of coalitions.

For example, for the value function given in figure 4.4 we start with $E^0(1,*) = E^0(2,*) = 0$. Then, at time 1 agent 1 can update $E^1(1,(1)) = 1$ and $E^1(1,(12)) = 3$ and then $A^1(1,(1)) = 3$ and $A^1(1,(12)) = 1$, while agent 2 updates $E^1(2,(2)) = 3$ and $E^1(2,(12)) = 3$ and then $A^1(1,(1)) = 3$ and $A^1(1,(12)) = 3$.

It has been shown that this basic algorithm does not always converge to a fixed point, however, variations of it have been proposed which do converge, such as PACT (Goradia and Vidal, 2007). In PACT the agents calculate their own E values and exchange them with others under the assumption that agents will not lie about these values. The algorithm ensures that the process will stop and a solution will be found.

Notice that equal excess is a procedural solution to the problem so we do not know which specific outcome the agents will converge to, other than to say that they will converge to the outcome that is found when we use equal excess.

4.1.2 Finding the Optimal Coalition Structure

As multiagent system designers we often simply want to find the outcome that maximizes the sum of values. That is, we want to find the **utilitarian** solution. When the characteristic function is super-additive then the grand coalition will have the highest value and thus finding the utilitarian solution is trivial. However, if the characteristic is not super-additive—as is often the case in multiagent systems—then we will want an algorithm for finding it. Notice that under this formulation we are no longer interested in the specific outcome (that is, individual payments to agents) we are now only interested in finding best coalition structure, ignoring the problem of dividing up the value of each coalition among its participants.

UTILITARIAN

Centralized Algorithm

One proposed approach is to a perform a complete search of the complete set of possible coalition structures, but in a specified order. Figure 4.5 shows all the possible coalition structures for four agents. Notice that the bottom two rows contain all possible coalitions. This means that after searching those two rows we have seen all possible coalitions. If we let S^* be the value of the highest valued coalition (*not* coalition structure) found after searching those two rows then we know that the best coalition structure cannot be more than $A \cdot S^*$. As such, after searching the bottom two levels we can say that the optimal solution is no more than A times better than the best solution we have found thus far.

Figure 4.6 shows the bounds that can be calculated after examining each of the levels in the graph. One simple algorithm consists of first searching the bottom two levels then continue searching down from the top level (Sandholm et al., 1999). In this way, the bound from optimal is reduced as indicated in the figure. Note that searching the levels in some other order will not guarantee these bounds. Notice also that the number of coalition structures in the second level is given by its **Stirling** number

STIRLING

$$\text{Stirling}(A,2) = \frac{1}{2}\sum_{i=0}^{1}(-1)^i\binom{2}{i}(2-i)^A = 2^{A-1} - 1. \tag{4.5}$$

Level	Bound
A	$A/2$
$A-1$	$A/2$
$A-2$	$A/3$
$A-3$	$A/3$
$A-4$	$A/4$
$A-5$	$A/4$
:	:
2	A
1	none

Figure 4.6: Bounds on optimality after searching various levels.

```
FIND-COALITION(i)
1  L_i ← set of all coalitions that include i.
2  S_i^* ← arg max_{S∈L_i} v_i(S)
3  Broadcast S_i^* and wait for all other broadcasts, put these into S^* set.
4  S_max ← arg max_{s∈S^*} v_i(s)
5  if i ∈ S_max
6      then join S_max
7           return
8  for j ∈ S_max
9       do Delete all coalitions in L_i that contain j
10 if L_i is not empty
11     then goto 2
12 return
```

Figure 4.7: Distributed algorithm for coalition formation. Each agent i must execute this function. We let $v_i(S) = \frac{v(S)}{|S|}$

So it takes exponential time just to search the second level. In general, the number of coalition structures for all levels is equal to the Stirling number for that level.

There also exists an algorithm for finding the optimal coalition structure which has slightly better bounds than the ones we just presented, but running time remains exponential and unusable for large number of agents (Dang and Jennings, 2004).

Distributed Algorithm

find-coalition

While the previous algorithm found the optical coalition structure, it did so at the expense of a lot of computation and in a centralized manner. We now look at one possible way of finding a good, but possibly not optimal, coalition structure in a decentralized manner.

Figure 4.7 shows a distributed algorithm for coalition formation (Shehory and Kraus, 1998). The agents order all their possible coalitions based on how much each will get in that coalition, where each agent i gets $v_i(S) = v(S)/|S|$ if it joins coalition S. The agents then broadcast the name of their best coalition. The coalition with maximal $v(S)/|S|$ is chosen by the agents in S who join the coalition and drop out of the algorithm. The remaining agents take note of the missing agents by eliminating from consideration all coalitions that include them. The process is then repeated again with the new set of coalitions.

This is a classic example of a greedy or hill-climbing algorithm. As such, we know that it might get stuck on a local optima, which might or might not be the global optimum. Still, the algorithm should execute very fast as there are at most A steps and each step involves having each agent examine all the possible coalitions that it can be participate in.

A slight modification of the algorithm would be to, instead of broadcasting at each time step we could let the agents meet randomly and form a coalition if $v_i(s)$ is maximal for all agents in s. Imagine the agents moving around in a space and forming sets whenever a group of them happen to be close to each other, then

forming a coalition only if that set has a maximal value. This process is effectively the same as broadcasting except that it eliminates the need to broadcast at the expense of taking a longer time to converge (Sarne and Arponen, 2007).

Reduction to Constraint Optimization

We note that the problem of finding the optimal coalition structure can be reduced to a constraint optimization problem. The basic idea is that for n agents there will be at most n coalitions as, at worst, each agent will stay in the individual coalition. Thus, we can imagine the problem as consisting of n agents each one deciding which of n rooms to go into. The agents are the variables and the rooms are the domains. The agents in a room form a coalition and empty rooms are ignored. We set a constraint for each room equal to $-v(s)$ where s is the set of agents that choose that room, or 0 if no agents choose it. We then have a constraint optimization problem where we are trying to the set of values which minimizes the sum of the constraint violations, thus maximizing the sum of the valuations.

Note that this is a degenerate case of the constraint optimization problem in that all the n constraints are over all agents. Most constraint problems exhibit some degree of locality in that constraints are only over a small subset of the variables. Having all constraints be over all variables makes this problem harder than average. Thus this reduction is likely to be only of theoretical interest.

4.2 Coalition Formation

The **coalition formation** problem, as studied in multiagent systems, extends the basic characteristic form game in an effort to make it a better match for real world problems. A coalition formation problem consists of three steps.

1. Agents generate values for the $v(\cdot)$ function.

2. Agents solve the characteristic form game by finding a suitable set of coalitions.

3. Agents distribute the payments from these coalitions to themselves in a suitable manner.

Steps 2 and 3 can be thought of as part of the traditional characteristic form game. The coalition formation definition simply chooses to split the problem of finding a suitable outcome $\vec{u}$ into two parts: finding the coalitions and then dividing the payments. The split mirrors the requirements of many application domains. Step 1 is completely new. It is there because in many domains it is computationally expensive to determine the value of $v(S)$ for a given S. For example, if the agents are trying to form groups that solve particular tasks then calculating $v(S)$ requires them to determine out how well they can perform the task as a group, which requires considering how all their different skills can be brought to bear and, in some common scenarios, requires the development of a full plan—an exponential problem. The approaches at solving step 1 are thus generally dependent on the domain and do not generalize well.

Exercises

4.1 Give an example problem in which agents using equal excess and reporting their own E values will want to lie about their own E values.

4.2 Find the set of core solutions and the Shapley value of the grand coalition for

the following game:

S	$v(S)$
(1)	1
(2)	2
(3)	3
(12)	5
(13)	4
(23)	5
(123)	5

4.3 Say you have robots which live in a 2-dimensional grid and each one has a strength given by a number in the set $\{1, 2, 3, 4, 5\}$. There are boxes in this world, each one of which must be moved to a specified destination. The speed with which a set of robots S can move a box is given by $1 - 1/\sum_{i \in S} i$.strength.

- Formulate this problem as a characteristic form game and provide a $v(S)$ definition.
- Find a good algorithm for calculating the optimal coalition structure.

4.4 Modify the FIND-COALITION algorithm from figure 4.7 so that instead of the agents broadcasting their values they move around randomly in a two-dimensional space. After each time step the set of agents in a tile checks if they form a maximal coalition, that is, there is no other coalitions that gives one of the agents a higher $v_i(S)$ value. If so, they form that coalition and leave the game while the rest keep moving. Implement this algorithm in NetLogo and check how long it takes to find a solution.

4.5 We can extend the problem of coalition formation and make it more realistic by defining the value function over a set of possible agent abilities and then giving the agents sets of abilities (Yokoo et al., 2005). We then face the possibility of an agent pretending to be multiple agents, each with a different ability. Why would an agent do this? Give an example when an agent benefits from this technique.

Another problem might be agents that fail to mention to others that they have certain skills. Why would an agent do this? Give an example when an agent benefits from this technique.

Chapter 5

Learning in Multiagent Systems

Machine learning algorithms have achieved impressive results. We can write software that processes larger amounts of data than any human can and which can learn to find patterns that escape even the best experts in the field. As such, it is only reasonable that at some point we will want to add learning agents to our multiagent system. There are several scenarios in which one might want to add these learning agents.

Many multiagent systems have as their goal the exploration or monitoring of a given space, where each agent has only a local view of its own area. In these scenarios we can envision that each agent learns a map of its world and the agents further share their maps in order to aggregate a global view of the field and cooperatively decide which areas need further exploration. This is a form of cooperative learning.

Another scenario is in competitive environments each selfish agent tries to maximize its own utility by learning the other agents' behaviors and weaknesses. In these environments we are interested in the dynamics of the system and in determining if the agents will reach a stable equilibrium. At their simplest these scenarios are repeated games with learning agents.

To summarize, agents might learn because they don't know everything about their environment or because they don't know how the other agents behave. Furthermore, the learning can happen in a cooperative environment where we also want the agents to share their learned knowledge, or in a competitive environment where we want them to best each other. We present analysis and algorithms for learning agents in these various environment.

5.1 The Machine Learning Problem

Before delving into multiagent learning we first present a high level view of what we mean by **machine learning**. The word "learning" as used casually can have many different meanings, from remembering to deduction, but machine learning researchers have a very specific definition of the machine learning problem.

MACHINE LEARNING

The goal of machine learning research is the development of algorithms that increase the ability of an agent to match a set of inputs to their corresponding outputs (Mitchell, 1997). That is, we assume the existence of a large set of examples E. Each example $e \in E$ is a pair $e = \{a, b\}$ where $a \in A$ represents the input the agent receives and $b \in B$ is the output the agent should produce when receiving this input. The agent must find a function f which maps $A \to B$ for as many examples of A as possible. For example, A could be a set of photo portraits, B could be the set $\{\text{male}, \text{female}\}$, and each element e tells the program if a particular photo is of a man or of a woman. The machine learning algorithm would have to learn to differentiate between a photo of a man and that of a woman.

In a controlled test the set E is usually first divided into a training set which is used for training the agent, and a testing set which is used for testing the performance of the agent. However, in some scenarios it is impossible to first train the agent and then test it. In these cases the training and testing examples are interleaved. The agent's performance is assessed on an ongoing manner.

Figure 5.1 shows a graphical representation of the machine learning problem. The

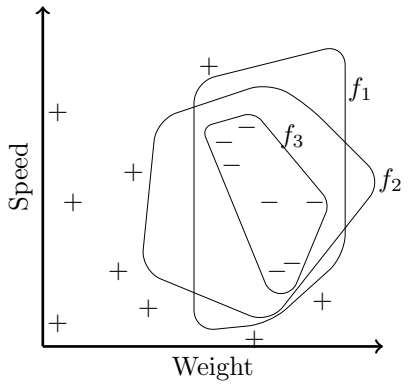


Figure 5.1: The machine learning problem. The input set A corresponds to the two axis: Weight and Speed. The outputs B are the set $\{+, -\}$. The lines represent three possible functions f which, in this case, map anything within the lines as a $-$ and anything outside as a $+$. Note that all functions have correctly solved the learning problem but in different ways.

learning problem is coming up with a function that maps all the points in the space to either $+$ or $-$ such that it will also correctly categorize any new examples that we have not seen. Our function must, therefore, extrapolate from the examples it has seen and generalize to all possible instances. That is, machine learning performs **induction** over the set of examples it has seen in order to categorize all future examples.

INDUCTION

Figure 5.1 shows three different functions, f_1, f_2, and f_3, each one of which correctly solves the learning problem. That is, they are all correct inductions since they correctly categorize all the examples. In fact, there are an infinite number of such functions. One might wonder which one is the best one to use. We don't have a general answer to that question. Each learning algorithm, from reinforcement learning to support vector machines, arrives at one learned function but the choice is arbitrary. That is, given the same examples two learning algorithms can learn to perfectly classify them but still have learned different functions. This effect is known as the **induction bias** of a learning algorithm. It is because of this induction bias that some learning algorithms appear to perform better than others in certain domains—their bias coincides with the implicit structure of the problem. However, we know that in general, that is, averages over all possible learning problems there is no learning algorithm that outperforms all others, a fact that has been formalized by the **no free lunch theorem** (Wolpert and Macready, 1995). Still, in practice we, as designers, do know a lot about any specific problem to be learned. You should always try to integrate this knowledge into the learning algorithm you are using.

INDUCTION BIAS

NO FREE LUNCH THEOREM

When a learning agent is placed in a multiagent scenario some of the fundamental assumptions of machine learning are violated. The agent is no longer learning to extrapolate from the examples it has seen of fixed set E, instead it's target concept keeps changing (the points in figure 5.1 keep moving), leading to a **moving target function** problem (Vidal and Durfee, 1998b). In general, however, the target concept does not change randomly; it changes based on the learning dynamics of the other agents in the system. Since these agents also learn using machine learning algorithms we are left with some hope that we might someday be able to understand the complex dynamics of these type of systems.

MOVING TARGET FUNCTION

Learning agents are most often selfish utility maximizers. These agents often face each other in encounters where the simultaneous actions of a set of agents leads to different utility payoffs for all the participants. For example, in a market-based setting a set of agents might submit their bids to a first-price sealed-bid auction. The outcome of this auction will result in a utility gain or loss for all the agents. In a robotic setting two agents headed in a collision course towards each other have to decide whether to stay the course or to swerve. The results of their combined actions have direct results in the utilities the agents receive from their actions. However, even agents that are trying to learn for themselves might find utility in sharing this knowledge.

		j	
		c	d
i	a	0,0	5,1
	b	-1,6	1,5

Figure 5.2: Sample two-player game matrix. Agent i chooses from the rows and agent j chooses from the columns.

5.2 Cooperative Learning

Imagine two robots equipped with wireless communication capabilities and trying to map an unknown environment. One of the robots could learn that the red rocks can be moved but the black rocks are too heavy to move. The robot could communicate this information to the other one so that it does not have to re-learn it. Similarly, once one robot has built a map of one area it could send this map to the other robot. Of course, this scenario assumes that the two robots are cooperating with each other in order to build the map.

This type of problem is easy to solve when the robots are identical. In this case they can simply tell each other everything that they learn knowing that it will be equally applicable to the other one. One challenge is trying to prevent the robots from telling each other things the other already knows. The problem gets much harder when the robots are heterogeneous. For example, one robot might have learned that the black rocks can be moved using its large arm but the other robot might not have an arm that large so this knowledge is useless to him. To solve this problem we need to somehow model the agents' capabilities so as to allow one agent to determine which parts of his learned knowledge will be useful to an agent with a different set of capabilities. To date, there is scant research on general approaches to the problem of sharing learned knowledge. Most systems that share learned knowledge among agents, such as (Stone, 2000), simply assume that all agents have the same capabilities.

5.3 Repeated Games

We now focus on the problem of learning in **repeated games** (Fudenberg and Levine, 1998). In these problems we have two players that face each other repeatedly on the same game matrix, like the one shown in figure 5.2, and each one tries to maximize the sum of its payoffs over time. You will remember that we already saw a specific version of this problem called the iterated prisoner's dilemma.

REPEATED GAMES

The theory of learning in games studies the equilibrium concepts dictated by various simple learning mechanisms. That is, while the Nash equilibrium is based on the assumption of perfectly rational players, in learning in games the assumption is that the agents use some kind of algorithm. The theory determines the equilibrium strategy that will be arrived at by the various learning mechanisms and maps these equilibria to the standard solution concepts, if possible. Many learning mechanisms have been studied. The most common of them are explained in the next few subsections.

5.3.1 Fictitious Play

A widely studied model of learning in games is the process of **fictitious play**. In it agents assume that their opponents are playing a fixed strategy. The agents use their past experiences to build a model of the opponent's strategy and use this model to choose their own action. Given that all agents are using fictitious play we try to determine if their learning will converge and, if so, to which strategy.

FICTITIOUS PLAY

Fictitious play uses a simple form of learning where an agent remembers everything the other agents have done and uses this information to build a probability distribution for the other agents' expected strategy. Formally, for the two agent case we say that agent i maintains a weight function $k_i : S_j \rightarrow \mathcal{R}^+$. The weight

		j	
		c	d
i	a	0,0	1,2
	b	1,2	0,0

s_i	s_j	$k_i(c)$	$k_i(d)$	$\Pr_i[c]$	$\Pr_i[d]$
a	c	1	0	1	0
b	d	1	1	.5	.5
a	d	1	2	1/3	2/3
a	d	1	3	1/4	3/4
a	d	1	4	1/5	4/5

Figure 5.3: Example of fictitious play. The matrix is shown above and the values at successive times, each on a different row, are shown on the table below. The first row corresponds to time 0. Note that only i is using fictitious play, j plays the values as in the s_j column. i's first two actions are stochastically chosen.

function changes over time as the agent learns. The weight function at time t is represented by k_i^t. It maintains a count of how many times each strategy has been played by each other player j. When at time $t-1$ opponent j plays strategy s_j^{t-1} then i updates its weight function with

$$k_i^t(s_j) = k_i^{t-1}(s_j) + \begin{cases} 1 & \text{if } s_j^{t-1} = s_j, \\ 0 & \text{if } s_j^{t-1} \neq s_j. \end{cases} \tag{5.1}$$

Using this weight function, agent i can assign a probability to j playing any of its $s_j \in S_j$ strategies with

$$\mathbf{Pr}_i^t[s_j] = \frac{k_i^t(s_j)}{\sum_{\tilde{s}_j \in S_j} k_i^t(\tilde{s}_j)}. \tag{5.2}$$

STOCHASTICALLY

That is, i assumes j will pick its action **stochastically** given the values in $k_i(s_j)$. Player i then determines the strategy that will give it the highest expected utility given that j will play each of its $s_j \in S_j$ with probability $\mathbf{Pr}_i^t[s_j]$. In other words, i determines its best response to a probability distribution over j's possible strategies. In effect, i is assuming that j's strategy at each time is taken from some fixed but unknown probability distribution.

An example of the best response dynamic at work is shown in figure 5.3. Here we see the values for agent i and its best responses to agent j's action. Note that in this example agent j is not using best response. Agent i first notices that j played c and thus sets $k_i^1(c) = 1$. It therefore predicts that j will play c with probability 1 so its best response at time 2 is to play b, as seen in the second row. Agent j then plays d which makes i have $\mathbf{Pr}_i^1[c] = \mathbf{Pr}_i^1[d] = .5$. Both of i's actions have the same expected payoff (1) so it randomly chooses to play b. After that, when j plays d again then i's best response is unequivocally b.

Several interesting results have been derived by research in repeated games. These results assume that all players are using fictitious play. For example, we know that the Nash equilibrium remains a powerful attractor.

Theorem 5.1 (Nash Equilibrium is Attractor to Fictitious Play)**.** *If s is a strict Nash equilibrium and it is played at time t then it will be played at all times greater than t (Fudenberg and Kreps, 1990).*

Intuitively, we can see that if the fictitious play algorithm leads all players to play the same Nash equilibrium then, afterward, they will all increase the probability that all others are playing the equilibrium because they just saw them play it. Since, by definition, the best response of a player when everyone else is playing a strict Nash equilibrium is to play the same equilibrium, then all players will play the same strategy and the next time. The same holds true for every time after that. More importantly, Nash is also where we will converge to.

Theorem 5.2 (Fictitious Play Converges to Nash)**.** *If fictitious play converges to a pure strategy then that strategy must be a Nash equilibrium (Fudenberg and Kreps, 1990).*

		j	
		c	d
i	a	0,0	1,1
	b	1,1	0,0

s_i	s_j	$k_i(c)$	$k_i(d)$	$k_j(a)$	$k_j(b)$
		1	1.5	1	1.5
a	c	2	1.5	2	1.5
b	d	2	2.5	2	2.5
a	c	3	2.5	3	2.5
b	d	3	3.5	3	3.5

Figure 5.4: A game matrix with an infinite cycle.

We can show this by contradiction. If fictitious play converges to a strategy that is not a Nash equilibrium then this means that the best response for at least one of the players is not the same as the convergent strategy. Therefore, that player will take that action at the next time, taking the system away from the strategy profile it was supposed to have converged to.

An obvious problem with the solutions provided by fictitious play can be seen in the existence of infinite cycles of actions. An example is illustrated by the game matrix in figure 5.4. If the players start with initial weights of $k_i^1(c) = 1$, $k_i^1(d) = 1.5$, $k_j^1(a) = 1$, and $k_j^1(b) = 1.5$ they will both believe that the other will play b or d and will, therefore, play a or c respectively. The weights will then be updated to $k_i^2(c) = 2$, $k_i^2(d) = 1.5$, $k_j^2(a) = 2$, and $k_j^2(b) = 1.5$. Next time, both agents will believe that the other will play a or c so both will play b or d. The agents will engage in an endless cycle where they alternatively play (a, c) and (b, d). The agents end up receiving the worst possible payoff.

This example illustrates the type of problems we encounter when adding learning to multiagent systems. Most learning algorithms can easily fall into cycles such as this one. One common strategy for avoiding this problem is the use of randomness. Agents will sometimes take a random action in an effort to exit possible loops and to explore the search space. It is interesting to note that, as in the example from figure 5.4, it is often the case that the loops the agents fall in often reflect one of the mixed strategy Nash equilibria for the game. That is, $(.5, .5)$ is a Nash equilibrium for this game. Unfortunately, if the agents are synchronized, as in this case, the implementation of a mixed strategy could lead to a lower payoff.

Games with more than two players require that we decide whether the agent should learn individual models of each of the other agents independently or a joint probability distribution over their combined strategies. Individual models assume that each agent operates independently while the joint distributions capture the possibility that the others agents' strategies are correlated. Unfortunately, for any interesting system the set of all possible strategy profiles is too large to explore—it grows exponentially with the number of agents. Therefore, most learning systems assume that all agents operate independently so they need to maintain only one model per agent.

5.3.2 Replicator Dynamics

Another widely studied learning model in repeated games is **replicator dynamics**. This model assumes that the fraction of agents playing a particular strategy will grow in proportion to how well that strategy performs in the population. A homogeneous population of agents is assumed. The agents are randomly paired in order to play a symmetric game, that is, a game where both agents have the same set of possible strategies and receive the same payoffs for the same actions. The replicator dynamics model is meant to capture situations where agents reproduce in proportion to how well they are doing and is inspired by biological evolution. In fact, the field that studies these type of solution concepts is known as **evolutionary game theory** (Weibull, 1997).

REPLICATOR DYNAMICS

EVOLUTIONARY GAME THEORY

Formally, we let $\phi^t(s)$ be the number of agents using strategy s at time t. We

can then define

$$\theta^t(s) = \frac{\phi^t(s)}{\sum_{s' \in S} \phi^t(s')} \tag{5.3}$$

to be the fraction of agents playing s at time t. The expected utility for an agent playing strategy s at time t is defined as

$$u^t(s) = \sum_{s' \in S} \theta^t(s') u(s, s'), \tag{5.4}$$

where $u(s, s')$ is the utility than an agent playing s receives against an agent playing s'. Notice that this expected utility assumes that the agents face each other in pairs and choose their opponents randomly. In the replicator dynamics the reproduction rate for each agent is proportional to how well it did on the previous step. Thus, the number of agents playing s at the next time step is given by


evolutionarygt

$$\phi^{t+1}(s) = \phi^t(s)(1 + u^t(s)). \tag{5.5}$$

Notice that the number of agents playing a particular strategy will continue to increase as long as the expected utility for that strategy is greater than zero. Only strategies whose expected utility is negative will decrease in population. As such, the size of a population will constantly fluctuate. However, when studying replicator dynamics we ignore the absolute size of the population and focus on the fraction of the population playing a particular strategy. We are also interested in determining if the system's dynamics will converge to some strategy and, if so, which one.

In order to study these systems using the standard solution concepts we view the fraction of agents playing each strategy as a mixed strategy for the game. Since the game is symmetric we can use that strategy as the strategy for both players, so it becomes a strategy profile. We say that the system is in a mixed Nash equilibrium if the fraction of players playing each pure strategy is the same as the probability of the corresponding strategy in the mixed Nash equilibrium. For a pure equilibrium all players must play that strategy. For example, if half the agents are playing a and half b then we can consider this a mixed Nash equilibrium where a and b are each played with .5 probability.

An examination of these systems quickly leads to the conclusion that

Theorem 5.3 (Nash equilibrium is a Steady State)**.** *Every Nash equilibrium is a steady state for the replicator dynamics (Fudenberg and Levine, 1998).*

We can prove this theorem by contradiction. If an agent had a pure strategy that would return a higher utility than any other strategy then this strategy would be a best response to the Nash equilibrium. If this strategy was different from the Nash equilibrium then we would have a best response to the equilibrium which is not the equilibrium, so the system could not be at a Nash equilibrium.

The reverse has also been shown to be true.

Theorem 5.4 (Stable Steady State is a Nash Equilibrium)**.** *A* stable steady state *of the replicator dynamics is a Nash equilibrium. A* stable steady state *is one that, after suffering from a small perturbation, is pushed back to the same steady state by the system's dynamics (Fudenberg and Levine, 1998) .*

These states are necessarily Nash equilibria because if they were not then there would exist some particular small perturbation which would take the system away from the steady state. This correspondence was further refined to show that

Theorem 5.5 (Asymptotically Stable is Trembling-Hand Nash)**.** *An asymptotically stable steady state corresponds to a Nash equilibrium that is trembling-hand perfect and isolated. That is, the stable steady states are a refinement on Nash equilibria—only a few Nash equilibria are stable steady states (Bomze, 1986).*

		j		
		a	b	c
i	a	1,1	2,0	0,2
	b	0,2	1,1	2,0
	c	2,0	0,2	1,1

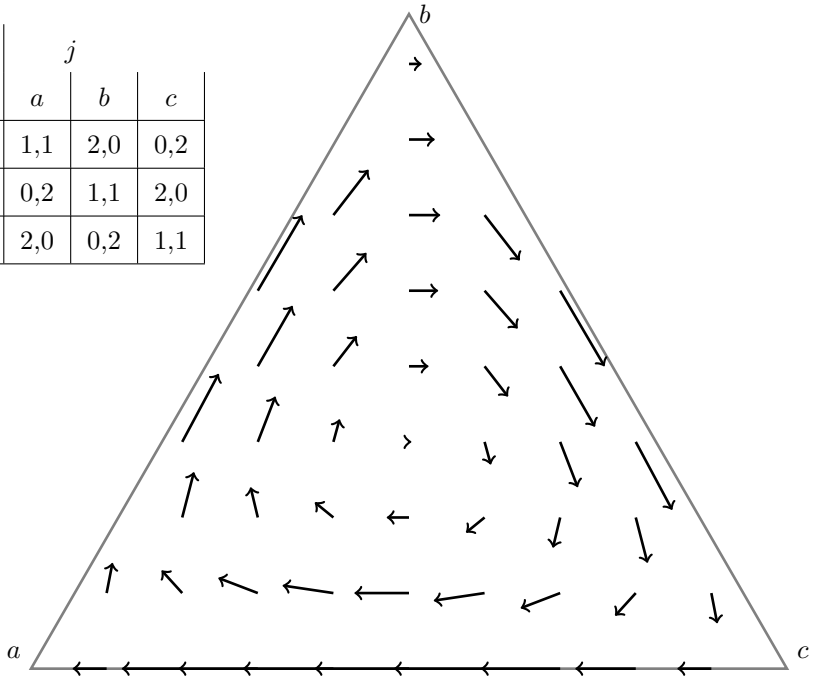


Figure 5.5: Visualization of the evolution of populations in replicator dynamics. The game is shown in the top left.

On the other hand, it is also possible that a replicator dynamics system will never converge. In fact, there are many examples of simple games with no asymptotically stable steady states.

While replicator dynamics reflect some of the most troublesome aspects of learning in multiagent systems some differences are evident. These differences are mainly due to the replication assumption. Agents are not usually expected to replicate, instead they acquire the strategies of others. For example, in a real multiagent system all the agents might choose to play the strategy that performed best in the last round instead of choosing their next strategy in proportion to how well it did last time. As such, we cannot directly apply the results from replicator dynamics to multiagent systems. However, the convergence of the systems' dynamics to a Nash equilibrium does illustrate the importance of this solution concept as an attractor of learning agent's dynamics.

Within replicator dynamics we can define a new solution concept inspired by the "survival of the fittest" idea from evolution. An **evolutionary stable strategy** (ESS) is one which, as a population, defeats any small number of invading mutants. That is, if everyone in the population plays that strategy then there is no way a small number of mutants can invade the population and receive greater utility than the agents there. More formally, we define it as

EVOLUTIONARY STABLE STRATEGY

Definition 5.1 (Evolutionary Stable Strategy)**.** *An* ESS *is an equilibrium strategy that can overcome the presence of a small number of invaders. That is, if the equilibrium strategy profile is ω and small number ϵ of invaders start playing ω' then ESS states that the existing population should get a higher payoff against the new mixture ($\epsilon\omega' + (1-\epsilon)\omega$) than the invaders.*

It has further been shown that

Theorem 5.6 (ESS is Steady State of Replicator Dynamics)**.** ESS *is an asymptotically stable steady state of the replicator dynamics. However, the converse need not be true—a stable state in the replicator dynamics does not need to be an* ESS *(Taylor and Jonker, 1978).*

This means that ESS is a further refinement of the solution concept provided by the replicator dynamics. ESS can be used when we need a very stable equilibrium concept.

While these convergence theorems are very usefull, it is also the case that most systems never converge to a fixed point. In these cases we need a way to visualize the system dynamics. Since replicator dynamics is deterministic—we know exactly how each population will evolve—we can plot a map showing how the population will vary over time. Figure 5.5 shows a sample game, at the top left, and its visualization. The triangle is known as a **simplex plot** and is used for games with three actions. Every point inside the triangle represents a possible population. The point at the bottom left is the population where all agents play a. Similarly, the top point is where all agents play b. The point in the exact middle of the triangle represents the population where 1/3 of the population play a, 1/3 play b, and 1/3 play c. Each arrow starts at some population and ends at the next population that would evolve from that one at the next time step. As can be seen, in this specific game the population can get into cycles and never converge to a fixed point.

SIMPLEX PLOT

5.3.3 The AWESOME Algorithm

While fictitious play and replicator dynamics do not always converge to an equilibrium, one way we can guarantee that agents will converge to an equilibrium is by having them calculate and play the same equilibrium strategy upon starting. Then, if an agent notices that the others are not playing the agreed upon equilibrium strategy it can play fictitious play instead. This is the basic idea of the **AWESOME** (Adapt When Everyone is Stationary, Otherwise Move to Equilibrium) algorithm (Conitzer and Sandholm, 2003; Conitzer and Sandholm, 2006). That is, if the other agents appear to be stationary the an AWESOME agent plays the best response to their strategy. On the other hand, if they appear to be adapting then the agents retreats to the equilibrium strategy.

AWESOME

The AWESOME algorithm starts by playing the equilibrium strategy π_i and keeping track of the actions each other player j has played. Every N rounds (an epoch) it uses these actions to build a, possibly mixed, strategy s_j for each player j. If s_j is the equilibrium strategy π_j for all players j then the algorithm keeps playing the equilibrium strategy. Otherwise, the algorithm plays the best response to s_j. It is easy to see that if all the other players are AWESOME players then they will play their equilibrium strategies and will never diverge from it. If, on the other hand, an AWESOME player is playing against some players who are, eventually, playing some other fixed strategy then it will play a best response strategy against those fixed strategies. Notice how the algorithm implements the reasoning behind the proof of the folk theorem.

While the basic idea is simple, in order to prove that the algorithm will converge some extra complexity had to be added to it. The algorithm, shown in figure 5.6, has two Boolean state variables *playing-equilibrium* which is true when all other agents played the equilibrium strategy during the last epoch and *playing-stationary* which is true when all the other agents played a stationary strategy during the last epoch. Also, *playing-equilibrium-just-rejected* is true when *playing-equilibrium* has just been set to false during the last check. The algorithm plays the same strategy ϕ for a whole epoch and then assesses the situation. If it turns out that either the players are not stationary or not in equilibrium the algorithm makes a note of this and changes its state. If the stationarity hypothesis is rejected then the whole algorithm is re-started again (back to line 2).

