Control of a multi-robot cooperative team guided by a human-operator

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Abstract

Cooperative manipulation with human guidance can be used to solve versatile tasks. A new approach to system modelling and control in cooperative manipulation is the use of port-Hamiltonian systems. Starting from modelling of a cooperative manipulation set-up a model-based controller in the framework of Intrinsically Passive Control is derived. In contrast to the quasi-static implementations for robot hands, the controller has fully dynamic impedance relations. The good dynamic performance of the proposed control scheme is shown by simulation and compared to simulations of state-of-the-art controllers in cooperative manipulation.

Zusammenfassung

Hier die deutschsprachige Zusammenfassung

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Chapter 1

Introduction

Cooperative manipulation involves two or more robot arms cooperatively moving or manipulating a common object [CM08]. Such a set-up overcomes many limitations seen by single robots. The prime example is the transportation of a large or heavy object, where a single manipulator would exert large local wrenches on the object. Back in the 1940s two remote manipulators were used to handle radioactive goods [Goe52]. Around 1990 the *NASA* researched cooperative manipulation in space construction applications [SC92]. Specifically in [LS05] a cooperative manipulation strategy for the repair of the Hubble Space Telescope is proposed. Other tasks of cooperative manipulation are for example:

- assembly of multiple parts without using special fixtures
- grasping an object without rigid fixture but by exerting a suitable squeezing force
- deforming a flexible object
- coordinated use of tools

A recent application has been presented at the Fukushima Daiichi Nuclear Power Station. By equipping a two-armed robot with different tools the robot is capable of grasping an item (e.g. piping) and cutting it with the other, see Fig. 1.1. For sensitive tools like an angle grinder it is favourable that movement and contact forces of both arms are coordinated.

Space and hazardous environments are fruitful environments for cooperative manipulators. Often they are controlled by a human operator from a safe location. Combining human reasoning and the enhanced flexibility of a cooperative set-up is a powerful combination in an unstructured environment. A human in the control loop comes with superior foresight and planning capabilities. To fully exploit this additional flexibility we require control architectures able to perform all the aforementioned tasks and operable in a comfortable and efficient manner.

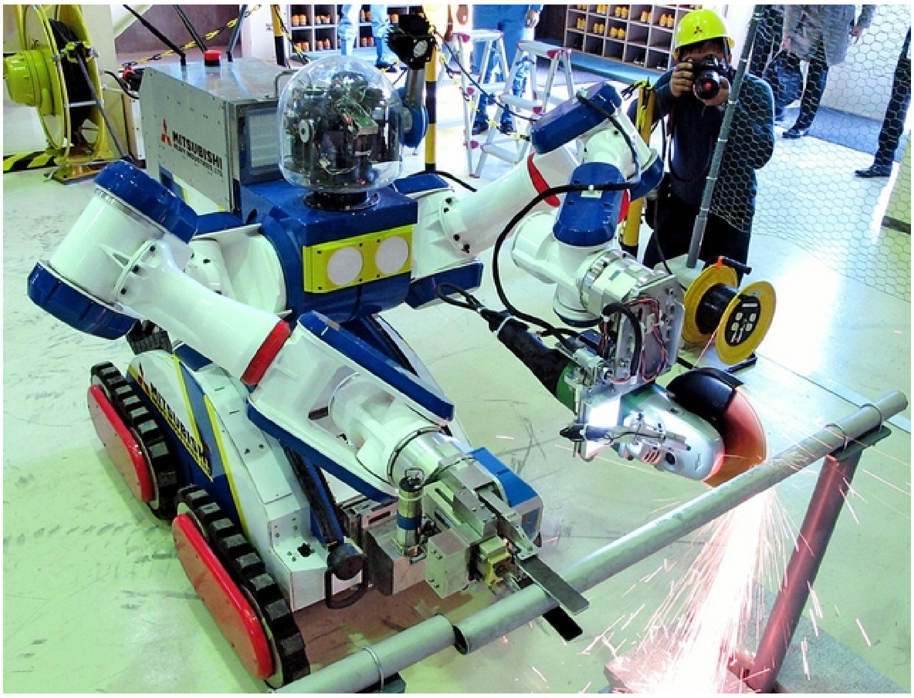

Figure 1.1: Demonstration of the tele-operated MHI MEISTeR robot at Fukushima Daiichi Nuclear Power Station [LTD14]

To address the first requirement the controller realizes changes of formation and relocates the constrained system. A change of formation first of all means the opening and closing of a grasp a around a common object. A virtual object and variable rest-length springs are introduced, the springs couple the virtual and the actual object. The size of virtual object specifies the rest-lengths, by choosing it smaller than the actual a grasping force is exerted. By choosing it bigger the grasp is opened and the actual object is released. Changing of rest-lengths means to change the potential energy stored in the springs. Since the controller itself is passive, the energy is exchanged with a distinct power port. In order to relocate the formation a spring is connected from the virtual object center to the desired new position/orientation. This ensures compliant behavior between (virtual) object and environment. Note that this concept is called *Intrinsically Passive Controller* (IPC) and was introduced by Stramigioli [Str01b], the control architecture described in the main part will closely stick to this concept.
The controller provides a reasonable layer of abstraction and takes responsibilities from the operator, in order to not overstrain her/his attention and let him focus on important elements of the task. To ensure a secure and stable grasp, the formation of robots is locally controlled without the help of the human. For dynamic manipulation tasks grasp force optimization can be utilized to limit internal to the absolute minimum. The control system provides appropriate feedback to provide better perception on the work environment and to make the job more intuitive. The user

specifies system motion and is provided with information about the necessary forces to accomplish the commanded motion. Thereby s/he gets a natural intuition of how much work has been done by the robots. To this end the controller shall never be a source of additional energy, the only way energy is fed into the system is by request of the operator, i.e. the controlled robots are passive. The maximum extractable energy is always bounded, the operator can estimate the stored energy and possible effects when interacting with the environment. Passive robotic systems are always stable when interconnected passive environments/humans and can be stable with some active systems. On the contrary, if the controlled robot is not passive, there is always a passive environment that destabilizes the interconnected system [Str15].

1.1 Problem Statement

Manipulating an unknown object in an unstructured environment is an interesting task for a human-guided cooperative manipulation set-up. Dropping the assumption of a rigid- connection between manipulator and object, the grasp has to be actively stabilized under varying circumstances. During dynamic manipulation a slipping of contact due to the inertia of the object has to be avoided using an automatic mechanism. On the other hand the operator must directly adjust grasp forces for heavy or fragile objects.
Position and velocity of the manipulated object are difficult to track in an everyday setting. Usually only end-effector position and velocity of the robots are available. Object position has to be estimated from the known data.
For grasping an object of unknown or even flexible shape, the determination of a fixed grasp map in advance is not useful. Size and shape of the grasp have to be determined by the operator during the grasp process. The controller has to be flexible to varying grasp geometries during the whole task execution.
Integrating the human into the control loop, energetic passivity of the closed loop system is a meaningful and intuitive way to ensure stability and a natural way of interaction. Energetic passivity limits the extractable mechanical energy from the closed loop system. This means that the potential damage is also limited. Energetic passivity of the robotic system ensures stability of the interconnection stability with any passive systems. Since humans and many relevant environments are passive the closed loop system will always be stable if the overall control scheme is designed energetically passive.

1.2 Related Work

A notable class of control architectures for cooperative manipulation is hybrid position/force control, with a motion control loop for trajectory tracking and a force control loop for internal forces [WKD92, Hsu93]. Their drawback is the inability to handle non-contact to contact transitions. Hogan introduced impedance control,

which enforces a relation between force and motion [Hog84]. Its first application in cooperative manipulation was in [SC92] for realizing compliant object-environment interaction (external impedance control). Bonitz and Hsia [BH96] applied the concept to the manipulator-object relation (internal impedance control).
More recent impedance control schemes can be classified in terms of the information and sensor data available for the problem. Frugal architectures are formation control [SMH15] and the static IPC [WOH06]. Both do not incorporate the object dynamics in control, thus very little knowledge about the object (e.g. dimensions) is necessary. The control loops depend only on relative positions and velocities of the manipulators and do not require object tracking. Neglecting considerable object dynamics is an obvious drawback of the schemes.
The concept of the *Intrinsically Passive Controller* (IPC), introduced by Stramigioli [Str01b] and called dynamic IPC by Wimböck et al. [WOH08], tries to overcome some of the limitations. In the controller the object is represented by a virtual pendent and simulated to reproduce its dynamics for control purpose. This has the advantage that still no tracking of the object is required, object velocity and even acceleration can be obtained from the simulation.
Techniques which rely on exact knowledge of the object motion were introduced by Caccavale et al. [CV01, CCMV08] and more recent [HKDN13] and [DPEZ+15]. It is common to them that they assume rigid fixtures to a rigid object, these conditions overcome the problem of object tracking. The approaches by Caccavale and coworkers use force/torque sensors at the manipulators to establish compliant object environment interaction, for this purpose Heck et al. [HKDN13] assume to have an exact model of the environment. Stramigioli's IPC implements a compliant relation between virtual object and environment.
The human operator must be able control the manipulators at a reasonable degree of complexity, therefore in an direct master-slave approach each robot is controlled independently by a human operator. Exactly coordinating their motions is a difficult task for humans, as a consequence a certain amount of autonomy is left to the robot system, enabling a single operator to control the cooperative system. Lee and Spong [LS05] apply the master-slave scheme but treat the constrained system as a single slave, while the formation is preserved by the robots autonomously. Many master-slave systems give the operator force-feedback, while s/he commands the motion. This helps the operator compensate for resistances and gives a natural feeling of the interaction with the environment. The structure then is fully bi-directional, ones refers to bilateral telemanipulation [NPH08]. Leader-follower [SMH15, SMP14] schemes differ in terms of feedback provided to the operator. Tactile and visual types are non-reactive, i.e. they do not induce operator movements reacting to a backdriving force [MT93]. A purely vision-based architecture is introduced by Gioioso et al. [GFS+14], hand gestures are used to both control the motion of the constrained system and the opening and closing of a grasp. Control architectures that leave even further autonomy to the robot system and possess a closed local, autonomous control loop, are categorized as supervisory control. They interacts with the opera-

tor by continuously sending information about the state and periodically receiving commands [She92].
The assumption of rigid fixtures between manipulators and object is very common in cooperative manipulation, Lee and Spong [LS05] are an exception. Friction grasps are mainly researched in robot hand literature ([WOH06, WOH08, Str01b]). For the stabilization of a friction grasp it is vital to choose appropriate forces depending on the dynamic state. Therefore the Coulomb friction constraints along with other criteria (safety margins, force limits) can be formulated as a cost function for optimization. Buss et al. [BHM96] realized that the Coulomb friction constraints can be formulated as positive definite matrices, Han et al. [HTL00] gave a linear matrix inequality problem. For this type of optimization problems very efficient, real-time solvers exist.

Chapter 2

port-Hamiltonian system modelling

In this chapter a modelling approach of the cooperative system based on the port-Hamiltonian framework is introduced. This aims for a consistent system description over different energetic domains, present in mechanical systems in the form of kinetic and potential energy. The general theory of port-Hamiltonian systems is presented in Section 2.1. The generalization for six-dimensional mechanical system leading to systems defined on manifolds is introduced in Section 2.2. In cooperative manipulation set-ups, manipulators and object are rigidly connected, the arising constraints are treated in Section ??. System modelling is performed by interconnecting standardized mechanical elements using network theory. The composition of a mechanical impedance, suitable for model-based control (see Chapter 3) is shown in Section 2.3.

2.1 port-Hamiltonian description of mechanical systems

For the derivation of the port-Hamiltonian description of a mechanical system we start from the classical *Euler-Lagrange* equations of motion

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = g(q)f, \tag{2.1}$$

where q is the vector of generalized configuration coordinates of the system. The *Lagrangian* $\mathcal{L} = V_k - V_p$ equals the difference between the kinetic co-energy V_k and the potential energy V_p. The kinetic co-energy is explicitly given as $V_k = \frac{1}{2}\dot{q}^T M(q)\dot{q}$, with a symmetric, positive definite inertia matrix $M(q)$. The generalized forces f act on the system with an input matrix $g(q)$. We define the generalized *momenta* for every Lagrangian $p := \frac{\partial \mathcal{L}}{\partial \dot{q}}$ and obtain $p = M(q)\dot{q}$. Introducing the *Hamiltonian* (energy) function $H(q,p) = p^T\dot{q} - \mathcal{L}(q,\dot{q})$ we can rewrite the Euler-Lagrange equation in form of the classical Hamiltonian equations of a mechanical system

$$\begin{aligned} \dot{q} &= \frac{\partial H}{\partial p}(q,p) \\ \dot{p} &= -\frac{\partial H}{\partial q}(q,p) + g(q)f \end{aligned} \tag{2.2}$$

The Hamiltonian describes the total energy stored in the system [vdS06], thus the energy balance is

$$\frac{d}{dt}H = \frac{\partial^T H}{\partial q}(q,p)\dot{q} + \frac{\partial^T H}{\partial p}(q,p)\dot{p} = f^T g^T(q)\dot{q} = f^T e \tag{2.3}$$

Hamiltonian systems are energy conservative, i.e. the energy supplied through the port is stored in the system. In the upper equation a new output $e = g^T(q)\dot{q}$ is introduced. Clearly the product $e^T f$ is the exchanged power and we call the pair (f,e) a *power port*. The general equations of a port-Hamiltonian system are

$$\begin{aligned} \dot{q} &= \frac{\partial H}{\partial p}(q,p) \\ \dot{p} &= -\frac{\partial H}{\partial q}(q,p) + g(q)f \\ e &= g^T(q)\frac{\partial H}{\partial p}(q,p) \end{aligned} \tag{2.4}$$

Port-Hamiltonian systems are suitable to describe a variety of physical systems including mechanical, electrical, thermal and hydraulic elements, see [DMSB09] for an overview. This motivates the more general input-output concept of flows f and efforts e forming the port variables (f,e).

