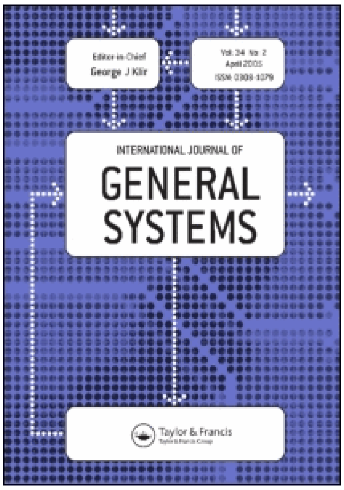

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FUZZY SETS AND SYSTEMS*

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FUZZY SETS AND SYSTEMS*

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The notion of fuzziness as defined in this paper relates to situations in which the source of imprecision is not a random variable or a stochastic process, but rather a class or classes which do not possess sharply defined boundaries, e.g., the "class of bald men," or the "class of numbers which are much greater than 10," or the "class of adaptive systems," etc.

A basic concept which makes it possible to treat fuzziness in a quantitative manner is that of a fuzzy set, that is, a class in which there may be grades of membership intermediate between full membership and non-membership. Thus, a fuzzy set is characterized by a membership function which assigns to each object its grade of membership (a number lying between 0 and 1) in the fuzzy set.

After a review of some of the relevant properties of fuzzy sets, the notions of a fuzzy system and a fuzzy class of systems are introduced and briefly analyzed. The paper closes with a section dealing with optimization under fuzzy constraints in which an approach to problems of this type is briefly sketched.

1. INTRODUCTION

This paper constitutes a very preliminary attempt at introducing into system theory several concepts which provide a way of treating fuzziness in a quantitative manner. Essentially, these concepts relate to situations in which the source of imprecision is not a random variable or a stochastic process but rather a class or classes which do not possess sharply defined boundaries. An example of such a "class" is the "class" of adaptive systems. A simpler example is the "class" of real numbers which are much larger than, say, 10. Still another example is the "class" of bald men. Clearly, such classes are not classes or sets in the usual mathematical sense of these terms, since they do not dichotomize all objects into those that belong to the class and those that do not.

One way of dealing with classes in which there may be intermediate grades of membership was described in a recent paper.[1] The approach in question is based on the use of the concept of a "fuzzy set," that is, a class in which there may be a continuous infinity of grades of membership, with the grade of membership of an object x in a set A represented by a number $\mu_A(x)$ in the inverval $[0,1]$. Thus, a fuzzy set A in a space of objects $X=\{x\}$ is characterized by a membership function μ_A which is defined on X and takes values in the interval $[0,1]$, such that the nearer the value of $\mu_A(x)$ to unity, the higher the grade of membership of x in A. As a simple example, let A be the fuzzy set of real numbers which are much

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greater than 10. In this case, a set of representative values of $\mu_A(x)$ may be: $\mu_A(10)=0$; $\mu_A(50)=0.6$; $\mu_A(100)=0.9$; $\mu_A(500)=1$; etc. In general, the values of $\mu_A(x)$ would be specified on a subjective rather than an objective basis.

2. CONCEPTS RELATING TO FUZZY SETS

As a preliminary to sketching a possible role for fuzzy sets in system theory, it will be helpful to summarize some of their main properties.* To begin with, it is clear that when there is no fuzziness in the definition of a class A, A is a set or a class in the ordinary sense of this term,† and its membership function reduces to the familiar two-valued characteristic (indicator) function for A, with $\mu_A(x)$ being one or zero according as x belongs or does not belong to A.

The notion of a fuzzy set provides a convenient way of defining *abstraction*—a process which plays a basic role in human thinking and communication. Specifically, suppose that A is a fuzzy set with an unknown membership function μ_A. Furthermore, suppose that one is given a set of n samples from A of the form

$$(x_1, \mu_A(x_1)), \ldots, (x_N, \mu_A(x_N)),$$

where $x_i, i=1,\ldots,N$, is an object in X and $\mu_A(x_i)$ is its grade of membership in A. Then, by abstraction on these samples is meant the formation of an estimate, $\tilde{\mu}_A$, of the membership function of A in terms of the specified values of $\mu_A(x)$ at the points $x_1,\ldots,x_N$.