In order for the algorithm to always converge, ϵ_e and ϵ_s must be decreased and N must be increased over time using a schedule where

1. ϵ_s and ϵ_e decrease monotonically to 0,
2. N increases to infinity,
3. $\prod_{t \leftarrow 1,\ldots,\infty} 1 - \frac{\sum_i |A_i|}{N^t (\epsilon_s^t)^2} > 0$
4. $\prod_{t \leftarrow 1,\ldots,\infty} 1 - \frac{\sum_i |A_i|}{N^t (\epsilon_e^t)^2} > 0.$

AWESOME

$\pi \leftarrow$ equilibrium strategy
repeat
  playing-equilibrium $\leftarrow$ TRUE
  playing-stationary $\leftarrow$ TRUE
  playing-equilibrium-just-rejected $\leftarrow$ FALSE
  $\phi \leftarrow \pi_i$
  $t \leftarrow 0$
  while *playing-stationary*
    do play ϕ for N^t times in a row (an epoch)
      $\forall_j$ update s_j given what they played in these N^t rounds.
      if *playing-equilibrium*
        then if some player j has $\max_a(s_j(a), \pi_j(a)) > \epsilon_e$
          then *playing-equilibrium-just-rejected* $\leftarrow$ TRUE
            $\phi \leftarrow$ random action
        else if *playing-equilibrium-just-rejected* = FALSE
          and some j has $\max_a(s_j^{\text{old}}(a), s_j(a)) > \epsilon_s$
          then *playing-stationary* $\leftarrow$ FALSE
        playing-equilibrium-just-rejected $\leftarrow$ FALSE
        $b \leftarrow \arg\max_a u_i(a, s_{-i})$
        if $u_i(b, s_{-i}) > u_i(\phi, s_{-i}) + n|A_i|\epsilon_s^{t+1}\mu$
          then $\phi \leftarrow b$
      $\forall_j s_j^{\text{old}} \leftarrow s_j$
      $t \leftarrow t + 1$

Figure 5.6: The AWESOME algorithm. Here π is the equilibrium strategy which has been calculated before the algorithm starts, n is the number of agents, $|A_i|$ is the number of actions the agent can take, μ is the difference between the player's best and worst possible utility values, and $s_j(a)$ gives the probability with which j will play action a under strategy s_j. Also, ϵ_e and ϵ_s must be decreased and N must be increased over time using a valid schedule.

Given such a valid schedule, it can be shown that

Theorem 5.7 (AWESOME converges)**.** *With a valid schedule, the* AWESOME *algorithm converges to best response if all the other players play fixed strategies and to a Nash equilibrium if all the other players are* AWESOME *players.*

Notice that the theorem says that, in self play, it converges to a Nash equilibrium which might be different from the originally agreed upon equilibrium strategy π. For example, say the agents agree on a mixed equilibrium strategy π but some of the actions played in π constitute a pure Nash equilibrium. In this case it could happen that the players end up, by chance, repeatedly playing the actions in the pure Nash equilibrium during the first epoch. They might then decide that everybody else is playing a stationary strategy which constitutes the pure Nash equilibrium. They will then play the best response to that pure Nash which, we know, is the same pure Nash equilibrium. As such, they will get stuck in that equilibrium.

The AWESOME algorithm is a simple way to force agents to converge to a Nash equilibrium while not letting them be exploited by other agents that are not using the same algorithm. In those cases where all agents are AWESOME agents then it converges from the first step. However, it remains to be seen exactly how much it can be exploited by clever opponents who know the equilibrium it wants to play but would rather play something else.

5.4 Stochastic Games

In many multiagent applications the agents do not know the payoffs they will receive for their actions. Instead, they must take random actions in order to first explore the world so that they may then determine which actions lead them to the best payoffs. That is, the agents inhabit a multiagent Markov decision problem.

Q-LEARNING

```
1  ∀_s ∀_a Q(s, a) ← 0; λ ← 1; ε ← 1
2  s ← current state
3  if RAND() < ε          ▷ Exploration rate
4     then a ← random action
5     else a ← arg max_a Q(s, a)
6  Take action a
7  Receive reward r
8  s' ← current state
9  Q(s, a) ← λ(r + γ max_a' Q(s', a')) + (1 − λ)Q(s, a)
10 λ ← .99λ
11 ε ← .98ε
12 goto 2
```

Figure 5.7: Q-LEARNING algorithm. Note that the .99 and .98 numbers are domain-dependent and need to be changed for each problem to ensure that the algorithm works. With $\epsilon \leftarrow 0$ the algorithm is still guaranteed to work, but in practice it might take longer to converge.

5.4.1 Reinforcement Learning

REINFORCEMENT LEARNING
Q-LEARNING

A very popular machine learning technique for solving these types of problems is called **reinforcement learning** (Sutton and Barto, 1998), a specific instance of it is known as **Q-learning** (Watkins and Dayan, 1992). Reinforcement learning assumes that the agent lives in a Markov process and receives a reward in certain states. The goal is to find the right action to take in each state so as to maximize the agent's discounted future reward. That is, find the optimal policy.

More formally, we define the reinforcement learning problem as given by an MDP (section 1.2) where the rewards are given on the edges instead of in the states. That is, the reward function is $r(s_t, a_t) \rightarrow \Re$. A reinforcement learning agent must find the policy π^* which maximizes his future discounted rewards (1.6).

LEARNING RATE
EXPLORATION RATE

The reinforcement learning problem can be solved using the Q-LEARNING algorithm shown in figure 5.7. Here λ is the **learning rate** and ϵ is the **exploration rate**. Both are always between 0 and 1. The learning rate guides how heavily we consider new rewards versus old values we have learned. When $\lambda = 1$ the algorithm completely re-writes the old $Q(s, a)$ value while when $\lambda = 0$ is completely ignores any new reward and instead uses the old $Q(s, a)$. The exploration rate ensures that we do not converge too quickly to a solution. When $\epsilon = 1$ all the actions taken by the agent are chosen randomly while when $\epsilon = 0$ all the actions taken maximize the Q values.

`qlearning`

It has been shown that Q-LEARNING will converge.

Theorem 5.8 (Q-LEARNING Converges)**.** *Given that the learning and exploration rates decrease slowly enough,* Q-LEARNING *is guaranteed to converge to the optimal policy (Watkins and Dayan, 1992) .*

Q-LEARNING differs from the value-iteration algorithm, from figure 1.2, in several respects. In Q-LEARNING the agent takes actions and learns at the same time. Of course, the agent's initial actions will be completely random as it has not learned anything about its expected rewards but, as it takes more actions it learns to choose better actions. Also, the value-iteration algorithm requires knowledge of the complete transition and reward functions of the MDP while the Q-LEARNING algorithm explores the parts of the MDP that it needs. However, to be sure that it has found the optimal policy, a Q-LEARNING agent will need to visit every state and take every action.

The convergence results of Q-LEARNING are impressive, but they assume that only one agent is learning. We are interested in multiagent systems where multiple agents are learning. In these games the reward function is no longer a function of the state and the agent's actions, instead it is a function of the state and all the agents' actions. That is, one agent's reward depends on the actions that other agents take, as captured by the multiagent MDP we discussed in section 1.2.1. In

NashQ-learning

1 $t \leftarrow 0$
2 $s \leftarrow$ current state
3 $\forall_{s \in S} \forall_{j \leftarrow 1,\ldots,n} \forall_{a_j \in A_j} Q_j^t(s, a_1, \ldots, a_n) \leftarrow 0$
4 Choose action a_i^t
5 $s \leftarrow s'$
6 Observe $r_1^t, \ldots, r_n^t$; $a_1^t, \ldots, a_n^t$; s'
7 **for** $j \leftarrow 1, \ldots, n$
8 **do** $Q_j^{t+1}(s, a_1, \ldots, a_n) \leftarrow$
$(1 - \lambda^t) Q_j^t(s, a_1, \ldots, a_n) + \lambda^t (r_j^t + \gamma\, Nash Q_j^t(s'))$
where $Nash Q_j^t(s') = Q_j^t(s', \pi_1(s') \cdots \pi_n(s'))$
and $\pi_1(s') \cdots \pi_n(s')$ are Nash EP calculated from Q values
9 $t \leftarrow t + 1$
10 **goto** 4

Figure 5.8: NashQ-learning algorithm

these multiagent MDPs it might be impossible for a Q-learning agent to converge to an optimal policy.

As we set out to study these multiagent learning problems, the first thing we need to do is choose a new equilibrium. Under the single agent problem definition we were simply looking for the policy that maximizes the agent's discounted rewards. However, when we have multiple agents we might want to maximize the sum of all agents' discounted future rewards (social welfare), or we might choose a more amenable (for convergence) equilibrium such as the **Nash equilibrium point**.

Nash equilibrium point

Definition 5.2 (Nash Equilibrium Point)**.** *A tuple of n policies $(\pi_1^*, \ldots, \pi_n^*)$ such that for all $s \in S$ and $i = 1, \ldots, n$,*

$$\forall_{\pi_i \in \Pi_i} v_i(s, \pi_1^*, \ldots \pi_n^*) \geq v_i(s, \pi_1^*, \ldots \pi_{i-1}^*, \pi_i, \pi_{i+1}^*, \ldots, \pi_n^*),$$

where $v_i(s, \pi_1^*, \ldots \pi_n^*)$ is the total rewards (value) that agent i can expect to receive starting from state s and assuming that agents use policies $\pi_1^*, \ldots \pi_n^*$.

The Nash equilibrium point is a set of policies such that no one agent i will gain anything by changing its policy from its Nash equilibrium point strategies to something else. As with the regular Nash equilibrium, it has been shown that the Nash equilibrium point always exists.

Theorem 5.9 (Nash Equilibrium Point Exists)**.** *Every n-player discounted stochastic game possesses at least one Nash equilibrium point in stationary strategies (Hu and Wellman, 2003).*

We can find the Nash equilibrium point in a system where all the agents use the NashQ-learning algorithm shown in figure 5.8. In this algorithm each agent must keep n Q-tables, one for each agent in the population. These tables are updated in line 7 using a formula similar to the standard Q-learning update formula but instead of using the Q values to determine future rewards it uses the *NashQ* tables. These tables hold the Q value for every agent given that all agents play the same Nash equilibrium policy on the multiagent MDP game induced by the current Q values. That is, the algorithm assumes that the MDP is defined by the Q tables it has and then calculates a Nash equilibrium point for this problem. We thus note that at each step the each agent must calculate the Nash equilibrium point given the current Q functions. This can be an extremely expensive computation—harder than finding the Nash equilibrium for a game matrix.

NashQ-learning

Still, the NashQ-learning algorithm is guaranteed to converge as long as enough time is given so that all state/action pairs are explored sufficiently, and the following assumptions hold.

Assumption 5.1. *There exists an adversarial equilibrium for the entire game and for every game defined by the Q functions encountered during learning.*

FRIEND-OR-FOE

1 $t \leftarrow 0$
2 $s_0 \leftarrow$ current state
3 $\forall_{s \in S} \forall_{a_j \in A_j} Q_i^t(s, a_1, \ldots, a_n) \leftarrow 0$
4 Choose action a_i^t
5 $s \leftarrow s'$
6 Observe $r_1^t, \ldots, r_n^t$; $a_1^t, \ldots, a_n^t$; s'
7 $Q_i^{t+1}(s, a_1, \ldots, a_n) \leftarrow$
$(1 - \lambda^t) Q_i^t(s, a_1, \ldots, a_n) + \lambda^t (r_i^t + \gamma \, NashQ_i^t(s'))$
where $NashQ_i^t(s') = \max_{\pi \in \Pi(X_1 \times \cdots \times X_k)} \min_{y_i, \ldots, y_l \in Y_1 \times \cdots \times Y_l}$
$\sum_{x_1, \ldots, x_k \in X_1 \times \cdots \times X_k} \pi(x_1) \cdots \pi(x_k) Q_i(s, x_1, \ldots, x_k, y_1, \ldots y_l)$
and X are actions for i's friends and Y are for the foes.
8 $t \leftarrow t + 1$
9 **goto** 4

Figure 5.9: FRIEND-OR-FOE algorithm. There are k friends with actions taken from $X_1, \ldots, X_k$, and l foes with actions taken from $Y_1, \ldots, Y_l$.

Where an adversarial equilibrium is one where no agent has anything to lose if the other agents change their policies. That is, if the other agents change their policies from equilibrium then the agent's expected reward will either stay the same or increase.

Assumption 5.2. *There exists a coordination equilibrium for the entire game and for every game defined by the Q functions encountered during learning.*

Where a coordination equilibrium is one where all the agents receive the highest possible value. That is, the social welfare solution.

Under these assumptions it can be shown that NASHQ-LEARNING converges.

Theorem 5.10 (NASHQ-LEARNING Converges)**.** *Under assumptions 5.1 and 5.2* NASHQ-LEARNING *converges to a Nash equilibrium as long as all the equilibria encountered during the game are unique (Hu and Wellman, 2003).*

These assumptions can be further relaxed by assuming that we can tell the agent whether the opponent is a "friend", in which case we are looking for a coordination equilibrium, or a "foe", in which case we are looking for an adversarial equilibrium (Littman, 2001). With this additional information we no longer need to maintain a Q table for each opponent and can achieve convergence with only one, expanded, Q table. The algorithm is thus called the FRIEND-OR-FOE algorithm and is shown in figure 5.9.

FRIEND-OR-FOE

In the FRIEND-OR-FOE algorithm the agent i has k friends with action sets represented by $X_1, \ldots, X_k$ and l foes with action sets represented by $Y_1, \ldots, Y_l$. The algorithm implements the idea that i's friends are assumed to work together to maximize i's value and i's foes are working to minimize it. We can show that FRIEND-OR-FOE converges to a stable policy. However, in general these do not correspond to a Nash equilibrium point. Still, we can show that it often converges to a Nash equilibrium.

Theorem 5.11. FOE-Q *learns values for a Nash equilibrium policy if the game has an adversarial equilibrium and* FRIEND-Q *learns values for a Nash equilibrium policy if the game has a coordination equilibrium. This is true regardless of opponent behavior (Littman, 2001).*

That is, if the game has one of the equilibria and we correctly classify all the other agents as either friends or foes then the FRIEND-OR-FOE algorithm is guaranteed to converge.

FRIEND-OR-FOE has several advantages over NASHQ-LEARNING. It is does not require the learning of Q functions for each one of the other agents and it is easy to implement as it does not require the calculation of a Nash equilibrium point at each

step. On the other hand, it does require us to know if the opponents are friends or foes, that is, whether there exists a coordination or an adversarial equilibrium.

Neither algorithm deals with the problem of finding equilibria in cases without either coordination or adversarial equilibria. Such cases are the most common and most interesting as they require some degree of cooperation among otherwise selfish agents.

5.5 General Theories for Learning Agents

The theory of learning in games provides the designer of multiagent systems with many useful tools for determining the possible equilibrium points of a system. Unfortunately, most multiagent systems with learning agents do not converge to an equilibrium. Designers often use learning agents because they do not know, at design time, the specific circumstances that the agents will face at run time. If a designer knew the best strategy, that is, the Nash equilibrium strategy, for his agent then he would simply implement this strategy and avoid the complexities of implementing a learning algorithm. Therefore, we will see a multiagent system with learning agents when the designer cannot predict that an equilibrium solution will emerge.

The two main reasons for this inability to predict the equilibrium solution of a system are the existence of unpredictable environmental changes that affect the agents' payoffs and the fact that on many systems an agent only has access to its own set of payoffs—it does not know the payoffs of other agents. These two reasons make it impossible for a designer to predict which equilibria, if any, the system would converge to. However, the agents in the system are still playing a game for which an equilibrium exists, even if the designer cannot predict it at design-time. But, since the actual payoffs keep changing it is often the case that the agents are constantly changing their strategy in order to accommodate the new payoffs.

As mentioned earlier, learning agents in a multiagent system are faced with a moving target function problem (Vidal and Durfee, 1998b). That is, as the agents change their behavior in an effort to maximize their utility their payoffs for those actions change, changing the expected utility of their behavior. The system will likely have non-stationary dynamics—always changing in order to match the new goal. While game theory tells us where the equilibrium points are, given that the payoffs stay fixed, multiagent systems often never get to those points. A system designer needs to know how changes in the design of the system and learning algorithms will affect the time to convergence. This type of information can be determined by using CLRI theory.

5.5.1 CLRI Model

CLRI MODEL

The **CLRI model** (Vidal and Durfee, 2003) provides a method for analyzing a system composed of learning agents and determining how an agent's learning is expected to affect the learning of other agents in the system. It assumes a system where each agent has a decision function that governs its behavior as well as a target function that describes the agent's best possible behavior. The target function is unknown to the agent. The goal of the agent's learning is to have its decision function be an exact duplicate of its target function. Of course, the target function keeps changing as a result of other agents' learning.

Formally, the CLRI model assumes that there is a fixed set of autonomous agents in the system. The system can be described by a set of discrete states $s \in S$ which are presented to the agent with a probability dictated by the fixed probability distribution $\mathcal{D}(S)$. Each agent i has a set of possible actions A_i where $|A_i| \geq 2$. Time is discrete and indexed by a variable t. At each time t all agents are presented with a new $s \in \mathcal{D}(S)$, take a simultaneous action, and receive some payoff. The scenario is similar to the one used by fictitious play except for the addition of state s.

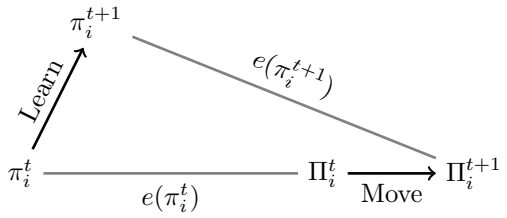


Figure 5.10: The moving target function problem.

Each agent i's behavior is defined by a policy $\pi_i^t(s) : S \to A$. When i learns at time t that it is in state s it will take action $\pi_i^t(s)$. At any time there is an optimal policy for each agent i, also known as its target function, which we represent as $\Pi_i^t(s)$. Agent i's learning algorithm will try to reduce the discrepancy between π_i and Π_i by using the payoffs it receives for each action as clues as clues as to what it should do given that it does not have direct access to Π_i. The probability that an agent will take a wrong action is given by its error $e(\pi_i^t) = \mathbf{Pr}[\pi_i^t(s) \neq \Pi_i^t(s) \,|\, s \in \mathcal{D}(S)]$. As other agents learn and change their decision function, i's target function will also change, leading to the moving target function problem, as depicted in figure 5.10.

An agent's error is based on a fixed probability distribution over world states and a Boolean matching between the decision and target functions. These constraints are often too restrictive to properly model many multiagent systems. However, even if the system being modeled does not completely obey these two constraints, the use of the CLRI model in these cases still gives the designer valuable insight into how changes in the design will affect the dynamics of the system. This practice is akin to the use of Q-learning in non-Markovian games—while Q-learning is only guaranteed to converge if the environment is Markovian, it can still perform well on other domains.

The CLRI model allows a designer to understand the expected dynamics of the system, regardless of what learning algorithm is used, by modeling the system using four parameters: Change rate, Learning rate, Retention rate, and Impact (CLRI). A designer can determine values for these parameters and then use the CLRI difference equation to determine the expected behavior of the system.

The change rate (c) is the probability that an agent will change at least one of its incorrect mappings in $\delta^t(w)$ for the new $\delta^{t+1}(w)$. It captures the rate at which the agent's learning algorithm tries to change its erroneous mappings. The learning rate (l) is the probability that the agent changes an incorrect mapping to the correct one. That is, the probability that $\delta^{t+1}(w) = \Delta^t(w)$, for all w. By definition, the learning rate must be less than or equal to the change rate, i.e. $l \leq c$. The retention rate (r) represents the probability that the agent will retain its correct mapping. That is, the probability that $\delta^{t+1}(w) = \delta^t(w)$ given that $\delta^t(w) = \Delta^t(w)$.

CLRI defines a volatility term (v) to be the probability that the target function Δ changes from time t to $t+1$. That is, the probability that $\Delta^t(w) \neq \Delta^{t+1}(w)$. As one would expect, volatility captures the amount of change that the agent must deal with. It can also be viewed as the speed of the target function in the moving target function problem, with the learning and retention rates representing the speed of the decision function. Since the volatility is a dynamic property of the system (usually it can only be calculated by running the system) CLRI provides an impact (I_{ij}) measure. I_{ij} represents the impact that i's learning has on j's target function. Specifically, it is the probability that $\Delta_j^t(w)$ will change given that $\delta_i^{t+1}(w) \neq \delta_i^t(w)$.

Someone trying to build a multiagent system with learning agents would determine the appropriate values for c, l, r, and either v or I and then use

$$E[e(\delta_i^{t+1})] = 1 - r_i + v_i \left(\frac{|A_i| r_i - 1}{|A_i| - 1} \right) + e(\delta_i^t) \left(r_i - l_i + v_i \left(\frac{|A_i|(l_i - r_i) + l_i - c_i}{|A_i| - 1} \right) \right) \quad (5.6)$$

in order to determine the successive expected errors for a typical agent i. This equation relies on a definition of volatility in terms of impact given by

$$\begin{aligned} \forall_{w \in W}\ v_i^t &= \mathbf{Pr}[\Delta_i^{t+1}(w) \neq \Delta_i^t(w)] \\ &= 1 - \prod_{j \in N_{-i}} (1 - I_{ji}\mathbf{Pr}[\delta_j^{t+1}(w) \neq \delta_j^t(w)]), \end{aligned} \tag{5.7}$$

which makes the simplifying assumption that changes in agents' decision functions will not cancel each other out when calculating their impact on other agents. The difference equation (5.6) cannot, under most circumstances, be collapsed into a function of t so it must still be iterated over. On the other hand, a careful study of the function and the reasoning behind the choice of the CLRI parameter leads to an intuitive understanding of how changes in these parameters will be reflected in the function and, therefore, the system. A knowledgeable designer can simply use this added understanding to determine the expected behavior of his system under various assumptions. An example of this approach is shown in (Brooks and Durfee, 2003).

For example, it is easy to see that an agent's learning rate and the system's volatility together help to determine how fast, if ever, the agent will reach its target function. A large learning rate means that an agent will change its decision function to almost match the target function. Meanwhile, a low volatility means that the target function will not move much, so it will be easy for the agent to match it. Thus, if the agents have no impact on each other, that is, $I_{ij} = 0$ for all i, j, then the agents will learn their target function and the system will converge. As the impact increases then it becomes more likely that the system will never converge.

Of course, this type of simple analysis ignores a common situation where the agent's high learning rate is coupled with a high impact on other agents' target function making their volatility much higher. These agents might then have to increase their learning rate and thereby increase the original agent's volatility. Equation (5.6) is most helpful in these type of feedback situations.

5.5.2 N-Level Agents

Another issue that arises when building learning agents is the choice of a modeling level. A designer must decide whether his agent will learn to correlate actions with rewards, or will try to learn to predict the expected actions of others and use these predictions along with knowledge of the problem domain to determine its actions, or will try to learn how other agents build models of other agents, etc. These choices are usually referred to as **n-level** modeling agents—an idea first presented in the recursive modeling method (Gmytrasiewicz and Durfee, 1995) (Gmytrasiewicz and Durfee, 2001).

N-LEVEL

A **0-level** agent is one that does not recognize the existence of other agents in the world. It learns which action to take in each possible state of the world because it receives a reward after its actions. The state is usually defined as a static snapshot of the observable aspects of the agent's environment. A **1-level** agent recognizes that there are other agents in the world whose actions affect its payoff. It also has some knowledge that tells it the utility it will receive given any set of joint actions. This knowledge usually takes the form of a game matrix that only has utility values for the agent. The 1-level agent observes the other agents' actions and builds probabilistic models of the other agents. It then uses these models to predict their action probability distribution and uses these distributions to determine its best possible action. A **2-level** agent believes that all other agents are 1-level agents. It, therefore, builds models of their models of other agents based on the actions it thinks they have seen others take. In essence, the 2-level agent applies the 1-level algorithm to all other agents in an effort to predict their action probability distribution and uses these distributions to determine its best possible actions. A 3-level agent believes that all other agents are 2-level, an so on. Using these guidelines

0-LEVEL

1-LEVEL

2-LEVEL

we can determine that fictitious play (section 5.3.1) uses 1-level agents while the replicator dynamics (section 5.3.2) uses 0-level agents.

These categorizations help us to determine the relative computational costs of each approach and the machine-learning algorithms that are best suited for that learning problem. 0-level is usually the easiest to implement since it only requires the learning of one function and no additional knowledge. 1-level learning requires us to build a model of every agent and can only be implemented if the agent has the knowledge that tells it which action to take given the set of actions that others have taken. This knowledge must be integrated into the agents. However, recent studies in layered learning (Stone, 2000) have shown how some knowledge could be learned in a "training" situation and then fixed into the agent so that other knowledge that uses the first one can be learned, either at runtime or in another training situation. In general, a change in the level that an agent operates on implies a change on the learning problem and the knowledge built into the agent.

Studies with n-level agents have shown (Vidal and Durfee, 1998a) that an n-level agent will perform better in a society full of (n-1)-level agents, and that the computational costs of increasing a level grow exponentially. Meanwhile, the utility gains to the agent grow smaller as the agents in the system increase their level, within an economic scenario. The reason is that an n-level agent is able to exploit the non-equilibrium dynamics of a system composed of (n-1)-level agents. However, as the agents increase their level the system reaches equilibrium faster so the advantages of strategic thinking are reduced—it is best to play the equilibrium strategy and not worry about what others might do. On the other hand, if all agents stopped learning then it would be very easy for a new learning agent to take advantage of them. As such, the research concludes that some of the agents should do some learning some of the time in order to preserve the robustness of the system, even if this learning does not have any direct results. That is, there appear to be decreasing marginal returns for strategic thinking.

5.6 Collective Intelligence

We can also try to use machine learning as a way to automatically build multiagent systems where each one of the agents learns to do what we want. For example, imagine a factory floor with boxes that need to be moved and robots of different abilities. Depending on the size and weight of the boxes, different robots, or combinations of robots, are able to move them. We know that our goal is for all boxes to be placed against the South wall. Then, rather than trying to come up with specific plans or rules of behavior for each robot, we instead install a reinforcement learning algorithm in each one of them and set them off to learn how best to coordinate. The question we must then ask is: what is the reward function for each agent? As we saw in the CLRI model, one agent's actions can have an impact on another agent's target function thus making it hard, or even impossible, for the other agent to converge in its learning. For example, if one robot keeps changing its behavior and moving around randomly then it will be impossible for another agent to learn to predict when they will both be in front of a large box so that they can move it together.

COIN

Collective intelligence (**COIN**) aims to formalize these ideas and determine what kind of rewards we should provide reinforcement-learning agents in order to achieve a desired global behavior (Wolpert and Tumer, 1999). Specifically, we are given a global utility function $U(s, \vec{a}) \rightarrow \Re$ which tells us the value for every vector of actions $\vec{a} = \{a_1, a_2, \ldots, a_n\}$ of the agents and state of the world s. The agents are assumed to use some reinforcement learning algorithm, such as Q-learning. Our problem is then to define the agents' reward functions $u_i(s, \vec{a})$ such that the agents will end up with policies π_i^* that maximize U. Notice that simply setting $u_i(s, \vec{a}) = U(s, \vec{a})$ for all i can lead to agents receiving an uninformative reward. For example, if a confused agents throws itself against a wall while, at the same time, all the other agents cooperate to move the box correctly then the confused agent will receive a high reward and will thus likely continue to throw itself against the wall.

Still, in general we do want to align each agent's rewards with the global utility. We can quantify this alignment by defining agent i's preference over $s, \vec{a}$ as

$$P_i(s, \vec{a}) = \frac{\sum_{\vec{a}' \in \vec{A}} \Theta[u_i(s, \vec{a}) - u_i(s, \vec{a}')]}{|\vec{A}|}, \tag{5.8}$$

where $\Theta(x)$ is the Heaviside function which is 1 if x is greater than or equal to 0, otherwise it is 0. Similarly, we can define the global preference function as

$$P(s, \vec{a}) = \frac{\sum_{\vec{a}' \in \vec{A}} \Theta[U(s, \vec{a}) - U(s, \vec{a}')]}{|\vec{A}|}. \tag{5.9}$$

Both of these functions serve to rank the agents', or the global, preferences over the set of all possible states and actions vectors. For example, if a particular $(s, \vec{a})$ is preferred over all others then $P(s, \vec{a}) = 1$.

In general, we might want an agent's preferences to be similar to the system's preferences so that the agent's learning mechanism will try to converge towards the desired system's preferences. A system where, for all agents i it is true that $P_i(s, \vec{a}) = P(s, \vec{a})$ is called **factored**. These systems are nice in that they provide the correct incentives to the agents in all situations. Thus, factored systems are more likely to converge to the set of policies that maximizes U.

FACTORED

It is possible that factored systems might not converge because agents' action might have an impact on each other thus changing each other's target function. This idea is captured within the COIN framework as the **opacity** of $u_i(s, \vec{a})$. Specifically, we define the opacity Ω_i for agent i as

OPACITY

$$\Omega_i(s, \vec{a}) = \sum_{\vec{a}' \in \vec{A}} \mathbf{Pr}[\vec{a}'] \frac{|u_i(s, \vec{a}) - u_i(s, \vec{a}'_{-i}, \vec{a}_i)|}{|u_i(s, \vec{a}) - u_i(s, \vec{a}_{-i}, \vec{a}'_i)|}. \tag{5.10}$$

The opacity is zero when the agent's rewards are the same regardless of the actions taken by other, that is, when the other agents' actions have no impact on what the agent should do. On the other hand, the opacity gets larger as the other agents' actions change the reward the agent will get. Just like with the CLRI impact, if the opacity is zero for all states and action vectors then the multiagent learning problem is reduced to the single-agent learning problem as the agents have no effect on each other's target function. More generally, a decrease in the opacity amounts to an increase in the signal-to-noise ratio that the agent receives via its reward function.

Systems that are both factored and have zero opacity for all agents are extremely rare, but would be easy to solve. Systems where the opacity is zero for all agents amount to multiple learning problems that do not interfere with each other and are easy to solve, but they are also very rare. Our goal can now be re-stated as that of finding reward functions for each agent that have as low opacity as possible while also being as highly factored as possible.

One solution COIN proposes is the use of the **wonderful life** reward function which gives each agent a reward proportional to its own contribution to the global utility. It implements the same idea as the VCG payments (Chapter 8.4). Formally, agent i's wonderful life reward is given by

WONDERFUL LIFE

$$u_i(s, \vec{a}) = U(s, \vec{a}) - U(s, \vec{a}_{-i}), \tag{5.11}$$

where the $U(s, \vec{a}_{-i})$ represents the utility in state s when agent i does not exist and all other agents take actions as in $\vec{a}_{-i}$. Notice that this equation can be derived directly from the global utility function. Thus, it is applicable to any system where we have a pre-defined utility function U. It has been shown that this reward function performs better than using $u_i = U$ and other seemingly appropriate reward functions (Wolpert et al., 1999; Tumer and Wolpert, 2004).

Another solution is the **aristocrat utility**, which is defined as

ARISTOCRAT UTILITY

$$u_i(s, \vec{a}) = U(s, \vec{a}) - \sum_{\vec{a}' \in \vec{A}} \mathbf{Pr}[\vec{a}'] U(s, \vec{a}_{-i}, \vec{a}'_i), \tag{5.12}$$

where $\mathbf{Pr}[\vec{a}']$ is the probability that $\vec{a}'$ happens. The aristocrat utility measures the difference in the global utility between the agent's action and its average or expected action. It has been shown that this reward function also performs well, sometimes better than the wonderful life utility (Wolpert and Tumer, 2001) .

5.7 Summary

We have seen how the theory of learning in games provides us with various equilibrium solution concepts and often tells us when some of them will be reached by simple learning models. On the other hand, we have argued that the reason learning is used in a multiagent system is often because there is no known equilibrium or the equilibrium point keeps changing due to outside forces. We have also shown how the CLRI theory and n-level agents are attempts to characterize and predict, to a limited degree, the dynamics of a system given some basic learning parameters.

5.8 Recent Advances

The GAMUT (Nudelman et al., 2004) software can produce game matrices from thirty-five base classes of games. These matrices can be used to determine how well learning algorithms perform under all possible game types and sizes.

Exercises

5.1 In general, which game matrices have cycles for fictitious play?

Chapter 6

Negotiation

A negotiation problem is one where multiple agents try to come to an agreement or **deal**. Each agent is assumed to have a preference over all possible deals. The agents send messages to each other in the hope of finding a deal that all agents can agree on. These agents face an interesting problem. They want to maximize their own utility but they also face the risk of a break-down in negotiation, or expiration of a deadline for agreement. As such, each agent must negotiate carefully, trading off any utility it gains from a tentative against a possibly better deal or the risk of a breakdown in negotiation.

