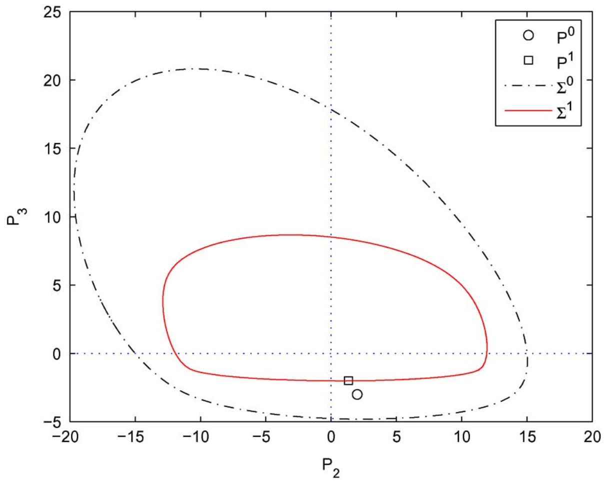

Fig. 3. Three-bus system: nominal and modified power flow solution space boundary as defined by the solution in Table I.

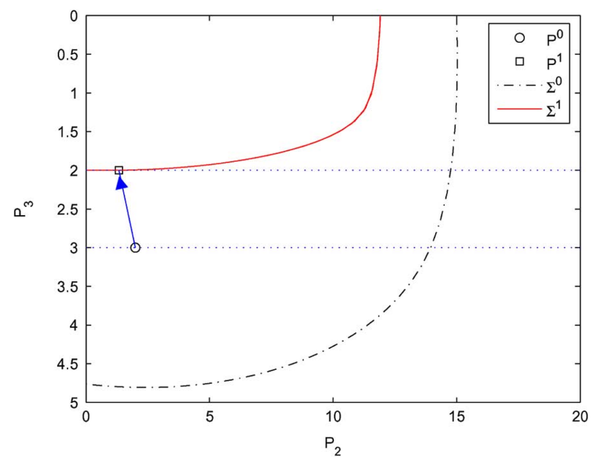

Fig. 4. Fourth quadrant of Fig. 3.

TABLE I
THREE-BUS SYSTEM SOLUTION: USING THE FORMULATION (26)–(41)

Lines identified		Load shed and generation redispatch		
Line #	γ	Bus #	P^0	P^1
3	0.67	1	1.0000	0.6667
5	1.00	2	2.0000	1.3333
		3	-3.0000	-2.0000

bus 1 is simply $-(P_2+P_3)$ and consideration of another axis P_1 is unnecessary. When all the lines are in service, the power flow solution space boundary can be traced by a continuation technique [19] and is identified as Σ^0. The region enclosed by Σ^0 contains all possible power flow solutions that the present network topology (defined by line parameters) supports. The part of this region relevant to us is the quadrant having $P_2 \geq 0$ and $P_3 \leq 0$, given that the devices connected to buses 2 and 3 are a generator and load, respectively. Note that the nominal operating point P^0 lies within this region.

First we perform our analysis by avoiding voltage constraints, setting $V_{\min} = 0.5$ p.u. We then raise the limit to examine the effect of a binding voltage constraint. The generator buses 1 and 2 maintain voltages at 1.0 p.u., and no maximum voltage constraints are included (or would be needed).

Using the problem formulation (26)–(41), a few critical lines in this system are identified while ensuring that their removal will cause a failure having a severity of at least 1 p.u., or equivalently an event that will necessitate at least 1 p.u. of load shedding at bus 3. The corresponding parameter $S_{\min}$ is defined as 1. The initial guess for the solution process was obtained as described in Appendix B. The nonzero values of γ_3 and γ_5 in the solution, summarized in Table I, identifies lines 3 and 5 as important. The values for γ_1, γ_2, and γ_4 are identically zero.

The first-stage relaxation solution using the values for γ_3 and γ_5 shown in Table I, yield a load bus voltage of 0.55 p.u.; the phase angles across the lines (that still remain in service) are within $\pm\pi/2$. With both voltage and angle constraints (38)–(39) inactive, the power flow Jacobian J is nontrivially singular, as discussed in Section II-B. This is shown pictorially in Fig. 3. With the new network topology, the original boundary Σ^0 moves to Σ^1. Note that the original operating point P^0 lies outside this new boundary, and is now infeasible. The solution identifies point P^1 which achieves feasibility again by shedding the least possible load at bus 3. As the power flow Jacobian J is nontrivially singular, this point lies on Σ^1.

For clarity, the relevant (fourth) quadrant of Fig. 3 is redrawn in Fig. 4. The arrow represents the movement of the operating point from P^0 to P^1. Its projection onto the P_3 axis corresponds to the amount of load that is shed at bus 3. Note in Table I that due to the distributed slack bus mechanism, the generators have been redispatched in a constant proportion to their nominal values so as to accommodate the reduction of load at bus 3.

This first stage solution gives a severity of minimum lost load equal to 1.0 p.u. The second stage analysis is performed by removing both line 3 and line 5 from service. The resulting severity is 1.5582 p.u.

Now we use formulation (44)–(45) to maximize the severity while limiting the number of line outages to be no more than two (i.e., with $L_{\max} = 2$). The values for γ obtained by the optimization algorithm again identify lines 3 and 5 as the most important. To obtain this solution, which is summarized in Table II, the same initialization procedure as before is employed. The load bus voltage is again about 0.55 p.u. The optimization algorithm completely removes lines 3 and 5 from service in order to achieve the maximum of the objective function. (The values of γ for the other three lines are identically zero.) This corresponds to a severity of 1.5582 p.u. load shed at bus 3. Since this first stage analysis resulted in an integer only solution and no phase angle limits were reached, the second stage analysis is not required.

The voltage and angle constraints are inactive for this solution too, thus making the Jacobian J nontrivially singular. It follows that the new (post load shedding) operating point P^2, as identified in Fig. 5, lies on the power flow solution space boundary Σ^2 resulting from a complete removal of lines 3 and 5 from service. The operating points P^0 and P^1, and corresponding boundaries Σ^0 and Σ^1 are also shown for comparison. Note that first stage