Example 2.1:

Consider a simple one-dimensional spring-mass system described by $m\ddot{x} = -kx + F$, where m,k,F denote the mass, stiffness and external force acting on the mass respectively. We can give a state space formulation of the system

$$\begin{pmatrix} \dot{x} \\ \ddot{x} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{pmatrix} \begin{pmatrix} x \\ \dot{x} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \frac{F}{m} \tag{2.5}$$

In the Hamiltonian approach the system is described based on the Hamiltonian energy functions, being $H_s(q) = \frac{1}{2}kq^2$ for the spring and $H_m(p) = \frac{1}{2m}p^2$ for the

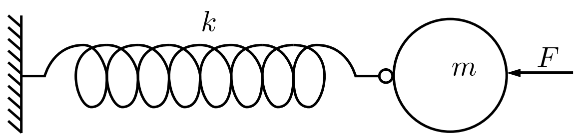

Figure 2.1: Spring-mass example.

Table 2.1: port-Hamiltonian system variables of mechanical elements

	Spring	**Mass**	**Damper**
Effort variable	Force F	Velocity $\dot{x}$	Force F
Flow variable	Velocity $\dot{x}$	Force $F = \dot{p}$	Velocity $\dot{x}$
State variable	Position x	Momentum p	-
Energy function	$E(x) = \frac{1}{2}kx^2$	$E(p) = \frac{p^2}{2m}$	$E(\dot{x}) = D\dot{x}^2$ (diss. co-energy)

mass. The total energy is thus $H(q,p) = H_s + H_m$ The state variables are thus replaced by the energy states, the *configuration* $q = x$ accounting for the spring and the *momentum* $p = m\dot{x}$ accounting for the mass.

$$\begin{aligned} \begin{pmatrix} \dot{q} \\ \dot{p} \end{pmatrix} &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial H}{\partial q}(q,p) \\ \frac{\partial H}{\partial p}(q,p) \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} F \\ e &= \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\partial H}{\partial q}(q,p) \\ \frac{\partial H}{\partial p}(q,p) \end{pmatrix} \end{aligned} \tag{2.6}$$

Key aspect of port-Hamiltonian system is the division into atomic energy storing elements (spring, mass) and their proper interconnection. In the example we have implicitly done this by defining energy functions and states for both elements. The rules of interconnection are explicitly given by:

- equal velocity of spring tip and mass $\dot{x} = \frac{\partial V}{\partial p}$(rigid connection)
- opposite forces at spring tip and mass $\dot{p} = -\frac{\partial V}{\partial q} + F$ (principle of action-reaction)

Due to the *second law of thermodynamics* real mechanical systems are never energy conservative. Thus we require energy dissipating elements (since thermal energy is "lost" w.r.t. the mechanical domain). A mechanical system is thus described by its basic elements: springs, masses and dampers. Table 2.1 gives an overview of the elements and their characterizing quantities. Note that dissipation elements do not have a state because they are static. It is worth pointing out that the *flows* are the time derivatives of the *states*.

Next to *energy-storing* and *-dissipating* there is a third class of elements, namely *energy-conservative* structures. Elements within this group are transformers, gyrators and ideal constraints. They are used to redirect the power flow in the system. It is possible to merge all *energy-storing* elements into a single object representation (see Fig. 2.2). Analogously this can be done for the dissipation elements. The interconnecting structure (denoted by $\mathcal{D}$ in Fig. 2.2), consisting of the *energy-conservative* elements, formalizes the energy routing and geometric dependencies of the system. A detailed explanation is given in the next subsection.

Complex physical systems can be modelled as a network of energy storing and dissipating elements, similar to representation of electrical networks consisting of

resistors, inductors and capacitors. The rules of interconnection are Newton's third law (action-reaction), Kirchhoff's laws and power-conserving elements like transformers or gyrators. The aim of PHS-modelling is to describe the power-conserving elements with the interconnection laws as a geometric structure and to define the Hamiltonian function as the total energy of the system.
The power flowing between the system's portions is described by the flow-effort product $e^T f$. The external port (f_P, e_P) describes the energy flow to/from environment or a controller.

2.1.1 Dirac structures and interconnection ports

The energy-routing structure forms the basis of every port-Hamiltonian system. It can be compared to the printed circuit board in electronics, where capacitors, inductors and resistors are the energy-storing and damping elements. Mathematically it has the form of a *Dirac* structure [vdS06]. The main property of a Dirac structure is power conservation, i.e. the power flowing into and out of it always sums to zero. We can define the set of ports (f, e) connecting to the Dirac structure $\mathcal{D}$, thus

$$e^T f = 0 \ \ \forall (f, e) \in \mathcal{D} \tag{2.7}$$

Where $\mathcal{D}$ is a subspace of the space of flow and effort $\mathcal{D} \subset \mathcal{E} \times \mathcal{F}$. The space of flows is $f \in \mathcal{F}$ the space of efforts is its dual linear space $e \in \mathcal{E} = \mathcal{F}^*$. The Dirac structure has the same dimension than the space of flow $dim\mathcal{D} = dim\mathcal{F}$. Further mathematical requirements can be found in literature [vdS06, vdSJ14]. The interconnection of ports is defined by the Dirac structure matrix D

$$\begin{pmatrix} e_P \\ f_S \\ e_R \end{pmatrix} = D \begin{pmatrix} f_P \\ e_S \\ e_R \end{pmatrix} \tag{2.8}$$

with the internal elements

$$D := \begin{pmatrix} D_P & G_1 & G_2 \\ -G_1^T & D_S & G_3 \\ -G_2^T & -G_3^T & D_R \end{pmatrix} \tag{2.9}$$

and D_P, D_S, D_R are skew-symmetric, G_* are arbitrary matrices. Thus $D = -D^T$ is skew-symmetric, which is necessary for $e^T f = 0$. The power exchanged through a port is given by $e_*^T f_*$. In Fig. 2.2 the port variables entering the Dirac structure have already been split.

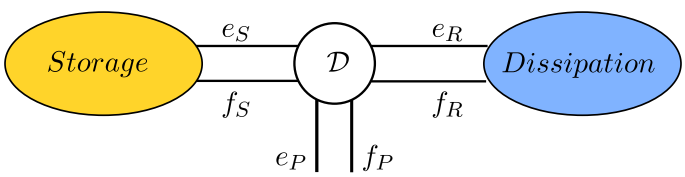

Figure 2.2: port-Hamiltonian system structure

Energy storage port

The port accounts for the internal storage of the system, its port variables are (f_S, e_S). The power supplied through this port is stored in the Hamiltonian energy function $H(x)$ of the system. Here x denotes a general energy state variable. The resulting energy balance is:

$$\frac{d}{dt}H = \frac{\partial^T H}{\partial x}(x)\dot{x} \tag{2.10}$$

The flow variable is the energy rate $f_S = -\dot{x}$ and the effort variable is $e_S = \frac{\partial H}{\partial x}(x)$.

Energy dissipation port

The port corresponds to internal dissipation and can be used to model resistive elements. The port variables are described by the general resistive relation

$$F(f_R, e_R) = 0 \tag{2.11}$$

with the property $e_R^T f_R \leq 0$ (energy dissipation). An important special case is the input-output resistive relation $f_R = -F(e_R)$, for linear elements simply

$$f_R = -Re_R\ ,\ R = R^T \geq 0 \tag{2.12}$$

For an uncontrolled system that does not interact with the environment, i.e. no energy exchange through the external port, the energy balance is:

$$\frac{dH}{dt} = -e_S^T f_S = e_R^T f_R \leq 0 \tag{2.13}$$

External port

The external port (f_P, e_P) can be further split into an environment *interaction* (f_I, e_I) and a *control* port (f_C, e_C), satisfying $e_P^T f_P = e_I^T f_I + e_C^T f_C$. The power balance of the whole system then is

$$e_S^T f_S + e_R^T f_R + e_I^T f_I + e_C^T f_C = 0 \tag{2.14}$$

or by using (2.10)

$$\frac{dH}{dt} = e_R^T f_R + e_I^T f_I + e_C^T f_C \tag{2.15}$$

Interconnection of port-Hamiltonian systems

It is important to notice that the interconnection of two port-Hamiltonian systems is again a port-Hamiltonian system [vdSJ14]. Consider two general systems $(i = 1, 2)$ with open control and environment interaction ports:

$$\begin{aligned} \dot{x}_i &= (J_i - R_i)\frac{\partial H_i}{\partial x_i} + \begin{pmatrix} g_i^C & g_i^I \end{pmatrix} \begin{pmatrix} f_i^C \\ f_i^I \end{pmatrix} \\ \begin{pmatrix} e_i^C \\ e_i^I \end{pmatrix} &= \begin{pmatrix} (g_i^C)^T \\ (g_i^I)^T \end{pmatrix} \frac{\partial H_i}{\partial x_i} \end{aligned} \tag{2.16}$$

where J_i, R_i are a skew-symmetric structure matrix and a positive semi-definite symmetric dissipation matrix. $(g_i^C g_i^I)$ is a general input matrix respectively. For notational convenience the usual dependencies on the states have been omitted. The control inputs and outputs are now connected by setting $f_1^C = e_2^C$ and $f_2^C = -e_1^C$. Note that the minus sign is necessary for power conservation. The power exchanged by the i-th system is $P_i = (e_i^C)^T f_i^C$, therefore the total exchanged energy fulfils $P_1 + P_2 = 0$. The resulting interconnected system has still the environment interaction ports open:

$$\begin{aligned} \dot{x} &= (J - R)\frac{\partial H}{\partial x} + (g_1^I \quad g_2^I)\begin{pmatrix} f_1^C \\ f_2^I \end{pmatrix} \\ \begin{pmatrix} e_1^I \\ e_2^I \end{pmatrix} &= \begin{pmatrix} (g_1^I)^T \\ (g_2^I)^T \end{pmatrix}\frac{\partial H}{\partial x} \end{aligned} \tag{2.17}$$

where $x = (x_1, x_2)^T$ and $H = H_1 + H_2$ is the sum of the two energies. The structure and dissipation matrix become:

$$J = \begin{pmatrix} J_1 & g_1^C (g_2^C)^T \\ -g_2^C (g_1^C)^T & J_2 \end{pmatrix}, \; R = \begin{pmatrix} R_1 & 0 \\ 0 & R_2 \end{pmatrix}$$

2.2 3D-space modelling of mechanical systems

2.2.1 Euclidean space and motions

Coordinate frames

A coordinate frame of the three-dimensional Euclidean space is a 4-tuple of the form $\Psi = (o, \hat{x}, \hat{y}, \hat{z})$. Where o is the three-dimensional vector of the origin and $\hat{x}, \hat{y}, \hat{z}$ are the linear independent, orthonormal coordinate vectors. Consider two coordinate frames Ψ_1, Ψ_2 which share the same origin but differ in orientation due to different choices of $\hat{x}_i, \hat{y}_i, \hat{z}_i,\ i = 1, 2$. The change of orientation from Ψ_i to Ψ_j is described by the rotation matrix R_i^j. The set of rotation matrices is called *special orthonormal* group ($SO(3)$) [Str01a] and is defined as:

$$SO(3) = \{R \in \mathbb{R}^{3\times 3} \mid R^{-1} = R^T, detR = 1\} \tag{2.18}$$

Usually coordinate frames are defined with respect to an inertial frame, and the coordinate vectors $\hat{x}, \hat{y}, \hat{z}$ are chosen equal for all frames, deviations of orientation are represented by a rotation matrix relative to the inertial frame. In general a change of coordinate frames from Ψ_i to Ψ_j can be expressed with the homogeneous matrix

$$H_i^j := \begin{pmatrix} R_i^j & p_i^j \\ 0_{1\times 3} & 1 \end{pmatrix}$$

where $p_i^j = o_j - o_i$ denotes the distance between the origins. A point $p^i \in \mathbb{R}^3$ expressed in Ψ_i is cast into Ψ_j by

$$\begin{pmatrix} p^j \\ 1 \end{pmatrix} = H_i^j \begin{pmatrix} p^i \\ 1 \end{pmatrix} \tag{2.19}$$

. The inverse transformation H_j^i is given by

$$H_i^j = (H_i^j)^{-1} = \begin{pmatrix} (R_i^j)^T & -(R_i^j)^T p_i^j \\ 0_{1\times 3} & 1 \end{pmatrix}$$

and is still a homogeneous matrix. The set of homogeneous matrices is called the *special Euclidean* group:

$$SE(3) := \{\begin{pmatrix} R & p \\ 0 & 1 \end{pmatrix} \mid R \in SO(3), p \in \mathbb{R}^3\} \tag{2.20}$$

The $SE(3)$ is a matrix Lie group, composed of the set of homogeneous matrices H_i^j and the matrix multiplication being the group operation. For more information on Lie groups see e.g. [Str01b].

