UFSM - Droop $\qquad \dot{x} = \tilde{f}(x)$ $\qquad$ 1

$$\begin{cases} \dot{x}_1 = -w_f x_1 + \dfrac{w_f\left[V\left(E_r - n(x_1 - P_r)\right)\cos x_3 - V^2\right]}{R_o} \\ \dot{x}_2 = -w_f x_2 - \dfrac{w_f V\left[E_r - n(x_1 - P_r)\right]\operatorname{sen} x_3}{R_o} \\ \dot{x}_3 = m(x_2 - Q_r) \end{cases}$$

em que

$$x_1 = P_f \in [0, 25K]$$

$$x_2 = Q_f \in [-70K, 5K]$$

$$x_3 = \delta \in [-0{,}02, 0.1]$$

e $\quad Q_r = 0$

Ponto de equilíbrio

$$\dot{x} = \tilde{f}(x) = 0 \iff x_e = [\underbrace{1.6.10^4}_{x_{e1}} \;\; 0 \;\; 0]'$$

Mudanças de variáveis (var. de desvio ou incrementais)

$$x_\delta = x - x_e \Rightarrow x_{\delta 1} = x_1 - x_{e1}, \quad x_{\delta 2} = x_2, \quad x_{\delta 3} = x_3$$

Logo,

$$x_{\delta 1} \in [-x_{e1}, 25K - x_{e1}]$$

$$x_2 \in [-70K, 5K]$$

$$x_3 \in [-0{,}02, 0.1]$$

Reescrevendo tq $f(\bar{x})=0$:

$$x_{\delta 1} = x_1 - x_{e1} \Rightarrow x_1 = x_{\delta 1} + x_{e1}$$

$$\dot{x}_{\delta 1} = -w_f x_{\delta 1} - w_f x_{e1} + \frac{w_f}{R_0} V E_r \cos x_3$$

$$- \frac{w_f}{R_0} V n x_{\delta 1} \cos x_3 - \frac{w_f}{R_0} V n x_{e1} \cos x_3$$

$$+ \frac{w_f}{R_0} V n P_r \cos x_3 - \frac{w_f}{R_0} V^2$$

$$\dot{x}_{\delta 1} = -w_f x_{\delta 1} - \frac{w_f}{R_0} V n x_{\delta 1} \cos x_3 + \frac{w_f}{R_0} V (E_r - n(x_{e1} - P_r)) \cos x_3$$

$$- w_f \left(x_{e1} + \frac{V^2}{R_0} \right)$$

$$\dot{x}_{\delta 1} = -w_f x_{\delta 1} - \frac{w_f}{R_0} V n x_{\delta 1} \cos x_3 + \varphi(x_3)$$

$$\varphi(x_3) = \frac{w_f}{R_0} \left(V (E_r - n(x_{e1} - P_r)) \cos x_3 - V^2 \right) - w_f x_{e1}$$

Observe que, $\varphi(0) = 0$ e $\varphi'(0) = 0$ então

$$\lim_{x_3 \to 0} \frac{\varphi(x_3)}{x_3} = 0 \quad \text{(por L'Hôpital)}$$

Logo,

$$\dot{x}_{\delta_1} = -\omega_f \, x_{\delta_1} - \frac{w_f}{R_0} V_n \, x_{\delta_1} \cos x_3 + \frac{\varphi(x_3)}{x_3} \cdot x_3$$

$$\dot{x}_{\delta_1} = \left[-w_f - \frac{w_f}{R_0} V_n \cos x_3 \; , \; 0 \; , \; \frac{\varphi(x_3)}{x_3} \right] \begin{bmatrix} x_{\delta_1} \\ x_2 \\ x_3 \end{bmatrix}$$

$$\dot{x}_{\delta_1} = \left[z_{11}(x_3) \; , 0 \; , \; z_{12}(x_3) \right] x_\delta \, ,$$

em que

$$z_{11}(x_3) = -w_f - \frac{w_f}{R_0} V_n \cos x_3$$

$$z_{12}(x_3) = \frac{\varphi(x_3)}{x_3}$$

$$\text{e} \quad x_\delta = \begin{bmatrix} x_{\delta_1} \\ x_2 \\ x_3 \end{bmatrix}$$

ou

$$\dot{x}_{\delta_1} = \left[-w_f - \frac{w_f}{R_0} V_n \, z_{11}(x_3) \, , 0 , \; z_{12}(x_3) \right] x_\delta$$

em que

$$z_{11}(x_3) = \cos x_3$$

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$$\dot{x}_2 = -w_f\, x_2 + \frac{w_f}{R_o} V n\, x_{\delta_1} \operatorname{sen} x_3$$

$$- \frac{w_f}{R_o} V \left[E_r - n\,(x_{e_1} - P_r) \right] \operatorname{sen} x_3$$

$$\dot{x}_2 = \left[\frac{w_f}{R_o} V n \operatorname{sen} x_3 \;,\; -w_f \;,\; -\frac{w_f}{R_o} V \left[E_r - n\,(x_{e_1} - P_r) \right] \frac{\operatorname{sen} x_3}{x_3} \right] x_\delta$$

$$\dot{x}_2 = \left[\frac{w_f}{R_o} V n\, z_{21}(x_3) \;,\; -w_f \;,\; -\frac{w_o}{R_o} V \left[E_r - n\,(x_{e_1} - P_r) \right] z_{22}(x_3) \right] x_\delta$$

em que

$$z_{21}(x_3) = \operatorname{sen} x_3$$

$$z_{22}(x_3) = \frac{\operatorname{sen} x_3}{x_3} \quad , \text{obs.: } z_{22}(0) = 1$$

(por L'Hôpital)

Finalmente,

$$\dot{x}_3 = m\,(x_2 - Q_f)$$

$$\dot{x}_3 = m \cdot x_2 \quad , \quad Q_f = 0$$

$$\dot{x}_3 = [\,0 \;,\; m \;,\; 0\,]\, x_\delta$$