DEAL

decide whether the current deal is good enough or whether it should ask for more and risk agreement failure.

Automated negotiation can be very useful in multiagent systems as it provides a distributed method of aggregating distributed knowledge. That is, in a problem where each agent has different local knowledge negotiation can be an effective method for finding the one global course of action which maximizes utility without having to aggregate all local knowledge in a central location. In fact, the metaphor of autonomous agents cooperating in this manner to solve a problem that cannot be solved by any one agent, due to limited abilities or knowledge, was the central metaphor from which the field of distributed artificial intelligence, later known as multiagent systems, emerged (Davis and Smith, 1983). The metaphor is based on the observation that teams of scientists, businesses, citizens, and others regularly negotiate over future courses of action and the result of these negotiations can incorporate more knowledge than any one individual possesses (Surowiecki, 2005). For example, in a NASA rover mission to Mars the various engineering teams and science teams negotiate over what feature to include in the rover. The scientists are concerned with having the proper equipment in Mars so that they can do good science while the engineers try to ensure that everything will work as expected. Often it is the case that one side does not understand exactly why the other wants or rejects a particular feature but by negotiating with each other they arrive at a rover that is engineered solidly enough to survive the trip to Mars and has enough equipment to do useful science while there. Thus, negotiation results in the aggregation of knowledge from multiple individuals in order to make decisions which are better, for the whole, than if they were made by any one individual.

See (Squyres, 2005) for the full story on the MER mission to Mars. An exciting read.

6.1 The Bargaining Problem

A specific version of the negotiation problem has been studied in game theory. It is known as the **bargaining problem** (Nash, 1950). In the bargaining problem, we say that each agent i has a utility function u_i defined over the set of all possible deals Δ. That is, $u_i : \Delta \rightarrow \Re$. We also assume that there is a special deal δ^- which is the no-deal deal. Without loss of generality we will assume that for all agents $u_i(\delta^-) = 0$ so that the agents will prefer no deal than accepting any deal with negative utility. The problem then is finding a protocol f which will lead the agents to the best deal. But, as with all the game theory we have studied, it is not obvious which deal is the best one. Many solutions concepts have been proposed. We provide an overview of them in the next sections.

BARGAINING PROBLEM

See (Osborne and Rubinstein, 1999, Chapter 7) or (Osborne and Rubinstein, 1990) for a more extended introduction to bargaining.

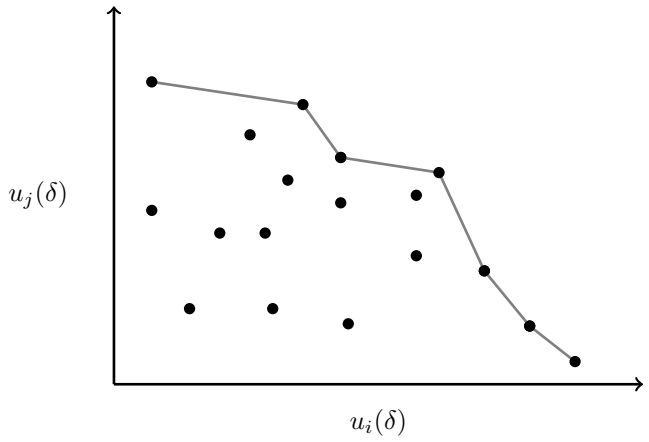


Figure 6.1: The dots represent possible deals. The deals on the gray line are Pareto optimal. The line is the Pareto frontier.

6.1.1 Axiomatic Solution Concepts

AXIOMATIC

An **axiomatic** solution concept is one which we can formally describe and then simply declare it to be our most desirable solution concept simply because we believe it satisfies all the important requirements. There are many possible requirements we might want to impose on a possible solution deal. The most obvious one is Pareto optimality.

Definition 6.1 (Pareto optimal)**.** *A deal δ is Pareto optimal if there is no other deal such that everyone prefers it over δ. That is, there is no δ' such that*

$$\forall_i u_i(\delta') > u_i(\delta).$$

This is an obvious requirement because if a deal is not Pareto optimal then that means that there is some other deal which all of the agents like better. As such, it does not make sense to use the non-Pareto deal when we could find this other deal which everyone prefers.

With only two agents, i and j, we can visualize the set of Pareto-optimal deals by using an xy-plot where the x and y coordinates are the utilities each agent receives for a deal. Each deal then becomes a point in the graph, as seen in figure 6.1. The **pareto frontier** is represented by the line shown in the upper-right which connects all the Pareto deals.

PARETO FRONTIER

We might also want a solution that remains the same regardless of the magnitude of an agent's utility values.

Definition 6.2 (Independence of utility units)**.** *A negotiation protocol is independent of utility units if when given U it chooses δ and when given $U' = \{(\beta_1 u_1, \ldots, \beta_I u_I) : u \in U\}$ it chooses δ' where*

$$\forall_i \; u_i(\delta') = \beta_i u_i(\delta).$$

That is, if an agent in a protocol that is independent of utility units used to get a utility of 10 from the result deal and now has multiplied its utility function by 5 then that agent will now get a utility of 50 from the new resulting deal. So, the utilities the agents receive remain proportional under multiplication.

Definition 6.3 (Symmetry)**.** *A negotiation protocol is symmetric if the solution remains the same as long as the set of utility functions U is the same, regardless of which agent has which utility.*

That is, if two agents swapped utility function then they also end up swapping the utilities they get from the resulting deal. In other words, the specific agents do not matter, the only thing that matters are the utility functions.

Definition 6.4 (Individual rationality)**.** *A deal δ is individually rational if*

$$\forall_i \; u_i(\delta) \geq u_i(\delta^-).$$

So, a deal is individual rational if $u_i(\delta) \geq 0$ since we will be assuming that $u_i(\delta^-) = 0$. That is, a deal is individually rational if all the agents prefer it over not reaching an agreement.

Definition 6.5 (Independece of irrelavant alternatives)**.** *A negotiation protocol is independent of irrelevant alternatives if it is true that when given the set of possible deals Δ it chooses δ and when given $\Delta' \subset \Delta$ where $\delta \in \Delta'$ it again chooses δ, assuming U stays constant.*

That is, a protocol is independent of irrelevant alternative if the deal it chooses does not change after we remove a deal that lost. Only removal of the winning deal changes the deal the protocol chooses.

Given these requirements we can now consider various possible solutions to a negotiation problem. In the **egalitarian solution** the gains from cooperation are split equally among the agents. That is, the egalitarian deal is the one where all the agents receive the same utility and the sum of their utilities is maximal, that is

EGALITARIAN SOLUTION

$$\delta = \arg\max_{\delta' \in E} \sum_i u_i(\delta') \tag{6.1}$$

where E is the set of all deals where all agents receive the same utility, namely

$$E = \{\delta \,|\, \forall_{i,j} u_i(\delta) = u_j(\delta)\}.$$

egalitarian n: one who believes in the equality of all people.

We can find the egalitarian solution visually for the two agent case by simply drawing the line $y = x$ and finding the deal on this line that is farther away from the origin, as seen in figure 6.2. Note that the egalitarian deal in this case is not Pareto optimal. However, if we allowed all possible deals (the graph would be solid black as every pair of x, y coordinates would represent a possible deal) then the egalitarian deal would also be a Pareto deal. Specifically, it would be the point on the $y = x$ line which marks the intersection with the Pareto frontier. Also note that the egalitarian deal does not satisfy the independence of utility units requirement since it assumes the utility units for the agents are comparable, in fact, it assumes that all agents' utility functions use the same units.

A variation on the pure egalitarian deal is the **egalitarian social welfare** solution which is the deal that maximizes the utility received by the agent with the lowest utility. That is, it is the deal δ that satisfies

EGALITARIAN SOCIAL WELFARE

$$\delta = \arg\max_{\delta} \min_i u_i(\delta). \tag{6.2}$$

The egalitarian social welfare solution is especially useful in scenarios where no deal exists which provides all agents with the same utility since every problem is guaranteed to have an egalitarian social welfare solution. However, the solution itself can in some cases seem very un-egalitarian. For example, a deal where two agents receive utilities of 10 and 100 respectively is the egalitarian social welfare solution even when another deal exists which gives the agents 9 and 11 respectively.

The **utilitarian solution** is the deal that maximizes the sum of the utilities, that is

UTILITARIAN SOLUTION

$$\delta = \arg\max \sum_i u_i(\delta). \tag{6.3}$$

The utilitarian deal is, by definition, a Pareto optimal deal. There might be more than one utilitarian deals in the case of a tie. The utilitarian deal violates the independence of utility units assumption as it also assumes utilities are comparable.

utilitarian n : someone who believes that the value of a thing depends on its utility.

We can find the utilitarian deal visually for the two agent case by drawing a line with slope of -1 and, starting at the top right, moving it perpendicular to $y = x$ until it intersects a deal. The first deals intersected by the line are utilitarian deals, as shown in figure 6.3.

The utilitarian and egalitarian solutions both seem like fairly reasonable solutions. Unfortunately, both violate the independence of utility units assumption. It

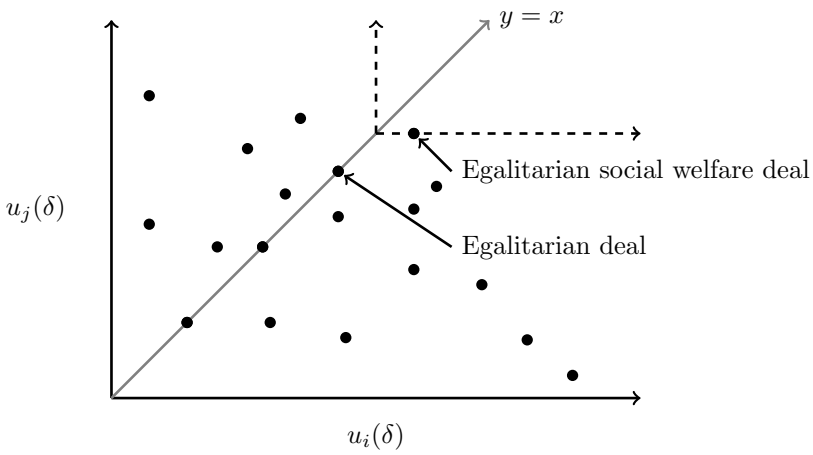


Figure 6.2: Egalitarian and egalitarian social welfare deals. The egalitarian social welfare deal can be found by moving the perpendicular dashed lines from a point high in the $y = x$ line down to $0, 0$ until they touch the first deal.

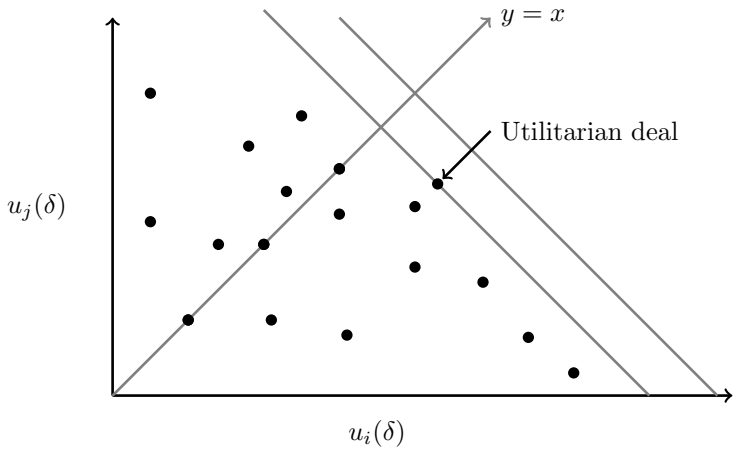


Figure 6.3: Utilitarian deal. The line $y = b - x$ starts with a very large b which is reduced until the line intersects a deal.

was Nash who proposed a solution which does not violate this assumption. The **Nash bargaining solution** is the deal that maximizes the product of the utilities. That is,

NASH BARGAINING SOLUTION

$$\delta = \arg\max_{\delta'} \prod u_i(\delta'). \tag{6.4}$$

The Nash solution is also Pareto efficient (Definition 6.1), independent of utility units (Definition 6.2), symmetric (Definition 6.3), independent of irrelevant alternatives (Definition 6.5). In fact, it is the *only* solution that satisfies these four requirements (Nash, 1950). This means that if we want a solution that satisfies these four requirements then our only choice is the Nash bargaining solution.

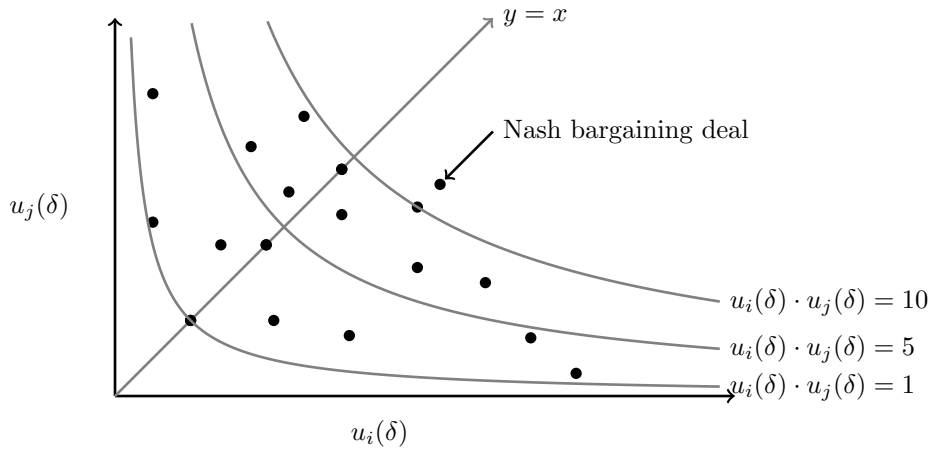


Figure 6.4: Nash bargaining deal.

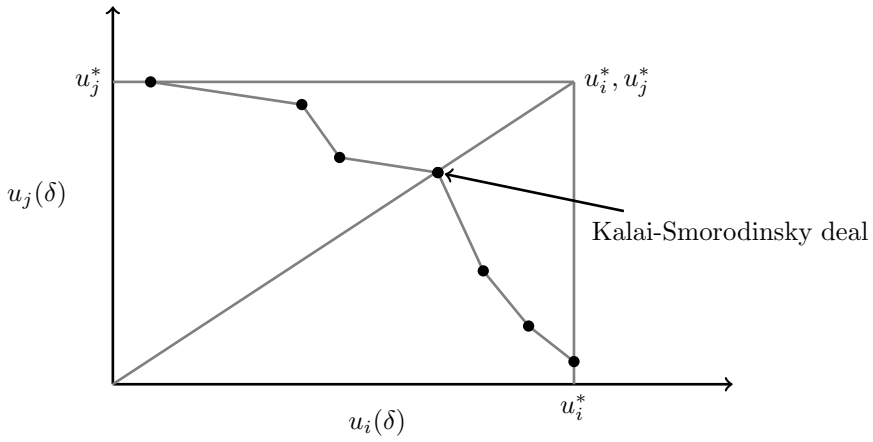


Figure 6.5: Kalai-Smorodinsky bargaining deal. We only show deals on the Pareto frontier.

Figure 6.4 shows a visualization of the Nash bargaining deal. Each curve represents all pairs of utilities that have the same product, that is, the line $y = c/x$. As we move northeast, following the line $y = x$, we cut across indifference curves of monotonically higher products. As such, the last deal to intersect a curve is the one which maximizes the product of the utilities, so it is the Nash bargaining solution.

Yet another possible solution is to find the deal that distributes the utility in proportion to the maximum that the agent could get (Kalai and Smorodinsky, 1975). Specifically, let u_i^* be the maximum utility that i could get from the set of all deals in the Pareto frontier. That is, the utility that i would receive if he got his best possible deal from the Pareto frontier. Then, find the deal that lies in the line between the point δ^- and the point (u_i^*, u_j^*). This solution is known as the **Kalai-Smorodinsky solution**. If all deals are possible then this solution is Pareto optimal. However, if there are only a finite set of possible deals then this solution might not exist. That is, there might not be a point in the specified line. This is a serious drawback to the use of the Kalai-Smorodinsky solution in a discrete setting.

KALAI-SMORODINSKY SOLUTION

The Kalai-Smorodinsky solution, like the Nash bargaining solution, is also independent of utility units requirement. On the other hand, it is not independent of irrelevant alternatives.

There is no general agreement as to which one of these axiomatic solutions is better. The social sciences try to develop experiments that will tell us which one of the solutions, if any, is arrived at by humans engaged in negotiation. Others try to justify some of these solutions as more fair than others. We, as multiagent designers, will probably find that all of them will be useful at some point, depending on the requirements of the system we are building.

6.1.2 Strategic Solution Concepts

Another way to think about what solution will be arrived at in a bargaining problem is to formalize the bargaining process, assume rational agents, and then determine their equilibrium strategies for their bargaining process. That is, define the solution concept to be the solution that is reached by automated rational agents in a bargaining problem. This method does raise one large obstacle: we need to first formally define a bargaining process that allows all the same "moves" as real-life bargaining. If you have ever haggled over the price of an item then you know that it is impossible to formalize all the possible moves of market-vendor. We need a negotiation protocol that is simple enough to be formally analyzed but still allows the agents to use most of their moves.

One such model is the **Rubinstein's alternating offers** model (Rubinstein, 1982). In this model two agents try to reach agreement on a deal. The agents can take actions only at discrete time steps. In each time step one of the agents proposes a deal δ to the other who either accepts it or rejects it. If the offer is rejected then

RUBINSTEIN'S ALTERNATING OFFERS

we move to the next time step where the other agent gets to propose a deal. Once a deal has been rejected it is considered void and cannot be accepted at a later time. The agents, however, are always free to propose any deal and to accept or reject any deal as they wish. We further assume that the agents know each other's utility functions.

The alternating offers models, as it stands, does not have a dominant strategy. For example, imagine that the two agents are bargaining over how to divide a dollar. Each agent wants to keep the whole dollar to itself and leave the other agent with nothing. Under the basic alternating offers protocol the agents' best strategy is to keep proposing this deal to the other agent. That is, each agent keeps telling the other one "I propose that I keep the whole dollar" and the other agent keeps rejecting this proposal. This scenario is not very interesting.

In order to make it more interesting, we further assume that time is valuable to the agents. That is, the agents' utility for all possible deals is reduced as time passes. For example, imagine that instead of haggling over a dollar the agents are haggling over how to split an ice cream sundae which is slowly melting and the agents hate melted ice cream. Formally, we say that agent i's utility at time t for deal δ is given by $\lambda_i^t u_i(\delta)$ where λ_i is i's discount factor, similarly the utility for j is $\lambda_j^t u_j(\delta)$. Thus, the agents' utility for every possible deal decreases monotonically as a function of time with a discount factor given by λ. Note that if $\lambda_i = 0$ then the agent must agree to a deal at time 0 since after that it will receive a utility of 0 for any deal. Conversely, if $\lambda_i = 1$ then the agent can wait forever without any utility loss.

Furthermore, lets assume that the agents' utilities are linear and complementary. Specifically, imagine that the deal δ is simply a number between 0 and 1 and represents the amount of utility an agent receives so that $u_i(\delta) = \delta$ and $u_j(\delta) = 1 - \delta$. In this scenario, it can be shown that a unique subgame perfect equilibrium strategy exists.

Theorem 6.1 (Alternating Offers Bargaining Strategy)**.** *The Rubinstein's alternating offers game where the agents have complementary linear utilities has a unique subgame perfect equilibrium strategy where*

- *agent i proposes a deal*
$$\delta_i^* = \frac{1 - \lambda_j}{1 - \lambda_i \lambda_j}$$
and accepts the offer δ_j from j only if $u_i(\delta_j) \leq u_i(\delta_j^*)$,
- *agent j proposes a deal*
$$\delta_j^* = \frac{1 - \lambda_i}{1 - \lambda_i \lambda_j}$$
and accepts the offer δ_i from i only if $u_j(\delta_i) \leq u_j(\delta_i^*)$.

(Rubinstein, 1982; Muthoo, 1999).

We can understand how these deals are derived by noting that since both agents have utilities that decrease with time the best deal will be reached in the first step. That means that each agent must propose a deal that the other will accept. Specifically, agent i must propose a deal δ_i^* such that $u_j(\delta_i^*) = \lambda_j u_j(\delta_j^*)$ because if it proposes a deal that gives j lower utility then j will reject it and if it proposes a deal that gives j higher utility then i is needlessly giving up some of its own utility. Conversely, j must propose a deal δ_j^* such that $u_i(\delta_j^*) = \lambda_i u_i(\delta_i^*)$. We thus have two equations.

Since $u_i(\delta) = \delta$ and $u_j(\delta) = 1 - \delta$, we can replace these definitions into the above equations to get

$$1 - \delta_i^* = \lambda_j (1 - \delta_j^*) \tag{6.5}$$
$$\delta_j^* = \lambda_i \delta_i^*. \tag{6.6}$$

```
MONOTONIC-CONCESSION
1  δ_i ← arg max_δ u_i(δ)
2  Propose δ_i
3  Receive δ_j proposal
4  if u_i(δ_j) ≥ u_i(δ_i)
5      then Accept δ_j
6      else  δ_i ← δ'_i such that u_j(δ'_i) ≥ ε + u_j(δ_i) and u_i(δ'_i) ≥ u_i(δ^-)
7  goto 2
```

Figure 6.6: The monotonic concession protocol.

Upon solving these two equations for δ_i^* and δ_j^* we get the equilibrium values from Theorem 6.1. More generally, it has been shown that a unique subgame perfect equilibrium exists even when the utility functions are not linear but are simply monotonic in δ.

The theorem tells us that the best strategy for these agents is propose a bid on the first time step which will be accepted by the other agent. This action makes sense because we know that utilities only decrease with time so the best possible deal will be had on the first time step. The value of the proposed deal, on the other hand, is very interesting. Notice that the deal i proposes depends only on the agents' discount factor, not on their utility values. The important thing is how fast each agent loses utility over time. For example, notice that if $\lambda_j = 0$ then i will propose $\delta_i^* = 1$. That is, i will propose to keep all the utility to himself. Agent j will accept this proposal since he knows that given that his $\lambda_j = 0$ if he waits for the next time step then he will receive utility of 0 in every possible deal.

Similar techniques have been used to show the existence of equilibrium in other types of bargaining games (Kraus, 2001). These games are all alternating offer games but assume different utility discounts such as a fixed loss utility function that is reduced by a constant amount each time and an interest rate utility which models the opportunity loss, among others. Depending on the type of utility discount and the type of game, some of these games can be proven to have a unique perfect equilibrium strategy that involves only one actions, others have multiple equilibria and thus would require some other coordination method, and yet others can be shown to be NP-complete in the time it takes to find the unique equilibrium. In general, however, the technique used to solve these bargaining games is the same.

1. Formalize bargaining as a Rubinstein's alternating offers games.
2. Generate extended-form game.
3. Determine equilibrium strategy for this game.

In practice, step 3 is often computationally intractable, at least for the general case of the problem.

6.2 Monotonic Concession Protocol

The simplest negotiation protocol following the Rubinstein's model of alternating offers is the **monotonic concession protocol**. In this protocol the agents agree to always make a counter offer that is slightly better for the other agent than its previous offer. Specifically, in monotonic concession the agents follow the algorithm shown in figure 6.6.

MONOTONIC CONCESSION PROTOCOL

Each agent starts by proposing the deal that is best for itself. The agent then receives a similar proposal from the other agent. If the utility the agent receives from the other's proposal is bigger than the utility it gets from its own proposal then it accepts it and negotiation ends. If no agreement was reached then the agent must propose a deal that is at least an increase of ϵ in the other agent's utility. If neither agent makes a new proposal then there are no more messages sent and

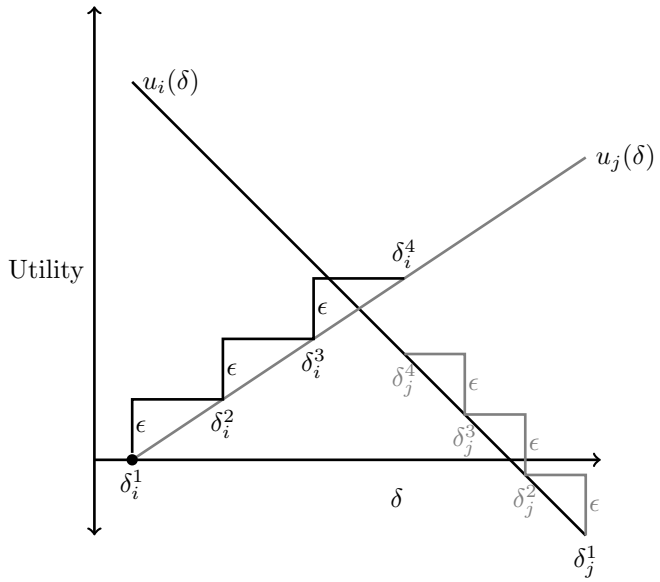


Figure 6.7: Monotonic concession protocol visualization. Both agents have linear utility functions over the set of deals. The superscripts indicate the time so δ_i^1 is i's proposal at time 1. At time 4 we have that $\delta_i^4 = \delta_j^4$ so $u_i(\delta_j^4) = u_i(\delta_i^4)$ and the agents agree on this deal.

negotiations fail, so they implicitly agree on the no-deal deal δ^-. The monotonic concession protocol makes it easy for the agents to verify that the other agent is also obeying the protocol. Namely, if i ever receives a proposal whose utility is less than a previous proposal then it knows that j is not following the protocol.

The protocol can be visualized as in figure 6.7. Here we see a simple example with linear utility functions where the agents reach an agreement (δ^4) after four time steps. Notice that the agreement reached is *not* the point at which the lines intersect. The monotonic concession protocol is not guaranteed to arrive at any particular axiomatic solution concept. All that it guarantees is that it will stop.

A common workaround is to assume its a zero-sum game and use the agent's own utility function.

Monotonic concession has several drawbacks. It can be very slow to converge. Convergence time is dictated by the number of possible deals, which is usually very large, and the value of ϵ. It is also impossible to implement this algorithm if the agents do not know the other agents' utility function. In practice, it is rare for an agent to know its opponent's utility function. Finally, the monotonic concession protocol has a tricky last step. Namely, the two agents could make simultaneous offers where each one ends up preferring the other agent's offer to the one it just sent. This is a problem as both of them now want to accept different offers. This can be solved by forcing the agents to take turns. But, if we did that then neither will want to go first as the agent that goes first will end up with a slightly worse deal.

6.2.1 The Zeuthen Strategy

The monotonic concession protocol we presented implements a simplistic strategy for the agent: always concede at least ϵ to the other agent. However, if an agent knows that the other one will always concede then it might choose to not concede at all. Smarter agents will examine their opponent's behavior and concede in proportion to how much they are conceding.

Zeuthen strategy

This idea is formalized in the **Zeuthen strategy** for the monotonic concession protocol. We start by defining the willingness to risk a breakdown in negotiations to be

$$\text{risk}_i = \frac{u_i(\delta_i) - u_i(\delta_j)}{u_i(\delta_i)}. \tag{6.7}$$

The agent can then calculate the risks for both agents. The Zeuthen strategy tells us that the agent with the smallest risk should concede just enough so that it does not have to concede again in the next time step. That is, the agent that has the least

```
ZEUTHEN-MONOTONIC-CONCESSION
 1  δ_i ← arg max_δ u_i(δ)
 2  Propose δ_i
 3  Receive δ_j proposal
 4  if u_i(δ_j) ≥ u_i(δ_i)
 5      then Accept δ_j
 6  risk_i ← (u_i(δ_i) − u_i(δ_j)) / u_i(δ_i)
 7  risk_j ← (u_j(δ_j) − u_j(δ_i)) / u_j(δ_j)
 8  if risk_i < risk_j
 9      then δ_i ← δ'_i such that risk_i(δ'_i) > risk_j(δ'_j)
10           goto 2
11  goto 3
```

Figure 6.8: The monotonic concession protocol using the Zeuthen strategy.

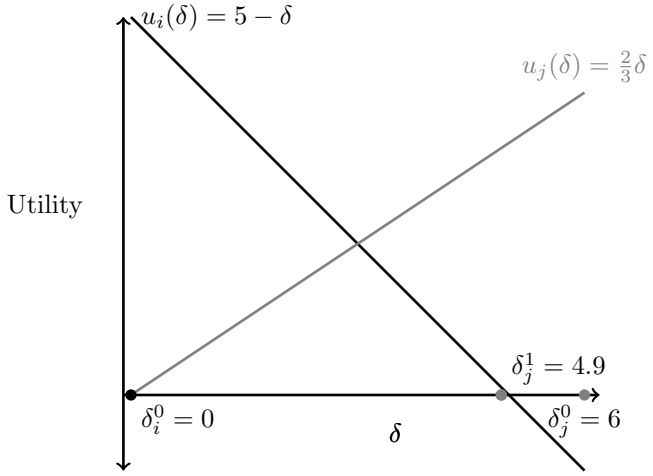


Figure 6.9: Visualization of the Zeuthen strategy at work. The initial proposals are $\delta_i^0 = 0$ and $\delta_j^0 = 6$.

to lose by conceding should concede. More formally, we define the new protocol as shown in figure 6.8.

Figure 6.9 shows a graphical representation of the first step in a Zeuthen negotiation. After the initial proposal the agents calculate their risks to be

$$\text{risk}_i^0 = \frac{5 - (-1)}{5} = \frac{6}{5}$$

and

$$\text{risk}_j^0 = \frac{4 - 0}{4} = 1.$$

Since j has a lower risk it must concede. The new deal must be such that j will not be forced to concede again. That is, it must insure that

$$\text{risk}_i = \frac{5 - (5 - \delta_j)}{5} < \frac{\frac{2}{3}\delta_j - 0}{\frac{2}{3}\delta_j} = \text{risk}_j$$

which simplifies to $\delta_j < 5$. As such, j can pick any deal δ less than 5. In the figure the agent chooses $\delta_j^1 = 4.9$.

The Zeuthen strategy is guaranteed to terminate and the agreement it reaches upon termination is guaranteed to be individually rational and Pareto optimal. Furthermore, it has been shown that two agents using the Zeuthen strategy will converge to a Nash bargaining solution.

Theorem 6.2 (Zeuthen converges to Nash solution)**.** *If both agents use the Zeuthen strategy they will converge to a Nash bargaining solution deal, that is, a deal that maximizes the product of the utilities (Harsanyi, 1965).*

ONE-STEP-NEGOTIATION

```
1  E ← {δ | ∀δ' u_i(δ)u_j(δ) ≥ u_i(δ')u_j(δ')}
2  δ_i ← arg max_{δ∈E} u_i(δ)
3  Propose δ_i
4  Receive δ_j
5  if u_i(δ_j)u_j(δ_j) < u_i(δ_i)u_j(δ_i)
6      then Report error, j is not following strategy.
7  Coordinate with j to choose randomly between δ_i and δ_j.
```

Figure 6.10: The one step negotiation protocol (Rosenschein and Zlotkin, 1994).

The proof of the theorem shows how the maximum of both agents' utilities monotonically increases until the protocol reaches an agreement. As such, if there is any other agreement that has higher utility than the one they agreed upon then that agreement would have been proposed in the negotiation. Therefore, there can be no deal with higher utility for any agent than the one they agreed on.

A problem with the Zeuthen strategy might arise in the last step if the agents' risks are exactly the same. Specifically, if both agents realize that their risks are the same and that the next proposal that either of them makes will be accepted by their opponent then both agents will want to wait for the other agent to concede. This is an example of the game of chicken. As such, selfish agents should play the Nash equilibrium strategy which, in this case, is a mixed strategy. Since it is a mixed strategy it means that there remains a possibility for both of them to decide to wait for the other one, thereby breaking down the negotiations when a solution was possible.

The Zeuthen strategy is also attractive because, when combined with the mixed Nash strategy we described, it is in a Nash equilibrium. That is, if one agent declares that it will be using the Zeuthen strategy then the other agent has nothing to gain by using any other negotiation strategy. As such, and agent that publicly states that it will be using the Zeuthen strategy can expect that every agent it negotiates with will also use the Zeuthen strategy.

6.2.2 One-Step Protocol

Once we have a multi-step protocol that (usually) reaches a particular solution, we immediately think about the possibility of skipping all those intermediate steps and jumping right to the final agreement. Specifically, we could define a protocol that asks both agents to send a proposal. Each agent then has two proposals: the one it made and the one it received. The agents must then accept the proposal that maximizes the product of the agents' utilities. If there is a tie then they coordinate and choose one of them at random.