Twists and wrenches

Consider any point p not moving in coordinate frame Ψ_i, i.e. $\dot{p}^i = 0$. If p is moving in another coordinate frame Ψ_j, the two frame move with respect to each other. The trajectory can be described as a function of time: $H_i^j(t) \in SE(3)$. By differentiating (2.19) one obtains

$$\begin{pmatrix} \dot{p}^j(t) \\ 1 \end{pmatrix} = \dot{H}_i^j(t) \begin{pmatrix} p^i \\ 1 \end{pmatrix}$$

$\dot{H}_i^j$ describes both motion and a change of the reference frame. A separated representation is

$$\begin{pmatrix} \dot{p}^j(t) \\ 1 \end{pmatrix} = \tilde{T}_i^{j,j} \left(H_i^j \begin{pmatrix} p^i \\ 1 \end{pmatrix} \right) \tag{2.21}$$

$\dot{H}_i^j$ is a tangential vector along the trajectory $H_i^j(t)$ and thus in the tangent space of the $SE(3)$: $\dot{H}_i^j \in T_{H_i^j} SE(3)$. To obtain a representation of motion which is referenced to a coordinate frame, we can map $\dot{H}_i^j$ to the identity of the $SE(3)$. At the identity e of the $SE(3)$ the tangent space $T_e SE(3)$ has the structure of a Lie algebra. The Lie algebra of the $SE(3)$ is denoted by $\mathfrak{se}(3)$. This is done either by left or right translation, for a definition see [Str01b]. The right translation is used in (2.21) and is written compactly

$$\dot{H}_i^j = \tilde{T}_i^{j,j} H_i^j \tag{2.22}$$

The left translation leads to

$$\dot{H}_i^j = H_i^j \tilde{T}_i^{i,j} \tag{2.23}$$

We call $\tilde{T} \in T_eSE(3)$ a twist and the $\mathfrak{se}(3)$ the space of twists. Let us look more closely at this representation by calculating the twist from the elements of the homogeneous matrix

$$\begin{aligned} T_i^j = \dot{H}_i^j H_j^i = \begin{pmatrix} \dot{R}_i^j & \dot{p}_i^j \\ 0 & 0 \end{pmatrix} \begin{pmatrix} R_j^i & p_j^i \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \dot{R}_i^j & \dot{p}_i^j \\ 0 & 0 \end{pmatrix} \begin{pmatrix} (R_i^j)^T & -(R_i^j)^T p_i^j \\ 0 & 1 \end{pmatrix} = \\ = \begin{pmatrix} \dot{R}_i^j (R_i^j)^T & -\dot{R}_i^j (R_i^j)^T p_i^j + \dot{p}_i^j \\ 0 & 0 \end{pmatrix} =: \begin{pmatrix} \tilde{\omega}_i^j & v_i^j \\ 0 & 0 \end{pmatrix} \end{aligned} \tag{2.24}$$

It is clear from this equation that the linear velocity part v_i^j is not the velocity of the frame Ψ_i with respect to Ψ_j, identified by $\dot{p}_i^j$. This twist representation is described by the *screw theory* (see e.g. [WS08]). It can be visualized by the angular velocity around an axis and the linear velocity along this axis.
Next to the 4×4 matrix $\tilde{T}$ there exists also a vector representation $T \in \mathbb{R}^6$

$$\tilde{T} = \begin{pmatrix} \tilde{\omega} & v \\ 0 & 0 \end{pmatrix} \, , \, T = \begin{pmatrix} \omega \\ v \end{pmatrix} \tag{2.25}$$

wherein v is the velocity and ω is the angular velocity. $\tilde{\omega}$ is the skew-symmetric representation of the vector ω

$$\omega = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \Rightarrow \tilde{\omega} = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & \omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix} \tag{2.26}$$

Left and right translations lead to different representations of a twist:

- $T_i^{k,j}$ is the twist of Ψ_i with respect to Ψ_j expressed in the frame Ψ_k
- $T_i^j = T_i^{j,j}$ is the twist of Ψ_i with respect to Ψ_j expressed naturally in Ψ_j

Changes of coordinates for twists are of the form

$$\tilde{T}_i^{j,j} = \tilde{T}_i^j = H_i^j \tilde{T}_i^{i,j} H_j^i \tag{2.27}$$

or for the vector representation

$$T_i^j = Ad_{H_i^j} T_i^{i,j} \, , \, Ad_{H_i^j} = \begin{pmatrix} R_i^j & 0 \\ \tilde{p}_i^j R_i^j & R_i^j \end{pmatrix} \tag{2.28}$$

The dual vector space of $\mathfrak{se}(3)$ is the space of linear operations from $\mathfrak{se}(3)$ to $\mathbb{R}$. It is denoted by $\mathfrak{se}^*(3)$ and represents the space of wrenches W. Wrenches decompose to moments m and forces f.

$$W = (m \;\; f) \, , \, \tilde{W} = \begin{pmatrix} \tilde{m} & f \\ 0 & 0 \end{pmatrix} \tag{2.29}$$

Again $\tilde{W} \in \mathbb{R}^{4\times4}$ is a matrix while $W \in \mathbb{R}^6$ is the row vector representation. The change of coordinates for twists is similar to the case of twists:

$$(W_i^{k,i})^T = Ad^T_{H_k^i}(W_i^i)^T \tag{2.30}$$

Here the mapping is in the opposite direction, from Ψ_j to Ψ_i, what is a consequence of the fact that wrenches are duals to twists. Again there are different representations of wrenches:

- $W_i^{k,j}$ is the wrench applied to a spring connecting Ψ_i to Ψ_j on the side of Ψ_i expressed in the coordinate frame Ψ_k.
- W_i^k is the wrench applied to a body attached to Ψ_i expressed in the coordinate frame Ψ_k.

These definitions lead to the following rules of interconnection: Connecting a body and spring in the point Ψ_i and applying the principle of action and reaction we get $W_i^{k,j} = -W_i^k$. Due to the nodicity of a spring we have $W_i^{k,j} = -W_j^{k,i}$

We define power flowing through a port by the duality product of flow and effort, in the mechanical domain this twist and wrench. On a vector space level the power port $\mathcal{P}$ is defined by the Cartesian product of the Lie algebra $\mathfrak{se}(3)$ and its dual $\mathfrak{se}^*(3)$: $\mathcal{P} = \mathfrak{se}(3) \times \mathfrak{se}^*(3)$.

General input-state-output port-Hamiltonian systems

The simple relation $\dot{q} = \frac{\partial H}{\partial p}$ does not hold for a 3D-mechanical system. With reference to the next section we find $\frac{\partial V}{\partial P_i^i} = T_i^{i,0} = H_0^i \dot{H}_i^0$, with the H_0^i being the configuration variable and P_i^i the momentum. For reasons of notational clarity the Hamiltonian is denoted with V. The interconnecting Dirac structure in 3D-mechanical system is dependent on the configuration and we call it a *modulated* Dirac structure. This motivates the more general concept of port-Hamiltonian systems on *manifolds*. By introducing local coordinates x on the state manifold $\mathcal{X}$, we can associate the flows towards the energy storage with $f_s = -\dot{x}$ and the efforts with $e_s = \frac{\partial V}{\partial x}$. The flows are elements of the tangent space of the state manifold $T_x\mathcal{X}$ and the efforts are elements of the co-tangent space $T_x^*\mathcal{X}$. A general port-Hamiltonian system defined on $\mathcal{X}$ is of the form

$$\begin{aligned} \dot{x} &= [J(x) - R(x)]\frac{\partial V}{\partial x}(x) + g(x)u \\ y &= g^T(x)\frac{\partial V}{\partial x} \end{aligned} \tag{2.31}$$

with a skew-symmetric matrix $J(x)$ and a resistive structure matrix $R(x)$ which is symmetric and positive semi-definite. Clearly u and y denote the input and output respectively and we call this representation *input-state-output* form [vdSJ14]. In the following section we introduce the corresponding representations of atomic mechanical elements.

2.2.2 Dynamics of physical components

Springs

A *spring* is the ideal, lossless element storing potential energy. It is connected to two bodies and is defined by a potential energy function. The energy is a function of the relative displacement of the attached bodies. Consider a spring between the two bodies B_i and B_j, with coordinate frames Ψ_i and Ψ_j fixed to the respective body. The stored potential energy is positive definite function $V_{i,j}$ of the form

$$V_{i,j} : SE(3) \to \mathbb{R};\ H_i^j \mapsto V_{i,j}(H_i^j) \tag{2.32}$$

For explicit energy functions for different types of springs see [Str01b]. The input-state-output form is defined by the relative displacement H_i^j (state variable), the wrench $W_i^{j,j}$ (effort) and the twist T_i^j (flow).

$$\begin{aligned} \dot{H}_i^j &= T_i^j H_i^j \\ W_i^{j,j} &= \frac{\partial V_{i,j}}{\partial H_i^j}(H_i^j)^T \end{aligned} \tag{2.33}$$

Note that $V_{i,j}$ is an energetic minimum when H_i^j is the identity matrix I_4. An energetic minimum is physically necessary, otherwise infinite energy would be extractable from the spring. With $H_i^j = I_4$ the frames Ψ_i and Ψ_j coincide. It is possible to define springs with non-zero rest-length by introducing coordinate frames Ψ_{ic} and Ψ_{jc} rigidly attached to Ψ_i and Ψ_j respectively. The spring is now between the new frames, thus the energetic minimum is $H_{ic}^{jc} = I_4$. The displacements H_i^{ic}, H_j^{jc} are the resulting rest-lengths. Frames Ψ_{ic} and Ψ_{jc} can be chosen to represent the *center of stiffness*, where translation and rotation are maximally decoupled [SMA99].

Inertias

Inertias are special since they in general store two types of energy: kinetic and potential energy due to gravitation. At first we exclude the gravitational terms and focus on motion. Kinetic energy is a function of the relative motion w.r.t. an inertial reference. When expressing motion in non-inertial or accelerated reference frames, fictitious forces such as the *Coriolis* or the *centrifugal* force need to be considered. Let us start from *Newton's* second law of dynamics, the time derivative of a body's momentum is equal to the applied wrench.

$$\dot{P}_b^0 = W_b^0 \tag{2.34}$$

The momentum of the inertia b and the wrench acting on it are both expressed in the inertial reference frame Ψ_0. Let us consider the non-inertial frame Ψ_b, fixed to the inertia. We start by changing coordinates, clearly we have $W_b^0 = Ad_{H_0^b}^T W_b^b$. It is detailed in [Str01b] that $P_0^b \in \mathfrak{se}^*(3)$ and thus the same transformation as for wrenches applies $P_b^0 = Ad^T_{H_0^b} P_b^b$. Expressing the *Newton's* second law in the non-inertial frame Ψ_b we have

$$\frac{d}{dt}(Ad^T_{H_0^b} P_b^b) = Ad^T_{H_0^b} W_b^b \tag{2.35}$$

The evolution of the accelerated body frame Ψ_b w.r.t to the inertial frame is time dependent. The time derivative of the transformation is $\frac{d}{dt} Ad^T_{H_0^b} = -Ad^T_{H_0^b} ad^T_{T_b^{b,0}}$, with the *adjoint* representation (see for example [Str01a]):

$$ad^T_{T_b^{b,0}} = \begin{pmatrix} -\tilde{\omega}_b^{b,0} & -\tilde{v}_b^{b,0} \\ 0 & -\tilde{\omega}_b^{b,0} \end{pmatrix} \tag{2.36}$$

The second law of dynamics expressed in the body's frame is then

$$\begin{aligned} Ad^T_{H_0^b} \dot{P}_b^b - Ad^T_{H_0^b} ad^T_{T_b^{b,0}} P_b^b = Ad^T_{H_0^b} W_b^b \\ \dot{P}_b^b = ad^T_{T_b^{b,0}} P_b^b + W_b^b \end{aligned} \tag{2.37}$$

This formulation is split into its rotational and translational components, then we can exchange twist and momentum by the following operation

$$\begin{pmatrix} \dot{P}_{b,\omega}^b \\ \dot{P}_{b,v}^b \end{pmatrix} = \begin{pmatrix} -\tilde{\omega}_b^{b,0} & -\tilde{v}_b^{b,0} \\ 0 & -\tilde{\omega}_b^{b,0} \end{pmatrix} \begin{pmatrix} P_{b,\omega}^b \\ P_{b,v}^b \end{pmatrix} + W_b^b = \begin{pmatrix} \tilde{P}_{b,\omega}^b & \tilde{P}_{b,v}^b \\ \tilde{P}_{b,v}^b & 0 \end{pmatrix} \begin{pmatrix} \omega_b^{b,0} \\ v_b^{b,0} \end{pmatrix} + W_b^b \tag{2.38}$$

This clearly corresponds to the classical description of a rigid body's dynamics of the form

$$\dot{P}_b^b = M_b \dot{T}_b^{b,0} = C_b T_b^{b,0} + W_b^b \tag{2.39}$$

Here M_b describes the body's inertia and C_b accounts for Coriolis and centrifugal terms.

