Fundamental Concepts in Computational and Applied Mathematics
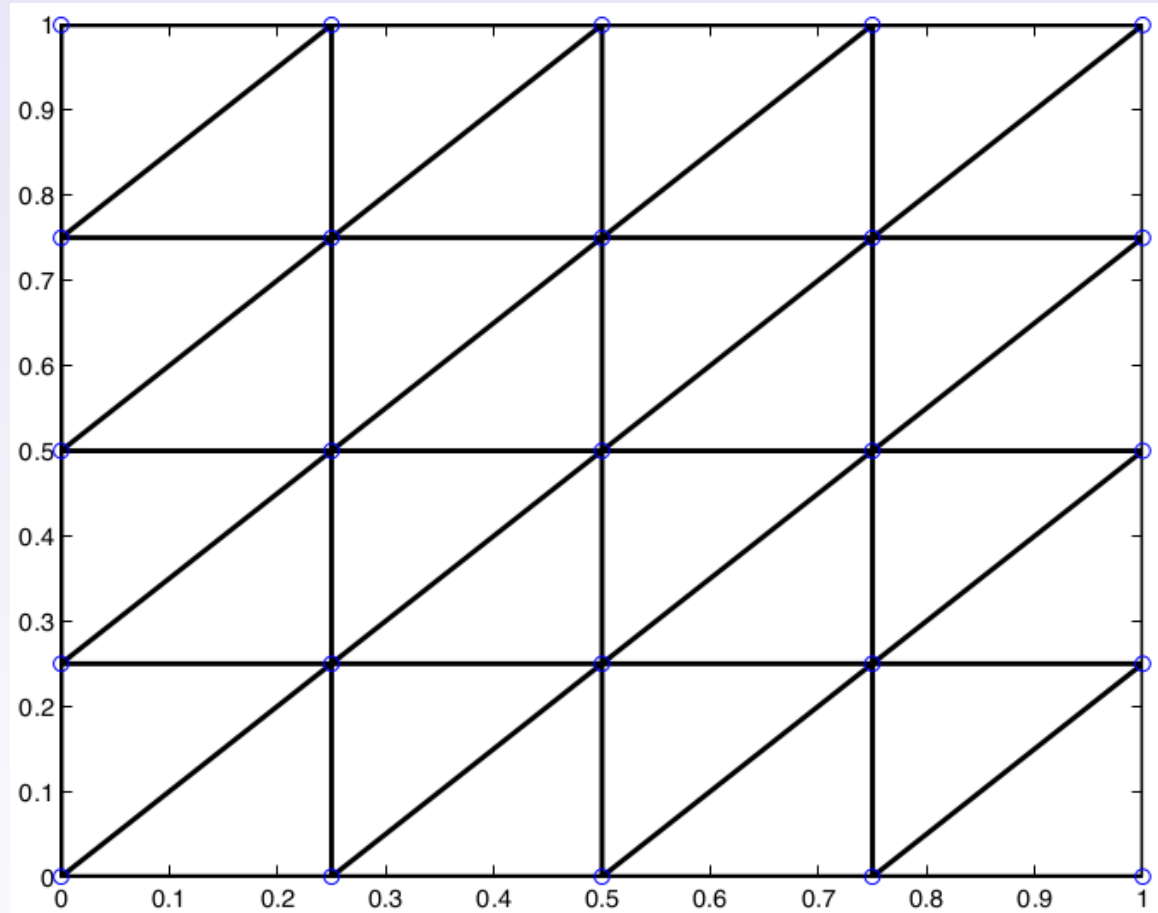
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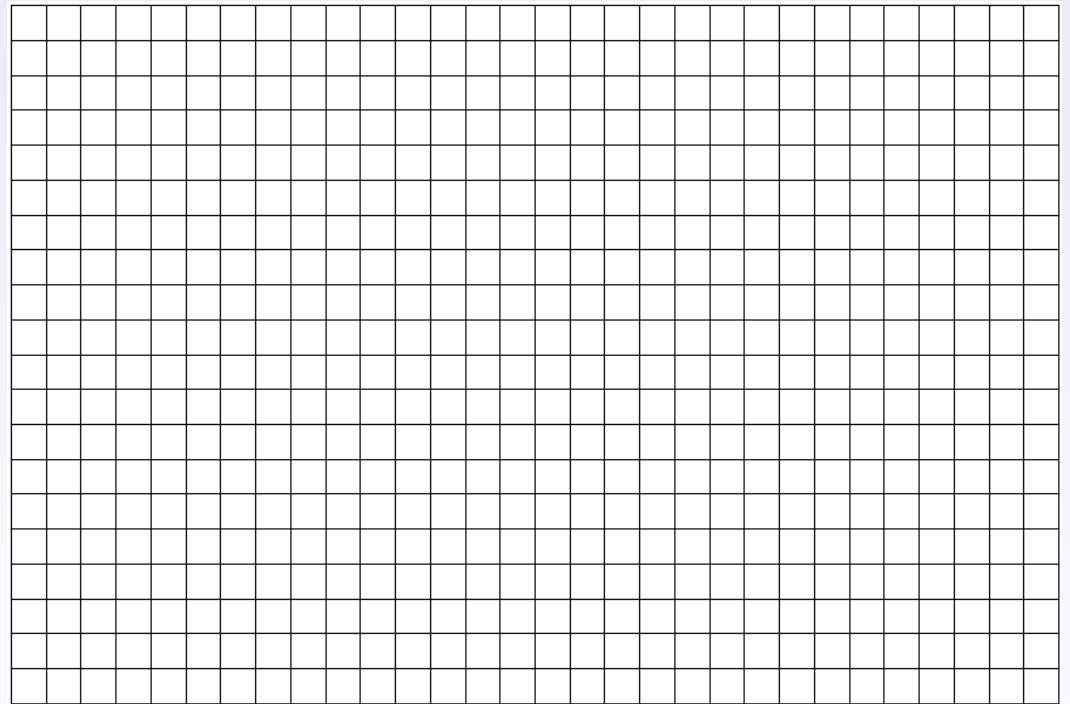# Homework 1 Discussion