Given the rules established by this protocol, an agent's best strategy is to find of the deals that maximize the product of the utilities and then, from these deals, propose the one that is best for the agent. The agent's algorithm for this **one-step negotiation** protocol is shown in figure 6.10. The strategy implemented by this algorithm is a Nash equilibrium. That is, if one agent decides to implement this algorithm then the other agent's best strategy is to also implement the same algorithm. Thus, when using perfectly rational agents it is always more efficient to use one-step negotiation rather than the more long winded monotonic concession protocol.

ONE-STEP NEGOTIATION

6.3 Negotiation as Distributed Search

If we take a step back and look at the bargaining problem, we realize that the problem of finding a chosen solution deal is in effect a distributed search problem. That is, via their negotiations the agents are effectively searching for a specific solution. A simple and natural way to carry out this search is by starting with some

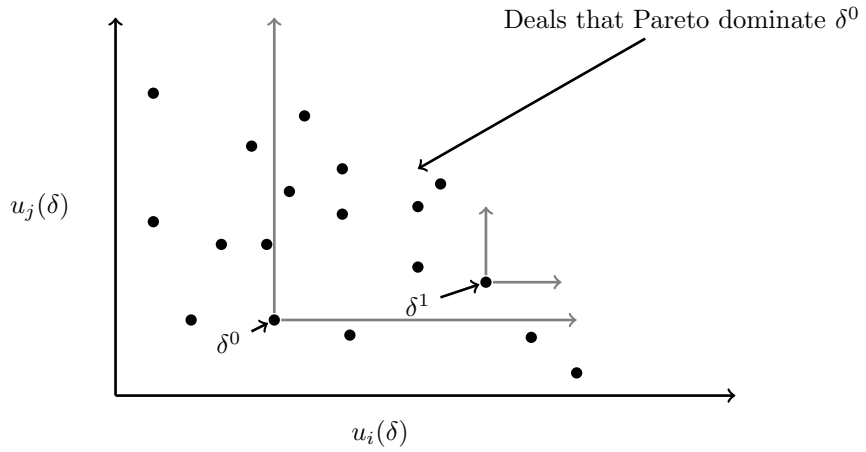


Figure 6.11: Hill-climbing on the Pareto landscape. The quadrant to the top and right of δ^0 contains all the deals that dominate δ^0, similarly for δ^1. There are no deals that dominate δ^1 and yet it is not the social welfare or Nash bargaining solution.

initial deal and at each time step moving to another deal. The process is repeated until there is no other deal which the agents can agree on.

For example, figure 6.11 shows an initial deal δ^0 along with all the other possible deals for a negotiation problem. Any deal that is above and to the right of δ^0, as delineated by the gray lines, dominates δ^0. This means that any deal in this quadrant is preferred by all the agents. Thus, we can expect selfish agents to cheerfully accept any one of those deals. A simple hill-climbing search process will continue moving in this up-and-right direction.

However, the search might get stuck in a local optima. For example, suppose that we go directly from δ^0 to δ^1 as shown in the figure. This new deal is not dominated by any other deal, thus our simple hill-climbing algorithm is now stuck at that deal. Notice also that this deal is not the social welfare maximizing deal nor is it the Nash bargaining deal. Thus, simple hill-climbing among selfish agents can lead us to sub-optimal solutions.

In real applications this search problem is further compounded by the fact that it is often impossible to move directly from any one deal to any other deal because the number of possible deals is simply too large to consider or there are other external limitations imposed on the agents. That is, most applications overlay an undirected graph to the bargaining problem. In this graph the nodes represent the possible deals and an edge between two nodes denotes the fact that it is possible to move between those two deals. Given such a graph, it is possible that the agents would find themselves in one deal that is pareto dominated by a lot of other deals but which are unreachable in one step because there is no edge between those deals and the current deal. Thus, we have even more local optima.

Researchers have tried to circumvent this problem of local optima in bargaining via a number of different methods, as we shall see in the next few sections.

6.4 Ad-hoc Negotiation Strategies

ADEPT

TACTICS

Deployed multiagent negotiation systems, such as **ADEPT** (Faratin et al., 1998; Binmore and Vulkan, 1999), have implemented agents with ad-hoc negotiation strategies. In ADEPT, a handful of selected negotiation **tactics** were tested against each other to determine how they fared against each other. The basic negotiation model used was an alternating offers model with time discounts. The tactics the agents could use included: a *linear* tactic that concedes a fixed amount each time, a *conceder* tactic that concedes large amounts of utility on earlier time steps, and an *impatient* tactic which conceded very little at first and requested a lot. We can think of each one of these tactics as stylized versions of the negotiation strategies people might use in a negotiation.

This ad-hoc approach is often used by multiagent developers who need to build a negotiating multiagent system which achieves a good-enough solution and where

all the agents are owned by the same party and, thus, do not need to behave truly selfishly. It allows us to use somewhat selfish agents that encapsulate some of the system's requirements and then use the negotiation protocol to aggregate these individual selfish interests into a global solution which, although not guaranteed to be the utilitarian solutions, can nonetheless be shown via experimentation to be a good-enough solutions. Nonetheless, we must always remember that these systems are not to be opened up to outside competition. Once we know the other agents' ad-hoc tactics, it is generally very easy to implement an agent that can defeat those tactics, usually at the cost of a much lower system utility. That is, one renegade agent can easily bring down a system of ad hoc negotiating agents, unless the system has been shown to be at an evolutionary stable equilibrium.

6.5 The Task Allocation Problem

TASK ALLOCATION PROBLEM

A common problem in multiagent systems is deciding how to re-allocate a set of tasks among a set of agents. This is known as the **task allocation problem**. In this problem there is a set of tasks T, a set of agents, and a cost function $c_i : s \rightarrow \Re$ which tells us the cost that agent i incurs in carrying out tasks $s \subseteq T$. In some simplified versions of the problem we assume that all agents share the same cost function. Each agent starts out with a set of tasks such that all tasks are distributed amongst all agents. We can think of this initial allocation as δ^- since, if negotiations break down then each agent is responsible for the tasks it was originally assigned. Similarly, every allocation of tasks to agents is a possible deal δ where $s_i(\delta)$ is the set of tasks allocated to i under deal δ. The problem we then face is how to design a negotiation protocol such that the agents can negotiate task re-allocations and arrive at a final re-allocation of the tasks that is one of the axiomatic solution concepts, such as the utilitarian deal.

POSTMAN PROBLEM

An example of this type of problem is the **postman problem** in which a set of postmen are placed around a city each with a bag of letters that must be delivered. A postman prefers to have all his letters with delivery addresses that are very close to each other so as to minimize his delivery costs. The postmen can negotiate with each other in order to make this happen. That is, a postman can trade some of its letters with another postman. In this example the tasks are the letters to be delivered and the cost function is the distance traveled to deliver a set of letters. Different cost functions arise when the postmen have different final destinations, for example, if they are required to go to their respective homes after delivering all the letters, or if they prefer certain areas of town, etc.

Once we are given task allocation problem we must then decide how the agents will exchange tasks. The simplest method is for agents to pair up and exchange a single task. This means that not all deals are directly accessible from other deals since, for example, we can't go from the deal where j does 2 tasks and i does nothing to the deal where i does 2 tasks and j does nothing in one step. We can represent this constraint graphically by drawing an edge between every pair of deals that are reachable from one another by the exchange of a single task. An example of such a graph is shown in figure 6.12. The table in this figure shows the tasks assignments that each deal represents and the costs that each one of the agents incurs for carrying out various subsets of tasks. In order to have positive utility numbers, we let the utility to the agent be 8 minus the cost of carrying out the tasks. Utility is often simply the negative of the cost so these two are used interchangeably. The graph shows all the possible deals as points and the edges connect deals that can be reached from one another by moving a single task between agents.

If the agents are only willing to accept deals that are better for them than their current deal, that is, the accept only Pareto-dominant deals, then every deal in figure 6.12 forms a local maximum. For example, if the agents are in δ^1 then, while j would prefer any one of δ^2, δ^3, or δ^4, agent i would not accept any one of these because u_i is lower for all of them than for δ^1. Thus, if the agents find themselves in δ^1 and they are not willing to lower their utility they will remain there. That is,

δ	$s_i(\delta)$	$s_j(\delta)$	$c_i(\delta)$	$c_j(\delta)$	$u_i(\delta) = 8 - c_i(\delta)$	$u_j(\delta) = 8 - c_j(\delta)$
δ^1	$\emptyset$	$\{t_1, t_2, t_3\}$	0	8	8	0
δ^2	$\{t_1\}$	$\{t_2, t_3\}$	1	4	7	4
δ^3	$\{t_2\}$	$\{t_1, t_3\}$	2	5	6	3
δ^4	$\{t_3\}$	$\{t_2, t_3\}$	4	7	4	1
δ^5	$\{t_2, t_3\}$	$\{t_1\}$	6	4	2	4
δ^6	$\{t_1, t_3\}$	$\{t_2\}$	5	3	3	5
δ^7	$\{t_1, t_2\}$	$\{t_3\}$	3	1	5	7
δ^8	$\{t_1, t_2, t_3\}$	$\emptyset$	7	0	1	8

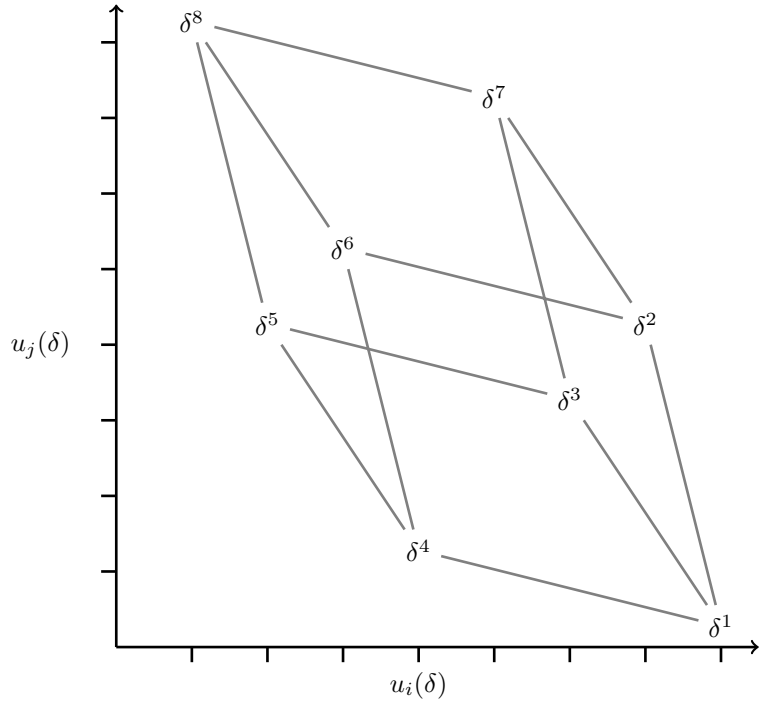


Figure 6.12: An example task allocation problem is shown in the table and its graphical representation as a bargaining problem is shown in the graph. The edges on the graph connect deals that can be reached by moving a single task between the agents. $s_i(\delta)$ is the set of tasks i has under deal δ.

δ^1 is a local maximum, as are all the other deals in this example.

6.5.1 Payments

One possible way to allow the agents to find deals that are closer to the utilitarian deal is by allowing them to use **monetary payments**. For example, agent i might convince j to take a deal that gives j less utility if i can sweeten the deal by giving j some money to take that deal. This basic idea was implemented in the **contract net protocol** (Smith, 1981; Davis and Smith, 1983). In contract net each agent takes the roles of either a contractor or contractee. The contractor has a task that it wants someone else to perform. The contractor is also willing to pay to get that task done. In the protocol, the contractor first announces that it has a task available by broadcasting a call for bids. All agents receive this call for bids and, if they want, reply to the contractor telling him how much they charge for performing the task. The contractor then chooses one of these bids, assigns the task to the appropriate agent and pays him the requested amount.

MONETARY PAYMENTS

CONTRACT NET PROTOCOL

For example, given the current task allocation δ agent i might find that one of his current tasks $t \in s_i(\delta)$ costs him a lot. That is, $c_i(s_i(\delta)) - c_i(s_i(\delta) - t)$ is a large number. In this case i will be willing to pay up to $c_i(s_i(\delta)) - c_i(s_i(\delta) - t)$ in order to have some other agent perform t—the agent will pay up to the utility he will gain from losing the task, any more than that does not make sense as the agent can simply do the task himself. Similarly, any other agent j will be willing

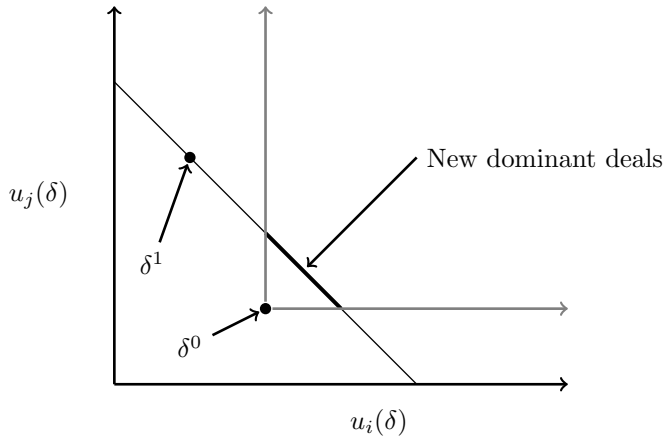


Figure 6.13: Deal δ^1 is turned into an infinite number of deals, represented by the line that intersects δ^1, with the use of payments. Some of those new deals Pareto dominate δ^0, as shown by the thicker part of the line.

to perform t as long as it gets paid an amount that is at least equal to any cost increase he will endure by performing the task, that is, the payment must be at least $c_j(s_j(\delta)) - c_j(s_j(\delta) + t)$. Note that in the general case some of these numbers could be negative, for example, an agent could actually have lower costs from performing an extra task. However, these sub-additive cost functions are rare in practice.

The use of monetary payments has the effect of turning one deal into an infinite number of deals: the original deal and the infinite number of payments, from $-\infty$ to ∞ that can be made between the agents. Figure 6.13 shows a graphical representation of this process. Here we see deal δ^1 transformed into the set of deals represented by the line going thru δ^1. The figure also shows us how this transformation creates new deals that dominate another existing deal δ^0. Thus, if the system was in δ^0 and we were doing hill-climbing then we would be stuck there as δ^1 does not dominate δ^0. But, if we used the contract net then agent j could make a payment to i, along with the task transfer, which would give i a higher total utility. This new task allocation plus payment is a deal that lies within the thick area of the line that intersects δ^1. Thus, contract net allows us to reach a task allocation that is the utilitarian solution. Unfortunately, the addition of payments does not guarantee that we will always reach the utilitarian solution, this depends on the particular characteristics of the cost function.

One type of cost functions that have been found to be easy to search over are additive cost functions where the cost of doing a set of tasks is always equal to the sum of the costs of doing each task individually.

ADDITIVE COST FUNCTION

Definition 6.6. *A function $c(s)$ is an **additive cost function** if for all $s \subseteq T$ it is true that*

$$c(s) = \sum_{t \in s} c(t).$$

In these scenarios it has been shown that the contract net protocol, or any other protocol that uses payments and always moves to dominant deals, will eventually converge to the utilitarian social welfare solution.

Theorem 6.3. *In a task allocation problem where every agent has an additive cost function c and where we only allow exchange of one task at a time, any protocol that allows payments and always moves to dominant deals will eventually converge to the utilitarian solution (Endriss et al., 2006).*

The proof follows from the fact that when using an additive cost function we can say that if we move from δ to δ' by moving task t from i to j then the total change in utility $(\sum_i u_i(\delta') - \sum_i u_i(\delta))$ is just $u_j(t) - u_i(t)$ because j gains the new task and i loses it and everyone else remains the same. Thus, the utilitarian deal under an additive cost function is always the deal that gives each task to the agent that wants it the most. Also, if a deal δ' is not the utilitarian deal then a task t and agent j must exist such that moving that task from the agent that has it in δ'

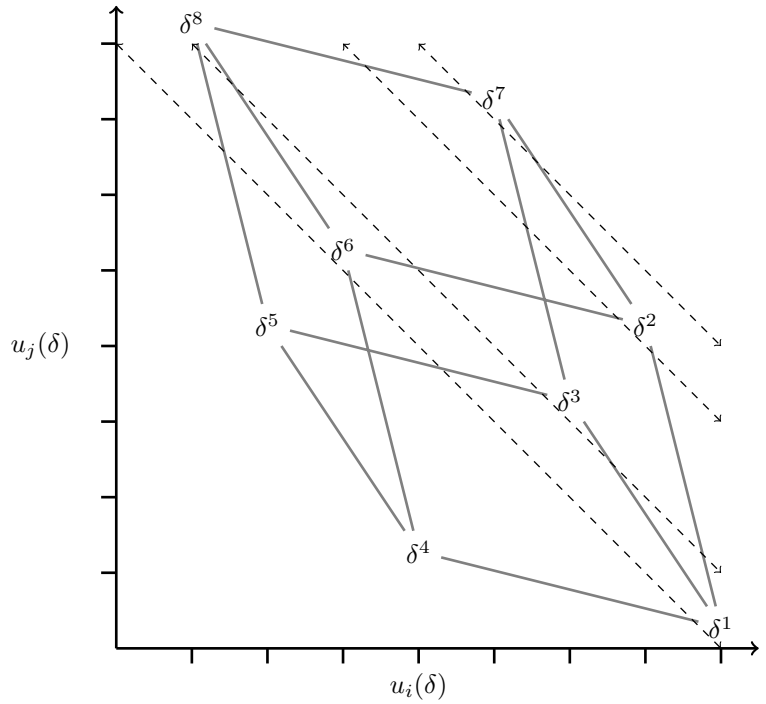


Figure 6.14: Monetary payments with an additive cost function. The deals expand to an infinite set of deals, represented by dashed lines intersecting the original deals. We show lines for deals δ^7, δ^2, δ^8, and δ^1 only. Note that the utilitarian deal δ^7 is the only one not dominated by any other deal.

to agent j results in a net positive utility because if no such tasks exists then this means that all tasks are with the agents that prefer them the most, so we are at the utilitarian deal, a contradiction.

For example, figure 6.14 reproduces the previous figure 6.12 but now we draw some lines to represent all the possible deals that are possible from the various deals using payments. The new deals are represented by the parallel lines which intersect each of the original deals. It is easy to confirm that all deals except for the utilitarian deal δ^7 are dominated by an adjacent deal. Thus, if we continue moving to successively dominant deals we are guaranteed to end up at δ^7. The proof of Theorem 6.3 follows the same line of thought; it shows that any deal (with payments) that is not dominated by some other deal must be the utilitarian social welfare solution.

If we allow arbitrary cost functions then there is very little we can say about which solution a hill-climbing protocol will reach, except for one scenario. In the case where every pair of deals is connected by an edge, we can get to any deal from every other deal in one step, imagine that the graph in figure 6.12 fully connected, then using payments guarantees we will converge to the utilitarian social welfare (Sandholm, 1997; Sandholm, 1998; Andersson and Sandholm, 1998). We can easily verify this by looking again at figure 6.14. Here we can see how each deal is converted into a line of slope -1 and the utilitarian deal will always be the one with the line that is furthest to the top right. Thus, any protocol that successively moves via dominant deals (hill-climbing towards the top right) will always arrive at the utilitarian solution because, as we assumed, every deal is always reachable from every other deal. We can directly connect every deal to every other deal by using a very flexible contracting language, such as **OCSM contracts** which allows all possible task transfer combinations.

OCSM contracts

Unfortunately, being able to move from every deal to every other deal in one step means that the agents will need to consider a vast number of possible deals at each time step. Thus, this approach merely replaces one intractable problem with another. In general, what we want is to limit the deal accessibility—the number of edges in the graph—such that there is a manageable number of deals accessible from every deal and there are few local optima where a hill-climbing negotiation could get stuck. We currently lack any general tools for achieving this result but,

δ	$s_i(\delta)$	$s_j(\delta)$	$u_i(\delta)$	$u_j(\delta)$
δ^1	$\emptyset$	$\{t_1\}$	1	3
δ^2	$\{t_1\}$	$\emptyset$	2	1

δ	$s_i(\delta)$	$s_j(\delta)$	$u_i(\delta)$	$u_j(\delta)$
δ^1	$\emptyset$	$\{t_1, t_2\}$	1	5
δ^2	$\{t_1\}$	$\{t_2\}$	2	3
δ^3	$\{t_2\}$	$\{t_1\}$	2	3
δ^4	$\{t_1, t_2\}$	$\emptyset$	8	1

Figure 6.15: Lying by task creation example. The top table shows the original utility values, where δ^1 is the Nash bargaining solution. The bottom table shows the values after agent i creates phony task t_2. Here, δ^4 is the Nash bargaining solution.

for any specific problem it is likely that a good multiagent designer can engineer a viable solution.

6.5.2 Lying About Tasks

In some cases it might be worthwhile for an agent to hide from other agents some of the tasks it has to perform, or to make up tasks and tell others that it has perform them, or to make up these tasks and, if someone else offers to perform the phony tasks then actually create the new tasks. All these methods of cheating result in the modification of the set of existing deals. For example, if an agent creates a new task then we now have to consider all the possible ways in which the new bigger set of tasks can be distributed among the agents. If an agent hides a task from the agents then the system has fewer possible allocations to consider. We can then ask the question, when is it in an agent's best interest to lie?

That is the question asked in *Rules of Encounter* (Rosenschein and Zlotkin, 1994). In it, the authors assume that the agents will use a bargaining protocol which will arrive at the Nash bargaining solution, presumably by either using alternating offers with the Zeuthen strategy or the ONE-STEP-NEGOTIATION protocol from figure 6.10. Once an agent knows that the final agreement deal will be the Nash bargaining solution then all it has to do is check each possible lie to see if the solution reached when it tells that lie gives him a higher utility than he would have received by telling the truth. Fortunately for the agent, and unfortunately for us, it has been shown that such lies do generally exist and will benefit the agent.

Figure 6.15 shows an example where an agent has an incentive to create a phony task. The table on the left shows the initial utility values for a simple game with only one task. In this game the Nash bargaining solution is δ^1 in which i does not perform any task and receives a utility of 1. Noticing that it would get a utility of 2 if it did perform t_1, i decides to create a phony task t_2 whose utility values are given on the table. In this new game the Nash bargaining solution is δ^4 which gives both tasks to agent i. Thus, by creating this task i was able to get a utility of 2 instead of the original 1 by getting t_1 allocated to itself. That is, since t_2 was assigned to i, the agent does not have to worry about perform this task nor does he have to worry about other agents trying to perform a task he made up. Thus, it can lie and get away with it.

6.5.3 Contracts

In our discussion of payments we have thus far assumed that if one agent says that he will pay another agent to perform some tasks then both of them will abide by that contract. The tasks will be performed and the money will be paid. Of course, things are not so simple for real applications. Specifically, in a dynamic environment an agent that is given a set of tasks to perform might find that right after he agrees to perform the tasks a new deal appears which will give him much higher utility. In this case the agent will be tempted to de-commit on his current contract and establish a new one. Similarly, it is possible that right after an agent agrees to pay

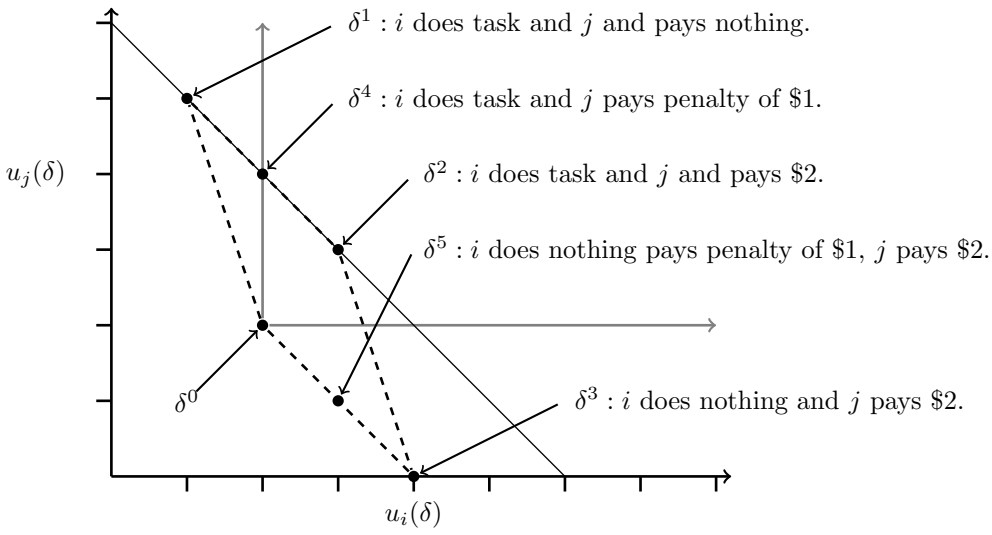


Figure 6.16: Leveled commitment contracts illustration. The agents start at δ^0 where j is performing the one available task and i is idle. δ^1 and δ^3 are the two de-commitment contracts. When we add a de-commitment penalty of 1 then these two become δ^4 and δ^5 respectively.

a certain amount to get some tasks done that a new agent appears which is willing to perform those tasks for less. Again, the agent will be tempted to de-commit on his payment.

For example, in figure 6.16 the agents start out with δ^0. Lets say that in this scenario there is only one task to perform and, under δ^0, agent j is going to perform it. Both agents get a utility of 2 under this initial deal. However, j notices that if he can give the task to i then they will be at δ^1 which gives j a utility of 4 and i a utility of 1. Thus, j reasons that he could pay i the amount of \$2 to perform the task. This new deal is δ^2 which gives both agents a utility of 3. δ^2 dominates δ^0 and will thus be accepted by both. The only problem in moving from δ^0 to δ^2 is the risks involved with this deal. If j fails to pay i then we are at δ^1 where i loses a utility of 1 as compared to δ^0. Similarly, if j pays but i fails to perform the task then j has to perform the task and we end up at δ^3 where j has lost 2 in utility.

A simple way to fix this problem is to enforce all contracts. If the agents agree to a deal then each one must deliver on that promise. However, there are cases when this might be impossible or undesirable. Specifically, in dynamic environments where new opportunities can appear at any time allowing agents to de-commit might increase social welfare, in other applications the cost of policing the systems might be higher than the gains from having enforcable rules. In these cases we might wish to use more sophisticated contracts.

LEVELED COMMITMENT CONTRACTS

One solution is to use **leveled commitment contracts** (Sandholm and Lesser, 2002; Sandholm and Zhou, 2002) which have the added stipulation that if an agent breaks the contract then it must pay a penalty. For example, we could add a penalty of \$1 to anyone who breaks the contract in figure 6.16. By adding this penalty we have that, instead of the de-commitments leading us to δ^1 and δ^3 they will lead us to δ^4 and δ^5. Note how these new deals are closer to δ^0 thus reducing the risks of loss for both agents. Specifically, if j were to de-commit and fail to pay the \$2, he must still pay the \$1 penalty to i so i's final utility is 2 even if j de-commits, the same as $u_i(\delta^0)$. This means that there is no risk for i to take this contract. Similarly, if i de-commits and fails to do the task then we end up at δ^4 because i pays a penalty of \$1 to j who must perform the task. In this case, j's utility is 1 which is greater than $u_j(\delta^3) = 0$ so j has reduced his risk. Such risk reduction means that in dynamic situations where the agents are constantly trying to tradeoff current income against future possibilities—deciding whether or not to wait for a better deal to come along—the agents will be able to accept more contracts and, hopefully, the system will perform better.

Note that the de-commitment decision itself is not obvious, even when the agent receives a better offer from somewhere else. For example, say that after agreeing

to δ^2 agent i receives another offer that will give him a utility of 10 but in order to perform that task he must de-commit on the task from j. Clearly i wants to de-commit but, a strategic agent i would also realize that there is a change that j will de-commit first. If j de-commits then i gets the penalty payment from j and still gets to perform to other task. The agents thus face a version of the game of chicken (figure 3.5) with uncertainty about the other agent's payoff. Namely, i does not know if j wants to de-commit or not. If i has some probabilistic data on j's offers then these could be used to generated an extended-form game. Agent i can then find the Nash equilibrium of this game and play that strategy.

Another solution is to extend the negotiation protocol to include tentative contracts. For example, we could extend the contract net protocol to include pre-bid and pre-assignment steps (Aknine et al., 2004). This allows agents to make tentative commitments which they can make binding at a later time when they are more certain that no new offers will arrive from other agents.

6.6 Complex Deals

Thus far we have considered deals to be atomic units that cannot be broken down into smaller pieces. In contrast, real world deals are known to be composed of many different items such as price, warranty, delivery time, color, etc. For example, two agents negotiating over the purchase of a car will need to agree on the price to be paid, the color of the car, the number of years in the warranty, the value of the trade-in car, the type of financing and interest rate provided, and many other possible parameters. We describe these complex preference languages in Section 6.7. These dimensions inevitably lead to an explosion in the space of possible deals.

MULTI-DIMENSIONAL DEAL

More formally we define a **multi-dimensional deal** as one that is composed of a set of variables $x_1, x_2, \ldots, x_n$ with domains $D_1, D_2, \ldots D_n$, respectively. For example, one variable could correspond to price and its domain could be all integer numbers in the range 0 to 100. We can also re-define the agent's utility function so that it explicitly represents each variable. For example, instead of an opaque $u_i(\delta)$, we could have a more expressive $u_i(\delta) = c_1 u_i^1(x_1) + c_2 u_i^2(x_2) + \cdots + c_n u_i^n(x_n)$ or some other combination of the individual variables. The negotiation problem remains that of finding a deal that corresponds to a chosen solution concept, such as the utilitarian deal or the Nash bargaining deal.

The astute reader will notice that this is nearly the exact same problem as the distributed constraint optimization problem from Chapter 2. In fact, the only difference between multi-dimensional negotiation and the constraint optimization problem is that they assume different data distribution requirements. In distributed constraint optimization it is assumed that each agent owns one or more variables whose value it can change and the agents generally only care about the value of their variable and a few other ones. In negotiation the variables are not owned by any agent and the agents generally have preferences over all the variables. Finally, in constraint optimization there is a clear solution concept—minimize constraint valuations—while in negotiation each agent is assumed to have a utility function and there is no one obviously better solution concept.

When the problem uses multi-dimensional deals we often find that the total number of deals is extremely large—the number of possible deals grows exponentially with the number of variables. Thus, it becomes even more likely that a hill-climbing negotiation algorithm will get stuck in a local optima. For example, figure 6.17 shows how we could converge to a non-Pareto deal via protocol similar to monotonic concession. The agents start out by proposing deals that are the best possible for them and then concede to the other agent. However, the agents only consider deals that differ from the last proposed deal by changing only a few variables—they are "near" the last proposed deal if we define the distance to be the number of variables that have changed. Since they only consider a subset of the deals they might end up ignoring better deals. Specifically, the agents in the figure ignore the deals on the right part of the chart. As such, they end up converging on a deal which is

The Hamming distance, a similar concept from information theory, between two binary strings is the number of positions which are occupied by different values. Thus the distance between 101 and 110 is 2.

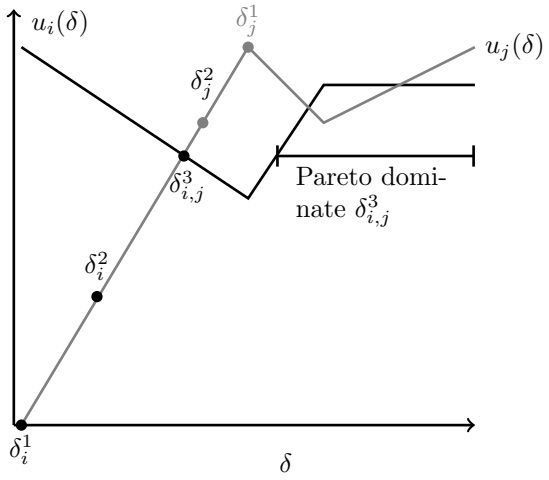


Figure 6.17: Example of convergence to non-Pareto deal. The agents converge to deal $\delta_{i,j}^3$ which is Pareto-dominated by all the deals indicated on the right.

Pareto-dominated by many other possible deals.

The reason for ignoring the deals is often the simple fact that the space of possible deals is so large that there is not enough time for the agent to sample it well enough to find other deals which are better than the ones it has proposed. In these cases agents choose a few attributes and change their values monotonically. For example, when negotiating a complex car deal you might wish to keep all other attributes at fixed values and negotiate only over price. In that case you might end up at a non-Pareto deal if your chosen values for the other attributes are not the right ones, such as the case where the dealer would be willing to accept a much lower price for a car with a one-year warranty rather than the three-year warranty you are asking for.

We can formalize this strategy by having the agents agree on one dimension at a time. For example, we could declare that the agents first agree on the price at which the car will be sold, then go on to haggle on the warranty, then on the color, and so on until all the dimensions are covered. The order in which these negotiations are carried out is called an **agenda**. As you might expect, ordering the negotiation in this way can sometimes lead to sub-optimal outcomes but does make the negotiation problem tractable for a lot more cases. Comparison of the results from agenda-based negotiations versus full negotiation, using specific negotiation strategies have shown that a social welfare deal is reached under certain combination of strategies (Fatima et al., 2004) .