Many of the problems in pattern classification involve abstraction in the sense defined above. For example, suppose that we are concerned with devising a test for differentiating between handwritten letters O and D. One approach to this problem would be to give a set of handwritten letters and indicate their grades of membership in the fuzzy sets O and D. On performing abstraction on these samples, one obtained the estimates $\tilde{\mu}_O$ and $\tilde{\mu}_D$ of μ_O and μ_D, respectively. Then, given any letter x which is not one of the given samples, one can calculate its grades of membership in O and D, and, if O and D have no overlap, classify x in O or D.

To make abstraction mathematically meaningful, it is necessary to have enough *a priori* information about the membership function μ_A to make it possible to assess or at least place bounds on the error in the estimate of μ_A. As stated in reference 2, a disconcerting aspect of the problem of abstraction is that the human mind can perform abstraction very effectively even when the problems involved are not mathematically well defined. It is this lack of understanding of the way in which humans can abstract in mathematically ill-defined situations and our consequent inability to devise abstracting devices which can perform even remotely as well as the human mind, that lie at the root of many unresolved problems in heuristic programming, pattern recognition and related problem areas.

There are several concepts relating to fuzzy sets which will be needed in later discussions. These are:

Equality Two fuzzy sets A and B in a space X are equal, written $A=B$, if and

*A more detailed exposition of fuzzy sets and their properties is given in references 1 and 2.

†When it is necessary to emphasize the distinction between fuzzy sets and ordinary sets, T. Cover has suggested that the latter be referred to as *crisp* sets. We shall follow his suggestion in this paper.

only if $\mu_A(x)=\mu_B(x)$ for all x in X. (In the sequel, to simplify the notation we shall follow the convention of suppressing an argument to indicate that an equality or inequality holds for all values of that argument.)

Containment A fuzzy set A is *contained* in a fuzzy set B, written as $A \subset B$, if and only if $\mu_A \leqq \mu_B$. In this sense, the fuzzy set of "very tall men" is contained in the fuzzy set of "tall men." Similarly, the fuzzy set of real numbers which are "much greater than 10" is contained in the crisp set of real numbers which are "greater than 10."

Complementation A fuzzy set A' is the *complement* of A if and only if $\mu_{A'} = 1 - \mu_A$.

Union The *union* of two fuzzy sets A and B is denoted by $A \cup B$ and is defined as the smallest fuzzy set containing both A and B. An immediate consequence of this definition is that the membership function of $A \cup B$ is given by

$$\mu_{A \cup B}(x) = \max[\mu_A(x), \mu_B(x)]. \tag{1}$$

Thus, if at a point x, $\mu_A(x)=0.8$, say, and $\mu_B(x)=0.5$, then at that point, $\mu_{A \cup B}(x)=0.8$.

Intersection The intersection of two fuzzy sets A and B is denoted by $A \cap B$ and is defined as the largest fuzzy set contained in both A and B. The membership function of $A \cap B$ is given by

$$\mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)]. \tag{2}$$

In this case, if $\mu_A(x)=0.8$ and $\mu_B(x)=0.5$, then $\mu_{A \cap B}(x)=0.5$.

So far, we have not made any restrictive assumptions either about X or μ_A (other than $0 \leqq \mu_A(x) \leqq 1$ everywhere on X). In what follows, we shall assume for concreteness that $X=E^n$ (Euclidean n-space) and define a few more notions* relating to fuzzy sets which will be of use at later points.

Shadow Consider a fuzzy set A in E^n which is characterized by a membership function $\mu_A(x_1, \ldots, x_n)$, with $x=(x_1, \ldots, x_n)$. Let H be a hyperplane in E^n. Then, the *orthogonal shadow* (or simply *shadow*) of A on H is a fuzzy set $S_H(A)$ in H which is related to A in the following manner.