Grid Terminology

- *mesh*

- *cells, elements,*
  e.g. triangles, quads,
  tetrahedrons, hex

- *node, vertex*

- *edge, face*

- Grid properties
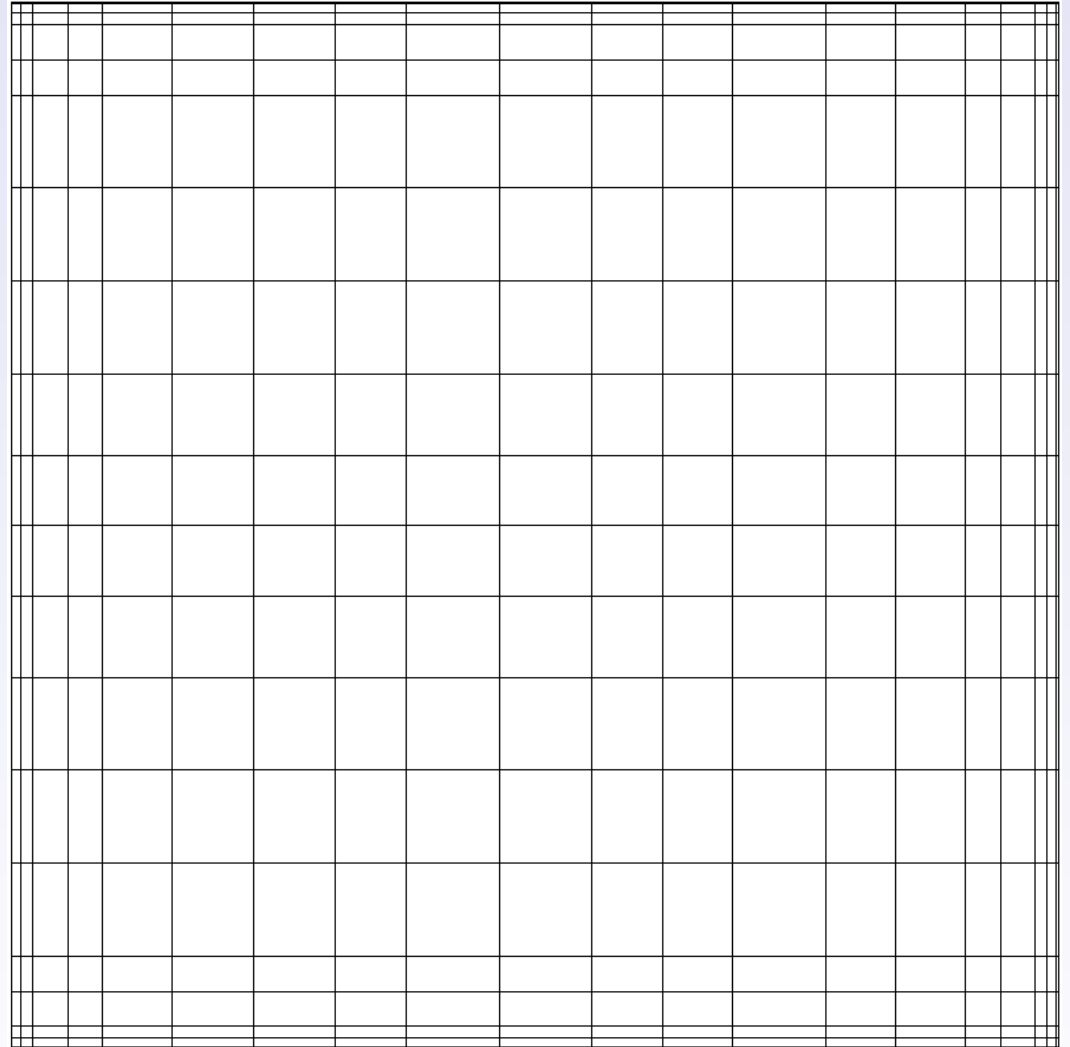 - quality of mesh
  - degeneracy
  - dof