Towards port-Hamiltonian representation we start from the kinetic (co-)energy given by $V_k^*(T_b^{b,0}) = \frac{1}{2}(T_b^{b,0})^T M_b T_b^{b,0}$. Formally speaking the kinetic energy is a function of the momentum. By using the relation of twist and momentum $P_b^b = M_b T_b^{b,0}$ we get the kinetic energy

$$V_k(P_b^b) = \frac{1}{2}(P_b^b)^T M_b^{-1} P_b^b \tag{2.40}$$

By differentiating the kinetic energy w.r.t to the state variable P_b^b we obtain

$$\frac{\partial V_k(P_b^b)}{\partial P_b^b} = M_b^{-1} P_b^b = T_b^{b,0} \tag{2.41}$$

Recall from Table 2.1 that the twist is the effort variable in the port-Hamiltonian representation of an inertia. The flow is the externally supplied wrench W_b^b, thus we obtain the port-Hamiltonian representation of a rigid body, neglecting gravity

$$\begin{aligned} \dot{P}_b^b &= C_b \frac{\partial V_k(P_b^b)}{\partial P_b^b} + I_6 W_b^b \\ T_b^{b,0} &= I_6 \frac{\partial V_k(P_b^b)}{\partial P_b^b} \end{aligned} \tag{2.42}$$

In cooperative manipulation we often deal with heavy objects, it is thus inevitable to include the potential energy resulting from the gravitational field. One can think of a spring connecting the body and an inertial frame associated with the ground. This spring can be formulated in port-Hamiltonian structure using the left translation (2.2.1)

$$\begin{aligned}\dot{H}_b^0 &= H_b^0 T_b^{b,0} \\ W_b^{b,0} &= (H_b^0)^T \frac{\partial V_g}{\partial H_b^0}\end{aligned} \tag{2.43}$$

Where V_g is a suitable energy function. For a combined description the potential and kinetic energy add up: $V_{kg} = V_k + V_g$. Since there are two types of energy stored by *one* body, the twists in both energy systems are equal. The wrenches on the body add up

$$W_{kg}^b = W_b^b + C_b \frac{\partial V_{kg}}{\partial P_b^b} - (H_b^0)^T \frac{\partial V_{kg}}{\partial H_b^0}$$

Note that the negative sign in the upper equation comes from $W_b^b = -W_b^{b,0}$. With this knowledge we can write the combined port-Hamiltonian representation

$$\begin{aligned}\begin{pmatrix}\dot{H}_b^0 \\ \dot{P}_b^b\end{pmatrix} &= \begin{pmatrix}0 & H_b^0 \\ -(H_b^0)^T & C_b\end{pmatrix}\begin{pmatrix}\frac{\partial V_{kg}}{\partial H_b^0} \\ \frac{\partial V_{kg}}{\partial P_b^b}\end{pmatrix} + \begin{pmatrix}0 \\ I_6\end{pmatrix} W_b^b \\ T_b^{b,0} &= \begin{pmatrix}0 & I_6\end{pmatrix}\begin{pmatrix}\frac{\partial V_{kg}}{\partial H_b^0} \\ \frac{\partial V_{kg}}{\partial P_b^b}\end{pmatrix}\end{aligned} \tag{2.44}$$

Dampers

Dampers do not have a state since they do not store energy, they only dissipate it. Note that energy is not "destroyed" in the dampers but transformed into thermal energy. This can be modelled with a thermal port connected to the environment, for reasons of simplicity we discard the generated thermal energy. The easiest way to achieve damping is a linear resistive element R, such that the wrench is directly proportional to twist. Consider for example a body's motion with respect to the inertial frame

$$W_b^b = R T_b^{b,0} \tag{2.45}$$

Or a damper in parallel with spring

$$W_i^{j,j} = R T_i^j \tag{2.46}$$

The dissipated co-energy is $E_d = \frac{1}{2} T^T R T$.

2.3 Spring-mass-damper systems

Recall the motivating example from Section 2.1 of a simple spring-mass system. We can add a damper d to the port-Hamiltonian representation and rewrite eq. (2.6)

$$\begin{pmatrix} \dot{q} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & -d \end{pmatrix} \begin{pmatrix} \frac{\partial V}{\partial q}(q,p) \\ \frac{\partial V}{\partial p}(q,p) \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} F_e \tag{2.47}$$

Clearly $\dot{p} = -\frac{\partial V}{\partial q}(q,p) - d\frac{\partial V}{\partial p}(q,p) + F_e$ equals the mechanical impedance relation $m\ddot{x} + d\dot{x} + kx = F_e$. Thus we have a port-Hamiltonian representation of the impedance control scheme. This is the basis for the control architecture presented below. Therefore the mass-spring-damper system, given as a one-dimensional example, is formulated in the $SE(3)$. We start from eqs. (2.42,2.43) and add another spring to the body associated with Ψ_b. This spring connects to a desired object position assigned to Ψ_v. Its port-Hamiltonian representation is given by

$$\begin{aligned} \dot{H}_b^v &= H_b^v T_b^{b,v} \\ W_b^{b,v} &= (H_b^v)^T \frac{\partial V_s}{\partial H_b^v} \end{aligned} \tag{2.48}$$

The spring's deformation twist is decomposed by $T_b^{b,v} = T_b^{b,0} - T_v^{b,0}$. The damping along this spring is $W_b^b = D_b T_b^{b,v}$. Body, spring and damper move uniformly with the twist $T_b^{b,0}$ and the wrenches add up. Combing all components we arrive at

$$\begin{aligned} \begin{pmatrix} \dot{H}_b^0 \\ \dot{H}_b^v \\ \dot{P}_b^b \end{pmatrix} &= \begin{pmatrix} 0 & 0 & H_b^0 \\ 0 & 0 & H_b^v \\ -(H_b^0)^T & -(H_b^v)^T & C_b - D_b \end{pmatrix} \begin{pmatrix} \frac{\partial V_{kgs}}{\partial H_b^0} \\ \frac{\partial V_{kgs}}{\partial H_b^v} \\ \frac{\partial V_{kgs}}{\partial P_b^b} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ -H_b^v Ad_{H_0^b} & 0 \\ D_b Ad_{H_0^b} & I_6 \end{pmatrix} \begin{pmatrix} T_v^0 \\ W_b^b \end{pmatrix} \\ \begin{pmatrix} W_v^{0,0} \\ T_b^{b,0} \end{pmatrix} &= \begin{pmatrix} 0 & -Ad_{H_0^b}^T (H_b^v)^T & Ad_{H_0^b}^T D_b \\ 0 & 0 & I_6 \end{pmatrix} \begin{pmatrix} \frac{\partial V_{kgs}}{\partial H_b^0} \\ \frac{\partial V_{kgs}}{\partial H_b^v} \\ \frac{\partial V_{kgs}}{\partial P_b^b} \end{pmatrix} \end{aligned} \tag{2.49}$$

In a cooperative manipulation set-up this clearly accounts for an *external* impedance relation, used to establish compliant behaviour between (virtual) object and environment. Analogously we can define impedance relations between manipulators and virtual object. To this purpose we define a manipulator inertia and connect to the object with a spring and a damper. Here we consider the manipulator masses to be gravity pre-compensated and omit the spring connecting to the ground. The i-th manipulator inertia is given by

$$\begin{aligned} \dot{P}_i^i &= C_i \frac{\partial V_{k(i)}(P_i^i)}{\partial P_i^i} + I_6 W_i^i \\ T_i^{i,0} &= I_6 \frac{\partial V_{k(i)}(P_i^i)}{\partial P_i^i} \end{aligned} \tag{2.50}$$

It is important that the spring connecting b and i does not connect to the center of the object b but to a point $b(i)$ on the surface of b. Clearly the distance $p^b_{b(i)}$ corresponds to the extent of the object. The spring's twist decomposes as follows

$$T^i_{b(i)} = T^i_b + T^{i,b}_{b(i)} = Ad_{H^i_b} T^{b,0}_b - T^{i,0}_i + Ad_{H^i_b} \underbrace{T^b_{b(i)}}_{=0} \tag{2.51}$$

The spring is given by

$$\begin{aligned} \dot{H}^i_{b(i)} &= H^i_{b(i)} \begin{pmatrix} Ad_{H^{b(i)}_b} & -Ad_{H^{b(i)}_i} \end{pmatrix} \begin{pmatrix} T^{b,0}_b \\ T^{i,0}_i \end{pmatrix} \\ \begin{pmatrix} W^{b,0}_b \\ W^{i,0}_i \end{pmatrix} &= \begin{pmatrix} Ad^T_{H^{b(i)}_b} \\ -Ad^T_{H^{b(i)}_i} \end{pmatrix} (H^i_{b(i)})^T \frac{\partial V_{s(i)}}{\partial H^i_{b(i)}} \end{aligned} \tag{2.52}$$

and the damper along the spring exerts a wrench on the body i

$$W^i_i = D_i T^{i,b}_i = D_i T^{i,0}_i - D_i Ad_{H^i_b} T^{b,0}_b. \tag{2.53}$$

We can combine spring, inertia and damper, the twist $T^{i,0}_i$ is the common quantity.

$$\begin{aligned} \begin{pmatrix} \dot{H}^i_{b(i)} \\ \dot{P}^i_i \end{pmatrix} &= \begin{pmatrix} 0 & -H^i_{b(i)} Ad_{H^{b(i)}_i} \\ Ad^T_{H^{b(i)}_i} (H^i_{b(i)})^T & C_i - D_i \end{pmatrix} \begin{pmatrix} \frac{\partial V_{ks(i)}}{\partial H^i_{b(i)}} \\ \frac{\partial V_{ks(i)}}{\partial P^i_i} \end{pmatrix} + \\ &\quad + \begin{pmatrix} H^i_{b(i)} Ad_{H^{b(i)}_b} & 0 \\ D_i Ad_{H^i_b} & I_6 \end{pmatrix} \begin{pmatrix} T^{b,0}_b \\ W^i_i \end{pmatrix} \\ \begin{pmatrix} W^{b,0}_b \\ T^{i,0}_i \end{pmatrix} &= \begin{pmatrix} Ad^T_{H^{b(i)}_b} (H^i_{b(i)})^T & Ad^T_{H^i_b} D^T_i \\ 0 & I_6 \end{pmatrix} \begin{pmatrix} \frac{\partial V_{ks(i)}}{\partial H^i_{b(i)}} \\ \frac{\partial V_{ks(i)}}{\partial P^i_i} \end{pmatrix} \end{aligned} \tag{2.54}$$

2.4 Imposing constraints

The interconnection of a spring and an inertia is the the ideal pair in terms of input-output causality. The spring expects a twist-input and outputs a wrench. The inertia has a wrench-input and outputs a twist. It can be seen from eq. (2.44) that the interconnection of inertia and spring gives a set of ordinary differential equations (ODE). Many mechanical systems cannot be modelled by an interconnection of springs and masses. The prime example is the contact of two rigid objects, rigid means there is no elastic deformation which could be modelled by a spring (for an extensive treatment of hard and soft contact see [DS09]). Rigidly connected objects cannot move with respect to each other, i.e. they move with the same velocity. In cooperative manipulation we often assume the manipulators rigidly connected to the common object. The attempt to move the bodies individually results in *internal* forces. We call a force internal if it produces no *virtual work* with the system's velocity (see [EH16] for a formal definition). The motion-limiting conditions are called kinematic *constraints* and are expressed in the form

$$A^T(q)\dot{q} = 0 \tag{2.55}$$

We call $A(q)$ the *constraint* matrix and start from the Euler-Lagrange equations of constrained motion [DMSB09]

$$\begin{aligned} \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} &= g(q)f + A(q)\lambda \\ A^T(q)\dot{q} &= 0 \end{aligned} \tag{2.56}$$