Let L be a line orthogonal to H and let h be its point of intersection with H. Then,

$$\mu_{S_H(A)}(h) = \sup_{x \in L} \mu_A(x) \tag{3}$$

and

$$\mu_{S_H(A)}(x) = 0 \text{ for } x \notin H.$$

*The notions of shadow, convexity, concavity, etc., can be defined, of course, in the context of more general spaces than E^n. For our purposes, however, it is sufficient to assume that $X=E^n$.

The fuzzy set $S_H(A)$ is called a shadow of A because it is suggestive of a shadow cast by a cloud on a plane.

If H_1 is a coordinate hyperplane $H_1=\{x|x_1=0\}$, then the membership function of the shadow of A on H_1 is given by

$$\mu_{S_{H_1}(A)}(x_2,\dots,x_n)=\sup_{x_1}\mu_A(x_1,\dots,x_n),\ x_1=0$$

$$=0,\ x_1\neq 0. \tag{4}$$

Note that $\mu_{S_{H_1}(A)}$ is analogous to a marginal distribution of a probability distribution in E^n. However, whereas in the case of a marginal distribution, an argument of a distribution is eliminated by integration, in the case of a shadow the elimination occurs through taking the supremum of the membership function.

Let $S_H^*(A)$ denote a cylindrical fuzzy set defined by

$$\mu_{S^*_{H_1}(A)}(x)=\mu_{S_{H_1}(A)}(x_2,\dots,x_n),\ x\in E^n. \tag{5}$$

Then, clearly, $A\subset S^*_{H_1}(A)$. Thus, if

$$H_i=\{x|x_i=0\}, i=1,\dots,n,$$

then A is bounded from above by the intersection of $S^*_{H_1}(A),\dots,S^*_{H_n}(A)$. In symbols,

$$A\subset\bigcap_{i=1}^{n}\cdot S^*_{H_i}(A). \tag{6}$$

Complementary shadow The *complementary shadow* of A on H is denoted by $C_H(A)$ and is defined as the complement (on H) of the shadow of the complement of A. More specifically,

$$\mu_{C_H(A)}(h)=\inf_{x\in L}\mu_A(x),\ x\in H$$

$$=0,\ x\notin H. \tag{7}$$

In terms of complementary shadow, A is bounded from below by the union of the cylindrical fuzzy sets $C^*_{H_1}(A),\dots,C^*_{H_n}(A)$. Thus,

$$\bigcup_{i=1}^{n} C^*_{H_i}(A)\subset A\subset\bigcap_{i=1}^{n} S^*_{H_i}(A). \tag{8}$$

As will be seen later, these bounds are of some use in problems involving optimization under fuzzy constraints.

Convexity A fuzzy set A is *convex* if and only if the sets $\Gamma_\alpha=\{x|\mu_A(x)\geqq\alpha\}$ are

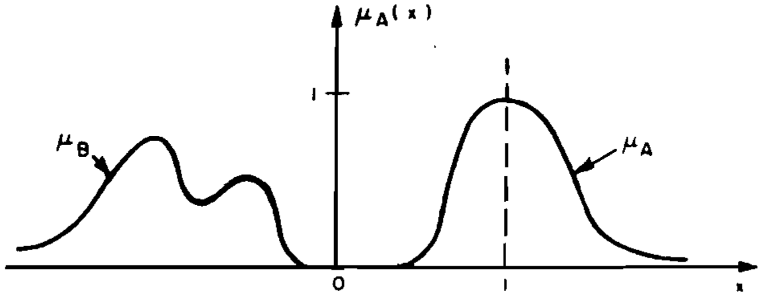

Figure 1 Convex and non-convex sets in E^1.

convex for all α in the interval $(0, 1]$. Equivalently, A is convex if and only if for any pair of points x_1 and x_2 in E^n and any λ in $[0, 1]$, we have

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geqq \min[\mu_A(x_1), \mu_A(x_2)]. \tag{9}$$

For example, the fuzzy set of real numbers which are "approximately equal to 1" is a convex fuzzy set in E^1. The membership function of this set is depicted in Figure 1. In the same figure, μ_B is the membership function of a non-convex fuzzy set in E^1.

Concavity A fuzzy set A is *concave* if and only if its complement A' is convex. The notions of convexity and concavity are duals of one another, as are A and A', union and intersection, $\supset$ and $\subset$, shadow and complementary shadow, etc.