Structured Grid Examples – Uniform

- (i, j, k) indexing
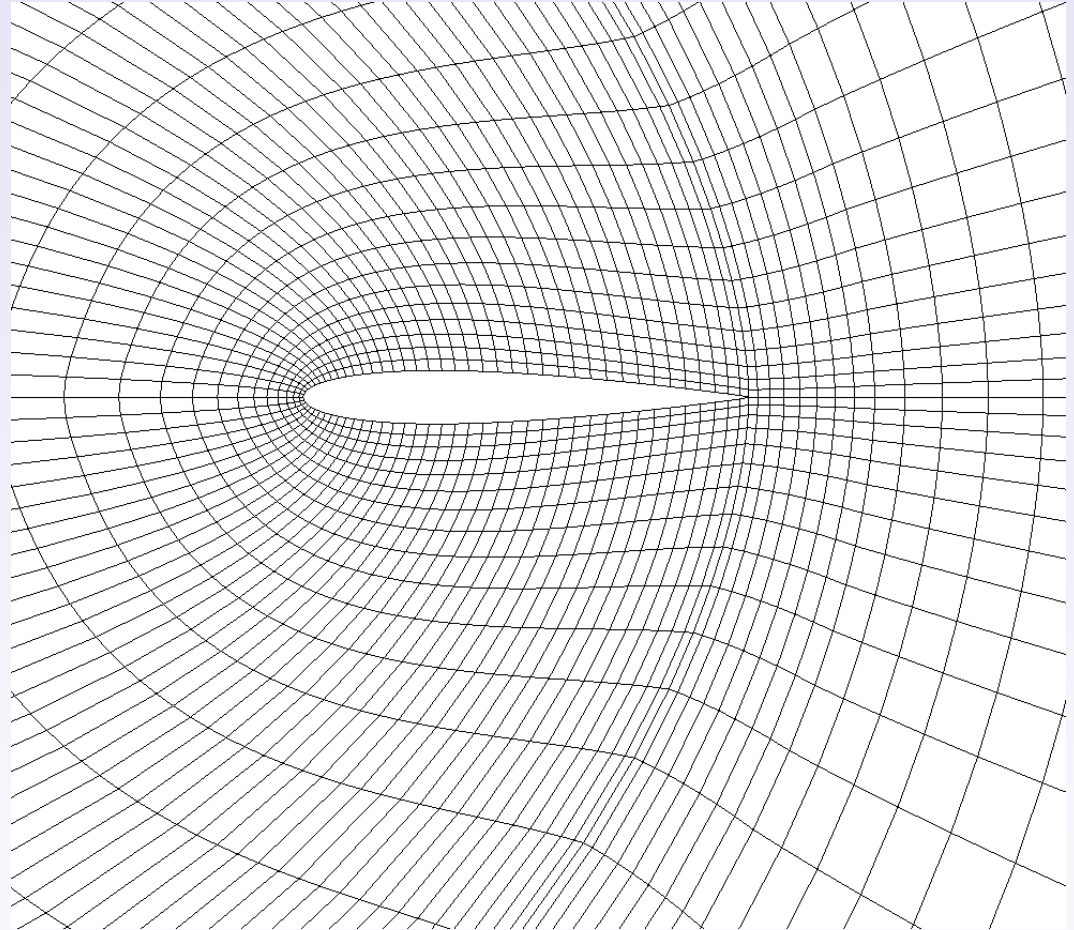- What are the advantages/disadvantages?