The associated constraint forces are given by $A(q)\lambda$, where we call λ the *Lagrange* multipliers. They are uniquely determined if the constraints are satisfied, i.e. are given by the requirement $A^T(q)\dot{q} = 0$. In this case the constraint forces do not influence the energy of the system since $\lambda^T(A^T(q)\dot{q}) = 0$, this corresponds to the requirement of a zero virtual work for internal forces. Similarly to Section 2.1, the Euler-Lagrange equations can be transformed to a port-Hamiltonian equivalent, which is a mixed set of differential and algebraic equations (DAE).

$$\begin{aligned} \begin{pmatrix} \dot{q} \\ \dot{p} \end{pmatrix} &= \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial V}{\partial q} \\ \frac{\partial V}{\partial p} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ A(q) & g(q) \end{pmatrix} \begin{pmatrix} \lambda \\ f \end{pmatrix} \\ \begin{pmatrix} 0 \\ e \end{pmatrix} &= \begin{pmatrix} 0 & A^T(q) \\ 0 & g^T(q) \end{pmatrix} \begin{pmatrix} \frac{\partial V}{\partial p} \\ \frac{\partial V}{\partial p} \end{pmatrix} \end{aligned} \tag{2.57}$$

The system is no longer described in the input-state-output form, but in an implicit form. Several approaches to solve the algebraic equations and restore the desired input-state-output form exist. Most of them are designed for generalized configuration q and momentum p coordinates, see e.g. [vdS13],[DS09]. Due to non-linearity of $\dot{q} = \frac{\partial V}{\partial p}$ in mechanical systems not all are feasible for six dimensional systems.
The following method (described e.g. in [DS09]) uses the time-derivative of the constraints

$$0 = \frac{d}{dt}\left(A^T(q)\frac{\partial V}{\partial p}\right) \tag{2.58}$$

We use the property $\frac{\partial V}{\partial p} = M^{-1}p$ for an energy function of the form $V = \frac{1}{2}p^T M^{-1} p + V_q$. Since $q(t)$ is time-dependent *indirect dependencies* arise in $A(q), p(q)$ when calculating the total time-derivative

$$\begin{aligned} 0 &= \frac{d}{dt}(A^T M^{-1} p) = \frac{\partial (A^T M^{-1} p)}{\partial q}\dot{q} + A^T M^{-1}\dot{p} = \\ &= \frac{\partial (A^T M^{-1} p)}{\partial q} M^{-1} p + A^T M^{-1}\left(-\frac{\partial V}{\partial q} + A(q)\lambda + g(q)f\right) \end{aligned} \tag{2.59}$$

Solving this equation for λ gives an analytic expression for the constrained forces

$$\lambda = (A^T M^{-1} A)^{-1} \left(-\frac{\partial (A^T M^{-1} p)}{\partial q} M^{-1} p + \frac{\partial V}{\partial q} - gf \right) \tag{2.60}$$

Then λ is re-inserted into eq. (2.57) and we obtain a set of ODEs in input-output form. Clearly the term $A(q)\lambda$ generates compensation forces that oppose relative motions of the bodies and keep the constraints $A^T(q)\dot{q}$ fulfilled. Since we computed the constraint forces starting from the time-derivative of the constraint, $A^T(q)\dot{q}$ stays constant. To guarantee $A^T(q)\dot{q} = 0$, this must be already fulfilled in the beginning.

Constraints for 6D-motion

Consider two rigidly connected bodies, associated with the frames Ψ_b and Ψ_i and a distance between them $p_i^b = p_i^0 - p_b^0$. Clearly, in the setting of cooperative manipulation, one can think of an object b and the i-th manipulator attached to it. Now let the body b rotate with the angular velocity ω_b^0. Being rigidly attached the body i rotates in the same manner, $\omega_i^0 = \omega_b^0$. The translational velocity of body i is expressed dependent on body b by

$$\begin{aligned} \dot{p}_i^0 &= \dot{p}_b^0 + \omega_b^0 \times p_i^b = \dot{p}_b^0 + \omega_b^0 \times (p_i^0 - p_b^0) \\ \dot{p}_i^0 - \omega_b^0 \times p_i^0 &= \dot{p}_b^0 - \omega_b^0 \times p_b^0 \end{aligned} \tag{2.61}$$

Recall the definition of the linear velocity component of a twist from eq. (2.24) being $v_i^j = \dot{p}_i^j - \omega_i^j \times p_i^j$. Thus we have $v_i^0 = v_b^0$ and consequently $T_i^0 = T_b^0$. For a system of $i = 1...N$ manipulators we write the constraints

$$0 = A^T T = \begin{pmatrix} -I_3 & 0_3 & I_3 & 0_3 & & & \\ 0_3 & -I_3 & 0_3 & I_3 & & & \\ \vdots & \vdots & & & \ddots & & \\ -I_3 & 0_3 & & & & I_3 & 0_3 \\ 0_3 & -I_3 & & & & 0_3 & I_3 \end{pmatrix} \begin{pmatrix} T_b^0 \\ T_1^0 \\ \vdots \\ T_N^0 \end{pmatrix} \tag{2.62}$$

We start by differentiating the constraints, here A is not time or configuration dependent, thus $0 = A^T \dot{T}$. Consider the simple example of two rigidly connected bodies b and i, the constraint equation is

$$\begin{aligned} 0 = \dot{T}_i^0 - \dot{T}_b^0 = \begin{pmatrix} \dot{\omega}_i^0 - \dot{\omega}_b^0 \\ \dot{v}_i^0 - \dot{v}_b^0 \end{pmatrix} &= \begin{pmatrix} \dot{\omega}_i^0 - \dot{\omega}_b^0 \\ \frac{d}{dt}(\dot{p}_i^0 - \omega_b^0 \times p_i^0) - \frac{d}{dt}(\dot{p}_b^0 - \omega_b^0 \times p_b^0) \end{pmatrix} = \\ = \begin{pmatrix} \dot{\omega}_i^0 - \dot{\omega}_b^0 \\ \ddot{p}_i^0 - \ddot{p}_b^0 - \dot{\omega}_b^0 \times p_i^b - \omega_b^0 \times (\omega_b^0 \times p_i^b) \end{pmatrix} \end{aligned} \tag{2.63}$$

Kinematic constraints are expressed very compactly in the twist notation. Second-order dynamics including *centripetal* terms are inherent and equivalent to classic representations (see e.g.[EH16]).

Towards solving the set of port-Hamiltonian DAEs, recall from eq. (2.57) that $0 = A^T \frac{\partial V}{\partial p}$. At this point it is necessary to distinguish different twist representations, e.g. $\frac{\partial V}{\partial P_b^b} = T_b^{b,0} = Ad_{H_0^b} T_b^0$. Continuing the two mass example we re-write the constraints

$$0 = A^T \begin{pmatrix} T_b^0 \\ T_i^0 \end{pmatrix} = \underbrace{A^T \begin{pmatrix} Ad_{H_b^0} & 0 \\ 0 & Ad_{H_i^0} \end{pmatrix}}_{\bar{A}^T} \begin{pmatrix} T_b^{b,0} \\ T_i^{i,0} \end{pmatrix} = A^T \begin{pmatrix} Ad_{H_b^0} & 0 \\ 0 & Ad_{H_i^0} \end{pmatrix} \begin{pmatrix} \frac{\partial V}{\partial P_b^b} \\ \frac{\partial V}{\partial P_i^i} \end{pmatrix} \tag{2.64}$$

Replacing the twists with momenta $T = M^{-1}P$ and differentiating w.r.t to time leads to

$$0 = \bar{A}^T \begin{pmatrix} M_b^{-1} & 0 \\ 0 & M_i^{-1} \end{pmatrix} \begin{pmatrix} \dot{P}_b^b \\ \dot{P}_i^i \end{pmatrix} + \bar{A}^T \begin{pmatrix} ad_{T_b^{b,0}} & 0 \\ 0 & ad_{T_i^{i,0}} \end{pmatrix} \begin{pmatrix} M_b^{-1} & 0 \\ 0 & M_i^{-1} \end{pmatrix} \begin{pmatrix} P_b^b \\ P_i^i \end{pmatrix} \tag{2.65}$$

Consider now the last part of this equation and recall the adjoint representation ad_T from eq. (2.36). We obtain for body b

$$ad_{T_b^{b,0}} M_b^{-1} P_b^b = \begin{pmatrix} \tilde{\omega}_b^{b,0} & 0 \\ \tilde{v}_b^{b,0} & \tilde{\omega}_b^{b,0} \end{pmatrix} \begin{pmatrix} \omega_b^{b,0} \\ v_b^{b,0} \end{pmatrix} = 0 \tag{2.66}$$

Here we assume that the inertias are not configuration dependent, i.e. not time-dependent. Now we insert the port-Hamiltonian system equation for $\dot{P}$ and obtain

$$0 = \bar{A}^T M^{-1} \dot{P} = \bar{A}^T M^{-1} (W + CT + \bar{A}\lambda) \tag{2.67}$$

and solve for λ

$$\lambda = -(\bar{A}^T M^{-1} \bar{A})^{-1} \bar{A}^T M^{-1} (W + CT) \tag{2.68}$$

Consider again the example of two rigidly connected bodies b and i. Let us first examine the part $\bar{A}^T M^{-1} CT$. Using eq. (2.38) and assuming that the bodies share the orientation $R_0^b = R_0^i$, we obtain

$$\begin{aligned} \bar{A}^T M^{-1} \begin{pmatrix} C_b T_b^{b,0} \\ C_i T_i^{i,0} \end{pmatrix} &= \bar{A}^T M^{-1} \begin{pmatrix} 0 \\ m_b \tilde{v}_b^{b,0} \omega_b^{b,0} \\ 0 \\ m_i \tilde{v}_i^{i,0} \omega_b^{b,0} \end{pmatrix} = \begin{pmatrix} -I_6 & I_6 \end{pmatrix} \begin{pmatrix} 0 \\ R_b^0 \tilde{v}_b^{b,0} \omega_b^{b,0} \\ 0 \\ R_b^0 \tilde{v}_i^{i,0} \omega_b^{b,0} \end{pmatrix} = \\ &= \begin{pmatrix} 0 \\ R_b^0 \tilde{\omega}_b^{b,0} (\tilde{p}_0^b R_b^0 \omega_b^0 + R_0^b v_b^0 - \tilde{p}_0^i R_0^b \omega_b^0 - R_0^b v_i^0) \end{pmatrix} = \begin{pmatrix} 0 \\ R_b^0 \tilde{\omega}_b^{b,0} \tilde{p}_i^b \omega_b^{b,0} \end{pmatrix} \end{aligned} \tag{2.69}$$

This example shows how kinematic constraints can be solved by calculating the constraint forces. The results are equivalent to approaches based on the *Gauss' principle of least constraint* ([EH16]) or *Euler-Lagrange* representations ([LL02]).

2.5 A model-based controller for cooperative manipulation

The idea of the presented controller is to model an impedance controlled cooperative manipulation set-up. This includes compliant relations between the object and the manipulators as well as an impedance relation between the object and a desired set-point trajectory. The concept was introduced by Stramigioli [Str01b] and is called the *Intrisically Passive Controller* (IPC). The controller is composed of geometric interconnection of mechanical elements and is thus passive. It features two energy ports, one to allow a high-level controller, e.g. a human operator, specify a reference trajectory. The other port connects to the robot side. Energy is exchanged only through these ports.

2.5.1 Compliant reference trajectory generation

Starting from the mechanical impedance equations derived in subsection 2.3, a controller based on the structure of the cooperative manipulation set-up is designed. The starting point is the impedance equation (2.49) accounting for the relation between object and reference trajectory. The inertia M_b represents the common object, spring and damper establish a relation between desired and *actual* object twist. In this context the actual twist is the twist of the modelled object (no object tracking information from the real set-up is used). Analogously to a real cooperative manipulation set-up, the i-manipulators connect to the virtual object. In the controller these connections are compliant (not rigid), i.e. springs are between object and manipulators. The manipulator-object impedance equation (2.54) defines the springs, mass and dampers of the modelled manipulators. One spring hinge-point is connected to the surface of the object, the other hinge point is connected to the impedance relation's inertia M_i, which clearly represents the i-th manipulator. In summary a simple geometric interconnection of the impedance equations (2.49,2.54) forms the controller, the structure can be seen from figure 2.4. In place of a full cooperative manipulation set-up with n manipulators we derive the controller for a single manipulator i, the equations for $i = 1...N$ manipulators can be found in the Appendix.