This concludes our brief introduction to some of the basic notions pertaining to fuzzy sets. .In the following, we shall merely indicate a few of the possible applications of these notions in system theory, without any attempt at detailed analysis or exploration.

3. FUZZY SYSTEMS

We begin with the notion of a fuzzy system, by which we mean the following. Let S be a system, with the input, output and state of S at time t denoted by $u(t)$, $y(t)$, and $x(t)$, respectively. Then S is a *fuzzy system* if $u(t)$ or $y(t)$ or $x(t)$ or any combination of them ranges over fuzzy sets. For example, if the input* to S at time t is specified to be "considerably in excess of 5," then this input is a fuzzy set and a system which can act on such imprecisely defined inputs is a fuzzy system. Similarly, if the states of S are described by such fuzzy adjectives as light, heavy, not very heavy, very light, etc., that is, if they are fuzzy sets, then S is a fuzzy system.

Assuming for simplicity that S is a discrete time system, with t ranging over the integers, we can characterize S by the usual state equations

$$x_{t+1} = f(x_t, u_t) \tag{10}$$

*It should be recognized, of course, that such inputs admit of precise definition in terms of fuzzy sets.

$$y_t = g(x_t, u_t) \tag{11}$$

where u_t, y_t and x_t denote, respectively, the input, output and state of S at time t, and f and g are functions of the indicated arguments. The difference between fuzzy and non-fuzzy systems, then, lies in the nature of the ranges of the variables, u_t, y_t and x_t. Thus, in the case of a fuzzy system, one or more of these variables range over spaces where elements are fuzzy sets. Since a fuzzy set is characterized by a scalar-valued membership function, that is, by a functional, this implies in effect that, in the case of a fuzzy system, the values of one or more of the variables u_t, y_t and x_t are functionals.

The situation described above is similar to that encountered in the case of stochastic systems which can be described as deterministic systems in terms of the probability distribution of the input, output and state variables.[3] In the case of such a system, a relation such as (10) would signify that the probability distribution of the states at time $t+1$ is uniquely determined by the probability distribution of the states at time t and the probability distribution of the inputs at time t.

The difference between a stochastic and a fuzzy system is that in the latter the source of imprecision is nonstatistical in nature and has to do with the lack of sharp boundaries of the classes entering into the descriptions of the input, output or state. Mathematically, however, these two types of systems are substantially similar and present comparable and, unfortunately, great difficulties in their analyses. These difficulties stem from the fact that, at present, we do not possess effective computational techniques for dealing with functionals, so that equations such as (10) and (11) are easy to write in symbolic form but difficult to translate into explicit computer programs when the variables involved in them are functionals rather than points in spaces of fairly low dimensionality—which is usually the case with conventional non-fuzzy systems.

4. FUZZY CLASSES OF SYSTEMS

A notion which is related to and yet distinct from that of a fuzzy system is that of a *fuzzy class* of systems. For instance, the "class" of systems which are "approximately equivalent" to a given system S is a fuzzy class of systems; so is the "class" of systems which are "approximately linear"; and so is the "class" of systems which are "adaptive." In fact, one may argue that most of the adjectives used in system theory to describe various types of systems, such as: linear, nonlinear, adaptive, time-invariant, stable, etc., are in reality names for fuzzy classes of systems. If one accepts this point of view, then, one is freed from the necessity of defining these terms in a way that dichotomizes the class of all systems into two classes, e.g., systems that are linear and systems that are nonlinear. More realistically, then one would regard, say, the "class" of adaptive systems as a fuzzy class, with each system having a grade of membership in it which may range from zero to one.

How can a fuzzy class of systems be characterized? The answer to this question depends on the mode of characterization of the systems which form a fuzzy class. More specifically, if one starts with the definition of a system as a set of input-output pairs—which is the point of departure in reference 4—then a fuzzy class of systems would be a fuzzy set in the product space $U \times Y$, where U is the space of inputs and Y is the space of outputs.