AGENDA

6.6.1 Annealing Over Complex Deals

A common solution to the problem of searching for an optimal solution over a very large space of possible answers is to use an **annealing** method. Simulated annealing algorithms start out with a randomly chosen deal which becomes the best deal found thus far. New possible deals are generated by mutating this deal and are accepted as the new best deal if they are better than the current best deal or, with a certain probability, they are accepted even if they are worse than the current best. The probability of accepting a worse deal and the severity of the mutations both decrease with time. In this way the algorithms is guaranteed to converge to a locally optimal solution. In practice it has been found that this solution is often also the global optimum.

ANNEALING

Simulated annealing has been used to implement several simple negotiation protocols (Klein et al., 2003). The protocol uses a **mediator** agent instead of having agents negotiate with each other. At each step the mediator presents a deal to both agents. The agents can either accept the deal or reject it. If both of them accept the deal the mediator mutates the deal slightly and offers the new deal to both agents, who can once again either accept it or reject it. If one or more of the agents

MEDIATOR

ANNEALING-MEDIATOR

```
1  Generate random deal δ.
2  δ_accepted ← δ
3  Present δ to agents.
4  if both accept
5     then δ_accepted ← δ
6          δ ← mutate(δ)
7          goto 3
8  if one or more reject
9     then δ ← mutate(δ_accepted)
10 goto 3
```

Figure 6.18: Procedure used by annealing mediator.

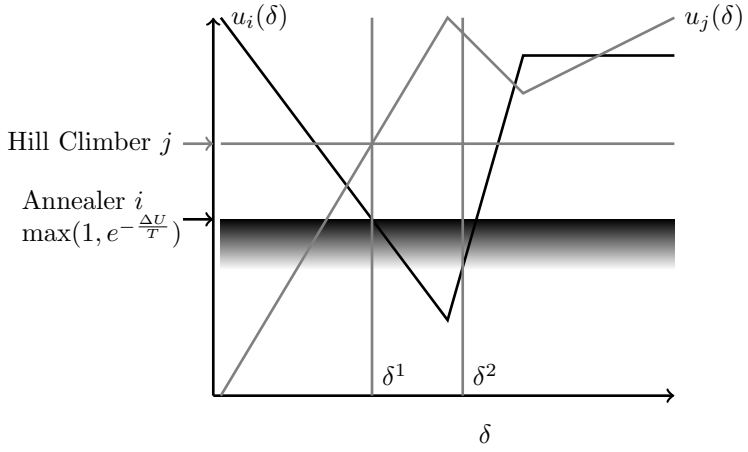


Figure 6.19: Two steps in the annealing algorithm with one hill climber j and one annealing agent i. After the mediator presents δ^1 both agents set up their new reservation prices shown by the line, for j, and the fuzzy bar for i.

rejected the proposed deal then a mutation of the most recently accepted deal is used instead. The protocol implemented by the mediator is shown in figure 6.18.

We can then implemented two types of agents.

Hill Climber Accepts a deal only if it gives him a utility higher than its reservation price $u_i(\delta^-)$ and higher than that of the last deal it accepted. That is, it monotonically increases it reservation price as it accepts deals with higher utility.

Annealer Use a simulated annealing algorithm. That is, it maintains a temperature T and accepts deals worse than the last accepted deal with probability $\max(1, e^{-\frac{\Delta U}{T}})$, where ΔU is the utility change between the deals.

Annealer agents, along with the annealing mediator, effectively implement a simulated annealing search over the set of possible deals. Meanwhile, hill climbing agents implement a hill climbing search over the set of possible deals.

For example, in figure 6.19 agent j is a hill climber and i is an annealer. After the mediator presents δ_1 both of them accept since its utility is higher than their reservation prices. The hill climber sets a new reservation price below which he will not accept any deal. The annealer sets up a probability of accepting deals that are below his new reservation price. When δ_2 appears, j will accept it because it is above its reservation price and i might also accept it, but with a small probability as $u_i(\delta_2)$ is slightly below its reservation price.

This type of annealing algorithm is a good example of how we can take a standard search technique like annealing and apply it to a distributed negotiation problem. Annealing works because there is some randomness in the sequence of deals that are explored. The annealing negotiation protocol places this randomness within a negotiation. Unfortunately, generating proposal deals purely at random means ignoring any knowledge that the agents have about the shape of their utility function.

Also, an annealing agent is acting irrationally when it accepts a deal that is worse for it than a previous deal. We would need some way to justify why this agent occasionally acts irrationally.

6.7 Argumentation-Based Negotiation

We can further relax the idea of an agenda, where agents negotiate sequentially over each dimension of the deal, and instead define a more complex negotiation language which agents can use for negotiation. That is, thus far we have only considered the use of a **1-sided proposal** where a deal is proposed and it is either accepted or rejected. In **argument-based protocols** the agents use a more sophisticated language for communications. There are currently no standard languages for argumentation although there is a lot of work being done on preference languages, as we will see in Chapter 7. Still, we can categorize the various types of utterances that an argumentation language might support (Jennings et al., 2001).

1-SIDED PROPOSAL
ARGUMENT-BASED PROTOCOLS

Specifically, an agent might be able to **critique** the proposal submitted by the other agent. Critiques provide the agents with information about others' utility functions, specifically with respect to the previous proposal. This information can be used to rule out some deals from consideration. For example, agent i and j might engage in the following negotiation:

CRITIQUE

i**:** I propose that you provide me with $x_1 = 10$ under conditions $x_2 < 20$ and delivery $x_3 < 20061025$.

j**:** I am happy with the price of $x_2 < 20$ but the delivery date x_3 is too late.

i**:** I propose that I will provide you with service $x_1 = 9$ if you provide me with $x_1 = 10$.

j**:** I don't want $x_1 = 9$.

An agent might come back with a **counter-proposal** which are new deals, just like in the alternating offers models, but which are generally assumed to be related to the last offer. For example,

COUNTER-PROPOSAL

i**:** I propose that you provide me with service $x_1 = 10$ under conditions $x_2 < 20$ and delivery $x_3 < 20061025$.

j**:** I propose that you provide me with service $x_1 = 10$ under conditions $x_2 < 30$ and delivery $x_3 \geq 20061025$.

An agent might be able to **justify** his negotiation stance with other statements. Statements it hopes will convince the other agent to accept his proposal. These statements are just ways of giving more knowledge to the other agent in the hopes that it will use that knowledge to eliminate certain deals from consideration. For example,

JUSTIFY

i**:** My warehouses is being renovated and it will be impossible to deliver anything before the end of the month, that is $x_3 > 20061031$.

An agent might try to **persuade** the other agent to change its negotiation stance. Persuasion is just another way of giving more knowledge tot the other agent. For example,

PERSUADE

i**:** Service $x_1 = 9$ is much better than you think, look at this report.

Finally, an agent might also employ **threats**, **rewards**, and **appeals** in order to convince the others to accept his proposal. These techniques help agents build better models of the others' utility functions, eliminate sets of deals from consideration, and change the agents utility functions in order to make them better reflect the reality of the situation.

THREATS
REWARDS
APPEALS

Argument-based negotiation closely matches human negotiation. Unfortunately, that also means that it is very hard to build agents that can understand these complex negotiation languages. A common approach is to build the agent using Prolog or some other logic-based language. The agent can then keep a database of the messages (facts) sent by the opponent and try to infer the set of possible contracts. Even when exploiting the inference powers of a logic-based programming language the problem of implementing correct argument-based negotiators is still very hard. No one has been able to implement a general argument-based negotiator. Even when limiting the problem to a specific domain, there are very few examples of successful programs that do argument-based negotiation. Still, there is a growing research community developing new preference description languages to be used by auctions, as we will see in Chapter 7. These languages could just as easily be used by an argumentation-based agent as by a centralized auctioneer.

There is a community of agent-based argumentation researchers working on developing a standardized language for arguments (nevar et al., 2006; Rahwan et al., 2004). They distinguish between the communication and domain languages used for representing the arguments and the negotiation protocol which constraints which type of messages can be sent at which time and what they mean. For example, a protocol could have rules on when a negotiation must end such as when one of the agents says "that is my final offer". Similarly, a protocol could have commitement rules which specify whether or not an agent can withdraw from a previous commitment it made.

6.8 Negotiation Networks

CONCURRENT NEGOTIATIONS

It is possible that an agent might be involved in **concurrent negotiations** with several other agents where the resulting deals might not be compatible with each other. For example, you might be negotiating with several car dealers at the same time, as well as negotiating a car loan with several lenders and car insurance with several insurance agencies. Once you make a purchasing deal with a specific dealer then you can stop negotiating with the other dealers. Furthermore, if the deal you made includes a loan agreement you can also stop negotiating with the lenders. Thus, as you negotiate with the various parties their last offers will tend to have an effect on how you negotiate with the others. For example, if a lender offers you a very low interest rate then you might be able to afford a more expensive car and you will not be swayed by the dealer's offer that includes a higher interest rate loan.

More formally, we can define a negotiation network problem as one were a set of agents negotiate over a set of deals such that all agents end up agreeing to a set of deals that are compatible with each other.

NEGOTIATION NETWORK

Definition 6.7. *A **negotiation network** problem involves a set of agents A and set of sets of deals. Each set of deals Δ_i involves only a subset of agents $\Delta_i^a \subseteq A$ and always includes the no-deal deal δ^-. A solution $\vec{\delta}$ to the problem is a set of deals, one from each Δ_i set, such that all the deals that each agent is involved in are compatible with each other. Compatibility is captures by the c function, where*

$$c_i(\delta, \delta') = \begin{cases} 1 & \text{if } \delta \text{ and } \delta' \text{ are compatible} \\ 0 & \text{otherwise.} \end{cases}$$

Figure 6.20 shows a graphical representation of a negotiation network problem with three agents: i, j and k, and where i and j can enter into any of the deals in Δ_1, similarly i and k can enter into a deal from Δ_2 and j and k can enter into a deal from Δ_3. We also need to define the boolean functions c_i, c_j, and c_k which tell us which pair of deals are compatible.

In the negotiation networks the standard negotiation problem is further exacerbated by the fact that each agent must maintain simultaneous negotiations with several other agents and these negotiations impact each other. The approaches at solving these type of problem has thus far consisted of using ad-hoc negotiation

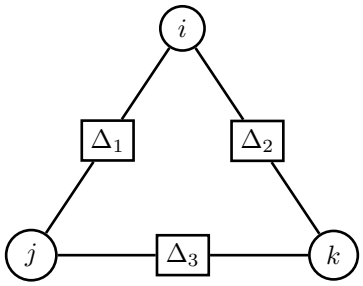


Figure 6.20: Graphical representation of a negotiation network with agents i, j, and k, where Δ_1 is a set of deals that i and j can agree upon.

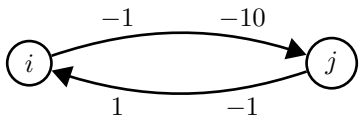


Figure 6.21: The coercion network.

strategies and running tests to determine how these fare against each other (Nguyen and Jennings, 2004; Zhang et al., 2005a; Zhang et al., 2005b).

6.8.1 Network Exchange Theory

Another way to approach the problem of negotiation networks is to look at what humans do and try to imitate it. This is known as the **descriptive approach**. Luckily for us, researchers in Sociology have developed and studied a model which they call **network exchange theory** (NET) that is very similar to the negotiation networks problem (Willer, 1999). In this model humans are represented by nodes in a graph. The annotated edges between nodes represent the possibility of a negotiation between those two humans. Sociologists have run tests with human subjects in these networks in order to record how they behave. They then found equations that can be used to predict the deals that people eventually agree on, on average, based on the initial structure of the problem.

DESCRIPTIVE APPROACH

NETWORK EXCHANGE THEORY

Figure 6.21 shows a simple NET network. The nodes represent agents i and j. The edges are directional and represent interaction possibilities from the source agent to the destination agent where only the source agent can decide whether to enact the interaction. Specifically, if i decides to enact its edge it would give i a utility of -1 and would give j a utility of -10. Imagine that i is threatening to harm j if i does not relinquish his wallet. Similarly j could decide to enact its edge which would give it a utility of -1 and give i a utility of 1. This particular network is known as a coercion scenario because i can threaten j to give him the 1 otherwise i will punish j by giving it -10. Even thought it is not rational for i to want go punish j for it is punishing itself at the same time, the threat works in the real world. Another type of edge used is the resource pool edge where an amount of utility is to be divided among the agents but the agents must decide how it is to be distributed. A sample is shown in figure 6.23. This network shows two agents, i and j, who are negotiating over how to divide 10 dollars.

NET can also represent various deal compatibility requirements. One such example is exclusive connections where one agent can exchange with any one of a number of other agents but exchange with one of them precludes exchange with any of the other ones. This is typically simply represented by just adding more nodes and edges to the graph, as shown in figure 6.22.

Based on test results, NET tells us that each person has a **resistance** to each particular payment p given by a resistance equation. i's resistance to payment p is

RESISTANCE

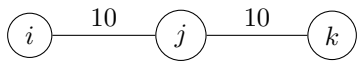


Figure 6.22: A network with three agents. Agent j can only split 10 dollars with either i or k, but not both.

Figure 6.23: A sample exchange network with 10 units to be distributed among two agents.

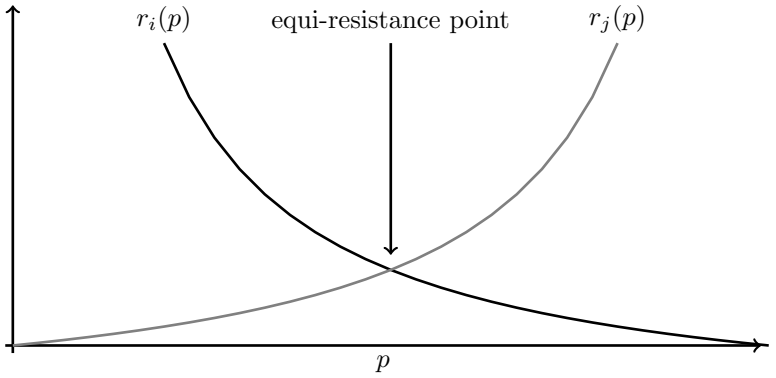


Figure 6.24: Resistance of two agents to each possible deal p.

given by

$$r_i = \frac{p_i^{\max} - p_i}{p_i - p_i^{\text{con,}}} \tag{6.8}$$

where $p_i^{\max}$ is the maximum i could get and p_i^{con} is the no-deal deal. j's resistance is similarly defined. If we further know that i and j are splitting 10 dollars then we know that $p_i + p_j = 10$.

The resistance equation is meant to capture the person's resistance to agreeing to a deal at any particular price. The higher the resistance the less willing the person is to agree to the deal. Note how this equation is not linear, as might be expected if people were rational, but has an exponential shape. This shape tells us a lot about our irrational behavior.

EQUI-RESISTANCE POINT

NET tells us that exchange happens at the **equi-resistance point** where both agents have equal resistance, that is where

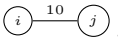
$$r_i = \frac{p_i^{max} - p_i}{p_i - p_i^{con}} = \frac{p_j^{max} - p_j}{p_j - p_j^{con}} = r_j. \tag{6.9}$$

Notice that the equi-resistance equation tells us the agreement that people will eventually reach via negotiation, but it does not tell us how the negotiation took place, that is, it does not tell us their negotiation tactics.

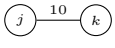
We can represent the equi-resistance point graphically by simply replacing p_j with $10 - p_i$ in j's resistance equation r_j and plotting the two curves r_i and r_j. The point at which the curves cross is the point of exchange, as shown in figure 6.24. Of course, the exchange does not always happen at the midpoint. For example, if i had an offer from some other agent for 6 dollars if it refused to negotiate with j, this would mean that $p_i^{con} = 6$ which would change the equi-resistance point.

ITERATED EQUI-RESISTANCE ALGORITHM

We can solve complex NETs using the **iterated equi-resistance algorithm**. The algorithm simply uses the equi-resistance equation repeatedly on each edge of the graph in order to calculate the payments that these agents can expect to receiver. This is repeated for all edges until payments stop changing. For example, for the graph in figure 6.22 we would follow these steps.

1. Apply Equi-resistance to 

. Gives us $p_j = 5$.

2. Apply Equi-resistance to 

. Let $p_j^{con} = 5$ and apply equi-resistance again.

3. Repeat until quiescence.

The iterated equi-resistance algorithm is not guaranteed to converge and it might converge to different solutions when the edges are sorted in differently. Even when it does converge the deal it reaches is not guaranteed to be the one that humans would reach. However, many experiments have been performed with humans in *small* networks (less than 12 nodes) which have shown that the iterated equi-resistance algorithm correctly predicts the resulting deal. This algorithm gives a new solution for the negotiation problem which, unlike the solutions in Section 6.1.1, is not based on some desirable properties of the solution but is based on evidence from human negotiation, that is, it gives us a descriptive solution to the negotiation problem. If you want to implement agents that reach the same solution as humans then you probably want to use the iterated equi-resistance solution.

Exercises

6.1 Given the following utility values for agents i and j over a set of possible deals δ:

δ	$u_i(\delta)$	$u_j(\delta)$
δ^1	1	0
δ^2	0	1
δ^3	1	2
δ^4	3	1
δ^5	2	2
δ^6	1	1
δ^7	8	1

1. Which deals are on the Pareto frontier?
2. Which one is the egalitarian social welfare deal?
3. Which one is the utilitarian deal?
4. Which one is the Nash bargaining deal?
5. Which one is the Kalai-Smorodinsky deal?

6.2 In a distributed workflow enactment application, a workflow is defined as a set of tasks all of which must be performed in order to complete the workflow and collect the payment. The following table lists the available workflow instances, their tasks and their payments:

Workflow	Tasks	Payment
w_1	t_1, t_1, t_1	6
w_2	t_1, t_1, t_2	5
w_3	t_1, t_1, t_3	3
w_4	t_1, t_2, t_2	1
w_5	t_2, t_2, t_3, t_3	8
w_6	t_2, t_2	1

There are three agents. An agent can perform at most two tasks at a time, as long as those tasks are **different**. Also, a task is performed for only a single workflow. That, if an agent performs t_1 for w_1 then this t_1 cannot be used for any other workflow. The goal is for the agents to find the set of workflows that maximizes the total payment.

1. Re-state this problem as a negotiation problem, show the set of possible deals. (Hint: note that the agents have identical capabilities, so you do not need to worry about which one does which task).
2. We further constrain this negotiation by only allowing agents to either drop one workflow, add one workflow, or exchange one workflow for another. But, they can only do so if this increases the total payment (thus,

you quickly conclude that they will never drop a workflow). Which deals become local optima under these constraints?

Chapter 7

Auctions

Auctions are a common and simple way of performing resource allocation in a multiagent system. In an auction, agents can express how much they want a particular item via their bid and a central auctioneer can make the allocation based on these bids. Of course, this generally requires the use of a centralized auctioneer but there are techniques for reducing this bottleneck. Still, even centralized auctions can be very complex and produce unexpected results if one does not understand all the details.

7.1 Valuations

Before we begin to talk about the various types of auctions we must first clarify how people value the items being sold. We have used the notation $u_i(s)$ to refer to the utility that agent i derives from state s. Similarly, if s is instead an item, or set of items, for sale we can say that $v_i(s)$ is the **valuation** that i assigns to s. We furthermore assume that this valuation is expressed in a common currency, thus $v_i(s)$ then becomes the maximum amount of money that agent i is willing to pay for s. When studying auctions we generally assume that all agents have a valuation function over all the items being sold.

VALUATION

In the simplest case this valuation function reflects the agent's utility of owning the given items. For example, if you plan to eat a meal then the amount you are willing to pay for each item in the menu depends solely on how hungry you are and how much you like each item. In these cases we say that the agent has a **private value** function.

PRIVATE VALUE

On the other hand, there are items which you cannot consume and gain no direct utility from but which might still have a resale value. The classic example are stocks. When you buy a share in some company you cannot do anything with that share, except sell it. As such, your valuation on that share depends completely on the value that others attribute, and will attribute, to that share. These are known as **common value** functions.

COMMON VALUE

Most cases, however, lie somewhere in the middle. When you buy a house you take into consideration the value that you will derive from living in that house as well as its appreciation prospects: the price you think others will pay when you finally sell it. This is an example of a **correlated value** function and is very common in the real world with durable high priced items.

CORRELATED VALUE

The type of valuation function that the agents use changes their optimal behavior in an auction. For example, if an agent has a common value function then it will likely pay more attention to what the other agents are bidding. Most multiagent implementations use agents with private value functions as most systems do not want to waste the time required to implement secondary markets for the items being sold. Still, in open multiagent systems it might be impossible to prevent secondary markets from appearing.

7.2 Simple Auctions

There are times when there are many agents and the only thing they need to negotiate over is price. In these occasions it makes sense to use an auction since they are fast and require little agent communication. However, auctions are not as simple as they might appear at first and there are many ways in which things can go wrong when using them.

English auction
first-price open-cry ascending

The actual mechanisms used for carrying out an auction are varied. The most common of all is the **English auction**. This is a **first-price open-cry ascending** auction. It is the standard one used in most auction houses. In it, the auctioneer raises the price as people yell higher bids. Once no one is willing to bid higher, the person with the highest bid gets the item and pays his bid price. These auctions sometimes have an initial or **reservation price** below which the seller is not willing to sell. The dominant strategy in an English auction, with private value, is to bid the current price plus some small amount until either the auction is won or one's reservation price is reached.

reservation price

winner's curse

If an English auction is common or correlated value then it suffers from the **winner's curse**. For example, when you buy a stock in an English auction it means that you paid more than anyone else was willing to pay. As such, your valuation of that share must now be less than what you paid for it. In the real world we gamble that at some point in the future the others will realize that this stock really is worth that much more.

First-price sealed-bid

A similar auction type is the **First-price sealed-bid** auction. In this auction each person places his bid in a sealed envelope. These are given to the auctioneer who then picks the highest bid. The winner must pay his bid amount. These auctions have no dominant strategy. The buyer's best strategy is to spy on the other bidders in order to determine what they are going to bid and then bid slightly higher than that, as long as that is less than one's reservation price. If spying is impossible then the agent has no clearly superior strategy. Because of the incentive for spying, these auctions lead to a lot of inefficiencies when paired with intelligent agents.

Ontario Flower Growers Co-op, an example of a Dutch auction at work. The two large circles in the back are used to show the descending price.

Dutch
open-cry descending price

The **Dutch** auction is an **open-cry descending price** auction. In it the seller continuously lowers the selling price until a buyer hits a buzzer, agreeing to buy at the current price. The auction's name comes from its use by the Dutch to sell flowers. The Dutch flower markets have been around for centuries and are still thriving. Every morning carts of flowers are paraded before eager flower shop owners who are equipped with a buzzer. Each cart stops before the buyers and a clock starts ticking backwards from a high number. When the first buyer hits his buzzer the flowers are sold to him at the current price. Analysis of the Dutch auction shows that it is equivalent to a first-price sealed-bid auction in terms of strategy. That is, it has no dominant strategy. However, it has the nice property of being real-time efficient. The auction closes quickly and the auctioneer can make it move even faster by lowering the price faster. This real-time efficiency makes it a very attractive auction for selling cut flowers as these lose their value quickly after being harvested.

Vickrey
second-price sealed-bid

The **Vickrey** auction is a more recent addition and has some very interesting properties. It is a **second-price sealed-bid** auction. All agents place their bids and the agent with the highest bid wins the auction but the price he pays is the price of the second highest bid. Analysis of this auction has shown that bidding one's true valuation, in a private value auction, is the dominant strategy. For example, let your valuation for the item being sold be v. If you bid less than v then you are risking the possibility that some other agent will bid $w < v$ and get the item even though you could have won it. Moreover, since w is less than v you could have bid v and paid only w. As such, you have nothing to gain by bidding less than v but risk the possibility of losing an auction that you could have won at an acceptable price. On the other hand, if you bid $v' > v$ then you are risking that some other agent will bid w, where $v' > w > v$, and thereby cause you to pay more than your reservation price v. At the same time, you do not gain anything by bidding higher

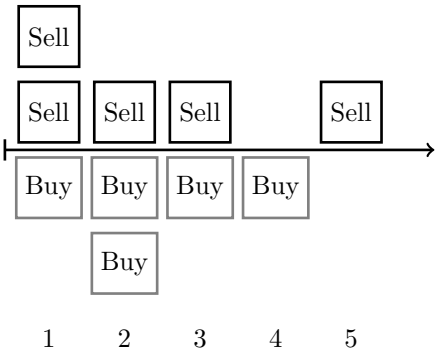


Figure 7.1: Graphical representation of a double auction. The x-axis represents prices. Each box represents one buy or sell order at the given price.

than v because the only auction that you might win by bidding v' instead of v are those where you have to pay more than v.

As such, the Vickrey auction eliminates the need for strategizing. Since there is an easy dominant strategy the agents do not have to think about what they should do. They just play their dominant strategy and bid their true valuation. Thus makes it a very attractive auction in terms of its efficiency but it is also for this reason that most people don't like Vickrey auctions. People are often hesitant about revealing their true valuations for an item because we know that the world is an iterated game and this information could be used against us in some future auction. As such, Vickrey auctions are seldom used in the real world.

William Vickrey. 1914–1996. Nobel Prize in Economics.

DOUBLE AUCTION

Finally, the **double auction** is a way of selling multiple units of the same item. It is the auction used in stock markets. Each buyer places either a buy or a sell order at a particular price for a number of instances of the item (number of shares in the stock-market). The buy and sell bids can be visualized in a simple graph such as the one shown in Figure 7.1. Here, the x-axis represents a price and each box represents an offer to buy or sell a share at the given price.

Once we have all the bids then it is time to clear the auction. There are many different ways to clear a double auction. For example, if figure 7.1 we could match the sell order for 1 with the buy for 5 then pocket the difference of $5 - 1 = 4$ for ourselves, or we could clear it at 3 and thus give both bidders a deal, or we could match the seller for 1 with the buy for 1, and so on. As can be seen, there are many different ways to match up these pairs and it is not clear which one is better.

One metric we might wish to maximize is the amount of surplus, known as the spread by traders. That is, the sum of the differences between the buy bids and the sell bids. In some auctions this surplus is kept by the auctioneer who then has an incentive to maximize it. Another option is to use it to enable more bids to clear. In the example above the total supply and demands are 12, therefore all bids should clear. One way to do this is to pair up all the small sell bids and give these sellers exactly what they asked for then give the surplus to the sell bid of 5 in order to clear it with the remaining buy bid.

Another metric we could use is a uniform price. That is, rather than giving each seller and buyer the exact price they asked for, we give them all one uniform clearing price. Clearly, the only buy bids that will clear are those that are above the clearing price and the only sell bids that clear are those below the clearing price.

7.2.1 Analysis

Now that we know the various auction types, there is an obvious question that we must ask ourselves. On which auction do sellers make more money? This question is answered by the following theorem.

Theorem 7.1 (Revenue Equivalence)**.** *All four single-item auctions produce the same expected revenue in private value auctions with bidders that are risk-neutral.*

Table 7.1: Example of inefficient allocation.

	tasks	Agent 1	Agent 2
Costs of Doing Tasks	t_1	2	1.5
	t_2	1	1.5
	t_1,t_2	2	2.5

We also know that if the bidders are risk-averse then the Dutch and first-price are better. A risk-averse bidder is willing to pay a bit more than their private valuation in order to get the item. In a Dutch or First-price auction a risk-averse agent can insure himself by bidding more than would be required.

In common or correlated value cases the English auction gives a higher revenue to the seller. The increasing price causes others to increase valuation. That is, once the agent sees others bidding very high for the item the agent realizes that the item is really worth more to the other agents so it also raises its valuation of the item.[1]

BIDDER COLLUSION

As it is often the case when money is involved, we have to be on the look out for ways in which the agents might cheat. The problem of **bidder collusion** affects all 4 auctions. In bidder collusion the bidders come to an a-priory agreement about what they will bid. They determine which one of them has the higher valuation and then all the other bidders refrain from bidding their true valuation so that the one agent can get it for a much lower price. The winner will then need to payback the others. The English and Vickrey auctions are especially vulnerable to bidder collusion as they **self-enforce collusion** agreements. For example, say there are 10 buyers and they realize that one of them has a valuation of 100 while the others have a valuation of 50 for the item. They agree to let him buy it for 1. In an English auction one of the 99 agents could defect and bid 2. However, this would only make the high-valuation agent bid 3, and so on until they get to 51. The high-valuation agent will get the item for 51 so the other agent gets nothing by defecting. The same logic applies in a Vickrey auction.

SELF-ENFORCE COLLUSION

LYING AUCTIONEER

Another problem might be that a **lying auctioneer** can make money from a Vickrey auction. All he has to do is to report a higher second-price than the one that was announced. The winner then pays this higher second price to the auctioneer who gives the buyer the real second price and pockets the difference. This requires that the bids are not revealed and that the buyer does not pay the seller directly. If the buyer paid the seller directly then a lying auctioneer and the seller could collude to charge the buyer a higher price. A lying auctioneer can also place a shill in an English auction. That is, assuming that the auctioneer gets paid a percentage of the sales price. If the auctioneer gets paid a fixed amount then there is no incentive for him to increase the sales price.

shill: a decoy who acts as an enthusiastic customer in order to stimulate the participation of others.

When auctioning items in a series when their valuations are interrelated, such as chairs in a dining set or bandwidth rights in adjacent counties, it is possible to arrive at inefficient allocations. For example, the problem in Table 7.1 leads to an inefficient allocation if we first auction t_1 and then t_2. Specifically, if we auction t_1 first then Agent 2 will get it as it has the lower cost. When we then auction t_2 both agents have the same cost (1) but, no matter who gets it the total cost is 2.5. If, on the other hand agent 1 had won both tasks then the total cost would be 2. matter who gets it the total cost is 2.5. This is the problem of **inefficient allocation**.

INEFFICIENT ALLOCATION

We could solve this problem if we made the agents use full lookahead effectively building an extended-form game from their possible interactions. With full lookahead the agents can build a tree where each level corresponds to a task being auctioned. In this way agent 1 can see that it will only cost him 2 to do t_1 and t_2 so it can reduce its cost for t_1 from 2 to 1. Of course, this puts agent 1 at risk of not getting t_2 since agent 1 generally will not know agent 2's cost for t2 so it does not

[1] An interesting example of this was a British auction for 3G bandwidth licenses. The standard English auction was modified so that everyone must agree to buy at the current price or leave the room. This led to the licenses selling for 1000 times the expected amount (Harford, 2005).

know if it will win that auction. Another much better way of solving the problem of inefficient allocations is to use a combinatorial auction, which we will learn about in Section 7.3.

7.2.2 Auction Design

When designing an auction for a multiagent system you must make many decisions. You must first determine what kind of control you have over the system. It is possible that you control only the bidding agent and the auction is already implemented, as when building agents to bid on Ebay. It is possible that you control only the auction mechanism, as when building an auction website. Finally, it is possible that you might control both agents and mechanism, as when building a closed multiagent system.

The case where you control the mechanism is especially interesting. You must then decide what bidding rules you will use: when bids are to be placed, when they can no longer be placed, what form can these bids takes, and what rules they must follow. For example, you might set a rule that a new bid has to always be for a higher value. You also set up clearing rules which determine when the items are sold. We explained some of the problems with various clearing rules in the double auction. The four standard auction types already have clearing rules but you might want to modify these. Finally, you must decide on information rules: how much information the agents are to know about what the other agents bid, whether to reveal information during the bidding process itself or after clearing (Wurman et al., 2002).

Currently all online auctions are implemented as centralized web applications but it is not hard to imagine a future where the auctions are freed from the constraints of a central hub and become a protocol enacted by buying and selling agents.

7.3 Combinatorial Auctions

Arguably, the **combinatorial auction** has been the most widely used auction in multiagent systems. In it agents can place bids for sets of items instead of just placing one bid for each item for sale. In many systems we have the problem that there is a set of tasks or jobs that needs to be distributed among the agents but the agents have complex preferences over the set of tasks. For example, in a workflow application we have a set of workflows, each composed of a set of web services, which must be performed by certain deadlines. Each agent can perform a subset of the services but each agent has different costs which might depend on the agent's type, its current load, the services it has done before, etc. Our problem as system designers is to allocate the workflows to agents so that we maximize the total number of workflows completed. Another example of combinatorial auctions is the selling of broadcasting rights by the federal government where cellular companies prefer to own the same frequencies in nearby locations, or at least to own some bandwidth in all the neighborhoods of a city. A final example is the buying of parts to put together a PC which requires a motherboard, CPU, ram, etc. Each part can be bought independently but only some bundles work together. These problems, and all problems of this type, can be solved by a combinatorial auction.