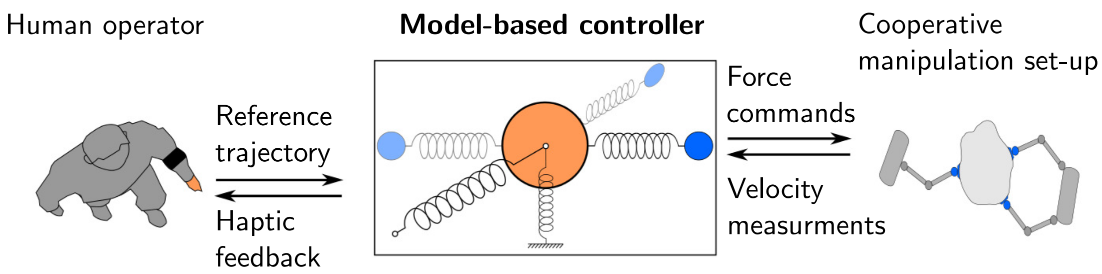

Figure 2.3: Overall set-up

$$
\begin{aligned}
\begin{pmatrix} \dot{H}_b^0 \\ \dot{H}_b^v \\ \dot{P}_b^b \\ \dot{H}_{b(i)}^i \\ \dot{P}_i^i \end{pmatrix} &= \begin{pmatrix} 0 & 0 & H_b^0 & 0 & 0 \\ 0 & 0 & H_b^v & 0 & 0 \\ -H_b^{0T} & -H_b^{vT} & C_b - D_b & -Ad^T_{H_b^{b(i)}} {H_{b(i)}^i}^T & -Ad^T_{H_b^i} D_i^T \\ 0 & 0 & H_{b(i)}^i Ad_{H_b^{b(i)}} & 0 & -H_{b(i)}^i Ad_{H_i^{b(i)}} \\ 0 & 0 & D_i Ad_{H_b^i} & Ad^T_{H_i^{b(i)}} {H_{b(i)}^i}^T & C_i - D_i \end{pmatrix} \begin{pmatrix} \frac{\partial V}{\partial H_b^0} \\ \frac{\partial V}{\partial H_b^v} \\ \frac{\partial V}{\partial P_b^b} \\ \frac{\partial V}{\partial H_{b(i)}^i} \\ \frac{\partial V}{\partial P_i^i} \end{pmatrix} + \\
&+ \begin{pmatrix} 0 & 0 \\ -H_b^v Ad_{H_0^b} & 0 \\ D_b Ad_{H_0^b} & 0 \\ 0 & 0 \\ 0 & I_6 \end{pmatrix} \begin{pmatrix} T_v^0 \\ W_i^i \end{pmatrix}
\end{aligned}
\tag{2.70}
$$

$$
\begin{pmatrix} W_v^{0,0} \\ T_i^{i,0} \end{pmatrix} = \begin{pmatrix} 0 & -Ad^T_{H_0^b} H_b^{vT} & Ad^T_{H_0^b} D_b^T & 0 & 0 \\ 0 & 0 & 0 & 0 & I_6 \end{pmatrix} \begin{pmatrix} \frac{\partial V}{\partial H_b^0} \\ \frac{\partial V}{\partial H_b^v} \\ \frac{\partial V}{\partial P_b^b} \\ \frac{\partial V}{\partial H_{b(i)}^i} \\ \frac{\partial V}{\partial P_i^i} \end{pmatrix}
$$

Note that the output to the robot side is a twist, the control scheme thus generates a compliant reference trajectory for the robots to follow. Similar to [CCMV08] we require a local a underlying force control layer to operate the robots.

2.5.2 Admittance-type control

Admittance-type control means (in contrast to the previous) specifying a reference trajectory on the user side and obtaining wrenches for the robots from the controller. The idea behind this control scheme is to connect the manipulator inertias rigidly to the object and leave the spring open to the actual robot side (see Figure 2.5). Rigid coupling of a manipulator with the object is treated with the approach presented in section 2.4. Once again we start from a single manipulator i in place of a full cooperative manipulation set-up. Bodies b and $b(i)$ are rigidly connected and their

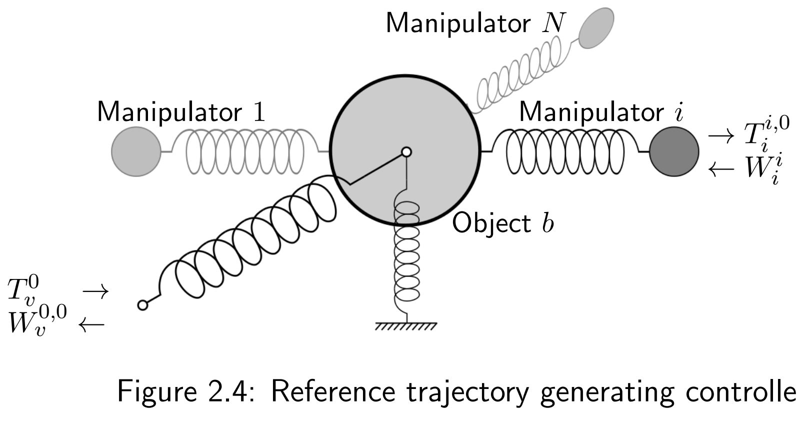

Figure 2.4: Reference trajectory generating controller

port-Hamiltonian equations are

$$
\begin{aligned}
\begin{pmatrix} \dot{P}_b^b \\ \dot{P}_{b(i)}^{b(i)} \end{pmatrix} &= \begin{pmatrix} C_b & 0 \\ 0 & C_{b(i)} \end{pmatrix} \begin{pmatrix} \frac{\partial V}{\partial P_b^b} \\ \frac{\partial V}{\partial P_{b(i)}^{b(i)}} \end{pmatrix} + \begin{pmatrix} W_b^b \\ W_{b(i)}^{b(i)} \end{pmatrix} + \bar{A}\lambda \\
0 &= \bar{A}^T \begin{pmatrix} \frac{\partial V}{\partial P_b^b} \\ \frac{\partial V}{\partial P_{b(i)}^{b(i)}} \end{pmatrix}
\end{aligned}
\tag{2.71}
$$

The Lagrange multipliers are eliminated as shown in section 2.4. In order to write the full control scheme in a compact notation we introduce four abbreviations accounting for the constrained forces

$$
\begin{aligned}
\mathcal{A}_{b,b} &= 1 - Ad^T_{H_b^0}(\bar{A}^T M^{-1}\bar{A})^{-1} Ad_{H_b^0} M_b^{-1} \\
\mathcal{A}_{i,b} &= Ad^T_{H_b^0}(\bar{A}^T M^{-1}\bar{A})^{-1} Ad_{H_{b(i)}^0} M_{b(i)}^{-1} \\
\mathcal{A}_{b,b} &= Ad^T_{H_{b(i)}^0}(\bar{A}^T M^{-1}\bar{A})^{-1} Ad_{H_b^0} M_b^{-1} \\
\mathcal{A}_{i,i} &= 1 - Ad^T_{H_{b(i)}^0}(\bar{A}^T M^{-1}\bar{A})^{-1} Ad_{H_{b(i)}^0} M_{b(i)}^{-1}
\end{aligned}
\tag{2.72}
$$

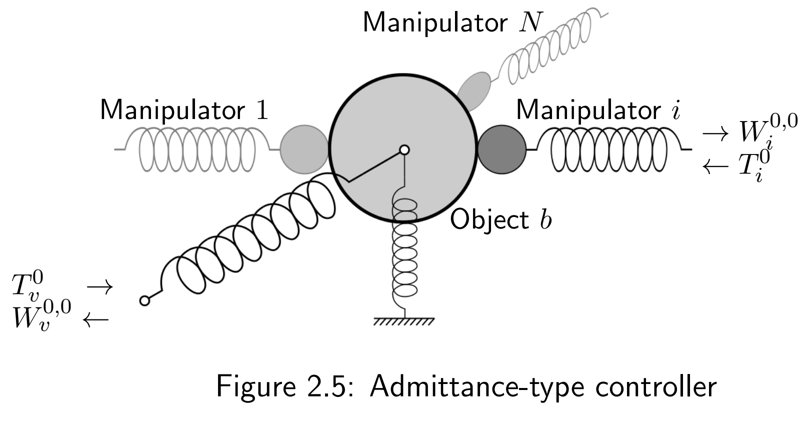

Figure 2.5: Admittance-type controller

$$\begin{pmatrix} \dot{H}_b^0 \\ \dot{H}_b^v \\ \dot{P}_b^b \\ \dot{H}_{b(i)}^i \\ \dot{P}_{b(i)}^{b(i)} \end{pmatrix} = \begin{pmatrix} 0 & 0 & H_b^0 & 0 & 0 \\ 0 & 0 & H_b^v & 0 & 0 \\ -\mathcal{A}_{b,b}{H_b^0}^T & -\mathcal{A}_{b,b}{H_b^v}^T & \mathcal{A}_{b,b}(C_b - D_b) & \mathcal{A}_{b,i}Ad^T_{H_b^{b(i)}}{H_{b(i)}^i}^T & \mathcal{A}_{b,i}(C_{b(i)} - D_{b(i)}) \\ 0 & 0 & 0 & 0 & H_{b(i)}^i \\ -\mathcal{A}_{i,b}{H_b^0}^T & -\mathcal{A}_{i,b}{H_b^v}^T & \mathcal{A}_{i,b}(C_b - D_b) & \mathcal{A}_{i,i}Ad^T_{H_i^{b(i)}}{H_{b(i)}^i}^T & \mathcal{A}_{i,i}(C_{b(i)} - D_{b(i)}) \end{pmatrix} \begin{pmatrix} \frac{\partial V}{\partial H_b^0} \\ \frac{\partial V}{\partial H_b^v} \\ \frac{\partial V}{\partial P_b^b} \\ \frac{\partial V}{\partial H_{b(i)}^i} \\ \frac{\partial V}{\partial P_{b(i)}^{b(i)}} \end{pmatrix} +$$

$$+ \begin{pmatrix} 0 & 0 \\ -H_b^v Ad_{H_0^b} & 0 \\ \mathcal{A}_{b,b}D_b Ad_{H_0^b} & \mathcal{A}_{b,i}D_{b(i)}Ad_{H_0^{b(i)}} \\ 0 & -H_{b(i)}^i Ad_{H_0^{b(i)}} \\ \mathcal{A}_{i,b}D_b Ad_{H_0^b} & \mathcal{A}_{i,i}D_{b(i)}Ad_{H_0^{b(i)}} \end{pmatrix} \begin{pmatrix} T_v^0 \\ T_i^0 \end{pmatrix} \tag{2.73}$$

$$\begin{pmatrix} W_v^{0,0} \\ W_i^{0,0} \end{pmatrix} = \begin{pmatrix} 0 & -Ad^T_{H_0^b}{H_b^v}^T & Ad^T_{H_0^b}D_b^T\mathcal{A}_{b,b}^T & 0 & Ad^T_{H_0^b}D_b^T\mathcal{A}_{i,b}^T \\ 0 & 0 & Ad^T_{H_0^{b(i)}}D_{b(i)}^T\mathcal{A}_{b,i}^T & -Ad^T_{H_0^{b(i)}}{H_{b(i)}^i}^T & Ad^T_{H_0^{b(i)}}D_{b(i)}^T\mathcal{A}_{i,i}^T \end{pmatrix} \begin{pmatrix} \frac{\partial V}{\partial H_b^0} \\ \frac{\partial V}{\partial H_b^v} \\ \frac{\partial V}{\partial P_b^b} \\ \frac{\partial V}{\partial H_{b(i)}^i} \\ \frac{\partial V}{\partial P_{b(i)}^{b(i)}} \end{pmatrix}$$

Chapter 3

Energy-aware control

Towards a conclusion on stability of the overall system, it is necessary to take the environmental conditions into consideration. Letting a human explicitly control a cooperative robot-team rises the question of her/his dynamic behaviour and the impact on the system. Furthermore the environment, the robotic system is interacting with, can change unexpectedly, stability considerations based on certain environment models may thus lose its basis. Preserving stability is especially crucial when the human operator is on-site to ensure her/his safety. The objective of this chapter is to introduce an energy tank with an energy controller to maintain a safe level of energy int the system, regardless of the dynamics of human and environment. The energy-based controller from chapter 2 integrates seamless with the tank concept.

3.1 Passivity-based control

...
Recall the power balance already given by eq. (2.15), in integrated form we obtain the energy balance of a pHs

$$\underbrace{H(x(t)) - H(x(0))}_{stored\, energy} = \underbrace{\int_0^t e_P^T f_P dt}_{ext.\, supplied} + \underbrace{\int_0^t e_R^T f_R dt}_{dissipated} \tag{3.1}$$

The dissipated co-energy is negative since $e_R^t f_R \leq 0$. Clearly the given energy balance corresponds to a *passive* system. The Hamiltonian energy functions of the systems presented in Chapter 2 are *candidate Lyapunov* functions. The interconnection of pH/passive systems is still a pH/passive system. By interconnection of pH-controller and passive plant we obtain a passive controlled-robotic system. This type of systems has an important advantage when it comes to interaction with humans and/or unknown environments. Usually whose dynamics are either uncertain or too complicated to be modelled. Passive systems are stable with any system, regardless its structure or complexity, that can provide only a bounded amount of energy (see [Str15] for details).