Structured Grid Examples – Rectilinear

- Similar to uniform grid
- What is the main advantage here as compared to a uniform grid?
- What are the disadvantages?

Structured Grid Examples – Curvilinear

- Note that each node still has the same number of neighbors
- What is the main advantage here?
- What are the disadvantages?

Structured Grid Examples - Block Structured

- Used in Adaptive Mesh Refinement methods
- Solves problem of having too much resolution in places that you don't need it
- Software is more complicated
- Error analysis more difficult
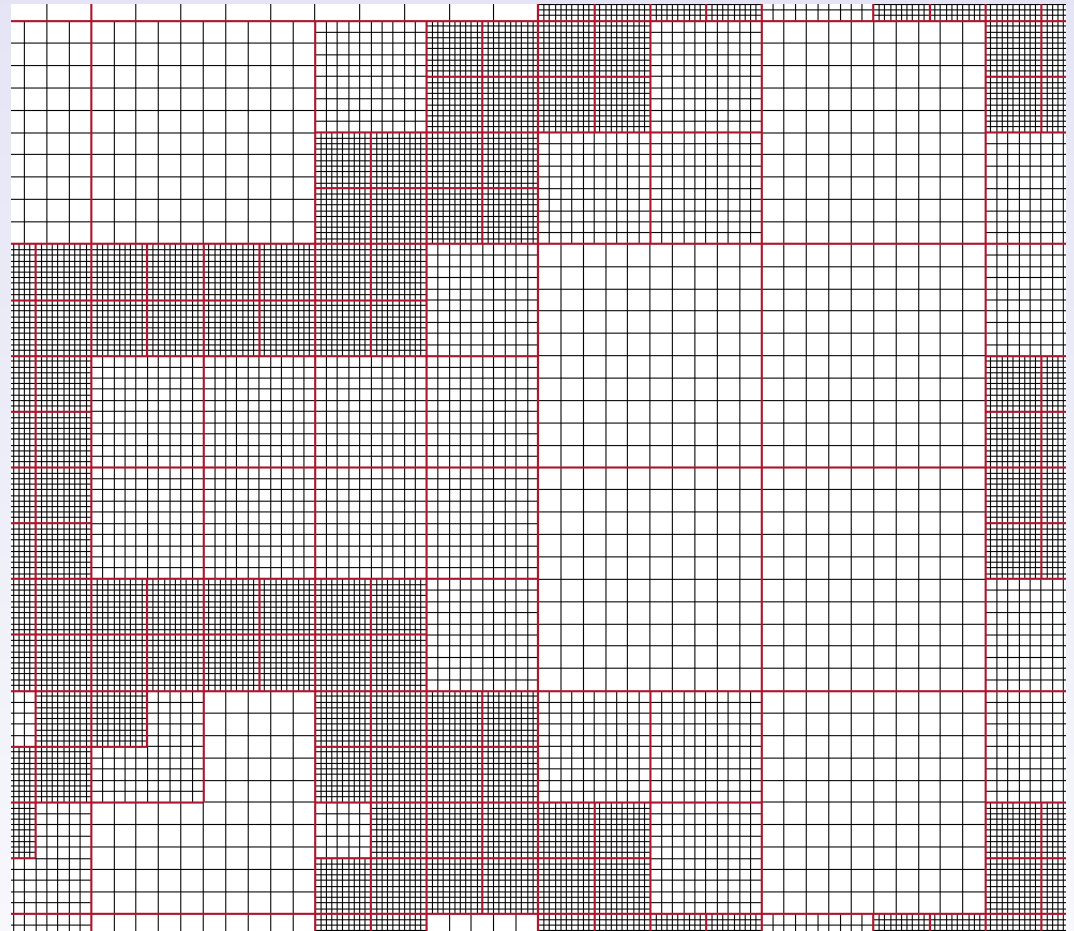
Main properties of structured grids