COMBINATORIAL AUCTION

Formally, we define a combinatorial auction over a set of items M as being composed of a set of bids, where each agent can supply many different bids for different subset of items. Each bid $b \in B$ is composed of b^{items}, which is the set of items the bid is over, b^{value} the value or price of the bid, and b^{agent} which is the agent that placed the bid.

For example, say you had a set of 5 figurines, one each of a different Teen Titan and you received 6 different combinatorial bids, as shown in Figure 7.2. The question you then face is how to determine which are the winning bids so as to maximize the amount of revenue you receive. Note that you can sell each item only once since you

Figure 7.2: Teen Titans figurines: (from top left) Beast Boy, Cyborg, Robin, Raven, and Starfire. The set of combinatorial bids received for them in on the table at the right.

Price	Bid items
\$1	Beast Boy
\$3	Robin
\$5	Raven, Starfire
\$6	Cyborg, Robin
\$7	Cyborg, Beast Boy
\$8	Raven, Beast Boy

only have one of each. This is the problem of winner determination. In the figure, the correct solution would be to accept the \$3, the \$5 and the \$7 bids.

7.3.1 Centralized Winner Determination

WINNER DETERMINATION

The **winner determination** problem is finding the set of bids that maximizes the seller's revenue. Or, more formally, find

$$X^* = \arg\max_{X \subseteq C} \sum_{b \in X} b^{\text{value}} \tag{7.1}$$

A variation on this problem is when agents can submit XOR bids. That is, when an agent can say that it wants only one of his bids to win. Computationally, both problems are similar.

where C is a set of all bid sets in which none of the bids share an item, that is

$$C = \{Y \subseteq B \,|\, \forall_{a,b' \in Y} a^{\text{items}} \cap b^{\text{items}} = \emptyset\}. \tag{7.2}$$

Unfortunately, this is not a simple problem as there are, in the worst case, many possible bidsets. Specifically, if bids exists for all subsets of items then X is a way of partitioning the set of items S into non-overlapping subset. That is, take the set of items S and figure out how many ways it can be split into smaller sets. We can calculate this number by remembering that the **Stirling number of the second kind** gives us the number of ways to partition a set of n elements into k non-empty sets. Namely,

STIRLING NUMBER OF THE SECOND KIND

$$S(n,k) = \frac{1}{k!} \sum_{i=0}^{k-1} (-1)^i \binom{k}{i} (k-i)^n. \tag{7.3}$$

Using this formula we can easily determine that the total number of allocations of m items is given by

$$\sum_{i=1}^{m} S(m,i), \tag{7.4}$$

which is bounded by

$$O(m^m) \text{ and } \omega(m^{m/2}).$$

This means that a brute force search of all possible allocations of items to agents is computationally intractable. In fact, no approach will work in polynomial time.

Theorem 7.2. *Winner Determination in Combinatorial Auction is NP-hard. That is, finding the X^* that satisfies* (7.1) *is NP-hard (Rothkopf et al., 1998).*

Even simplifying the problem does not make it easier to solve. For example, say that instead of trying to find the best allocation we simply want to check if there exists an allocation with total revenue of at least w. We call this the **decision version** of the winner determination problem. Lets also further restrict the types of bids the agents can submit. Even under these circumstances the problem remains hard.

DECISION VERSION

Theorem 7.3. *The decision version of the winner determination problem in combinatorial auctions is NP-complete, even if we restrict it to instances where every bid has a value equal to 1, every bidder submits only one bid, and every item is contained in exactly two bids (Cramton et al., 2006, Chapter 12).*

```
BUILD-BRANCH-ON-ITEMS-SEARCH-TREE
1  Create a singleton bid for any item that does not have one
2  Number items from 1 to m
3  Create empty root node
4  for n ∈ M in order
5        do Add as its children all bids that
6              include the smallest item that is not an ancestor of n but
7              that do not include any item that is an ancestor of n.
```

Figure 7.3: Algorithm for building a branch on items search tree. This algorithms does not find a solution, it only builds a tree for the purpose of illustration.

Thus, the problem is very hard, even when we try to limit its complexity. But, there is some hope. The winner determination problem in combinatorial auctions can be reduced to a **linear programming** problem and, therefore, solved in polynomial time with well-known algorithms but only if prices can be attached to single items in the auction (Nisan, 2000). That is, there needs to be a singleton bid for every item. In many cases we can satisfy this requirement by simply adding the missing singleton bids, each with a value of 0. Specifically, the linear program which models the winner determination problem is to find the x that satisfies the following:

LINEAR PROGRAMMING

Simplex is the most widely used linear programming algorithm. It has worst-case exponential time, but in practice it is much faster. Other algorithms exist that are guaranteed polynomial.

Maximize:

$$\sum_{b \in B} x[b] b^{\text{value}}$$

Subject to:

$$\sum_{b \mid j \in b^{\text{items}}} x[b] \leq 1, \forall j \in M$$

$$x[b] \in \{0, 1\}, \forall b \in B,$$

where $x[b]$ is a bit which denotes whether bid b is a winning bid. That is, maximize the sum of the bid values given that each item can be in, at most, one winning bid. It has also been shown that the linear programming problem will solve a combinatorial auction when the bids satisfy any one of the following criteria (Nisan, 2000):

1. All bids are for consecutive sub-ranges of the items.
2. The bids are hierarchical.
3. The bids are only OR-of-XORs of singleton bids.
4. The bids are all singleton bids.
5. The bids are downward sloping symmetric.

A different approach to solving the winner determination problem is to conduct one of the standard AI-searches over all possible allocations, given the bids submitted. The advantage over using a linear programming solver is that we can tweak our AI search algorithms and optimize them to solve our specific problem. That is, we can put some of our domain knowledge into the algorithm to make it run faster, as we shall see.

Before we can do search we need to define our search tree. One way we can build a search tree is by having each node be a bid and each path from the root to a leaf correspond to a set of bids where no two bids share an item. The algorithm for building this tree is shown in figure 7.3. We refer to this tree as a **branch on items** search tree. Figure 7.4 shows an example tree built in this fashion. In this case we have five items for sale, numbered 1–5. The column on the left lists all the bids received. We omit the bid amount and only show the set of items for each bid. The search algorithm uses these bids to build the tree shown on the right of the figure. We start at the first level from the top. All the children of the root are bids that have item 1 in them. Then, we proceed to add children to each node. The

BRANCH ON ITEMS

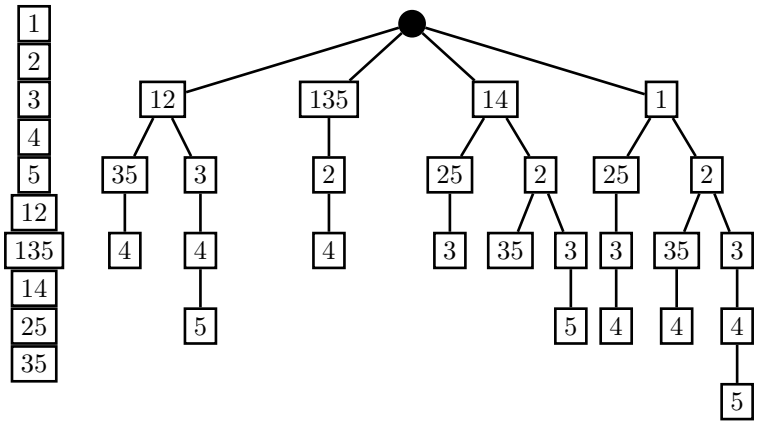


Figure 7.4: Branch on items search tree for winner determination in combinatorial auctions. Note that this tree has 9 leafs (9 possible ways of selling all items given the bids) while the total number of dividing 5 items into subsets is 52.

children of every node will be all the bids that contain the smallest number that is **not** on the path from the root to the node. Since the algorithm has the provision of adding a singleton bid with value 0 for every item, we are guaranteed to find a suitable bid as a children of every node. The only time we cannot find such a bid is when the path from the root to the node contains all items. In this case the node is a leaf and the set of bids from root to leaf constitutes a possible bid set.

The speedup of this search over the brute force method of considering all possible ways of breaking up 5 items into subsets can be confirmed by the fact that this tree has 9 leafs, therefore only 9 working bid sets exists. Meanwhile, the application of the Stirling formula gives us

$$\sum_{i=1\ldots5} S(5,i) = 52,$$

which means that there are 52 ways to break up 5 items into subsets. Clearly, fewer bids means faster run time which is the central idea of the search algorithm. In general, we know that the number of leafs in the tree is bounded.

Theorem 7.4. *The number of leaves in the tree produced by* BUILD-BRANCH-ON-ITEMS-SEARCH-TREE *is no greater than* $(|B|/|M|)^{|M|}$. *The number of nodes is no greater than* $|M|$ *times the number of leaves plus 1 (Sandholm, 2002).*

BRANCH ON BIDS

We can also build a binary tree where each node is a bid and reach edge represents whether or not that particular bid is in the solution. We refer to this tree as a **branch on bids** search tree, an example is shown in Figure 7.5. Each edge on the tree indicates whether the parent node (bid) is to be considered as part of the final bidset. For example, the rightmost branch of the tree consists of all "In" branches so the rightmost leaf node corresponds to the bidset (35)(14)(2) which forms a complete allocation. In practice, the branch on bids search tree is often faster than the previous tree because it gives us more control over the order of bids so we can better optimize the bid order. Also, the branch on bids search does not require us to add dummy singleton bids.

We now have to decide how to search our chosen tree. Since both trees have a number of nodes that is exponential on the number of bids a breadth first search would require too much memory. However, a depth first search should be possible, but time consuming. A branch and bound algorithm like the one we used for DCOP in Chapter 2.2 further helps reduce the search space and speed up computation. In order to implement it we first need a function h which gives us an upper bound on the value of allocating all the items that have yet to be allocated. One such function is h

$$h(g) = \sum_{j \in M - \bigcup_{b \in g} b^{\text{items}}} \max_{b | j \in b^{\text{items}}} \frac{b^{\text{value}}}{|b^{\text{items}}|}, \tag{7.5}$$

where g is the set of bids that have been cleared. The function h simply adds up the maximum possible revenue that each item not in g could contribute by using the

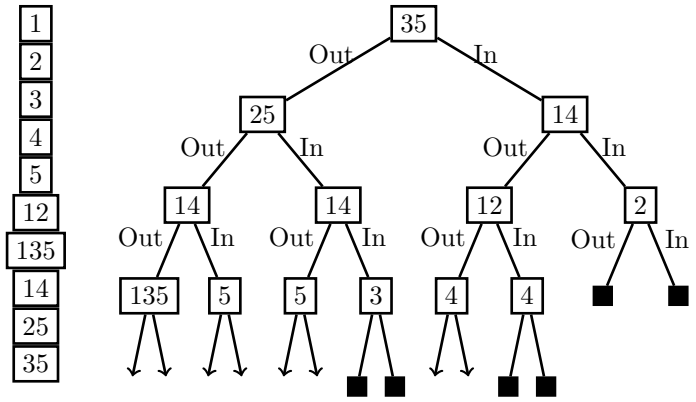


Figure 7.5: Branch on bids partial tree. The black boxes indicate search paths that have terminated because they denote a complete set of bids, that is, no more bids can be added because they contain items already sold.

```
BRANCH-ON-BIDS-CA()
1  r* ← 0        ▷ Max revenue found. Global variable.
2  g* ← ∅        ▷ Best solution found. Global variable.
3  BRANCH-ON-BIDS-CA-HELPER(∅, B)
4  return g*

BRANCH-ON-BIDS-CA-HELPER(g, available-bids)
 1  if available-bids = ∅
 2     then return
 3  if ⋃_{b∈g} b^items = M                          ▷ g covers all items
 4     then if Σ_{b∈g} b^value > r*                 ▷ g has higher revenue than r*
 5             then g* ← g
 6                  r* ← Σ_{b∈g} b^value
 7          return
 8  next ← FIRST(available-bids)
 9  if next^items ∩ ⋃_{b1∈g} b1^items = ∅}          ▷ next's items do not overlap g
10     then g' ← g + next
11          if Σ_{b1∈g'} b1^value + h(g') > r*
12             then BRANCH-ON-BIDS-CA-HELPER(g', REST(available-bids))
13  BRANCH-ON-BIDS-CA-HELPER(g, REST(available-bids))
```

Figure 7.6: A centralized branch and bound algorithm that searchers a branch on bids tree and finds the revenue maximizing solution given a set B of combinatorial bids over items M.

bid that pays the most for each item, divided by the number of items on the bid. This function provides an upper bound since no feasible bidset with higher revenue can exist.

Given the upper bound $h(g)$ we can then implement the branch and bound algorithm shown in figure 7.6. This algorithm searches the branch on bids tree. It maintains a partial solution g to which it adds one bid on each recursive call. Whenever it realizes that partial solution will never be able to achieve revenue that is higher than the best revenue it has already found then it gives up on that subtree, see line 8 of BRANCH-ON-BIDS-CA-HELPER. This algorithm is complete and thus guaranteed to find the revenue maximizing bidset.

We can also use the same heuristic function to do an A^* search. Unfortunately, since A^* acts much like a breadth first search it generally consumes too much memory. A viable solution is to use iterative deepening A^*. IDA^* guesses how much revenue we can expect and runs a depth-first search that prunes nodes that have used more than that. If a solution is not found then the guess is reduced and we try again. IDA^*, with some optimizations, was implemented by the **Bidtree** algorithm (Sandholm, 1999) on the branch on items search tree. In practice, this approach was found to often be slower than a branch and bound search.

BIDTREE

The BRANCH-ON-BIDS-CA algorithm is the basic framework for the Combinatorial Auction Branch on Bids (**CABOB**) algorithm (Sandholm et al., 2005). CABOB

CABOB

```
BRANCH-ON-ITEMS-CA()
1  r* ← 0        ▷ Max revenue found. Global variable.
2  g* ← ∅        ▷ Best solution found. Global variable.
3  BRANCH-ON-ITEMS-CA-HELPER(1, ∅)
4  return g*

BRANCH-ON-ITEMS-CA-HELPER(i, g)
1  if i = m                                          ▷ g covers all items
2     then if Σ_{b∈g} b^value > r*                   ▷ g has higher revenue than r*
3             then g* ← g
4                  r* ← Σ_{b∈g} b^value
5          return
6  for b ∈ {b ∈ B | i ∈ b^items ∧ b^items ∩ ⋃_{b1∈g} b1^items = ∅}  ▷ b's items do not overlap g
7        do g' ← g + b
8           if Σ_{b1∈g'} b1^value + h(g') > r*
9              then BRANCH-ON-ITEMS-CA-HELPER(i + 1, g')
```

Figure 7.7: A centralized branch and bound algorithm that searchers a branch on items tree and finds the revenue maximizing solution given a set B of combinatorial bids over items M.

improves the performance of the basic algorithm in several ways, one of which is by improving the search for new bids to add to the partial solution. Specifically, we note that a naive implementation of line 6 of BRANCH-ON-BIDS-CA-HELPER would mean that we would build this set on each recursive call to the function. That would be very time consuming as there are an exponential number of bids in B. CABOB handles this problem by maintaining graph data structure which has all the bids that can still be used given g. The nodes in the graph are the bids that are still available and the edges connect every pair of bids that share an item. In this way when a new bid is added to g it is removed from the graph as well as all the other bids that are connected to it.

ca

We can also perform the branch and bound search on the branch on items search tree, as shown in Figure 7.7. This algorithm is the basis for the **CASS** (Combinatorial Auction Structured Search) algorithm which also implements further refinements on the basic algorithm (Fujishima et al., 1999).

CASS

Most algorithms for centralized winner determination in combinatorial auction expand on the basic branch and bound search by using specialized data structures to speed up access to the information need—the viable bids given the current partial solution—and implement heuristics which have been shown to reduce the size of the search space, especially for certain popular bid distributions. Some heuristics that have been found useful include the following:

- Keep only the highest bid for any set. That is, if there is a bid of \$10 for items 1,2 and another bid of \$20 for 1,2 then we get rid of the \$10 bid.

- Remove provably noncompetitive bids, that is, those that are dominated by another bid or sets of bids. For example, if there is a bid for \$10 for item 1 and another bid for \$5 for items 1,2 then the \$10 bid dominates the \$5 bid—any situation in which we choose the \$5 bid would be made better if we changed that bid for the \$10 bid.

- Decompose bids into connected sets, each solved independently. If we can separate the set of bids into two or more sets of bids where all bids for any item are to be found in only one of the sets then this set of bids becomes a smaller, and independent, winner determination problem.

- Mark noncompetitive tuple of bids. For example, if there are bids \$1:(1,2), \$1:(3,4), \$10:(1,3), \$10:(2,4) then the pair of \$10 bids dominates the pair of \$1 bids, so we can get rid of them.

- In the branch-on-items tree place the items with the most bids first on the tree. This way the most constrained items are tried first thereby creating fewer leafs.
- If the remaining search subtree is the same for different nodes in the search tree, as can happen when different items are cleared but by different bids, then these subtrees can be cached. The subtree is solved once and the answer, that is, the best set of bids found in it, is saved for possible future use.

In general the best speed attainable by the best algorithms varies greatly depending on the type of bids submitted. For example, if the number of items in each bid is chosen from a flat probability distribution then we can solve problems with thousands of items and tens of thousands of bids in seconds. On the other hand, if each bid contains exactly five randomly chosen items and a price of 1 then we can only solve problems with tens of items and hundreds of bids in a minute. The Combinatorial Auction Test Suite (**CATS**) can generate realistic types of bid distributions so new algorithms can be compared to existing ones using realistic bid sets (Leyton-Brown et al., 2000). It generates these bids by using several sample scenarios. In one scenario there is a graph where the nodes represent cities and the edges are railroad tracks that connect these cities. The items for sale are the tracks between cities, that is, the edges. The agents are given pairs of host/destination cities and are told that they need to establish a train route between their city pairs. Thus, each agent determines all the possible sets of edges which connect his city pairs and submits XOR combinatorial bids on them. The value of each path depends on the total distance; shorter routes are preferred.

CATS

7.3.2 Distributed Winner Determination

One problem with the centralized winner determination algorithms, aside from the bottleneck, is that they require the use of a trusted auctioneer who will perform the needed computations. Another option is to build a peer-to-peer combinatorial auction protocol which lets the sellers themselves determine the winning set of bids and discourages them from cheating.

Incremental Auctions: Distribute over Bidders

One way to distribute the winner determination calculation is by offloading it on the bidding agents. We can do this by using an increasing price auction and making the bidders figure out which bids would be better than the current standing bid. This is the approach taken by the Progressive Adaptive User Selection Environment or **PAUSE** combinatorial auction (Kelly and Stenberg, 2000) (Cramton et al., 2006, Chapter 6).

PAUSE

A PAUSE auction for m items has m stages. Stage 1 consists of having simultaneous ascending price open-cry auctions for each individual item. During this stage the bidders can only place individual bids on items. At the end of this state we will know what is the highest bid for each individual item and who placed that bid. In each successive stage $k = 2, 3, \ldots, m$ we hold an ascending price auction where the bidders must submit sets of bids that cover all items but each one of the bids must be for k items or less. The bidders are allowed to use bids that other agents have placed in previous rounds. Also, any new bid set has to have a sum of bid prices which is bigger than the currently winning bid set.

At the end of each stage k all agents know the best bid for every subset of size k or less. Also, at any point in time after stage 1 has ended there is a standing bid set whose value increases monotonically as new bid sets are submitted. Since in the final round all agents consider all possible bid sets, we know that the final winning bid set will be one such that no agent can propose a better bidset. Note, however, that this bid set will generally not be X^* since we are using ascending price auction so the winning bid will be only slightly bigger than the second highest bid for the particular set of items.

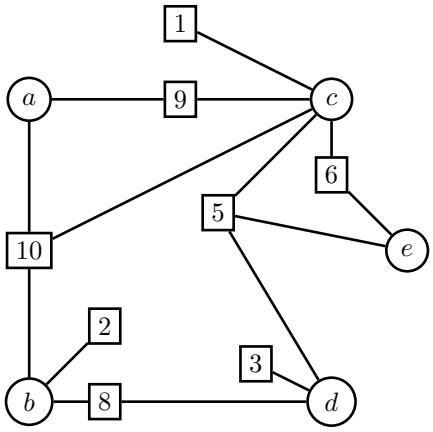


Figure 7.8: Graphical representation of a distributed winner determination problem. The circles represent agents/items while the squares represent combinatorial bids.

ENVY-FREE

In the general case, the PAUSE auction has been shown to be **envy-free** in that at the conclusion of the auction no bidder would prefer to exchange his allocation with that of any other bidder. However, it is not guaranteed to find the utilitarian solution (7.1).

The PAUSE auction makes the job of the auctioneer very easy. All it has to do is make sure each new bidset adds up to a number that is bigger than the current best as well as make sure that any bids an agent places that are not his do indeed correspond to other agents' bids. The computational problem shits from one of winner determination by the auctioneer to one of bid generation by the prospective buyer agents. Each agent must search over the space of all bid sets which contain at least one of its bids. The search is made easier by the fact that the agent need only consider the current best bids and that in stage k all bid sets must contain at least one bid of size k since they would have otherwise been bid in a previous stage. The **pausebid** algorithm uses the same branch and bound techniques used in centralized winner determination but expands them to include the added constraints an agent faces. As such, the pausebid algorithm can be used to find the myopically optimal bid for agents in the PAUSE auction (Mendoza and Vidal, 2007). The research also reveals that agents using pausebid reach the same allocation as the utilitarian solution, assuming all the agents bid their true valuations, at least 95% of the time.

PAUSEBID

VSA

Another way of distributing the winner determination problem among the bidders is provided by the Virtual Simultaneous Auction (**VSA**) (Fujishima et al., 1999) which is based on market-oriented programming ideas (Wellman, 1996). The VSA is an iterative algorithm where successive auctions for the items are held and the bidders change their bids based on the last auction's outcome. The auction is guaranteed to find the optimal solution when the bidding terminates. Unfortunately, there is no guarantee that bidding will terminate and experimental results show that in most cases bidding appears to go on forever.

Distribute over Sellers

Another way to distribute the problem of winner determination is to distribute the actual search for the winning bid set among the sellers. Imagine a distributed system where each person that wants to sell an item runs an agent. The agent advertises that the item is for sale. Every person who wants to place a, possibly combinatorial, bid does so by telling all the agents present in the bid about it. After some time the agents have gathered some bids and begin the clearing process. The set of agents and bids can be visualized in a graph such as Figure 7.8.

Here we see that agent b has received three bids. One of them is a singleton bid worth \$2, the other two are combinatorial bids one of them for \$8 and including agent d and the other for \$10 and including agents a and c. The problem we face is how can the agents a–e determine the set of winning bids.

The simplest solution is to do a complete search via sequentialized ordering. In

this method we use the same search tree as before but instead of implementing it centrally we pass messages among the agents. Agent 1 handles the top level of the tree. It tentatively sets one of its bids as cleared and sends a message to agent 2. In this way, each agent (roughly) handles one level of the tree. Note the agents are sequentialized so there is no parallelism, even though it is distributed.

Another option is to partition the problem into smaller pieces then have sets of agents to a complete search via sequentialized ordering on each of the parts. That is, we first partition the set of agents then do a complete search on each subset. This method means that each subset works in parallel. However, if there is any bid that contains agents from more than one subset then the solution found is no longer guaranteed to be optimal.

Another option is to maximize the available parallelism by having the agents do **individual hill-climbing**. Each agent starts by ordering all its bids based on their price divided by the number of agents in the bid under the assumption that the agent gets $1/n$ of a bid that includes n agents. The agent picks the first bid in the list (the highest valued) and tells all the other agents in the bid that it wants to clear it. If the agent receives similar messages from all the agents in the bid this means that they all wanted to clear it so the bid is considered cleared and all the agents in it are removed from the protocol. The remaining agents repeat the same process with their next highest bid and so on until they run out of bids. This algorithm is guaranteed to terminate but will only find, at best, a local optima.

INDIVIDUAL HILL-CLIMBING

Winner Determination as Constraint Optimization

It is interesting to note that we can reduce the winner determination problem to a constraint optimization problem as described in Chapter 2.2 in two different ways. One way is to let the variables $x_1, \ldots, x_m$ be the items to be sold and their domains be the set of bids which include the particular item. That is, each item k is represented by a variable x_k with domain D_k which contains all the bids that involve k. The constraints are given by the bids. Every bid is replaced with a constraint which returns the value of the bid if all the items in the bid have a value equal to that bid. That is, if all the items are cleared for that bid/constraint then that constraint returns its value, otherwise the constraints returns a value of zero. In this problem we now want to maximize the value returned by the constraints.

We can also reduce the winner determination problem by letting the variables be the bids themselves with binary domains which indicate whether the bid has been cleared or not. We then need two sets of constraints. One set of constraints has to be generated such that they give a very large penalty if any two bids with items in common have been cleared, so as to make sure that never happens. The other set simply gives a value for every cleared bid equal to the value of that bid.

Since both of these are standard constraint optimization problems we can use the algorithms from Chapter 2.2 to solve them in a distributed manner. However, as those algorithms are generic, it seems likely that they will not perform as well as the specialized algorithms.

7.3.3 Bidding Languages

We have thus far assumed that each buyer can submit a set of bids, where each bid b specifies that he is willing to pay b^{value} for the set of items b^{items}. Implicit in the set of bids is the fact that the agent is also willing to accept winning two or more of his bids. That is, if b and b' are two bids for non-overlapping sets of items then any agent that places them should also be happy to win both bids. This bidding language is known as **or bids**, because agents can place multiple **atomic bids** and they can win any feasible combination of the bids they submit. That is, the agent expresses his valuation as b_1 OR b_2 OR... OR b_k.

OR BIDS
ATOMIC BIDS

A limitation of OR bids is that they cannot represent sub-additive valuations. For example, a sub-additive valuation arises if you have a value of 10 for a red hat and 10 for a blue hat but together value them at 11 because you really only need

one hat. In this scenario if you placed the individual bids as OR bids it could be that you end up paying 10 for both hats. We thus say that OR bids are not a complete bidding language since they cannot represent all possible valuations.

XOR BIDS

Xor bids, on the other hand, can represent all possible valuations. An XOR bid takes the form of a series of atomic bids joined together by exclusive-or operations: b_1 XOR b_2 XOR... XOR b_k. This bid represents the fact that the agent is willing to win any *one* of the bids, but not more than one. Thus, you can use it to place a bid that says you are willing to pay 10 for a red hat or 10 for a blue hat or 11 for both but want to buy at most one of them.

One problem with XOR bids is that they can get very long for seemingly common valuations that can be more succinctly expressed using the OR bids. For example, say an agent has a purely additive valuation function over a set of items, that is if $s = s' \cup s''$ then $v(s) = v(s') + v(s'')$. This agent could have expressed this valuation by placing an OR bid where each atomic bid was just for one item. Implicit in this bid is the fact that the agent would be willing to accept any subset of the items as long as he gets paid the sum of the individual valuation. If the same agent had to place an XOR bid it would have to place an atomic bid for every subset of items, and there are $2^{|s|}$ such subsets.

OR* BIDS

Another practical problem with XOR bids is that most of the winner determination algorithms are designed to work with OR bids. The problem can be solved by adding dummy items to OR bids, these bids are known as **or* bids**. For example, if an agent wants to place a bid for item a and b, but not both, it could generate a dummy item d and place an OR bid for items $\{a, d\}$ and $\{b, d\}$. This way the agent guarantees that it will not win both a and b. OR* combines the completeness of XOR bids with the succinctness of OR bids without adding too many new dummy items. In fact, any bid that can be expressed by OR or XOR using x atomic bids can be represent using an OR* bids with at most x^2 dummy items (Nisan, 2000). Thus, all the winner determination algorithms we have studied can also be used with XOR bids as long as you first convert them to OR* bids.

7.3.4 Preference Elicitation

We can also try to reduce the amount of information the bidders must supply by trying to only ask them about those valuations that are important in finding the utilitarian solution (Hudson and Sandholm, 2004). This can best be achieved in the common case of **free disposal** where there is no cost associated with the disposal of an item, that is, if $S \subseteq T$ then $v(S) \leq v(T)$. For example, if we know that an agent values item a at 10 then we know that it must also value the set (a, b) at *least* at 10. Similarly, if the agent values items (a, b) at 5 then we know that its value for item a is at *most* 5.

FREE DISPOSAL

Given free disposal, an auctioneer can incrementally explore a network like the one in Figure 7.9 which shows all the possible subsets of items and associates with each one a upper bound (UB) and a lower bound (LB) on the valuation that the agent has for that particular set of items. The directed edges indicate which sets are preferred over other ones. For example, the agent will always prefer the set $\{a, b, c\}$ over the set $\{a, b\}$ and even, transitively, over the set $\{a\}$. The graph also makes it easy to see how the auctioneer can propagate the bounds he learns on one set to the others. Namely, the lower bounds can be propagated upstream and the upper bounds can be propagated downstream. For example, in the figure there is a lower bound of 5 the set $\{b\}$, knowing this the auctioneer can immediately set the lower bounds of $\{b, c\}$ and $\{a, b, c\}$ to 5. Similarly, the upper bound of 9 for the set $\{a, c\}$ can be propagated down to $\{a\}$, $\{c\}$, and $\emptyset$. Note also that if the agent tells the auctioneer its exact valuation for a particular set then the auctioneer will set both the upper and lower bounds of that set to the given value.

The goal of an elicitation auctioneer is to minimize the amount of questions that it asks the bidders while still finding the best allocation. If we limit the auctioneer to only asking questions of the type *"What is your n^{th} most preferred set?"* then

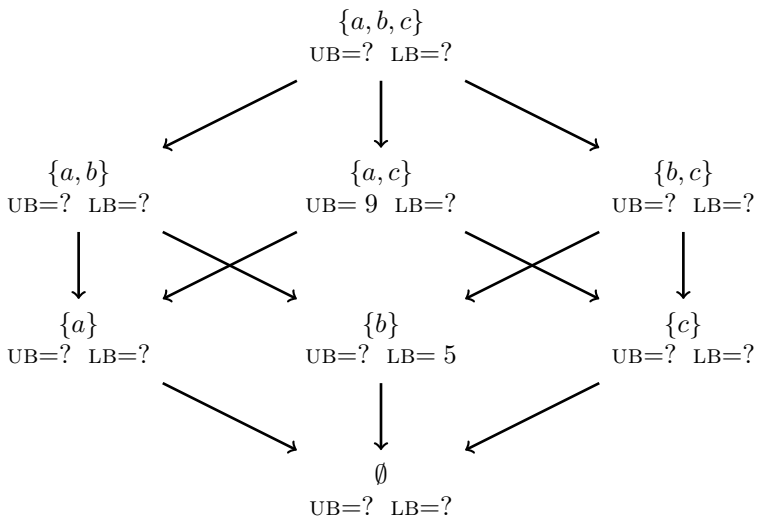


Figure 7.9: Constraint network for determining an agent's valuation. Directed edges indicate preferred sets.

```
PAR()
 1  fringe ← {{1, ..., 1}}
 2  while fringe ≠ ∅
 3        do c ← first(fringe)
 4           fringe ← rest(fringe)
 5           successors ← CHILDREN(c)
 6           if FEASIBLE(c)
 7              then pareto-solutions ← pareto-solutions ∪ c
 8              else for n ∈ successors
 9                         do if n ∉ fringe ∧UN-DOMINATED(n, pareto-solutions)
10                               then fringe ← fringe ∪ n

CHILDREN({k_1, ..., k_n})
1  for i ∈ 1...n
2        do c ← {k_1, ..., k_n}
3           c[i] ← c[i] + 1
4           result ← result ∪ c
5  return result
```

Figure 7.10: The PAR algorithm. The procedure FEASIBLE($\{k_1, \ldots, k_n\}$) asks each bidder i for its k_i most valued set, if we haven't asked before. It uses these sets to determine if, together, they form a feasible allocation. The CHILDREN procedure returns a set of possible solutions, by adding 1 to each position in $\{k_1, \ldots, k_n\}$.

we will be unable to find the revenue maximizing allocation. However, we can still find the Pareto optimal allocations.