3.2 Human guidance

An intuitive way of guiding the robotic system is to let it follow the hand of the operator. Tracking systems for either of the hand or a device held by the hand are available. These motion capture systems usually run at lower frequencies than the robotic control loop (which is assumed time-continuous). Therefore the tracking output is not a smooth twist trajectory but a time-discrete sequence of position and orientation data. The objective of this section mainly is the derivation of the reference twist T_v^0 from the discrete tracking results. From the previous sections we know $T_v^0 = \dot{H}_v^0 H_0^v$, in a discrete representation this is

$$\begin{aligned} T_v^0(k+1) &= \dot{H}_v^0(k+1) H_0^v(k+1) (H_0^v(k))^{-1} = \\ &= \dot{H}_v^0(k+1) (H_v^0(k+1))^{-1} H_v^0(k) \end{aligned} \tag{3.2}$$

We can calculate the change of position/orientation over the sampling time interval ΔT

$$\dot{H}_v^0(k+1) = \frac{H_v^0(k+1)(H_v^0(k))^{-1}}{T_s} \tag{3.3}$$

Thus we have obtained a discrete twist representation based on discrete pose inputs. It is a discontinuous sequence updated every T_s and thus exhibits steps in its value. The energy exchanged through the port during T_s is

$$\Delta V_k^{k+1} = \int_{kT_s}^{(k+1)T_s} W_v^{v,0} dt \; T_v^0(k+1) \tag{3.4}$$

Steps of the input result in discontinuities in the energy function, since the input(flow) is the time derivative of the energy state. Consider the time-discrete twist $T_v^0(k+1)$ assigned to the continuous system input, i.e. $T_v^0(t) := T_v^0(k+1)$. Thus the input changes its value every T_s. The input $T_v^0(t)$ is an external flow f_p, contributing to the energy balance given in eq. (2.15). Since the pHs is continuous the property of energy conservation holds for all f_P. The steps in the system energy are exclusively supplied by external power and are thus not passivity violating. In contrast to there are time-discrete pHs, described in [SSVdSF05]. The main difference are time-discrete energy states, i.e. the state of the next time step is computed from the actual flow and state value. In this class of systems passivity is violated if, throughout a time-step, more energy than stored is extracted. Therefore consider the example of an elongated spring. The relaxing twist is so high that its integration (i.e. the displacement) over the time-step is higher than the initial elongation. I.e. in one time-step the spring not only fully relaxes to its equilibrium but also elongates in the other direction, this would generate energy and violates passivity.

In our case assuming a continuous pHs, this cannot happen because the time-steps are infinitely small.
Using time-discrete input functions with a continuous pHs does not affect passivity. Thus they can be used as a minimum solution for connecting a human directly to the controller presented in the previous section.

3.3 Energy control

By incorporating a human in the control loop, who commands the system by hand motion, we introduce a source of energy. Unlike in many master-slave systems where the human motion is hindered by a reactive force-feedback, in our case the human's motion is free. Thus we cannot assume her/him to be passive any more, since the energy supply can be unbounded. One solution ensuring stability is assigning the operator with a certain *energy budget* s/he can use to change the energetic state of the system. The idea is that the combined total energy either used in the system or stored in a *energy tank* is always constant. Next to the naturally present types (kinetic, potential energy) we add the virtually stored energy. Being energetically conservative means that the energy is transformed between these three types. Clearly this demands that *dissipated* energy is regained into the virtual storage. From a purely physical point of view this not meaningful since every energy conversion increases a system's entropy. In control theory however this poses no problem, the lossless case leads to stable, passive results [Str15].

3.3.1 Energy tanks

The *energy tank* is a virtual storage element defined with a Hamiltonian energy function. Let x_t denote the (scalar) energy state and we have a simple, positive definite function $T(x_t) = \frac{1}{2}x_t^2$. The dynamic equations of the tank are

$$\begin{aligned} \dot{x}_t &= u_t \\ y_t &= \frac{\partial T(x_t)}{\partial x_t}(= x_t) \end{aligned} \tag{3.5}$$

Here u_t, y_t denote the input and output respectively. Towards the application of the energy tank it is useful to consider a standard port-Hamiltonian system of the form

$$\begin{aligned} \dot{x} &= [J(x) - R(x)]\frac{\partial V}{\partial x} + g(x)u \\ y &= g^T(x)\frac{\partial V}{\partial x} \end{aligned} \tag{3.6}$$

Now we seek to re-route the dissipated energy into the energy tank, this can be accomplished by choosing the tank input as $u_t = \frac{1}{x_t}\frac{\partial V^T}{\partial x}R(x)\frac{\partial V}{\partial x} + \tilde{u}_t$. With a new input $\tilde{u}_t$ we have a the tank's energy balance

$$\dot{T}(x_t) = y_t u_t = \frac{\partial V^T}{\partial x} R(x) \frac{\partial V}{\partial x} + x_t \tilde{u}_t \tag{3.7}$$

This corresponds to the systems dissipated energy plus some external supply. Next we use this external supply to interconnect tank and pHs. A power-preserving interconnection is established, here we use a combination of gyrator and transformer with ratio n

$$\begin{aligned} u &= n y_t \\ \tilde{u}_t &= -n^T y \end{aligned} \tag{3.8}$$

By construction and independent of a particular choice of n this relation is power-continuous

$$u^T y = y_t^T n^T y = -y_t^T \tilde{u}_t \tag{3.9}$$

The combined energy function of the interconnected system is $\mathcal{V}(x, x_t) = V(x) + T(x_t)$. The choice of n is not fixed but can be adjusted dynamically to shape the energy flow. In particular it is appealing to use n to replicate the original control signal by choosing $n = \frac{w}{x_t}$. The new control input w can effectively take the role of u, i.e. the pHs becomes

$$\dot{x} = [J(x) - R(x)] \frac{\partial V}{\partial x} + g(x) \frac{w}{x_t} y_t = [J(x) - R(x)] \frac{\partial V}{\partial x} + g(x) w \tag{3.10}$$

For a $x_t \to 0$, i.e. an empty energy tank, there is a singularity. Thus an complete depletion of the tank must be avoided. Introducing a switching parameter α, we set $n = \frac{\alpha w}{x_t}$ with

$$\alpha = \begin{cases} 1 & \text{if } T(x_t) \geq \epsilon > 0 \\ 0 & \text{if } T(x_t) < \epsilon \end{cases} \tag{3.11}$$

This means that a control input is executed unchanged as long as certain amount ϵ of energy is in the tank. Once the energy budget is exceeded and command execution possibly violates passivity the input is set to 0, effectively suspending energy exchange. This happens in both ways, neither is it possibly to re-transfer into the tank through the interconnection. At this point the tank can only be refilled by dissipation. Dissipation is associated with kinetic energy, as long as the system is moving it can recover from the input-suspended state. If the system is driven to a state of exclusively potential energy, there is no dissipation and a deadlock is reached.

Reaching a state of exclusively potential energy takes infinite time, this is because a state of higher potential energy can only be reached by motion. I.e. kinetic energy is a predecessor of potential energy. A fragment of the kinetic energy is dissipated and is available in the energy budget again. The available energy approaches ϵ asymptotically.

Consequently we can give a combined port-Hamiltonian representation of system and tank

$$\begin{pmatrix} \dot{x} \\ \dot{x}_t \end{pmatrix} = \left[\begin{pmatrix} J(x) & \frac{\alpha w}{x_t} \\ -\frac{\alpha w^T}{x_t} & 0 \end{pmatrix} - \begin{pmatrix} R(x) & 0 \\ -\frac{1}{x_t}\frac{\partial \mathcal{V}^T}{\partial x} R(x) & 0 \end{pmatrix} \right] \begin{pmatrix} \frac{\partial \mathcal{V}}{\partial x} \\ \frac{\partial \mathcal{V}}{\partial x_t} \end{pmatrix} \tag{3.12}$$

Note that there is no more an input-output port, this is because ports are defined by energy exchange.
However when a robotic system interacts with the environment energy is exchanged, this clearly interferes with the concept of a constant energy budget. On the other hand the possible impact on the environment is limited. This is beneficial when interacting with humans and requiring to ensure their safety. If we do not assume a model or certain properties for the environment, a basic strategy for coping with undefined energy exchange is to allow the operator to supply and withdraw energy from the tank. Practically of the tank reaches a maximum energy level, further supplies are discarded (or transferred to the operator). On the other hand if the budget is used up and no more actions on the environment are allowed the human can supply more energy to the tank.

3.3.2 Energy-adapted stiffness and damping

As soon as the energy tank is depleted, control inputs are discarded and trajectory following is not given. It is clear that trajectories draining the tank are not feasible with the given energy budget and need to be altered. Simply discarding the desired trajectory information leads to a permanent deviation of the position. Thus we demand a controller that returns onto the desired trajectory as soon as the energy level has recovered. Therefore we propose the use of an energy-adapted spring and damper. The idea is to relax the stiffness of the user-object spring as function of the available energy. This allows the system to "float" loosely and regain energy through damping. Moreover a change of stiffness affects the energy stored in the spring, relaxing stiffness sets energy free which is re-fed into the tank. With a rise of the tank level, stiffness is increased and the spring re-directs the system onto the desired trajectory. To illustrate the principle we start with a general spring given by

$$F = k(x_v - x_b)\ ,\ T(x_t) \geq T_{th} \tag{3.13}$$

This equation is valid if the tank level $T(x_t)$ is above a certain threshold level T_{th}. The spring's stiffness k is unaltered, x_v and x_b denote the reference and the actual position respectively. Below the threshold we propose the following spring function

$$F = k\frac{T(x_t)}{T_{th}}(x_v - x_b)\ ,\ T(x_t) < T_{th} \tag{3.14}$$

For ease of notation we define a new stiffness parameter

$$\kappa = \begin{cases} k & \text{if } T(x_t) \geq T_{th} \\ k\frac{T(x_t)}{T_{th}} & \text{if } T(x_t) < T_{th} \end{cases} \tag{3.15}$$

The corresponding energy function is then

$$V_\kappa(x_v, x_b, \kappa) = \frac{1}{2}\kappa(x_v - x_b)^2 \tag{3.16}$$

The exchanged power with respect to a change of stiffness is

$$\dot{V}_\kappa = \frac{\partial V_\kappa}{\partial \kappa}\frac{d\kappa}{dt} = \frac{1}{2}(x_v - x_b)^2\dot{\kappa} = e^T f, \tag{3.17}$$

which corresponds to the product of effort ($\frac{\partial V_\kappa}{\partial \kappa}$) and flow ($\dot{\kappa}$) and forms a power port (f, e) as defined in Section 2.1. The energy function of a 6-DoF spring is of the from

$$V_\kappa : SE(3) \times \mathcal{K} \to \mathbb{R}; (H_b^v, \kappa) \mapsto V_\kappa(H_b^v, \kappa), \tag{3.18}$$

which is equal to the previous spring energy function (eq. 2.32) but depends explicitly on the stiffness parameter κ. $\mathcal{K}$ is a parametric space containing the stiffness matrices. For more information on variable spatial springs see [**?**]. The port-Hamiltonian representation of a variable stiffness spring is given by

$$\begin{aligned} \begin{pmatrix} \dot{H}_b^v \\ \dot{\kappa} \end{pmatrix} &= \begin{pmatrix} H_b^v & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} T_b^{b,v} \\ u_k \end{pmatrix} \\ \begin{pmatrix} W_b^{b,v} \\ y_k \end{pmatrix} &= \begin{pmatrix} H_b^{vT} & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} \frac{\partial V_\kappa}{\partial H_b^v} \\ \frac{\partial V_\kappa}{\partial \kappa} \end{pmatrix}, \end{aligned} \tag{3.19}$$

where u_k, y_k is the input-output pair.

3.4 Performance Comparison of control strategies

To evaluate the presented controllers in an objective way, they are implemented in *Simulink* and compared in terms of:

- Trajectory tracking
- Dynamic behaviour
- Internal Forces

3.4.1 Internal impedance control with feed-forward of the object dynamics

De Pascali et al. [DPEZ+15] present combination of impedance control on manipulator/object level and feed-forward object dynamics. The internal impedance control relation (between object and manipulator) encompasses a spring and a parallel damper, inertia is used to feed-forward the desired acceleration:

$$M_i\ddot{x}_i^d + D_i(\dot{x}_i^d - \dot{x}_i) + K_i(x_i^d, x_i) = h^x \tag{3.20}$$

I know I have to use consistent notation and explain the variables, comes later ;) This avoids the necessity of either measuring manipulator acceleration or contact force. Object dynamics is represented with a feed-forward term, mapped to the manipulators with a weighted pseudoinverse G^{+} of the grasp matrix:

$$h^d = G^{+}(M_o \ddot{x}_o^d + C_o \dot{x}_o^d + g_o) \tag{3.21}$$

Note that this term is not an impedance relation and does not adjust if the environment hinders motion. The combined control law is $h^{\Sigma} = h^x + h^d$.
The set-up consists of four manipulators, distributed symmetrically around the object. In the first case translation in x-direction commanded. Results in Fig. **??** show good tracking behaviour: no position errors in steady state and only small deviations from the desired values during transient phase. No internal stress is exerted on the object. Due to fast translation high manipulator forces occur.
In the second test case the object is rotated around the z-axis. This is done at a significantly lower speed of at most 1 rad/s, thus the manipulator forces are smaller, desired and actual object trajectory cannot be distinguished in Fig. **??**. However some small internal forces can be seen. Interestingly they are proportional to the simulation step size (running *Simulink's ode3* solver), i.e. a ten-times smaller step size gives ten-times smaller internal forces. Internal forces are calculated based on the geometry in the last simulation step. The correlation between step size and values indicates that these forces are rather due to the discrete nature of the simulation than of the control law generating internal stress. Simulation of the constrained system dynamics as well as calculation of internal wrench is done as described in [**?**].