To illustrate this point, suppose that a system designer wishes to give a precise, albeit subjective, characterization of the fuzzy class, A, of time-invariant, approximately linear, systems. He could do this, at least in principle, by associating with each input-output pair (u, y) a number in the interval $[0, 1]$, representing its grade of membership in A. For example, he may assign the grade 0.8 to an input-output pair $(t, t+0.001\,t^2)$, $0 \leqq t \leqq 100$; the grade 0.3 to $(t, 0.1+t)$, $0 \leqq t \leqq 100$; and so forth. In this way, A would be characterized as a fuzzy set in the product space of input-output pairs. In practice, of course, the designer could assign grades of membership to only a finite number of input-output pairs (u, y). Thus, in general, one would have to estimate the membership function $\mu_A(u, y)$ from the values of this function over a finite set of sample input-output pairs (u_1, y_1), $(u_2, y_2), \ldots, (u_N, y_N)$. As was pointed out in the beginning of this paper, this is a problem in abstraction—a problem for which we do not possess as yet effective general methods of solution.

Alternatively, if one starts with a characterization of a system in terms of its input-output-state relation or state equations, and if each system is indexed by a parameter λ taking values in a space Λ, then a fuzzy class of systems may be characterized as a fuzzy set in Λ. For example, consider a family of discrete-time systems characterized by the state equations

$$x_{t+1} = f_\lambda(x_t, u_t) \tag{12}$$

$$y_t = g_\lambda(x_t, u_t) \tag{13}$$

in which λ is a real non-negative number. Then a fuzzy class, say A, of such systems would be defined by a fuzzy set, say B, in the parameter space $[0, \infty]$, with the grade of membership of the system with index λ in A being given by the grade of membership of λ in B, that is, by $\mu_B(\lambda)$.

One of the basic problems in the case of non-fuzzy (crisp) systems is that of deriving an input-output-state relation or, equivalently, state equations for a system which is defined by a set of input-output pairs. Recently, a solution to this problem in the context of finite-state automata was described by Tal.[5,6] The same basic, although much more difficult problem presents itself in the case of fuzzy classes of systems. More specifically, suppose that a fuzzy class of discrete-time systems, say A, is defined as a fuzzy set of input-output pairs (u, y) in the product space $U \times Y$. Then, the problem is to find a representation for this fuzzy class in the form of a family of state equations (12) and (13) and a fuzzy set B in the parameter Λ. It hardly needs saying that, in general, this would be an extremely difficult problem—a problem concerning which we know practically nothing at this time.

5. OPTIMIZATION UNDER FUZZY CONSTRAINTS

In the previous two sections, we focused our attention on ways in which fuzziness can enter into the definition of a system or a class of systems. In this section, we briefly touch on a different facet of fuzziness in system theory, namely, the optimization of crisp systems under fuzzy constraints.

Our consideration of this problem is motivated by the fact that in many practical optimization problems, particularly these involving man-machine systems, the constraints on variables are seldom sharply defined. Thus, in many instances the constraints are fuzzy or "soft," in the sense that the variables which they involve are only approximately—rather than precisely—constrained to fall within specified sets. For example, a constraint on a variable x may have fuzzy forms such as "x should not be significantly larger than 5," or "x should be close to 10," or "x should be approximately between 5 and 10," etc.

A standard approach to problems of this type is to idealize a fuzzy constraint by replacing it with an approximating "hard" (that is, crisp) constraint. For obvious reasons, this approach and its variants do not constitute a satisfactory way of handling problems in which the constraints are intrinsically fuzzy, and do not lend themselves to satisfactory approximation with hard constraints. In such cases, it is more natural—and perhaps more efficient computationwise—to deal with fuzzy constraints in the manner sketched below.

As a preliminary, consider the standard problem of maximizing a non-negative objective function $f(x_1,\ldots,x_n)$ over a crisp constraint set A in E^n, that is, subject to the condition that $x \in A$.