- Number of adjacent mesh elements is always the same
- Generally more accurate per unknown/dof than unstructured
- Convergence of algorithms (linear solvers) well understood
- Better data layout, which is good for computation

Example: 5-Point Stencil

5-point stencil

- Simplest 2-D case
- Leads directly to a sparse (penta-diagonal) matrix
- Iterative methods easy to apply
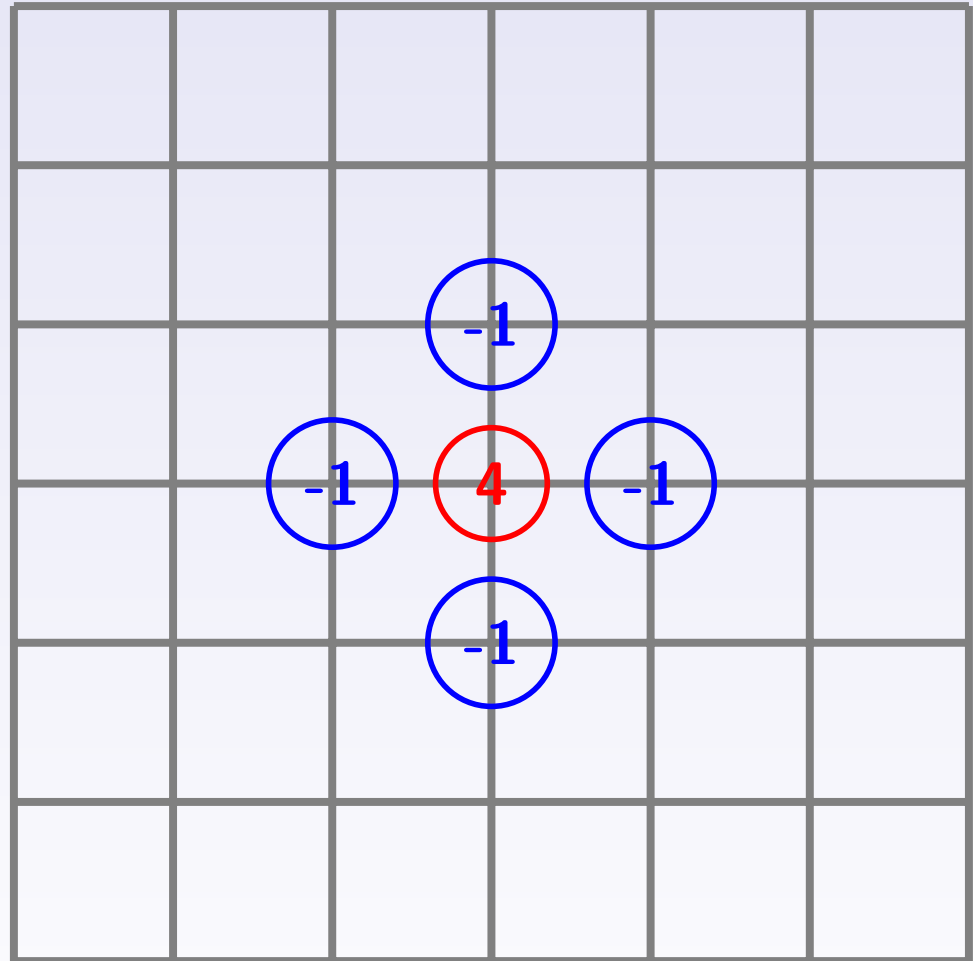
5-Point Stencil Matrix for $N = 3$ grid

