along with (10)–(13). Thus the vector z that provides the best load shedding strategy is obtained by solving (10) and (15)–(22), while satisfying the inequalities (11)–(13) and (23). The notation "$\cdot$" in (17)–(22) is used to indicate component-wise multiplication of associated vectors.

When the inequality constraints (11)–(13) are inactive, we have $\mu_1, \ldots, \mu_6 = 0$. Referring to (16), this results in $J^T\lambda = 0$, which in turn makes λ a left eigenvector of J corresponding to its zero eigenvalue. Note that J must have an extra (nontrivial) zero eigenvalue, as $\lambda = w_0 = e^T$ does not satisfy (15) and (8) simultaneously. In other words, λ equals w, where w is a left eigenvector of J corresponding to its nontrivial zero eigenvalue. For this case, it is insightful to appreciate the geometrical interpretation of (15)–(16). Consider a hyperplane tangent to the manifold defined by (10). Vectors $(\delta\theta, \delta V, \delta z)$ on that tangent hyperplane satisfy

$$J \begin{bmatrix} \delta\theta \\ \delta V \end{bmatrix} + \frac{\partial F}{\partial z}\delta z = 0. \tag{24}$$

Premultiplying with the eigenvector w^T results in

$$w^T J \begin{bmatrix} \delta\theta \\ \delta V \end{bmatrix} + w^T \frac{\partial F}{\partial z}^T \delta z = 0. \tag{25}$$

However, as the first term in (25) vanishes to zero, we must have $((\partial F)/(\partial z)w)^T \delta z = 0$. This implies that the normal to the power flow feasibility boundary at (θ, V, z) is given by $(\partial F)/(\partial z)^T w$. It also follows from (15) that this normal aligns with e when the inequality constraints are inactive.

C. Constrained Optimization Problem

Our formulation of the contingency screening problem employs the mechanisms of best load shedding strategy, distributed slack and binary line indicator variables, described in the previous sections. Note that we seek to move both the original operating point P^0 and the power flow feasibility boundary. This boundary is moved past P^0 such that the minimum load shedding required to move P^0 to a different operating point lying on the new boundary is greater than the minimum desired severity of the blackout. Mathematically, the problem takes the following form:

$$\min_{\theta, V, z, \gamma, \mu_1, \ldots, \mu_6, \lambda} \quad e^T\gamma \tag{26}$$

$$\text{s.t.} \quad F(\theta, V, z, \gamma) = 0 \tag{27}$$

$$e + \frac{\partial F}{\partial z}^T \lambda - \mu_1 + \mu_2 = 0 \tag{28}$$

$$J^T\lambda + \begin{bmatrix} -A^T\mu_5 + A^T\mu_6 \\ -\mu_3 + \mu_4 \end{bmatrix} = 0 \tag{29}$$

$$\mu_1 \cdot z = 0 \tag{30}$$

$$\mu_2 \cdot \left(P_{pq}^0 + z\right) = 0 \tag{31}$$

$$\mu_3 \cdot (V_{\min} - V) = 0 \tag{32}$$

$$\mu_4 \cdot (V - V_{\max}) = 0 \tag{33}$$

$$\mu_5 \cdot (\pi/2 + A\theta) = 0 \tag{34}$$

$$\mu_6 \cdot (A\theta - \pi/2) = 0 \tag{35}$$

$$\mu_1, \ldots, \mu_6 \geq 0 \tag{36}$$

$$P_{pq}^0 \leq P_{pq}^0 + z \leq 0 \tag{37}$$

$$V_{\min} \leq V \leq V_{\max} \tag{38}$$

$$-\pi/2 \leq A\theta \leq \pi/2 \tag{39}$$

$$\gamma \in \{0, 1\} \tag{40}$$

$$e^T z \geq S_{\min}. \tag{41}$$

Constraints (27) denotes the power flow equations (10), now with γ as an additional variable. Together, constraints (27)–(39) are the Karush–Kuhn–Tucker conditions as obtained in Section II-B; they are repeated here for clarity. These constraints help us avoid explicitly solving a bilevel optimization problem. Constraint (40) limits γ variables to binary values. Constraint (41) ensures the total amount of load shed is greater than $S_{\min}$, a positive-valued user-defined parameter that indicates the minimum severity of lost load.

By virtue of the distributed slack mechanism, all the generators contribute to the load shedding in the same proportion. This ensures that all the generators must reduce (not increase) their dispatches to account for load shedding, thus in turn ensuring that their upper dispatch capacity limits are never hit. This also guarantees that all generators will reach the lower dispatching limit (assumed as zero) simultaneously, when all the load in the system is shed. Thus a constraint that limits generator dispatches is not included in the set of constraints (27)–(41). Note, however, that if the lower dispatching limits are nonzero, constraints enforcing such limits can be added to the set (27)–(41). If we denote the minimum generation capacity of the ith PV bus by $(P_{\text{pv,min}}^0)_i$, then the new generations would need to satisfy [cf. (4)]

$$\left({P^0}_{pv}\right)_i + k\left({P^0}_{pv}\right)_i \geq \left(P_{pv,\min}^0\right)_i \tag{42}$$

$i = 1, \ldots, m_g$. Using the definition of the scalar k given by (3) in (42), it follows that the constraints to enforce the lower dispatching limits would be

$$e^T z \leq \left(\frac{1 - \left(P_{pv,\min}^0\right)_i}{\left({P^0}_{pv}\right)_i}\right) e^T {P^0}_{pv} \tag{43}$$

$i = 1, \ldots, m_g$. Note that only one such constraint would really need to be enforced, namely that for which $(1 - (P_{\text{pv,min}}^0)_i/({P^0}_{pv})_i)$ is smallest, since the remaining would then be necessarily satisfied. Moreover, when the generator corresponding to that constraint reaches its lowest capacity limit during optimization iterations, it is excluded from the distributed slack formulation (3), thus providing the freedom required for rest of the generators to participate in further load shedding. This can be achieved by placing an outer loop on the optimization problem (26)–(41) that gets activated when a generator lower dispatch limit given by (43) is reached and reinitializes the problem with the modified slack, that excludes the binding generator. Recall that our goal is to *quickly* screen multiple contingencies to identify the severe ones. A reasonably flexible and practical slack mechanism as the one we adopt, is sufficient to serve our goal.