Let $\mu_A(x)$ denote the characteristic function of A ($\mu_A(x)=1$ for $x \in A$ and $\mu_A(x)=0$ for $x \notin A$). Then, it is clear that the above problem is equivalent to the maximization of the modified objective function†

$$f^*(x)=f(x)\mu_A(x) \tag{14}$$

without any side conditions. This suggests that, when A is a fuzzy set and hence it is not meaningful to speak of x being constrained to A, the maximization of f over a fuzzy constraint set A be interpreted to mean the maximization of the modified objective function (14) over E^n. In this way, the maximization of $f(x)$, subject to a fuzzy constraint represented by a fuzzy set A, reduces to an unconstrained maximization of the function,

$$f^*(x_1,\ldots,x_n)=f(x_1,\ldots,x_n)\mu_A(x_1,\ldots,x_n). \tag{15}$$

In many optimal control problems, the constraints on x have the form $x_1 \in A_1$, $x_2 \in A_2,\ldots,x_n \in A_n$, where $A_1,\ldots,A_n$ are specified crisp sets in E^n. In such cases, the constraints in question can be replaced by the single constraint on x: $x \in A$, where A is the direct product of $A_1,\ldots,A_n$, that is, $A=A_1 \times A_2 \ldots \times A_n$. Note that the crisp sets $A_1,\ldots,A_n$ may be regarded as the shadows on the crisp set A on the coordinate axes.‡

A natural question that suggests itself at this point is: what if $A_1,\ldots,A_n$ are fuzzy sets? How can we derive A from $A_1,\ldots,A_n$, if the latter are taken to be shadows of A on the coordinate axes?

In the case of fuzzy sets, these questions do not have a unique answer, since A is

†More generally, the modified objective function can be taken to be $f^*(x)=f(x)[\mu_A(x)]^k$, where k is any positive real number. For our purposes, it will suffice to let $k=1$.

‡The membership functions of the shadow of A on the axis $0x_1$ is given by

$$\mu_{A_1}(x)=\sup_{x_2}\sup_{x_3}\ldots\sup_{x_n}\mu_A(x_1,\ldots,x_n)$$

on the axis; $\mu_{A_1}(x)=0$ elsewhere; and similarly for other axes.

not uniquely determined by its shadows.† However, as in (6), one can bound A from above by the intersection of the cylindrical fuzzy sets $A_1^*,\ldots,A_n^*$, where the membership function of A_i^*, $i=1,\ldots,n$, is given by

$$\mu_{A_i}^*(x)=\sup_{x_2}\sup_{x_3}\ldots\sup_{x_n}\mu_A(x_1,\ldots,x_n)=\mu_i(x_i). \tag{16}$$

More specifically

$$A\subset\bigcap_{i=1}^{n}\mu_i(x_i) \tag{17}$$

or equivalently,

$$\mu_A(x)\leqq\min[\mu_1(x_1),\ldots,\mu_n(x_n)]. \tag{18}$$

Then, if—as an approximation—$\mu_A(x)$ is identified with the right member of (18), the modified objective function becomes

$$f^*(x_1,\ldots,x_n)=f(x_1,\ldots,x_n)\min[\mu_1(x_1),\ldots,\mu_n(x_n)]. \tag{19}$$

In many practical situations, this approximation may be quite adequate. Unfortunately, no sharper estimates of A can be made when all we know about A are its shadows on coordinate axes.

The subject of optimization under fuzzy constraints has numerous additional ramifications, a few of which are now in process of exploration. Preliminary results seem to indicate that, in some cases, it may actually be advantageous to approximate to hard constraints by fuzzy constraints and employ steepest ascent techniques or other methods to maximize the modified objective functions. There are many other cases, however, in which optimization under fuzzy constraints is ineffective or computationally infeasible as an alternative to conventional optimization methods for dealing with problems involving crisp constraints.

CONCLUDING REMARKS

In the foregoing sections, we have not attempted to do more than merely touch upon the concept of fuzziness and point to some of its implications in system theory. Whether the particular concepts defined in this paper will prove to be of value in system design or analysis remains to be seen. It is clear, though, that in one form or another, the notion of fuzziness will come to play an important role in pattern classification, control, system optimization and other fields, since fuzziness is a basic and all-pervasive part of life that cannot be avoided merely because it is difficult to deal with precisely.

†As shown in references 1 and 2, a convex fuzzy set is uniquely determined by the totality of its shadows on all hyperplanes in E^n. Dually, a concave fuzzy set is uniquely determined by the totality of its complementary shadows.

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