Getting acquainted - nonlinear model and linearization

Since the state-space representation is $\dot{\boldsymbol{x}} = A_c\boldsymbol{x} + B_c\boldsymbol{u}$, and the vectors $\boldsymbol{x}, \boldsymbol{u}$ have the following dimensions: $\boldsymbol{x} \in \mathbb{R}^{12}$ and $\boldsymbol{u} \in \mathbb{R}^4$, it is normal that $A_c \in \mathbb{R}^{12\times 12}$ and $B_c \in \mathbb{R}^{12\times 4}$.

1. Interpretation of structure of matrices A_c and B_c

We have the following state-space representation:

Regarding the matrix A_c, the lines 1 to 3 are stating $[\dot{x}, \dot{y}, \dot{z}] = [\dot{x}, \dot{y}, \dot{z}]$. It is the same for the derivatives of the variables α, β and γ in the lines 7 to 9.
The lines 4 and 5 are stating $\ddot{x} = g \cdot \beta$ and $\ddot{y} = -g\alpha$, with g the gravitational constant – ¿ $\boldsymbol{WHY}$.

Regarding the matrix B_c:
- Line 6: We have $\ddot{z} = 3.5 \cdot \sum_{i=1}^{4} u_i$:
The acceleration of the non-rotational origin $\ddot{\boldsymbol{O}}_B$ is only present in the z coordinate and is $\ddot{z}_s = -g + \frac{F}{m}$, with $F = k_f \cdot [\sum_{i=1}^{4} u_i]$. By taking the value of k_f in the first row of `quad.K` and the value of m in `quad.mass`, this leads to $\frac{k_f}{m} = \frac{28}{8} = 3.5$.

- Lines 10 to 12: $\begin{bmatrix}\ddot{\alpha}\\ \ddot{\beta}\\ \ddot{\gamma}\end{bmatrix} = \begin{bmatrix}0 & 0.56 & 0 & -0.56\\ -0.56 & 0 & 0.56 & 0\\ 0.73 & -0.73 & 0.73 & -0.73\end{bmatrix}\boldsymbol{u}$:
We know that $\omega_s = \begin{bmatrix}\dot{\alpha_s}\\ \dot{\beta_s}\\ \dot{\gamma_s}\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$. We also know that

$$\dot{\omega}_s|_{\omega_s=\mathbf{0}} = \begin{bmatrix}\ddot{\alpha}\\ \ddot{\beta}\\ \ddot{\gamma}\end{bmatrix}\Bigg|_{\omega_s=\mathbf{0}} = \mathcal{I}^{-1}\begin{bmatrix}M_\alpha\\ M_\beta\\ M_\gamma\end{bmatrix} = \mathcal{I}^{-1}\begin{bmatrix}0 & k_F L & 0 & -k_F L\\ -k_F L & 0 & k_F L & 0\\ k_M & -k_M & k_M & -k_M\end{bmatrix}\boldsymbol{u}.$$

The matrix `quad.L` gives $L = 0.2$ and `quad.km` gives $k_m = 11$. Since `quad.I` $= diag(10, 15, 15)$ and $k_F = 28$, we have $\mathcal{I}^{-1} = diag(0.1, 0.1, \frac{1}{15})$. This leads to $\begin{bmatrix}\ddot{\alpha}\\ \ddot{\beta}\\ \ddot{\gamma}\end{bmatrix} = \begin{bmatrix}0 & 0.56 & 0 & -0.56\\ -0.56 & 0 & 0.56 & 0\\ 0.73 & -0.73 & 0.73 & -0.73\end{bmatrix}\boldsymbol{u}$.

First MPC Controller

1. Choice of tuning parameters and motivation for them

First, the matrices Q and R are set to identity matrices. Those parameters give the results in Fig. 1. We see that the roll and pitch angles take far too much time too reach steady-state. Thus, the states α and β have to be more penalized, namely the state penalty matrix is set to $Q = \text{diag}(1,\ 100,\ 100,\ 1,\ 1,\ 1,\ 1)$. The results are shown on Fig. 2.