PAR ALGORITHM

The **PAR algorithm**, shown in figure 7.10, allows an elicitation auctioneer to find a Pareto optimal solution by only using rank questions (Conen and Sandholm, 2002). It does this by incrementally building a solution lattice for the bidders. Figure 7.11 shows an example of a complete lattice for two bidders. The PAR algorithm maintains a set variable, called the *fringe*, of possible Pareto optimal allocations. At the first time step the auctioneer adds the solution {1,1} to the *fringe*, where {1,1} represents the solution formed by using both agents' most preferred solution. In every succeeding step the auctioneer picks some solution from the *fringe* and asks the agents for those sets, if it does not already know them. This communication with the bidders occurs within the FEASIBLE procedure. In the example figure both agents prefer the set $\{a, b\}$ the most so they will both respond with this set. Since the set of bids $(\{a, b\}, \{a, b\})$ is not feasible the algorithm checks to make sure that each one of the CHILDREN of {1,1}, in this case {1,2} and {2,1}, is not dominated by any other allocation in the set *pareto-solutions* and adds it to the *fringe* if its not already there. The algorithm continues in this way until the *fringe* is empty, at that point the variable *pareto-solutions* contains all the Pareto allocations for the

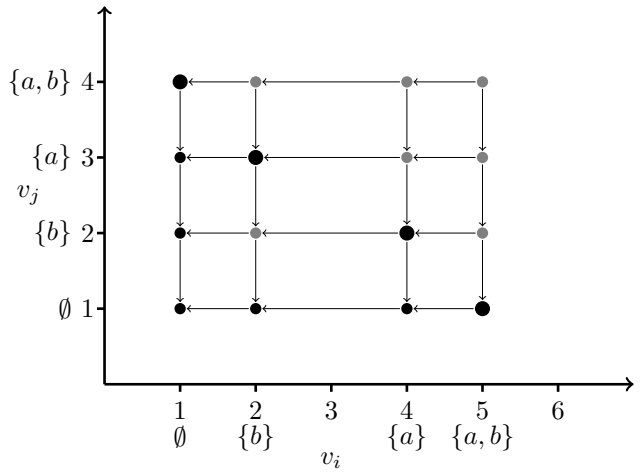


Figure 7.11: Rank lattice. The dots represent all possible allocations. Directed edges represent Pareto dominance. Grey dots are infeasible while black dots are feasible allocations. The PAR search starts at the top rightmost point and stops when it has identified the complete Pareto frontier of feasible allocations—the larger black dots.

```
EBF()
 1  fringe ← {{1, ..., 1}}
 2  if |fringe| = 1
 3     then c ← first(fringe)
 4     else  M ← {k ∈ fringe | v(k) = max_{d∈fringe} v(d)}
 5           if |M| ≥ 1 ∧ ∃_{d∈M} FEASIBLE(d)
 6              then return d
 7              else  c ← PARETO-SOLUTION(M)
 8  if FEASIBLE(c)
 9     then return c
10  successors ← CHILDREN(c)
11  for n ∈ successors
12        do if n ∉ successors
13              then fringe ← fringe ∪ {n}
14  goto 2
```

Figure 7.12: The EBF algorithm. FEASIBLE(d) returns true if d is a feasible allocation. PARETO-SOLUTION(M) returns one allocation from M which is not Pareto-dominated by any other allocation in M.

problem.

Since the PAR algorithm does not ask the agents for their valuation values it cannot determine which one of the Pareto allocations is the utilitarian allocation. Of course, once we start asking for valuations we have a better idea of which bids will likely be part of the utilitarian allocation, namely those sets that have the highest value per item.

EFFICIENT BEST FIRST

The **efficient best first** (EBF) algorithm performs a search similar to the one that PAR implements but it also asks for the values of the sets and always expands the allocation in the *fringe* which has the highest value (Conen and Sandholm, 2002). Figure 7.12 shows the algorithm. This algorithm will find the utilitarian allocation.

Unfortunately, both PAR and EBF have worst case running times that are exponential in the number of items. They also perform rather poorly in practice. PAR does not make any effort at trying to pick a item set to ask about, it simply chooses randomly, so its bad performance is not surprising. EBF's elicitation strategy has also been shown to perform poorly in experiments—it asks too many questions. Its performance also degrades as more agents are added.

A more general framework for elicitation is to maintain a set of allocations which are potentially optimal. Initially, this set would contain all allocations in the general case. In cases where we can assume some structure for the value function, such as free disposal, then it contains all those allocations that are not dominated by another. The elicitation algorithm can then choose one allocation from this set and ask the agents their values for the sets they receive in that allocation. These

values can then be propagated to other sets and a new allocation chosen (Conen and Sandholm, 2001b; Conen and Sandholm, 2001a).

Within this general framework there are various elicitation strategies we could try. The simplest one is to randomly choose an allocation from the set. This technique has been shown to require a number of elicitations that is proportional to $n2^m$ where n is the number of agents and m is the number of items. Another strategy is to choose the candidate with the highest lower bound value on the expectation that a candidate with a higher value is more likely to be the optimal choice, or at least will need to be eliminated from competition. Experiments have shown that this strategy performs better than random elicitations.

7.3.5 VCG Payments

VCG payments, which we will see in Chapter 8.2, can be applied to combinatorial auctions (MacKie-Mason and Varian, 1994). In a VCG combinatorial auction the bid set with maximum payment is chosen as the winner but the bidders do not have to pay the amounts they bid. Instead, each bidder pays the difference in the total value that the *other* bidders would have received if he had not bid (and the best set of bids was chosen from their bids) minus the total value the other bidders receive given his bid. Each bidder thus get a payment that is proportional to his contribution to everyone else's valuation. This has the desirable effect of aligning the bidders' incentives with the utilitarian allocation and thus eliminating the incentive to lie about their true valuation. However, it increases the computational requirements as we now have to also solve a winner determination problem for every subset of $n - 1$ agents in order to calculate the payments. That is, VCG payments increase the work by a factor of n, where n is the number of agents.

Exercises

7.1 The branch and bound algorithm for the branch on bids search tree, seen in figure 7.6, does not specify in which order the bids should be searched. Give a item heuristic for this ordering and explain why it should, on average, help find a solution quicker than expanding bids in a random order.

7.2 What is the set of winning bids given the following bids?

Price	Bid items
\$1	Beast Boy
\$3	Robin
\$5	Raven, Starfire
\$6	Cyborg, Robin
\$4	Cyborg, Beast Boy
\$3	Raven, Beast Boy

7.3 Show how we can reduce the problem of winner determination in a combinatorial auction to a constraint optimization problem.

7.4 Can the problem of winner determination in a combinatorial auction be reduced to a constraint satisfaction problem? Show your proof.

7.5 You are given a painting to sell at an auction and wish to maximize its sale price. What type of auction should you use? Explain.

7.6 In Chapter 6.5 we mentioned the postman problem. How can the postmen use a combinatorial auction to solve their problem? Explain how the bids are to be generated.

7.7 Why is a branch on bids search faster than a branch on items search for winner determination in combinatorial auctions?

7.8 In a combinatorial auction with 50 items and using a computer that takes 1 millisecond to explore each bidset, what is an upper bound on the amount of time it would take to find a solution to the winner determination problem?

7.9 The winner determination problem in combinatorial auction seeks to maximize revenue, that is, maximize the amount paid by the buyers. Provide three reasons or situations in which this solution might not be the most desirable on.

Chapter 8

Voting and Mechanism Design

Once you have a multiagent system composed of autonomous locally-aware agents, you will often desire a way to aggregate their knowledge for making a system-wide decision. That is, you will want to ask them to vote on some issue. Unfortunately, there are many different voting mechanisms, each one of which might lead to different answers. More importantly, it could be that selfish agents do not want to vote their true preferences. They might prefer to lie, in the same way you might not vote for your favorite political candidate if you think that she does not have any hope of winning and, instead, you vote for your second most favorite candidate. In this chapter we examine the problems with voting and mechanism design.

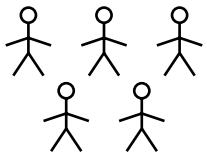
8.1 The Voting Problem

At first glance, the voting problem seems very simple. We ask all agents to proclaim their preferences over a set of candidates and then tally these votes to find the most preferred candidate. The problem comes if we want this result to match, in some way, the agents' preferences. It turns out that the common voting mechanisms all fail to aggregate the voters' preferences in some cases.

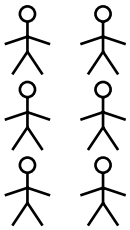
For example, figure 8.1 shows 15 mathematicians who are planning to throw a party. They must first decide which beverage the math department will serve at this party. There are three choices available to them: beer, wine, and milk. As the figure shows, 6 mathematicians prefer milk over wine and wine over beer, 5 prefer beer over wine and wine over milk and 4 prefer wine over beer and beer over milk. We then need to determine which drink they will choose.

One option is to have a **plurality vote** where each one votes for their favorite drink, the votes are tallied and the drink with the most votes is the winner. Under this voting scheme beer would get 5 votes, wine 4, and milk 6. Therefore, the

PLURALITY VOTE

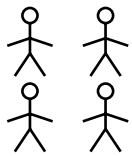


Beer
Wine
Milk



Milk
Wine
Beer

	Beer	Wine	Milk
Plurality	5	4	6
Runoff	5,9	4	6,6
Pairwise	Beer-Milk	Beer-Wine, Wine-Milk	0



Wine
Beer
Milk

Figure 8.1: Fifteen mathematicians trying to decide whether they should buy beer, wine, or milk.

mathematicians should clearly choose milk as the drink for the party.

RUNOFF

Another option is to have a **runoff** election (primaries) then pick the two winners and have another election with only those two (these technique can be easily extended to any number of runoff elections). Under this scheme the first election would lead to the same votes as before so the second election would consist of beer and milk. With only these two candidates beer would get 9 votes while milk would get 6 votes. Therefore, the mathematicians should clearly choose beer as the drink for the party.

PAIRWISE

Yet another option is to hold three elections each one with only two candidates (that is, implement all **pairwise** elections) and the candidate that wins the most elections is the chosen one. Under this scheme we would see that if beer and wine are paired then wine wins, if beer and milk are paired then beer wins, and if wine and milk are paired then wine wins. Wine wins two elections so it is the chosen one. Therefore, the mathematicians should clearly choose wine as the drink for the party. After realizing the complexity of this problem the mathematicians wisely decide to give up and have everyone bring their own drink.

8.1.1 Possible Solutions

We, on the other hand, will not give up that easily. We want to clearly define the best solution. Specifically, we want a fair solution. But, it is not clear what fairness means in this situation. One way to approach the fairness problem is to require **symmetry**. There are two different types of symmetry that we can identify in this problem.

SYMMETRY

- *Reflectional symmetry*: If one agent prefers A to B and another one prefers B to A then their votes should cancel each other out.
- *Rotational symmetry*: If one agent's preferences are A,B,C and another one's are B,C,A and a third one prefers C,A,B then their votes should cancel out.

If we look back at the three types of schemes presented in the previous section we notice that the plurality votes violates reflectional symmetry since, in the example from figure 8.1, there are 8 agents that prefer beer over milk and wine over milk, but only 6 that have the opposite preferences, and yet milk wins. Similarly, since the runoff election is just a series of plurality votes it also violates reflectional symmetry. It has also been shown that pairwise comparison violates rotational symmetry. In the example we can take one agent from each of the three groups and these three agents cancel each other's votes, so we can eliminate them. We can do this four times and end up with two agents with preferences milk, wine, beer and one agent with preference of beer, wine, milk. A plurality vote over these would lead to milk as the winner while the pairwise vote led to wine being the winner.

Jean-Charles de Borda.
1733–1799.

BORDA COUNT

Thus, none of the previous voting schemes satisfy both forms of symmetry. But there exists one voting mechanism which does satisfy them. It is known as the **Borda count** and it works as follows:

1. With x choices, each agent awards x to points to his first choice, $x-1$ points to his second choice, and so on.
2. The candidate with the most points wins.

The Borda count satisfies both reflectional and rotational symmetry. It is most useful when there are many candidates and we want to choose the best one by taking into account all agents' knowledge, equally. With the Borda count we do not have to worry about a minority winning the election because the majority is divided among a small number of choices.

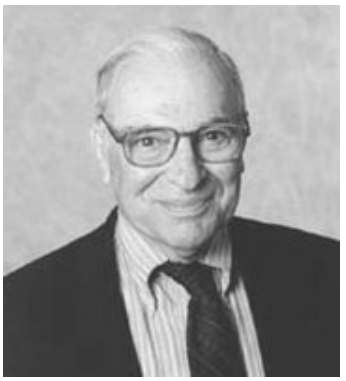
Now that you understand the intuitions behind the voting problem, we give a formal presentation of the problem.

Definition 8.1 (Voting Problem)**.** *We are given set of agents A and a set of outcomes O. Each agent $i \in A$ has a preference function $>_i$ over the set of outcomes. Let $>^*$ be the global set of social preferences – what we think the final vote should reflect.*

Using this notation we can clearly specify the kind of $>^*$ that we would like. Namely, we are probably interested in a $> *$ that is efficient, can be calculated, and is fair to everyone. After thinking about it for some time, you would probably come up with a set of voting conditions similar to the following.

Definition 8.2 (Desirable voting outcome conditions)**.** *A voting protocol is desirable if it obeys the following conditions:*

1. *$>^*$ exists for all possible inputs $>_i$,*
2. *$>^*$ exists for every pair of outcomes,*
3. *$>^*$ is asymmetric and transitive over the set of outcomes,*
4. *$>^*$ should be Pareto efficient. That is, if all agents prefer Beer over Milk then $>^*$ should also prefer Beer over Milk.*
5. *The scheme used to arrive at $>^*$ should be independent of irrelevant alternatives. That is, if in one world all agents prefer Beer to Milk and in another world all agents again prefer Beer to Milk then in both cases the rankings of Beer and Milk should be the same, regardless of how they feel about Wine.*
6. *No agent should be a dictator in the sense that $>^*$ is always the same as $>_i$, regardless of the other $>_j$.*



Kenneth Joseph Arrow. 1921–. Nobel prize in Economics.

Unfortunately, Arrow showed that no voting mechanism exists which satisfies all these conditions.

Theorem 8.1 (Arrow's Impossibility)**.** *There is no social choice rule that satisfies the six conditions (Arrow, 1951).*

Specifically, we can show that plurality voting violates conditions 3 and 5 when there are three or more candidates. Similarly, since runoff elections are just several plurality votes they also violate conditions 3 and 5. Pairwise voting can violate condition 5 as does the Borda count. We can show that the Borda count violates condition 5 with a simple example. Say there are seven agents whose preferences over choices a, b, c, d are as follows:

1. $a > b > c > d$
2. $b > c > d > a$
3. $c > d > a > b$
4. $a > b > c > d$
5. $b > c > d > a$
6. $c > d > a > b$
7. $a > b > c > d$

If we applied the Borda count to these agents we would find that c gets 20 points, b gets 19, a gets 18, and d 13. As such, c wins and d comes out last. If we then eliminate d we would then have the following preferences.

1. $a > b > c$
2. $b > c > a$

Table 8.1: List of individual desires for painting the house.

Name	Wants house painted?
Alice	Yes
Bob	No
Caroline	Yes
Donald	Yes
Emily	Yes

3. $c > a > b$
4. $a > b > c$
5. $b > c > a$
6. $c > a > b$
7. $a > b > c$

If we ran Borda on this scenario we would find that a gets 15 votes, b gets 14, and c gets 13, so a wins. So, originally c wins and d comes out last but by eliminating d we then get that a wins! This is a violation of condition 5.

8.1.2 Voting Summary

In practice we find voting mechanism often used in multiagent systems but without much thought given to which one might be best. Thus, plurality vote is most often used. In general, the Borda count should be the preferred voting mechanism as it can effectively aggregate multiple disparate opinions, but it does have some drawbacks. Namely, the Borda count requires the agents to generate a complete preference ordering over all items, which could be computationally expensive. For example, if each choice is an allocation of packages that the agent must deliver then the agent must solve a traveling salesman problem for each choice in order to determine how much it would cost him to deliver all those packages. One could try to reduce these costs by implementing a limited version of the Borda count where instead of voting for all choices the agents limit themselves to voting for only their best k options, for some small number k.

8.2 Mechanism Design

Mechanism design asks how we can provide the proper incentives to agents so that we can aggregate their preferences correctly. The mechanism design problem has been studied in Economics for some time. It is and interesting to us because it maps very well to open multiagent systems with selfish agents. In this chapter we present the standard mechanism design problem as studied in Economics as well as the distributed mechanism design extension which is an even better model for many multiagent system design problems.

8.2.1 Problem Description

Alice lives in a house with four other housemates. They are thinking about paying someone to paint the exterior of their house and have decided to hold a vote where everyone will vote either Yes, if they want the house painted, or No if they don't. The votes will be public and the set of people who vote for painting will share equally in the cost of the painters, as long as two or more people vote Yes. The people who voted against painting will pay nothing. We note that, since the paint covers the whole outside of the house everyone will be able to enjoy the new cleaner house. Each person knows whether or not they want the house to be painted. Their desires are shown in Table 8.1.

i	θ_i	$v_i(\text{Paint}, \theta_i)$	$v_i(\text{NoPaint}, \theta_i)$
Alice	WantPaint	10	0
Bob	DontWantPaint	0	0
Caroline	WantPaint	10	0
Donald	WantPaint	10	0
Emily	WantPaint	10	0

Table 8.2: Values for the house painting problem where $O = \{\text{Paint}, \text{NoPaint}\}$ and the agents are either of type WantPaint or type DontWantPaint.

Alice wants the house painted, but lets assume that she does not want to pay for it. She realizes that if only two people voted yes that the house will be painted. As such, she has an incentive to vote against painting – that is, lie about her true preferences – in the hope that some other two agents will vote for it and the house will get painted anyway. This means that Alice's strategy will be to try to determine what the others are likely to vote and see if there are enough Yes votes so that she can safely lie. Unfortunately, all that scheming is very inefficient from a system's perspective. It would be much easier if everyone wanted to tell the truth.

We would like to create a protocol where these types of incentives do not exist. That is, we would like for all agents to want to vote truthfully rather than lie or try to find out how the other agents are going to vote. If we could do that then the agents would not waste their resources trying to beat the system and instead use them to work for the system.

Mechanism design (Mas-Colell et al., 1995, Chapter 23) studies how private information can be elicited from individuals. It tells us how to build the proper incentives into our protocols such that the agents will want to tell the truth about their preferences. It also tells us about some circumstances when this is impossible.

More formally, we define a **mechanism design** problem as consisting of a set of agents with the following properties.

MECHANISM DESIGN

- Each agent i has a **type** $\theta_i \in \Theta_i$ which is private. That is, only the agent knows its type, no one else does.

  TYPE

- We let $\theta = \{\theta_1, \theta_2, \ldots, \theta_A\}$ be the set of types.
- The **mechanism** g we are to implement will map from the set of agents' actions to a particular **outcome** $o \in O$.

  MECHANISM
  OUTCOME

- Each agent i receives a value $v_i(o, \theta_i)$ for outcome o.
- The **social choice function** $f : \theta \to O$ tells us the outcome we want to achieve.

  SOCIAL CHOICE FUNCTION

For example, the social choice function

$$f(\theta) = \arg\max_{o \in O} \sum_{i=1}^{n} v_i(o, \theta_i) \tag{8.1}$$

is the social welfare solution. It tries to maximize the sum of everyone's utility. You can, however, choose to implement a different social choice function. Other popular choices include minimizing the difference in the agents' utility, maximizing the utility of the agent that receives the highest utility, and the **paretian** social choice function f such that for all θ there is no $o' \neq o = f(\theta)$ such that some agent i gets a higher utility from o' than it would have received under $f(\theta)$. That is, in the paretian social choice function f there does not exist an outcome o' such that there is some agent i for which $v_i(o', \theta_i) > v_i(f(\theta), \theta_i)$ and for all i $v_i(o', \theta_i) \geq v_i(f(\theta), \theta_i)$.

PARETIAN

Note also that since the agent's type are usually fixed – an agent cannot change its true type, only lie about it – then the v_i usually only needs to be defined for the agent's particular θ_i.

If we apply this notation to the example from Table 8.1 we get the values shown in Table 8.2. We have that $\Theta = \{WantPaint, DontNeedPaint\}$ since there are

only two types of agents: those that want the house painted and those that think it does not need paint. Also, $O = \{Paint, NoPaint\}$ since either the house gets painted or it doesn't. Notice that we had to add some arbitrary number for the agents' utilities for all possible actions. We decided that the agents that want the house painted would get a value of 10 from seeing it painted and 0 if it does not get painted while those who think the house is fine as it is get a value of 0 either way. Lets further assume that we want to maximize social welfare. That is, our social choice function is (8.1). Finally, we assume that the cost of painting the house is 20. We now face the problem of designing a protocol that will decide whether or not to paint the house and how to pay for it.

One possible way to solve this problem is to let all the agents vote Yes or No. We then count the votes and if a majority voted for painting the house then the house will be painted and the cost (20) will be divided evenly among the 5 agents. That is, each agent will have to pay 4 no matter what. This scheme works fine for all agents except for Bob who did not want the house painted and must now pay 4. We are imposing a tax on Bob for something he does not want. This might not be a desirable solution.

Another way is to let everyone vote Yes or No and then split the cost of painting the house among those who voted Yes, as we discussed earlier. This seems fairer but it has the problem that it gives all the agents, except Bob, and incentive to lie, as we explained before. They would want to lie in the hopes that someone else would vote Yes and spare them having to pay for it. In general, we are looking for a mechanism which implements the social choice function. This idea can be formalized as follows:

IMPLEMENTS

Definition 8.3 (g Implements f)**.** *A mechanism $g : S_1 \times \cdots \times S_A \to O$ **implements** social choice function $f(\cdot)$ if there is an equilibrium strategy profile $(s_1^*(\cdot), \ldots, s_A^*(\cdot))$ of the game induced by g such that*

$$\forall_\theta \; g(s_1^*(\theta_1), \ldots, s_A^*(\theta_A)) = f(\theta_1, \ldots, \theta_A)$$

where $s_i(\theta_i)$ is agent i's strategy given that it is of type θ_i.

The definition might sound a little bit circular but it isn't. Say you start out with a set of agents each one with a type – which you don't know about – and you tell them that you are going to use $g(\cdot)$ to calculate the final outcome. That is, you tell them how the function g works. The agents will use their knowledge of g to determine their best action and will take that action. You then input this set of actions into g to come up with the outcome. If the outcome is the same as $f(\theta_1, \ldots, \theta_A)$ then you just implemented f. As you can see, the tricky part is that you have to pick g such that it will encourage the agents to take the appropriate actions.

Another point of confusion might be that we have changed from types to actions. In the previous examples the agents actions – their votes – where merely the revelation of their types. That is, there was a one-to-one mapping from types to actions. However, in general this need not be the case. We could, for example, have a system with 20 different types but only 2 possible actions.

We have also not defined what we mean by an "equilibrium strategy" as used in Definition 8.3. As you will remember from Chapter 3, there are many equilibrium concepts that can be applied to a game. The equilibrium we will concern ourselves with is the dominant strategy equilibrium. A player has a dominant strategy (action) if the agent prefers to use this strategy regardless of what anyone else will do. In our mechanism design problem we formally define a dominant strategy as follows:

DOMINANT STRATEGY EQUILIBRIUM

Definition 8.4 (Dominant Strategy Equilibrium)**.** *We say that a strategy profile $(s_1^*(\cdot), \ldots, s_A^*(\cdot))$ of the game induced by g is a **dominant strategy equilibrium** if for all i and all θ_i,*

$$v_i(g(s_i^*(\theta_i), s_{-i}), \theta_i) \geq v_i(g(s_i', s_{-i}), \theta_i)$$

for all $s_i' \in S_i$ and all $s_{-i} \in S_{-i}$.

We can now specialize Definition 8.3 for dominant equilibria.

Definition 8.5 (g Implements f in Dominant Strategies)**.** *A mechanism $g : S_1 \times \cdots \times S_A \to O$ **implements social choice function** $f(\cdot)$ **in dominant strategies** if there is a dominant strategy equilibrium strategy profile $(s_1^*(\cdot), \ldots, s_A^*(\cdot))$ of the game induced by g such that $g(s_1^*(\theta_1), \ldots, s_A^*(\theta_A)) = f(\theta_1, \ldots, \theta_A)$ for all $\theta \in \Theta$.*

IMPLEMENTS SOCIAL CHOICE FUNCTION $f(\cdot)$ IN DOMINANT STRATEGIES

Before we go into how to find this magical g lets explore some simplifications of the problem. The first simplification is one we made in our first example. Namely, that the agents' strategies correspond to the revelation of their types. That is, lets assume that the agents' actions are simply type revelations. The only thing an agent can do is say "I am of type θ_x". Of course, he could be lying when he makes this statement. We call this a **direct revelation mechanism** because the agents directly reveal their types rather than taking an action that might or might not be correlated to their type.

DIRECT REVELATION MECHANISM

In these cases we would want to design a mechanism g which implements a social choice function f and encourages all agents to tell their true type. This might or might not be possible for a particular f. If it is possible then we say that f is strategy-proof.

Definition 8.6 (Strategy-Proof)**.** *The social choice function $f(\cdot)$ is **truthfully implementable in dominant strategies** (or **strategy-proof**) if for all i and θ_i we have that $s_i^*(\theta_i) = \theta_i$ is a dominant strategy equilibrium of the direct revelation mechanism $f(\cdot)$. That is, if for all i and all $\theta_i \in \Theta_i$,*

STRATEGY-PROOF

$$v_i(f(\theta_i, \theta_{-i}), \theta_i) \geq v_i(f(\hat{\theta}_i, \theta_{-i}), \theta_i)$$

for all $\hat{\theta}_i \in \Theta_i$ and all $\theta_{-i} \in \Theta_{-i}$.

That is, the value that each agent receives under the outcome prescribed by the social choice function when all tell the truth is bigger than or equal to the value it gets if it lied about its type. Notice how we have plugged in f directly as the mechanism instead of using g, in other words $g = f$. As you might guess, this would make g trivial to implement because we are given f. For example, if I ask you to find a mechanism that implements social function f and you look at f and realize that it is strategy-proof then all you have to do is directly use f. That is, you would ask the agents for their types, they would all tell the truth because telling the truth is their dominant strategy, you would then plug these values into f and out would come the desired outcome.

These strategy-proof social choice functions make it trivial to find a g that implements them, namely $g = f$. Still, we might worry that the particular f we have been given to implement is not strategy-proof but there might exist some mechanism g which implements f in dominant strategies. That is, g lets the agents take some action, which might be different from revealing their type, and uses these actions to come up with the same outcome that f would have resolved using the agents true types. Furthermore, the actions the agents take are dominant given that they know about g.

Fortunately, there is no need to worry about finding such g as it has been proven that no such g exists. This is known as the **revelation principle**.

REVELATION PRINCIPLE

Theorem 8.2 (Revelation Principle)**.** *If there exists a mechanism g that implements the social choice function f in dominant strategies then f is truthfully implementable in dominant strategies.*

That is, if there exists a complicated mechanism that implements a given social function then there is also a much simpler mechanism which just asks the agents to reveal their types. Of course, this simpler mechanism might have other problems, as we will see.

An Example Problem and Solution

Let's now use all this notation in an example. Imagine that you want to sell an item. There are a bunch of prospective buyer agents. You want to sell it to the agent that wants it the most, but you can't trust them to tell you the truth. More formally, we can describe this problem as consisting of the following variables.

- $\theta_i \in \Re$: types are the valuations.
- $o \in \{1, \ldots, n\}$: index of agent who gets the item.
- $v_i(o, \theta_i) = \theta_i$ if $o = i$, and 0 otherwise.
- $f(\theta) = \arg\max_i(\theta_i)$
- Each agent gets a $p_i(o)$ so that $u_i(o, \theta_i) = v_i(o, \theta_i) + p_i(o)$.

Given this problem we must now try to figure out how to implement the payments p as well as how to determine the outcome given the agents' reported types, both of these together constitute the desired mechanism g. That is, as with most research in mechanism design, we are only interested in mechanism that involve paying or taxing the agents some amount of money in order to change their utility valuation.

After thinking about this problem for a while, you suddenly realize that this is a problem you have already seen. The solution is to use a Vickrey auction. Set $p(o)$ such that the agent who wins must pay a tax equal to the second highest valuation. No one else pays or receives anything. This mechanism results in the agents having final utilities as follows.

$$u_i(o, \theta_i) = \begin{cases} \theta_i - \max_{j \neq i} \theta_j & \text{if } o = i \\ 0 & \text{otherwise.} \end{cases}$$

That is, we define g such that it returns an outcome o which contains the index of the agent that sent you the highest bid. The g also charges this winning agent an amount equal to the second highest bid and charges everyone else 0.

In fact, we can use the notation we have set up to prove that telling the truth is the dominant strategy in this scenario which, along with the fact that we implement the social choice function, makes the Vickrey auction strategy-proof for this social choice function.

Truth-Telling is Dominant in Vickrey Payments Example. We can prove that telling the truth is the dominant strategy by following these steps.

1. Let $b_i(\theta_i)$ be i's bid given that his true valuation is θ_i.
2. Let $b' = \max_{j \neq i} b_j(\theta_j)$ be the highest bid amongst the rest.
3. If $b' < \theta_i$ then any bid $b_i(\theta_i) > b'$ is optimal since
$$u_i(i, \theta_i) = \theta_i - b' > 0$$
4. If $b' > \theta_i$ then any bid $b_i(\theta_i) < b'$ is optimal since
$$u_i(i, \theta_i) = 0$$
5. Since we have that if $b' < \theta_i$ then i should bid $> b'$ and if $b' > \theta_i$ then i should bid $< b'$, and we don't know b' then i should bid θ_i.

□

Name	$v_i(o,\tilde{\theta})$	$v_i(o,\theta)+\sum_{j\neq i} v_j(\tilde{\theta})$
Alice	$10-\frac{20}{4}=5$	$5+15=20$
Bob	$0-0=0$	$0+20=20$
Caroline	$10-\frac{20}{4}=5$	$5+15=20$
Donald	$10-\frac{20}{4}=5$	$5+15=20$
Emily	$10-\frac{20}{4}=5$	$5+15=20$

Table 8.3: Groves-Clarke payments for house painting assuming that all agents tell the truth.

Theodore Groves.

The Groves-Clarke Mechanism

We have just shown how to check that a mechanism implements a particular social choice function can be truthfully implemented in dominant strategies. However, we did not show how we came up with the mechanism itself or how we decided to use Vickrey payments. We would like a general formula that can be used to calculate the agents' payments no matter what social choice function is given to us. Unfortunately, such a formula does not appear to exist.

However, if we instead assume that the social choice function is the social welfare solution and further assume that the agents have quasilinear preferences then we can use the **Groves-Clarke mechanism** to calculate the desired payments. Agents with quasilinear preferences are those with utilities in the form $u_i(o,\theta_i)=v_i(o,\theta_i)+p_i(o)$. Formally, the Groves-Clarke mechanism is defined as follows:

GROVES-CLARKE MECHANISM

Theorem 8.3 (Groves-Clarke Mechanism)**.** *If we have a social choice function*

$$f(\theta)=\arg\max_{o\in O}\sum_{i=1}^{n} v_i(o,\theta_i)$$

then calculating the outcome using

$$f(\tilde{\theta})=\arg\max_{o\in O}\sum_{i=1}^{n} v_i(o,\tilde{\theta}_i),$$

where $\tilde{\theta}$ are reported types, and giving the agents payments of

$$p_i(\tilde{\theta})=\sum_{j\neq i} v_j(f(\tilde{\theta}),\tilde{\theta}_j)-h_i(\tilde{\theta}_{-i}), \tag{8.2}$$

where $h_i(\theta_{-i})$ is an arbitrary function, results in a strategy-proof mechanism (Groves, 1973; Clarke, 1971)..

Edward H. Clarke. 1939–.

Notice that the payments that i receives are directly proportional to the sum of everybody else's value. This is the key insight of the Groves-Clarke mechanism. In order to get the agents to tell the truth so that we may improve the social welfare we must pay the agents in proportion to this social welfare. Another way to look at it, perhaps a bit cynically, is to say that the way to get individuals to care about how everyone else is doing is to pay them in proportion to how everyone else is doing. For example, companies give shares of their company to employees in the hope that this will make them want the company as a whole to increase its profits, even if it means they have to work longer or take a pay-cut. In effect, the Groves-Clarke mechanism places the social welfare directly into the agent's utility function.

Lets apply the Groves-Clarke Mechanism to the house painting example from Table 8.2. Remember that the second solution we tried, where the cost of painting was divided among those who voted to paint, was not strategy-proof. Perhaps we can add Groves-Clarke payments to make it strategy-proof. To do this we must first re-evaluate the agents' value which will be decreased from 10 since they might have

Table 8.4: Groves-Clarke payments for house painting assuming that Alice lies and all others tell the truth.

Name	$v_i(o, \tilde{\theta})$	$v_i(o, \theta) + \sum_{j \neq i} v_j(\tilde{\theta})$
Alice	$0 - 0 = 0$	$10 + (\frac{10}{3} \cdot 3) = 20$
Bob	$0 - 0 = 0$	$0 + (\frac{10}{3} \cdot 3) = 10$
Caroline	$10 - \frac{20}{3} = \frac{10}{3}$	$\frac{10}{3} + (\frac{10}{3} \cdot 2) = 10$
Donald	$10 - \frac{20}{3} = \frac{10}{3}$	$\frac{10}{3} + (\frac{10}{3} \cdot 2) = 10$
Emily	$10 - \frac{20}{3} = \frac{10}{3}$	$\frac{10}{3} + (\frac{10}{3} \cdot 2) = 10$

to pay for part of the painting, if they voted yes, and then calculate the agents' payments using Theorem 8.3. The set of payments the agents would receive if they all told the truth is shown in Table 8.3. As you can see, all the agents get the same utility (20) from telling the truth.