$$A_9 = \begin{bmatrix}
4 & -1 & & -1 & & & & & \\
-1 & 4 & -1 & & -1 & & & & \\
& -1 & 4 & & & -1 & & & \\
-1 & & & 4 & -1 & & -1 & & \\
& -1 & & -1 & 4 & -1 & & -1 & \\
& & -1 & & -1 & 4 & & & -1 \\
& & & -1 & & & 4 & -1 & \\
& & & & -1 & & -1 & 4 & -1 \\
& & & & & -1 & & -1 & 4
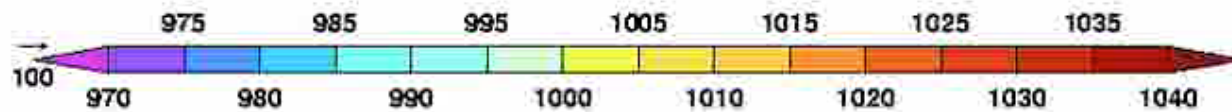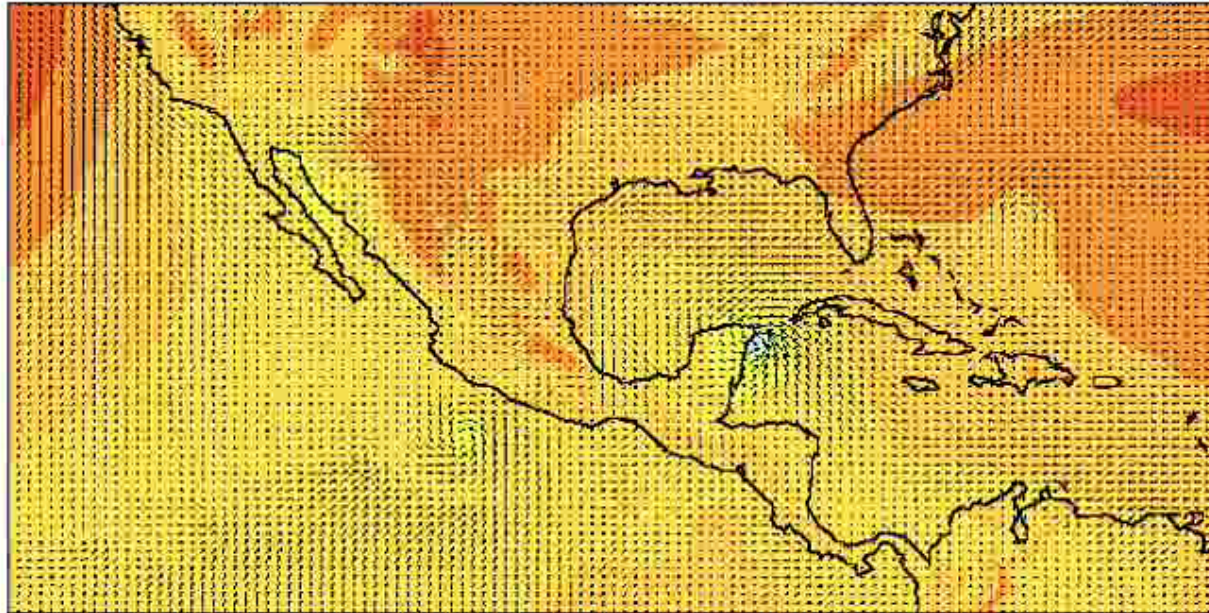\end{bmatrix}$$

Question

What matrix properties can you name?

Structured Grid Applications: Climate Modeling

AdGif - UNREGISTERED

Maximum surface wind speed = 76.703981490904894 mph

Minimum sea level pressure = 993.58273437499997 mb

Summary

- Structured grids exist in many shapes and forms
- Well understood methods
- Well developed software available
- Work well on parallel and other high performance computing environments
- Don't work as well with complex geometries
- Also harder to use with multi-material, multi-block problems