Mathematical model of cancer with competition

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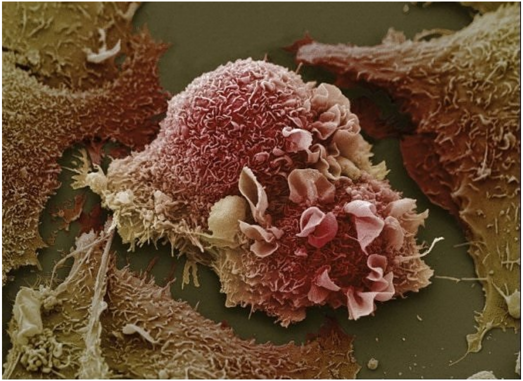
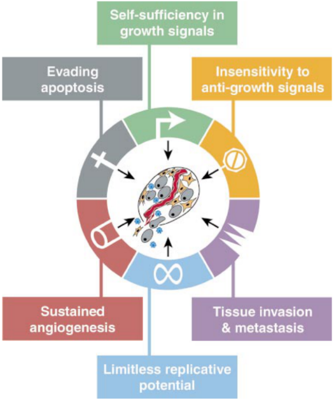
What is Cancer?

- Cancer (known medically as *malignant neoplasm*) is a group of diseases involving unregulated cell growth.
- Cancer is associated to both internal and external factors.
- Most forms of cancer have the same defining traits.



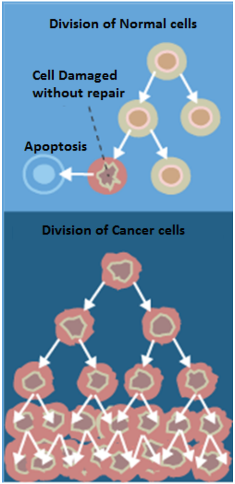
Common Cancer Traits



- Self-sufficiency in growth signals.
- Insensitivity to anti-growth signals.
- Evading apostosis.

Common Cancer Traits

- Limitless replicative growth potential.
- Sustained angiogenesis.
- Tissue invasion and metastasis.

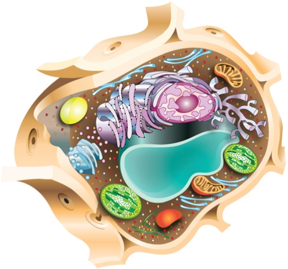


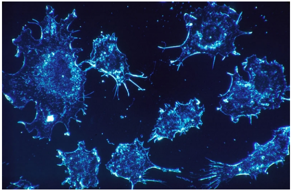
Cancer Statistics

- The National Cancer Institute estimates that there are about 13.7 million Americans with a history of cancer.
- In 2013, about 1,660,290 new American cancer cases are expected to be diagnosed. Roughly 580,350 of those cases will result in death.
- The National Institutes of Health (NIH) estimates that overall costs of cancer in 2008 was 201.5 billion dollars.

Models of Cancer

- Sub-Cellular Scale
- Cellular Scale - ODEs
- Macroscopic Scale - PDEs





The Model

$$\frac{dx_1}{dt} = -\gamma x_1 x_2,$$

$$\frac{dx_2}{dt} = \alpha x_2 - x_1 x_2 - x_2 x_3 - x_2^2,$$

$$\frac{dx_3}{dt} = \beta x_3 - f x_2 x_3,$$

where α, β, γ, f are positive constants and

x_1 : normal cells

x_2 : cancer cells

x_3 : the response of the immune system

Stability Analysis

- Using matrix notation, we rewrite our system of equations in the following form:
$$\mathbf{x}' = \mathbf{F}(\mathbf{x})$$
and calculate the Jacobian
$$D\mathbf{F}(\mathbf{x}) = \begin{pmatrix} -\gamma x_2 & -\gamma x_1 & 0 \\ -x_2 & \alpha - x_1 - x_3 - 2x_2 & -x_2 \\ 0 & -f x_3 & \beta - f x_2 \end{pmatrix}$$

Stability Analysis

- We find that the fixed points of $\mathbf{x}' = \mathbf{F}(\mathbf{x})$ are given by

$$P_1 = (a, 0, 0), \text{ where } a \neq 0,$$

$$P_2 = (0, 0, 0),$$

$$P_3 = (0, \alpha, 0),$$

$$P_4 = (0, \frac{\beta}{f}, \alpha - \frac{\beta}{f})$$

where we assume that $\alpha - \frac{\beta}{f} > 0$.

Stability Analysis

By looking at the fixed points of the equation in relation to the equation

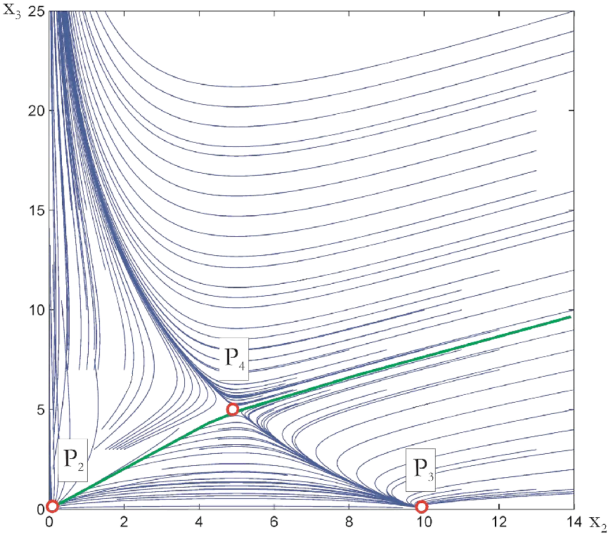
$$\det(D\mathbf{F}(P_i) - \lambda I) = 0, \text{ for } i = 1, 2, 3, 4,$$

where I is the identity matrix and λ is an eigenvalue, we can determine the stability of the fixed points.