Now, what if Alice lied? Would she get a higher utility? Table 8.4 shows the payments and utility values for the case where Alice lies and the rest tell the truth. As you can see, Alice still gets the same 20 of utility. As such, she has nothing to gain by lying (you should repeat these calculations for Bob to make sure you understand how the equations are used). It is interesting to note how everyone else's utilities have dropped due to Alice's lie. Of course Alice, being purely selfish, does not care about this.

Finally, notice how the payments add up to 80 on the first example and 40 in the second example. Where does this money come from? Note that we must give the agents real money otherwise the mechanism does not work. We cannot simply tell them to imagine we are giving them \$20. What would be really nice is if some of the agents payed us money and we payed some back to the other agents such that the total amount we pay equals the total amount we receive. We would thus achieve **revenue equivalence**. Unfortunately, as you have seen, the Groves-Clarke mechanism is not revenue equivalent.

REVENUE EQUIVALENCE

The Vickrey-Clarke-Groves Mechanism

VICKREY-CLARKE-GROVES

Another well-known payment mechanism is **Vickrey-Clarke-Groves**, which is just a small variation on Groves-Clarke but is closer to achieving revenue equivalence.

Theorem 8.4 (Vickrey-Clarke-Groves (VCG) Mechanism)**.** *If*

$$f(\theta) = \arg\max_{o \in O} \sum_{i=1}^{n} v_i(o, \theta_i)$$

then calculating the outcome using

$$f(\tilde{\theta}) = \arg\max_{o \in O} \sum_{i=1}^{n} v_i(o, \tilde{\theta}_i)$$

(where $\tilde{\theta}$ are reported types) and giving the agents payments of

$$p_i(\tilde{\theta}) = \sum_{j \neq i} v_j(f(\tilde{\theta}), \tilde{\theta}_j) - \sum_{j \neq i} v_j(f(\tilde{\theta}_{-i}), \tilde{\theta}_j) \tag{8.3}$$

results in a strategy-proof mechanism.

VCG is almost identical to Groves-Clarke except that the payments are slightly different. The payments in VCG measure the agent's contribution to the whole, what some call the "wonderful life" utility, which we saw in Chapter 5.6. You might have seen that old Frank Capra film "It's a Wonderful life". In it, George Bailey gets a chance to see how the world would have been had he never existed. He notices that

Name	$v_i(o, \tilde{\theta})$	$\sum_{j \neq i} v_j(f(\tilde{\theta}_{-i}), \tilde{\theta}_j)$	$\sum_{j \neq i} v_j(f(\tilde{\theta}), \tilde{\theta}_j) - \sum_{j \neq i} v_j(f(\tilde{\theta}_{-i}), \tilde{\theta}_j)$
Alice	$10 - \frac{20}{4} = 5$	$(10 - \frac{20}{3}) \cdot 3 = 10$	$15 - 10 = 5$
Bob	$0 - 0 = 0$	$(10 - \frac{20}{4}) \cdot 4 = 20$	$20 - 20 = 0$
Caroline	$10 - \frac{20}{4} = 5$	$(10 - \frac{20}{3}) \cdot 3 = 10$	$15 - 10 = 5$
Donald	$10 - \frac{20}{4} = 5$	$(10 - \frac{20}{3}) \cdot 3 = 10$	$15 - 10 = 5$
Emily	$10 - \frac{20}{4} = 5$	$(10 - \frac{20}{3}) \cdot 3 = 10$	$15 - 10 = 5$

Table 8.5: VCG payments for house painting assuming Alice tells the truth.

the social welfare in the world without him is lower than in the world with him in it. As such, he calculates that his existence has had a positive effect on social welfare and thus decides not to end his life, which makes us conclude that he wanted to increase social welfare. Nice guy that George.

That is exactly what the VCG payments calculate. In the first term of (8.3) the $f(\tilde{\theta})$ captures the outcome had agent i not existed. The first term thus captures the social welfare had i not existed while the second term captures the social welfare with i in the picture. We point out that, in practice, $f(\tilde{\theta})$ is very hard to compute. For many multiagent systems it is not clear what would happen if we took out one of the agents – imagine a soccer team without a player, or an assembly line without a worker, or workflow without an agent.

The advantage of VCG over Groves-Clarke is that it results in a lower net revenue gain or loss. We can see this if we re-calculate the payments for the house painting example, as seen in Table 8.5. The payments in this case add up to 20, as compared with 40 and 80 for the Groves-Clarke payments. Of course, 20 is not 0 so we have not yet achieved total revenue equivalence. The research literature tells us that revenue equivalence for many specific cases is impossible to achieve.

The main problem we face when trying to use both Groves-Clarke and VCG payments is that calculating these payments takes exponential time on the number of agents. This means that they can only be used when we have a small number of agents. Still, you might consider approximations of these payments which can be calculated quickly for your specific problem domain.

8.2.2 Distributed Mechanism Design

Computer scientists have taken the results from mechanism design and tried to apply them to multiagent systems problems, this required further specification of the problem. The first extension to mechanism design that appeared was **algorithmic mechanism design** which proposes that the mechanism should run in polynomial time (Nisan and Ronen, 2001). These types of mechanism are sorely needed because the calculation of VCG and Groves-Clarke payments takes exponential time. The second extension was **distributed algorithmic mechanism design** (DAMD) (Feigenbaum et al., 2001; Dash et al., 2003) which proposes that the computations carried out by the mechanism should be performed by the agents themselves and in polynomial time, with added limitations on the number of messages that can be sent. Note that if the agents themselves are performing the calculations for the mechanism then we must also guard against their interference with the calculation of the outcome. This can be done either by design – for example, by encrypting partial results – or by providing the agents with the proper incentives so they will not want to cheat.

ALGORITHMIC MECHANISM DESIGN

DISTRIBUTED ALGORITHMIC MECHANISM DESIGN

Thus far, DAMD algorithms have been almost exclusively developed for computer network applications. Unlike traditional network research, these algorithms treat the individual routers as selfish agents that try to maximize their given utility function. That is, the Internet is no longer viewed as composed of dumb and obedient routers but is instead viewed as composed of utility-maximizing autonomous entities who control specific routers or whole sub-networks. It is interesting to not that while

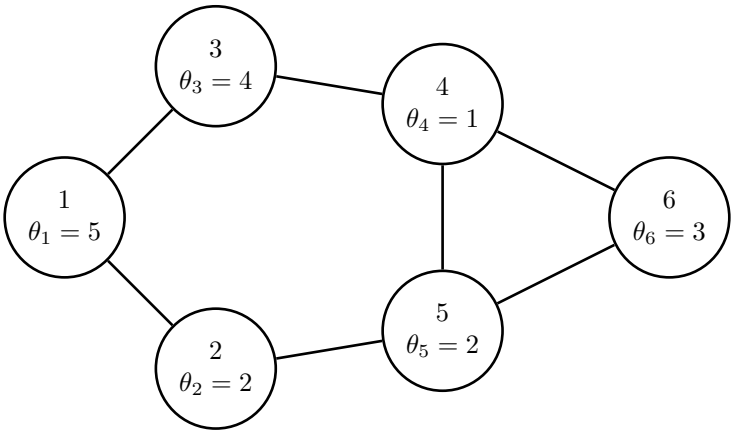


Figure 8.2: Example Inter-domain routing problem. The nodes represent networks. Each one shows the cost it would incur in transporting a packet through its network. The agents' types are their costs.

this model is a much more faithful representation of the real Internet, network researchers have largely ignored the fact that private interests exist in the Internet and instead assume that all parties want to maximize the social welfare.

Figure 8.2 shows a sample inter-domain routing problem. In this problem each node represents a computer network. The edges represent how these networks are connected to each other. Each network is willing to handle packets whose source or destination address is within the network. However, they are reticent to carry packets that are just passing thru. Each network in the figure also shows the cost it incurs in routing one of these passing-thru packets. These costs correspond to the agents' types. That is, only the agent knows how much it costs to pass a packet across its network. We further assume that we know how much traffic will travel from every start node to every destination node. The problem then is to find the routes for each source-destination pair such that the costs are minimized. This is a mechanism design problem which can be solved in a distributed manner (Feigenbaum et al., 2005), as we now show.

We can formally define the problem as consisting of n agents. Each agent i knows its cost to be θ_i. An outcome o of our mechanism design problem corresponds to a set of routing rules for all source-destination pairs. That is, the outcome o should tell us, for all i, j, which path a packet traveling from i to j should follow. The value $v_i(o)$ that each agent receives for a particular outcome o is the negative of the total amount of traffic that i must endure given the paths specified in o and the traffic amounts between every pair.

The solution we want is then simply

$$f(\theta) = \arg\max_o \sum_i v_i(o). \tag{8.4}$$

Since this is the social-welfare maximizing solution we are free to use VCG payments to solve the problem. Specifically, we can ask all agents to report their costs $\tilde{\theta}$ and then use (8.3) to calculate everyone's payments thus ensuring that telling the truth is the dominant strategy for all agents. Note that, in this case, calculating the outcome given $\tilde{\theta}$ amounts to calculating a minimum spanning tree for each destination node j. The problem of finding the routes that minimize the sum of all costs has already been solved, in a distributed manner, by the standard Internet **Border Gateway Protocol** (BGP). The protocol works by propagating back the minimum costs to a given destination. All agents tell each other their costs to destination. Each agent then updates its cost to a destination by adding its own transportation costs. The process then repeats until quiescence. The algorithm is, in fact, a variation on Dijkstra's algorithm for finding shortest paths in a graph (Dijkstra, 1959).

Border Gateway Protocol

Note, however, that the payment calculation (8.3) requires us to find the sum of everyone else's valuation for the case where i does not exist, the second term in (8.3). For this problem, eliminating i simply means eliminating all links to i. But,

it could be that the elimination of these links partitions the graph into two or more pieces thereby making it impossible for traffic from some nodes to reach other nodes. In such a case then it would be impossible to calculate the VCG payments. Thus, we must make the further assumption that the network is **bi-connected**: any one node can be eliminated and still every node can be reached from every other node.

BI-CONNECTED

We also note that the VCG payments can be broken down into a sum

$$p_i(\tilde{\theta}) = \sum_{a,b} p_i(a, b, \tilde{\theta}), \tag{8.5}$$

where $p_i(a, b, \tilde{\theta})$ is the payment that i receives for the traffic that goes from node a to node b. This payment can be further broken down into its constituent VCG parts: the value everyone else gets and the value everyone else would get if i did not exist. We can find these by first defining the total cost incurred in sending a packet from a to b. We let $\textit{lowest-cost-path}(a, b, \tilde{\theta})$ be the lowest cost path – a list of nodes – for sending a packet from a to b given $\tilde{\theta}$. The total cost associated with that path is given by

$$\textit{cost}(a, b, \tilde{\theta}) = \sum_{i \in \textit{lowest-cost-path}(a,b,\tilde{\theta})} \tilde{\theta}_i, \tag{8.6}$$

which is simply the sum of the costs in all the intervening nodes. We can then determine that the payment that i gets is given by

$$p_i(a, b, \tilde{(\theta)}) = \tilde{\theta}_i - \textit{cost}(a, b, \tilde{\theta}_i) + \textit{cost}_{-i}(a, b, \tilde{\theta}_i), \tag{8.7}$$

where $\textit{cost}_{-i}(a, b, \tilde{\theta}_i)$ is defined in the same as *cost* but for a graph without agent i. Note how this payment equation is really just the VCG payments once again. The first two terms capture the value that everyone else receives while the third term capture the value that everyone else receives if i did not exist; since i does not exist he does not contribute cost. The main difference is that the signs are reversed since we are dealing with costs and not value.

We now note that since the payments depend solely on the costs of the lowest cost path from i to j then some of these costs could be found by adding other costs. For example, if the lowest cost path from i to j involves first going from i to a directly and then taking the lowest cost path from a to j then the $\textit{cost}(i, j, \tilde{\theta}) = \tilde{\theta}_i + \textit{cost}(a, j, \tilde{\theta})$. A careful analysis of these type of constraints leads us to deduce that for all $k \in \textit{lowest-cost-path}(i, j, \tilde{\theta})$ where a is a neighbor of i, we have that:

1. If a is i's parent in $\textit{lowest-cost-path}(i, j, \tilde{\theta})$ then $p_k(i, j) = p_k(a, j)$.
2. If a is i's child in $\textit{lowest-cost-path}(i, j, \tilde{\theta})$ then $p_k(i, j) = p_k(a, j) + \tilde{\theta}_i + \tilde{\theta}_a$.
3. If i is neither a parent or a child of i and $k \in \textit{lowest-cost-path}(a, j, \tilde{\theta})$ then $p_i(i, j) = \tilde{\theta}_a + \textit{cost}(a, j) = \textit{cost}(i, j)$.
4. If i is neither a parent or a child of i and $k \notin \textit{lowest-cost-path}(a, j, \tilde{\theta})$ then $p_k(i, j) = \tilde{\theta}_k + \tilde{\theta}_a + \textit{cost}(a, j) - \textit{cost}(i, j)$.

We can then build an algorithm where the agents send each other payment tables and then update their own payment tables using the ones they received along with the above equations. Figure 8.3 shows such an algorithm. All agents start by executing the procedure INITIALIZE and then send HANDLE-UPDATE to their neighbors in order to get the process going. The algorithm has been shown to converge to the true prices for agents.

Note that the calculation of *lowest-cost-path* is itself a distributed calculation that is being carried out simultaneously by the BGP algorithm. This means that the values of *lowest-cost-path* are also changing and, thus, might be incorrect. However, it has been shown that the payment calculations will converge even in the presence of such changes.

```
INITIALIZE()
1  for j ← 1...n ▷ for each destination j
2      do calculate lowest-cost-path(i, j) and cost(i, j)
3          for k ∈ lowest-cost-path(i, j)
4              do p_k[i, j] ← ∞

HANDLE-UPDATE(a, j, cost-aj, path, payments)
    ▷ a is the agent sending the message (invoking this procedure).
    ▷ j is the destination node.
    ▷ cost-aj is the reported cost of sending a packet from a to j.
    ▷ path is a list of nodes on the lowest cost path from a to j.
    ▷ payments are the payments for each node on path.
1   modified ← FALSE
2   if a ∈ lowest-cost-path(i, j) ▷ Parent
3       then for k ∈ path
4                do if p_k[i, j] > p_k[a, j]
5                       then p_k[i, j] ← p_k[a, j]
6                            modified ← TRUE
7   elseif i ∈ lowest-cost-path(a, j) ▷ Child
8       then for k ∈ {All but last item in path}
9                do if p_k[i, j] > p_k[a, j] + θ̃_a + θ̃_i
10                      then p_k[i, j] ← p_k[a, j] + θ̃_a + θ̃_i
11                           modified ← TRUE
12  else
13          t ← position of last node for which path equals lowest-cost-path(i, j)
14          for k ∈ lowest-cost-path(i, j)[1..t]
15               do if p_k[i, j] > p_k[a, j] + θ̃_a + cost-aj − cost(i, j)
16                      then p_k[i, j] ← p_k[a, j] + θ̃_a + cost-aj − cost(i, j)
17                           modified ← TRUE
18          for k ∈ lowest-cost-path(i, j)[t + 1..]
19               do if p_k[i, j] > θ̃_k + θ̃_a + cost-aj − cost(i, j)
20                      then p_k[i, j] ← θ̃_k + θ̃_a + cost-aj − cost(i, j)
21                           modified ← TRUE
22  if modified
23      then my-payments ← p_k[i, j] for k ∈ lowest-cost-path(i, j)
24          for b is my neighbor
25               do b.HANDLE-UPDATE(i, j, cost(i, j),
                                    lowest-cost-path[i, j], my-payments)
```

Figure 8.3: Algorithm for calculating VCG payments for the inter-domain routing problem. i refers to the agent running the algorithm. All agents start by executing INITIALIZE and then send a HANDLE-UPDATE to all their neighbors.

8.2.3 Mechanism Design Summary

Mechanism design provides us with a solid mathematical framework for expressing many multiagent design problems. Specifically, we can represent problems involving selfish agents who are intelligent enough to lie if it will increase their utility. Furthermore, the Groves-Clarke and VCG payment equations give us a ready-made solution for these problems. Unfortunately, this solution only works if we can implements payments and, even then, the payments are not revenue-neutral so we need an infinite amount of cash to give the agents. Also, these solutions are centralized in that all types are reported to a central agent who is then in charge of calculating and distributing the payments.

Distributed mechanism design is a new research area: few effective algorithms exist and many roadblocks are visible. Different approaches are being taken to overcome these roadblocks. One approach is to extend the mechanism design problem by defining trust-based mechanisms in which agents take into account the degree of trust they have on each other when determining their allocations (Dash et al.,

2004). Another approach is to distribute the calculation of VCG payments by using redundantly asking agents to perform sub-problems (Parkes and Shneidman, 2004) and use encryption (Monderer and Tennenholtz, 1999).

Chapter 9

Coordination Using Goal and Plan Hierarchies

Goal, plan, and policy hierarchies have proven to be very successful methods for coordinating agents. In these approaches we assume the existance of one of these hierarchies and the problem then becomes that of determining which parts of the hierarchy are to be done by which agents. In this setting we assume that the agents are coopertive, that is, they will do exactly what we tell them to. The problem is one of finding an good-enough answer.

Hierarchies like the ones we show here are a way to solve the problem of finding an optimal policy in multiagent MDPs. That is, since the traditional mathematical tools for solving those problems are computationally intractable for real-world problems, researchers hoping to build real-world systems have had to find ways to add more of their domain knowledge into the problem description so as to make it easier to find a solution. In general, the more we tell the agent about the world the easier it is to solve the problems it faces. Unfortunately, if we want to tell the agent more information about its domain we need more sophisticated data structures. In this chapter we describe one of the most popular such data structures: TÆMS. Once we have these new data structures we will generally need new algorithms which take advantage of the new information. The algorithms used for TÆMS are captured in the GPGP framework which we also discuss.

9.1 tæms

tæms is a language for representing large task hierarchies that contain complex constraints among tasks. You can think of it as a data structure for representing very complicated constraint optimization problems (Lesser et al., 2004). At its simplest, TÆMS represents a goal hierarchy. As such, TÆMS structures are roughly tree-shaped. The root is the top-level goals that we want the system to achieve. The children are the sub-goals that need to be achieved in order for the top goal to be achieved—a form of divide and conquer. The leafs of the tree are either goals that can be achieved by a single agent or tasks that can be done by an agent. These goals and tasks, however, might require the use of some resources or data. TÆMS represents these requirements with an arrow from the data or resource to the goal.

TÆMS

Figure 9.1 shows a simple example TÆMS structure. It tells us that in order to achieve goal G_0 we must do G_1 and G_2 and G_3. But, in order to achieve G_3 we must achieve either G_{23} or G_{31}. Note also how G_{23} is a subgoal of both G_2 and G_3, which makes this a graph instead of a tree. The bottom row shows two data elements and two resources. Data elements are pieces of data that are needed in order to achieve the goals to which they are connected. Resources are consumable resources that are needed by their particular goals. The difference between data and resource is that a piece of data can be re-used as many times as needed while resources are consumed every time they are used and must therefore be replenished at some point. TÆMS also allows one to annotate links to resources to show how much of the resource is needed in order to achieve a particular goal. We can see that G_1 requires $data_1$ while G_{22} requires $data_1$ and $resource_1$.

Figure 9.1 also shows **non local effects**, namely the *enables* constraint. The directed link (in red) from G_1 to G_{22} indicates that G_1 enables G_{22} which means

NON LOCAL EFFECTS

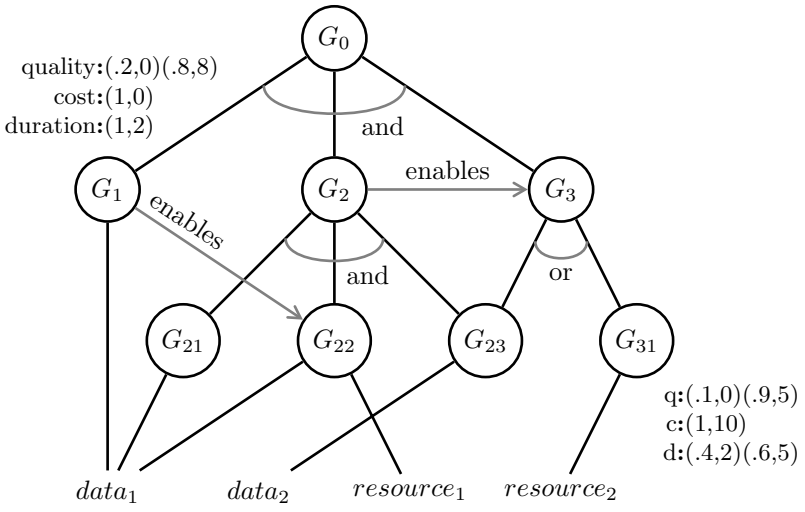


Figure 9.1: An example of a simple TÆMS structure.

Function	Description
q_{min}	minimum quality of all subtasks
q_{max}	maximum quality of all subtasks
q_{sum}	aggregate quality of all subtasks
q_{last}	quality of most recently completed subtask
q_{sum_all}	as with q_{sum} but all subtasks must be completed
q_{seq_min}	as with q_{min} but all subtasks must be completed in order
q_{seq_max}	as with q_{max} but all subtasks must be completed in order

Figure 9.2: Some of the quality accumulation functions available for a TÆMS structure.

that G_1 must be achieved before we can start to work on G_{22} if we want G_{22} to be achieved with quality.

The TÆMS formal model calls for each leaf goal or task to be described by three parameters: quality, cost, and duration. Figure 9.1 shows examples of these parameter values for only two nodes (G_1 and G_{31}) due to space constraints. In practice all leaf nodes should have similar values. These parameters represent the quality we expect to get when the goal is achieved (that is, how well it will get done), the cost the system will incur in achieving that goal, and the duration of time required to achieve the goal (or perform the task). The values are probabilistic. For example, G_{31} will with .1 probability be achieved with a quality of 0 (that is, it will fail to get achieved) while with a probability of .9 it will be achieved with quality of 5. G_{31} also has a fixed cost of 10 and it will take either 2 time units to finish, with probability .4, or 5 units to finish, with probability .6.

Figure 9.1 shows two ways of aggregating the quality of children nodes: the and and or functions. These are boolean functions and require us to establish a quality threshold below which we say that a goal has not been achieve. TÆMS usually assumes that a quality of 0 means that the task was not achieved and anything higher means that the task was achieved. For example, G_0 will only be achieved if (because of the *and*) G_1 is achieved, which only happens with probability .8 each time an agent tries.

QUALITY ACCUMULATION FUNCTIONS

TÆMS allows us to use many different functions besides simply *and* and *or*. Table 9.2 shows some of the **quality accumulation functions** (qafs) that are defined in TÆMS. Of course, the more different qafs we use the harder it will be, in general, to find a good schedule for the agents. Scheduling algorithms must always limit themselves to a subset of qafs in order to achieve realistic running times.

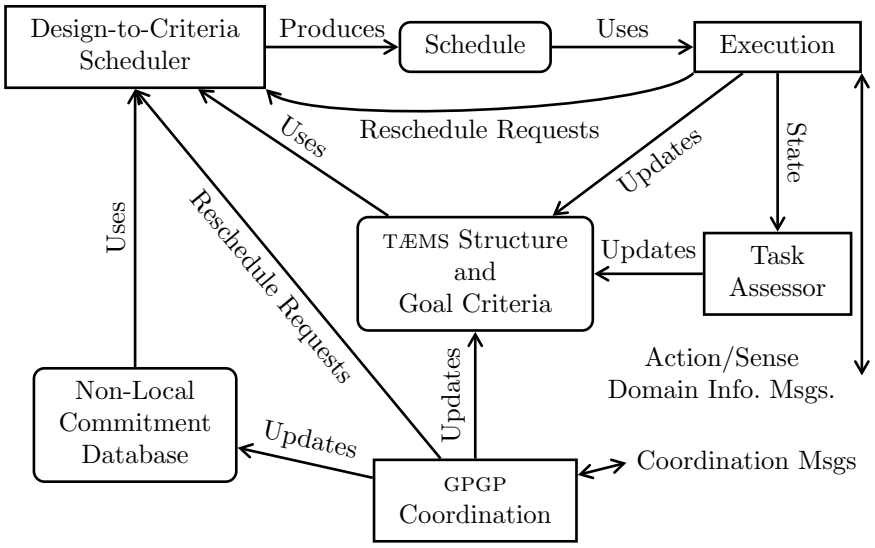


Figure 9.3: GPGP agent architecture. Components are in squares and data structures are in rounded squares.

9.2 GPGP

Generalized Partial Global Planning (GPGP) is the complete framework for multiagent coordination which includes, as one of its parts the TÆMS data structure. Over the years there have been many GPGP agent implementations each one slightly different from the others but all sharing some basic features. They have been applied to a wide variety of applications, from airplane repair scheduling, to supply chain management, to distributed sensor networks. The TÆMS data structure and the GPGP approach to coordination via hierarchical task decomposition are among the most successful and influential ideas in multiagent systems.

9.2.1 Agent Architecture

Figure 9.3 shows a typical agent architecture using GPGP. The red squares are software components. These typically operate in parallel within the agent. The black rounded squares are some of the most important data structures used by a GPGP agent.

The TÆMS data structure lies at the heart of a GPGP agent. This TÆMS structure is also annotated with the goal criteria. That is, it specifies which goals the agent should try to achieve. Remember that the basic TÆMS structure only lists all possible goals, it does not say which agent should try to achieve which goal. The TÆMS structure used by an agent does contain information that tells the agent who is trying to achieve which task. The agent gets this information from the GPGP coordination module which we will explain later.

The design to criteria scheduler is a component (program) which takes as input a TÆMS data structure, along with some commitments the agent has made, and generates a schedule. We will discuss the commitments in more detail later but, basically, the agent has the ability to make commitments to other agents that say that it will achieve a certain goal by a certain time. Once an agent has made a commitment the design to criteria scheduler does everything possible to ensure that the schedule it produces satisfies that commitment. The schedules produced by the scheduler are simply lists of tasks or goals the agent must execute and the times at which the agent will execute them.

The execution module handles plan execution as well as monitoring. That is, it uses the schedule generated by the scheduler and instructs the agent to perform the next task according to the schedule. It then uses the agent's sensors to make sure that the task was accomplished. In the case of a failure, or if it notices that the next task/goal is impossible to achieve in the current world state, it will ask the design to criteria scheduler to generate a new schedule. The execution module also sends

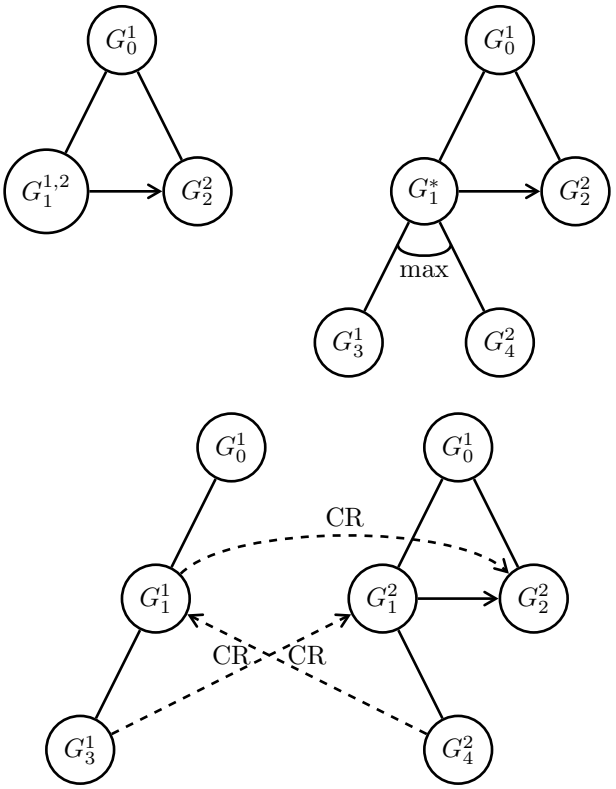


Figure 9.4: Example of how GPGP forms coordination relationships. Superscripts on the goals indicate the number of the agents that can achieve that goal. The original TÆMS structure, on the top left, is expanded, top right, and two graphs generated from it and given to agents 1 and 2, bottom. The coordination relationships are then added.

all the information it has about the world and the schedule's execution state to the task assessor module. The task assessor module has domain-specific knowledge and can determine if there are changes that need to be made to the TÆMS structure. For example, it might determine that one of the subgoals is no longer needed because of some specific change in the world. The execution module can also, in a more generalized manner, make changes to the TÆMS structure. These changes are all incorporated into the current TÆMS structure.

9.2.2 Coordination

In parallel with all these modules, the GPGP coordination module is always in communication with other agents and tries to keep the agent's commitments and TÆMS structure updated. The GPGP coordination module implements (mostly) domain independent coordination techniques and is the module that tells the agent which of the goals and tasks in its TÆMS structure it should achieve.

Figure 9.4 gives a pictorial view of the general steps the GPGP module follows in order to achieve coordination among agents. The coordination module starts with the TÆMS structure shown on the left. The structure is then modified so that every goal that can be achieved by more than one agent is replaced by a new dummy goal which has one child for every agent that could perform the goal. Each one of the children is a new goal that can only be performed by one child. The children are joined by a max qaf. In Figure 9.4 we see show $G_1^{1,2}$, which could be achieved by either 1 or 2 (as denoted by the superscript) is replaced by a new G_1 with two children G_3^1 and G_4^2.

The resulting graph is then divided among the agents such that each agent has the subgraph with all the goals that the agent can achieve plus all the ancestors of these goals (all the way to the root). This division can be seen in the last two graphs of Figure 9.4.

COORDINATION RELATIONSHIPS

The GPGP coordination module identifies **coordination relationships** (CR). These come from two different situations: when a non-local effect in the original graph now starts in one graph and ends in another, or when a non-local effect

or a subtask relationship has one end in one subgraph but the other end in both subgraphs.

Notice that these CRs denote situations where one of the agents needs to either wait for the other to finish or to determine whether the other one has failed. The GPGP module uses some mechanism to remove the uncertainty that these CRs cause. Specifically, it makes the agent commit to do those actions that are at the initiating end of the CR to the other agent. For example, in Figure 9.4 the GPGP module would make agent 1 send to agent 2 the messages: "Commit ($\mathbf{Do}(G_1)$)" and "Commit ($\mathbf{Do}(G_3)$)".

9.2.3 Design-to-Criteria Scheduler

The design-to-criteria scheduler is a complex piece of software which uses search along with a bunch of heuristics to find a good schedule given the current TÆMS structure and the set of commitments. The scheduler only tries to schedule the first few tasks since it knows that it is likely that the problem will change and it will need to re-schedule. The scheduler also sometimes needs to create temporary commitments and generate schedules using those commitments for the GPGP coordination module. That is, when the GPGP coordination module wants to know how a certain commitment might affect the agent's other commitments it will ask the design-to-criteria scheduler to create a new schedule using those commitments.

The job of the scheduler is very complicated. It must decide how to satisfy goals in the TÆMS() structure and which ones are likely to generate the maximum utility. As such, the scheduler is a large piece of code which is generally considered a black box by GPGP users. They just feed it the TÆMS structure and out comes a schedule. The scheduler solves this problem by using a number of search heuristics and other specialized techniques.

9.2.4 GPGP/tæms Summary

The GPGP/TÆMS framework utilizes some key concepts. It views the problem of coordination as distributed optimization, in fact, it reduces complex problems to something similar to the distributed constraint optimization problem of Chapter 2. It provides a family of coordination mechanism for situation-specific control. It uses TÆMS to provide proven domain-independent representation of agent tasks, with support for multiple goals of varying worth and different deadline. It also uses quantitative coordination relationships among tasks.

The general approach of GPGP has been widely successful and has proven itself in many domains. The TÆMS structure has been adopted not just as a programming tools but also as a design tool to be used to better understand the problem. However, one problem with GPGP has been that it has required modifications for each one of its applications. As such, GPGP is best thought of as a general approach to designing multiagent systems rather than as a specific tool/architecture.

Chapter 10

Nature-Inspired Approaches

(Abelson et al., 2000), (Nagpal, 2002)

10.1 Ants and Termites

(Kube and Bonabeau, 2000), (Bonabeau et al., 1999), (Parunak et al., 2002), (Sauter et al., 2002), (Parunak, 1997)

10.2 Immune System

(Kephart and White, 1991), (Kephart, 1994)

10.3 Physics

(Cheng et al., 2005)

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