2. Plots of the response to an appropriate initial condition

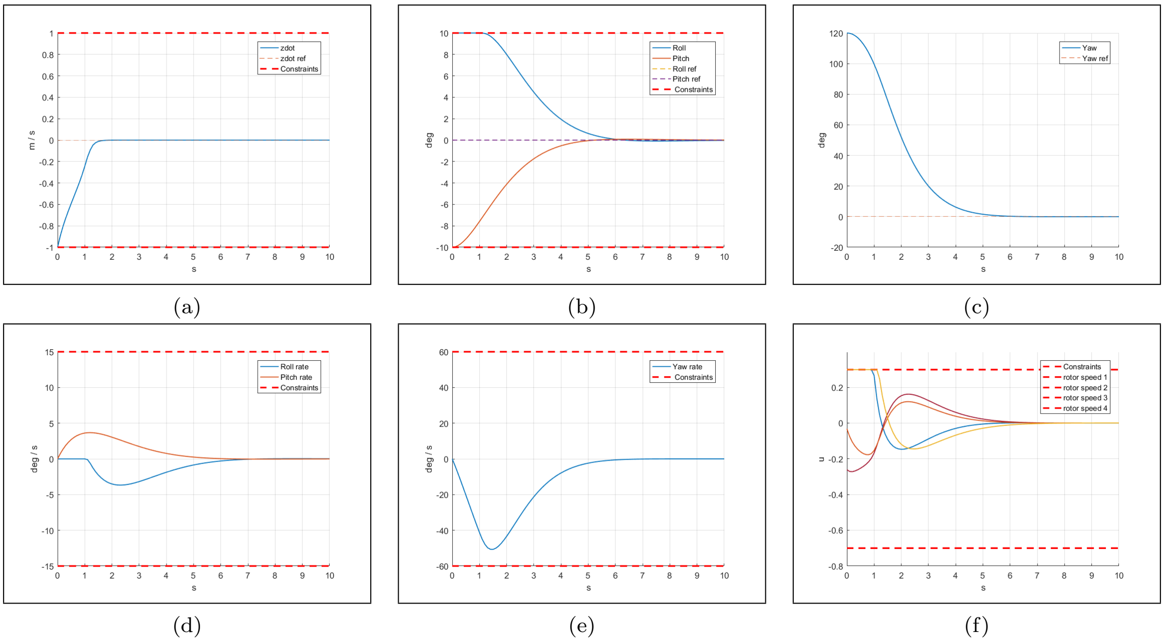

Figure 1: (a) $\dot{z}$, (b) $\alpha,\ \beta$, (c) γ, (d) $\dot{\alpha},\ \dot{\beta}$, (e) $\dot{\gamma}$, (f) u

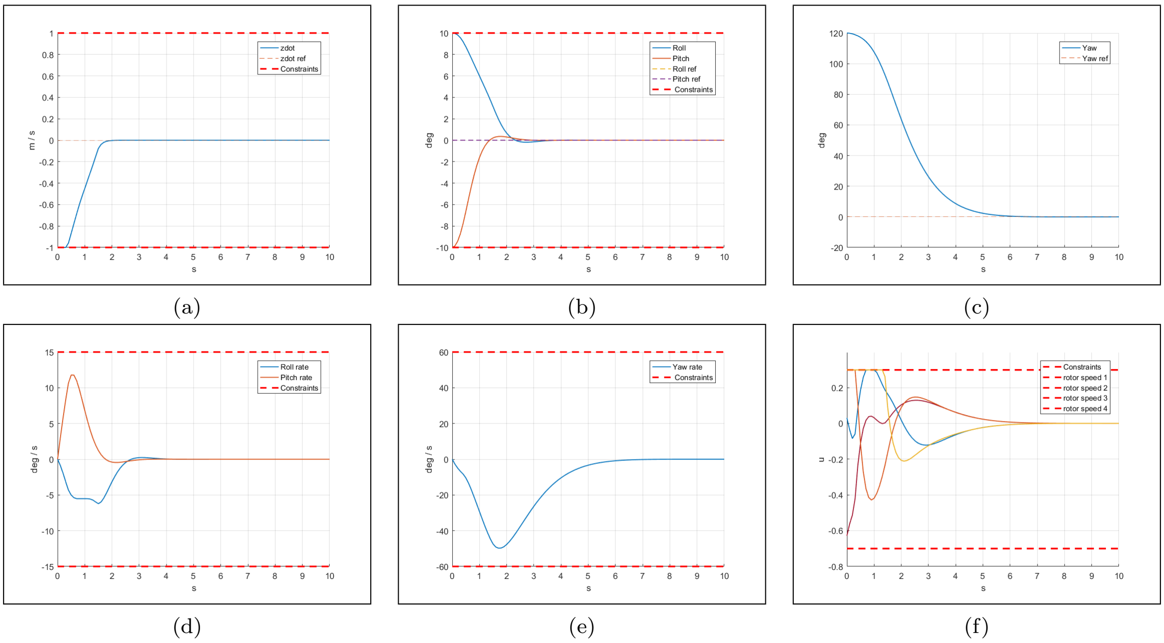

Figure 2: (a) $\dot{z}$, (b) $\alpha,\ \beta$, (c) γ, (d) $\dot{\alpha},\ \dot{\beta}$, (e) $\dot{\gamma}$, (f) u

Reference tracking

Deliverables

Interpretation of the solution $(\mathbf{x}_r; \mathbf{u_r})$ to system (4) for arbitrary $\mathbf{r}_1$

plots of the response to a constant reference signal

plots of the response to a slowly varying reference signal

First simulation of the nonlinear model

Deliverables

plots of a reference tracking response of the nonlinear model

Offset free MPC

Deliverables

motivation for the choice of the estimation error dynamics

First, we need to be sure that the system is observable. This is done by verifying that, in the initial system:

$$\begin{cases} x^+ = & A_1\, x + B\, u + & B_d\, \bar{d} \\ \bar{d}^+ = & & \bar{d} \\ y = & C\, x + & C_d\, \bar{d} \end{cases}$$

We can identify the matrices $B_d, C = diag(\mathbf{1}_7)$ and $C_d = diag(\mathbf{0}_7)$, with $\mathbf{1}_n$ is a vector of ones $\in \mathbb{R}^n$ (same for $\mathbf{0}$), and $diag(vec)$ is a matrix with the elements of vec in the diagonal $\in \mathbb{R}^{|vec| \times |vec|}$.

We have that the rank of the matrix $\begin{bmatrix} C \\ CA_1 \\ \vdots \\ CA_1^{N-1} \end{bmatrix}$ is $n_x = 7$ and the rank of the matrix $\begin{bmatrix} A_1 - I_7 & I_7 \\ C & C_d \end{bmatrix}$ is $n_x + n_d =$ 14.
The two conditions are verified using the "rank" command on Matlab.

We are choosing a dead-beat observer: this observer is taking the observability error to 0 in $n = n_x + n_d$ samples, which corresponds to $n \cdot T_s = 14 \cdot 0.1 = 1.4s$.

step reference tracking plots in the presence of disturbance

slowly-varying reference tracking plots in the presence of disturbance