Stability Analysis

We find that:
(1) $P_1 = (a, 0, 0)$ is a **saddle point** (if $a > \alpha$), or a **source** ($a \leq \alpha$).
(2) $P_2 = (0, 0, 0)$ is a **source**.
(3) $P_3 = (0, \alpha, 0)$ is a **sink**.
(4) $P_4 = (0, \frac{\beta}{f}, \alpha - \frac{\beta}{f})$ is a **saddle point**.

Stability Analysis

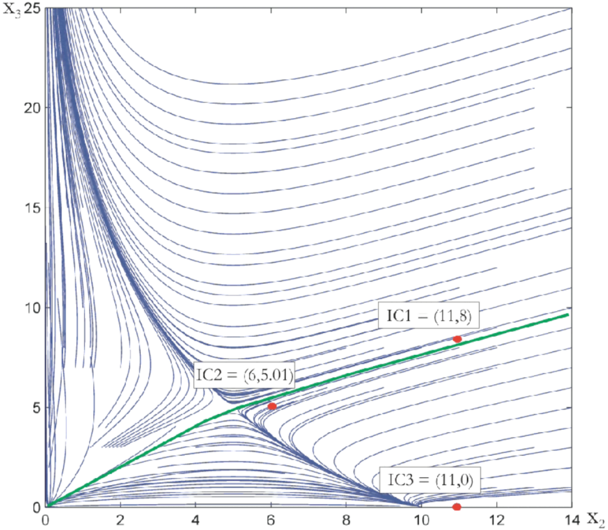


Results

If we have an immune response, the phase plane tells us that we have 3 situations:

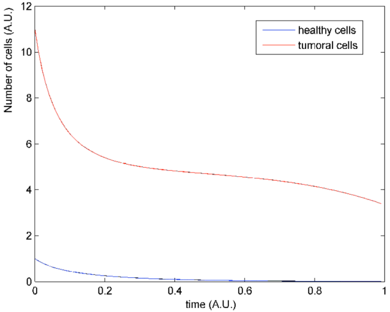
- **Elimination**
- **Equilibrium**
- **Escape**

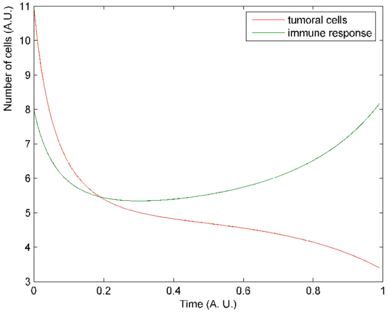
Results



Results

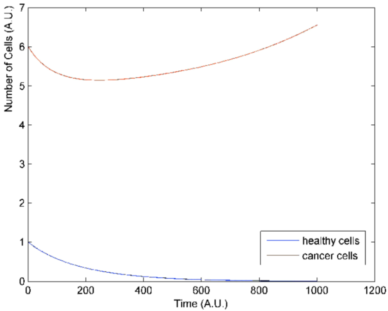
Initial condition $(1, 11, 8)$:

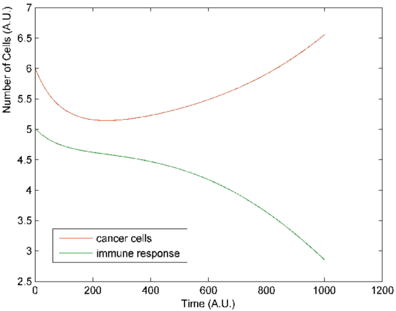




Results

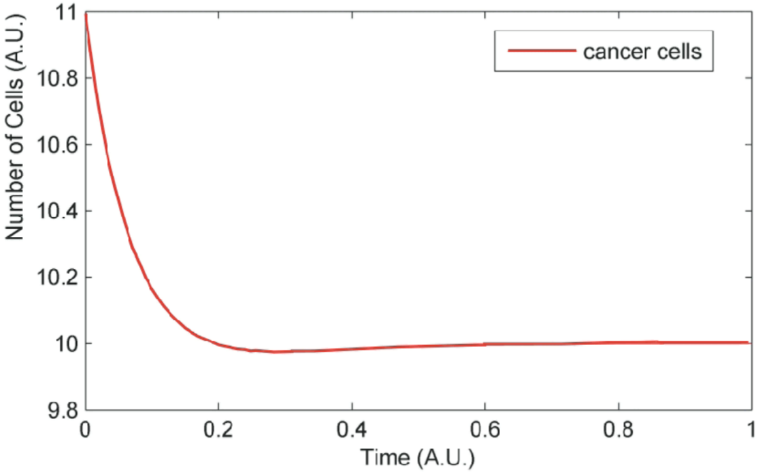
Initial condition $(1, 6, 5.01)$:





Results

Initial condition $(0.5, 11, 0)$:



Concluding Remarks

- Introduced an ODE model of cancer based on competition.
- Model yielded two groups of solutions possible.
- What are the implications of this model?

References

- J. M. Chrobak, H. Herrero, "Mathematical model of cancer with competition," in Mathematical Models in Engineering, Biology AND Medicine: International Conference on Boundary Value Problems 1124, 2009, pp. 79-88.
- "Cancer Facts and Figures 2013", National Cancer Institute, 2013. `http://www.cancer.org/acs/groups/content/@epidemiologysurveilance/documents/document/acspc-036845.pdf`
- D. Hanahan and R. A. Weinberg, "The Hallmarks of Cancer," in Cell 100, 2000, pp. 57-70.

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