3.4.2 Internal and external impedance based reference trajectory generation

Caccavale and Villani [CV01] combine both internal and external impedance control. With external we mean a compliant relation between object and (external) environment. The architecture is cascaded, consisting of a two level reference trajectory generation and a motion control loop below. On top-level an impedance relation between object and environment is used to generate a compliant trajectory subject to environmental forces:

$$\alpha M_o(\ddot{x}_o^d - \ddot{x}_o^r) + D_o(\dot{x}_o^d - \dot{x}_o^r) + K_o(x_o^d, x_o^r) = h_{env} \tag{3.22}$$

The constant α scales the object inertia proportionally to a desired value. The control output is the reference object acceleration $\ddot{x}_o^r$, h_{env} is an input. This is sometimes called admittance control, admittance being the inverse of impedance. h_{env} has to be known, but is not easily measured in a practical set-up. Recalling (**??**) the environmental forces can be expressed as:

$$h_{env} = M_o \ddot{x}_o^r + C_o \dot{x}_o^r + g_o - G^{\dagger} h \tag{3.23}$$

Herein $G^{\dagger}$ is a generalized inverse of the grasp matrix, selecting the motion inducing components from the measured contact wrench h. $\dot{x}_o^r, x_o^r$ are calculated from $\ddot{x}_o^r$ by integration. From the compliant object trajectory $(\ddot{x}_o^r, \dot{x}_o^r, x_o^r)$ the desired trajectories of the manipulator $(\ddot{x}_i^d, \dot{x}_i^d, x_i^d)$ using the kinematic constraints. The reference manipulator trajectory, enforcing compliant behaviour between manipulators and object, is calculated from manipulator dynamics and internal forces:

$$M_i(\ddot{x}_i^d - \ddot{x}_i^r) + D_i(\dot{x}_i^d - \dot{x}_i^r) + K_i(x_i^d, x_i^r) = VV^{\dagger}h \tag{3.24}$$

The control output is the reference acceleration of the i-th manipulator $\ddot{x}_i^r$, $\dot{x}_i^r, x_i^r$ are obtained from integration. These variables are the inputs the inner motion control loop (PD-type). The strategy of compliant trajectories allows for high gains in the motion controller. Knowledge of object dynamics and measurement of the contact wrenches is required.
Results for translational motion (see Fig. **??**) are very similar to that of the previous control scheme.
When it comes to rotation, higher internal forces can be observed in Fig. **??**. In this case they are not influenced by numerical parameters of the simulation.
This architecture in contrast to the previous makes use of measured contact wrenches. Contact wrench as measured can be obtained from the constrained system simulation. As described in [CM08] this wrench is than decomposed by kineostatic filters in internal and external components. In this simulation this not cancel out undesired internal stress but magnifies it: when the contact wrench is not fed back and set to zero in the internal force impedance controller, results are slightly better. Note that the simulation represents an ideal case, where all parameters are exactly known and no deviations in grasp positions occur. Behavior in a real experiment may be different and this observation does not mean that the kineostatic-filtered feed-back of contact wrench is unjustified in general.

3.4.3 Intrinsically Passive Controller (IPC)

Introduced by Stramigioli [Str01b] and implemented by Wimböck et al. [WOH08], the architecture has been detailed throughout this work. In contrast to Stramigioli dampers are used in parallel with the manipulator springs and in contrast to Wimböck all springs have 6-DoF. Simulation results for pure translation can be seen in Fig. **??**. The object trajectory falls slightly behind the reference input, the dynamic behaviour is inferior to the previous approaches. Despite very stiff springs the magnitude of force seen in the previous approaches is not reached. No internal forces can be observed.
When it comes to rotation (see Fig. **??**) again the dynamic behaviour falls short of the two other approaches, while the manipulator wrench is higher. Significant internal wrenches are present, they amount up to 50% of the manipulator wrenches.

Chapter 4

Conclusion

List of Figures

Bibliography

[BH96] R.G. Bonitz and T.C. Hsia. Internal force-based impedance control for cooperating manipulators. *Robotics and Automation, IEEE Transactions on*, 12(1):78–89, 1996.

[BHM96] M. Buss, H. Hashimoto, and J.B. Moore. Dextrous hand grasping force optimization. *Robotics and Automation, IEEE Transactions on*, 12(3):406–418, 1996.

[CCMV08] F. Caccavale, P. Chiacchio, A. Marino, and L. Villani. Six-dof impedance control of dual-arm cooperative manipulators. *Mechatronics, IEEE/ASME Transactions on*, 13(5):576–586, 2008.

[CM08] F. Caccavle and Uchiyama M. Cooperative Manipulation. In Bruno Siciliano and Oussama Khatib, editors, *Springer Handbook of Robotics*, chapter 29, pages 701–718. Springer Berlin Heidelberg, May 2008. ISBN 978-3-540-23957-4.

[CV01] F. Caccvale and L. Villani. An impedance control strategy for cooperative manipulation. *Advanced Intelligent Mechatronics, IEEE/ASME International Conference on*, 1:343–348, 2001.

[DMSB09] V. Duindam, A. Macchelli, S. Stramigioli, and H. Bruyninckx. *Modeling and Control of Complex Physical Systems*. Springer Verlag, Berlin Heidelberg, 2009.

[DPEZ+15] L. De Pascali, S. Erhart, L. Zaccarian, F. Biral, and S. Hirche. A decoupling scheme for force control in cooperative multi-robot manipulation tasks. *Manuscript submitted for publication*, 2015.

[DS09] V. Duindam and S. Stramigioli. *Modeling and Control for Efficient Bipedal Walking Robots*. Springer Tracts in Advanced Robotics. Springer-Verlag, Berlin Heidelberg, 2009.

[EH16] S. Erhart and S. Hirche. Model and analysis of the interaction dynamics in cooperative manipulation tasks. *Robotics, IEEE Transactions on*, 2016.

[GFS+14] G. Gioioso, A. Franchi, G. Salvietti, S. Scheggi, and D. Prattichizzo. The flying hand: A formation of UAVs for cooperative aerial tele-manipulation. *Robotics and Automation (ICRA), 2014 IEEE International Conference on*, pages 4335–4341, May 2014.

[Goe52] R.C. Goertz. Fundamentals of general-purpose remote manipulators. *Nucleonics*, 10(11):36–45, 1952.

[HKDN13] D. Heck, D. Kostic, A. Denasi, and H. Nijmeijer. Six-dof impedance control of dual-arm cooperative manipulators. *Control Conference (ECC), 2013 European*, pages 2299–2304, July 2013.

[Hog84] N. Hogan. Impedance control: An approach to manipulation. *American Control Conference*, pages 304–313, June 1984.

[Hsu93] P. Hsu. Coordinated control of multiple manipulator systems. *Robotics and Automation, IEEE Transactions on*, 9(4):400–410, 1993.

[HTL00] L. Han, J.C. Trinkle, and Z.X. Li. Grasp analysis as linear matrix inequality problems. *Robotics and Automation, IEEE Transactions on*, 16(6):663–674, 2000.

[LL02] Guanfeng Liu and Zexiang Li. A unified geometric approach to modeling and control of constrained mechanical systems. *IEEE Transactions on Robotics and Automation*, 18(4):574–587, Aug 2002.

[LS05] Dongjun Lee and M.W. Spong. Bilateral teleoperation of multiple cooperative robots over delayed communication networks: Theory. *Robotics and Automation (ICRA), International Conference on*, pages 360–365, April 2005.

[LTD14] Mitsubishi Heavy Industries LTD. "meister" remote control robot completes demonstration testing at fukushima daiichi nuclear power station, 2014. Press Information 1775, February 20, 2014; Online, accessed January 13, 2016. URL: `https://www.mhi-global.com/news/story/1402201775.html`.

[MT93] M.J. Massimino and Sheridian T.B. Sensory substitution for force feedback in teleoperation. *Presence, MIT Press Journals*, 2(4):344–352, 1993.

[NPH08] G. Niemeyer, C. Preusche, and G. Hirzinger. Telerobotics. In Bruno Siciliano and Oussama Khatib, editors, *Springer Handbook of Robotics*, chapter 31, pages 741–757. Springer Berlin Heidelberg, May 2008. ISBN 978-3-540-23957-4.

[NS04] G. Niemeyer and J.J.E. Slotine. Telemanipulation with time delays. *International Journal on Robotics Research*, 23(9):873–890, Sep 2004.

[SC92] S.A. Schneider and R.H. Cannon. Object impedance control for cooperative manipulation: theory and experimental results. *Robotics and Automation, IEEE Transactions on*, 8(3):383–394, 1992.

[She92] T.B. Sheridian. *Telerobotcis, Automation and Human Supervisory Control*. MIT Press, Cambridge, MA, 1992.

[SMA99] S. Stramigioli, Claudio Melchiorri, and S. Andreotti. A passivity-based control scheme for robotic grasping and manipulation. *Decision and Control, 1999. Proceedings of the 38th IEEE Conference on*, 3:2951–2956, 1999.

[SMH15] D. Sieber, S. Music, and S. Hirche. Multi-robot manipulation controlled by a human with haptic feedback. *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pages 2440–2446, Sep 2015.

[SMP14] S. Scheggi, F. Morbidi, and D. Prattichizzo. Human-robot formation control via visual and vibrotactile haptic feedback. *Haptics, IEEE Transactions on*, 7(4):499–511, Oct 2014.

[SSVdSF05] S. Stramigioli, C. Secchi, A.J. Van der Schaft, and C. Fantuzzi. Sampled data systems passivity and discrete port-hamiltonian systems. *IEEE Transactions on Robotics*, 21(4):574–587, Aug 2005.

[Str01a] Stefano Stramigioli. Geometric modeling of mechanical systems for interactive control. In Alfonso Banos, Francoise Lamnabhi-Lagarrigue, and FranciscoJ. Montoya, editors, *Advances in the control of nonlinear systems*, volume 264 of *Lecture Notes in Control and Information Sciences*, pages 309–332. Springer London, 2001.

[Str01b] Stefano Stramigioli. *Modeling and IPC Control of Interactive Mechanical Systems: A Coordinate-Free Approach*. Springer-Verlag London, London, UK, 2001.

[Str15] Stefano Stramigioli. Energy-aware robotics. In M. K. Camlibel, A. A. Julius, R. Pasumarthy, and J. Scherpen, editors, *Mathematical Control Theory I*, volume 461 of *Lecture Notes in Control and Information Sciences*, pages 37–50. Springer London, 2015.

[vdS06] Arjan van der Schaft. Port-hamiltonian systems: an introductory survey. In M. Sanz-Sole, J. Soria, J.L. Varona, and J. Verdera, editors, *Proceedings of the International Congress of Mathematicians Vol. III: Invited Lectures*, pages 1339–1365, Madrid, Spain, 2006. European Mathematical Society Publishing House (EMS Ph).

[vdS13] Arjan van der Schaft. Port-hamiltonian differential-algebraic systems. In A. Ilchmann and T. Reis, editors, *Surveys in Differential-Algebraic Equations I*, Differential-Algebraic Equations Forum, pages 173–226. Springer Berlin, 2013.

[vdSJ14] Arjan van der Schaft and Dimitri Jeltsema. *Port.Hamiltonian Systems Theory: An Introductory Overview.* Foundations and Trends in Systems and Control. NOW Publishing Inc., Hanover, MA, 2014.

[WKD92] J. Wen and K. Kreutz-Delgado. Motion and force control of multiple robotic manipulators. *Automatica*, 28(4):729–743, 1992.

[WOH06] T. Wimboeck, C. Ott, and G. Hirzinger. Passivity-based object-level impedance control for a multifingered hand. *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pages 4621–4627, Oct 2006.

[WOH08] T. Wimboeck, C. Ott, and G. Hirzinger. Analysis and experimental evaluation of the intrinsically passive controller (ipc) for multifingered hands. *Robotics and Automation, IEEE International Conference on*, pages 278–284, May 2008.

[WS08] K. Waldron and J. Schmiedeler. Kinematics. In Bruno Siciliano and Oussama Khatib, editors, *Springer Handbook of Robotics*, chapter 1, pages 9–33. Springer Berlin Heidelberg, May 2008. ISBN 978-3-540-23957-4.

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