Fundamental Concepts in Computational and Applied Mathematics

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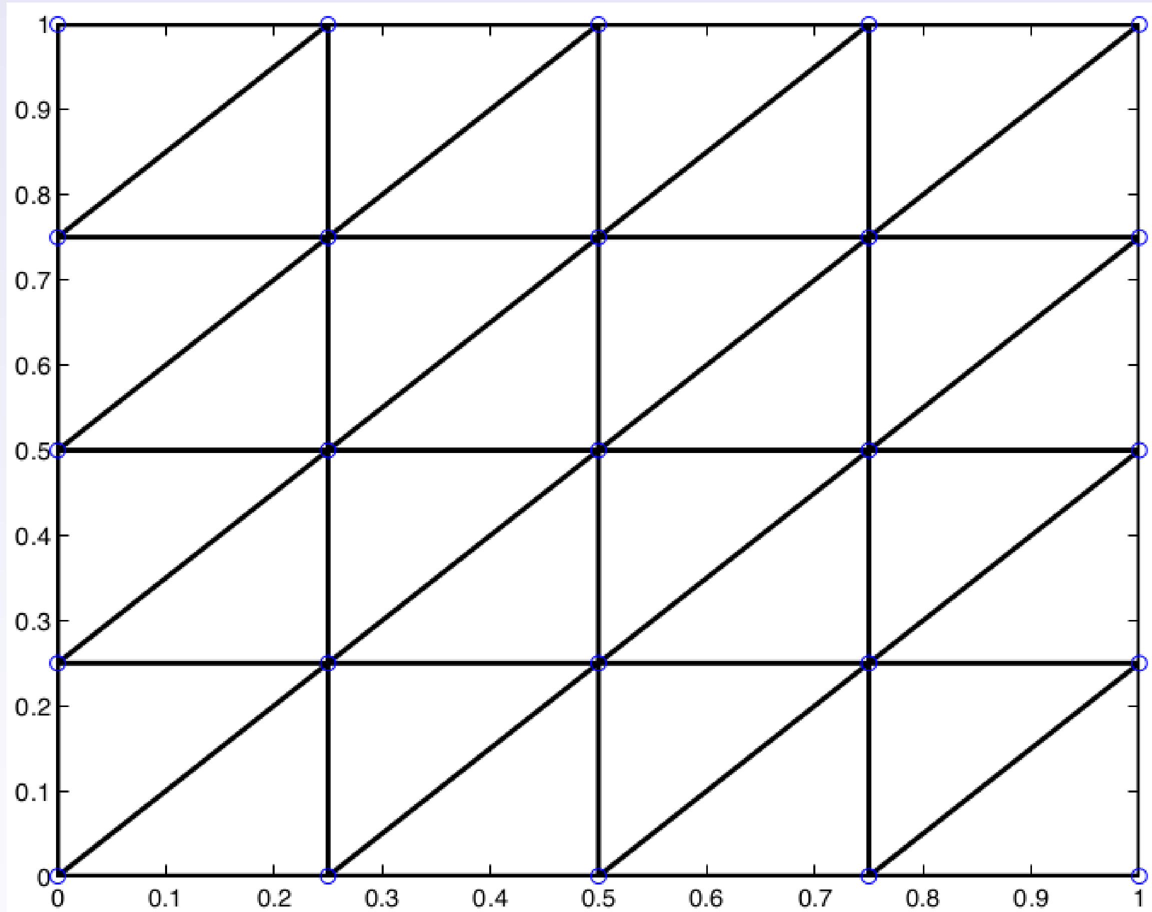
Homework 1 Discussion

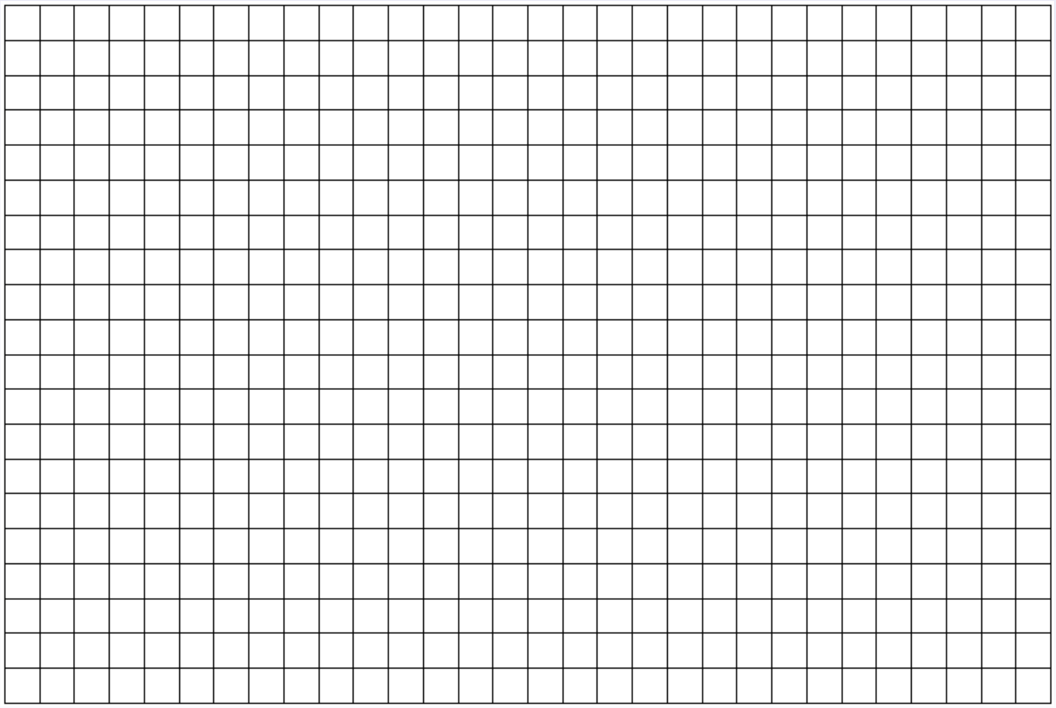
Grid Terminology

- *mesh*
- *cells, elements*,
  e.g. triangles, quads, tetrahedrons, hex
- *node, vertex, grid point*
- *cell center, edge, face*
- Grid properties
  - quality of mesh
  - degeneracy
  - dof



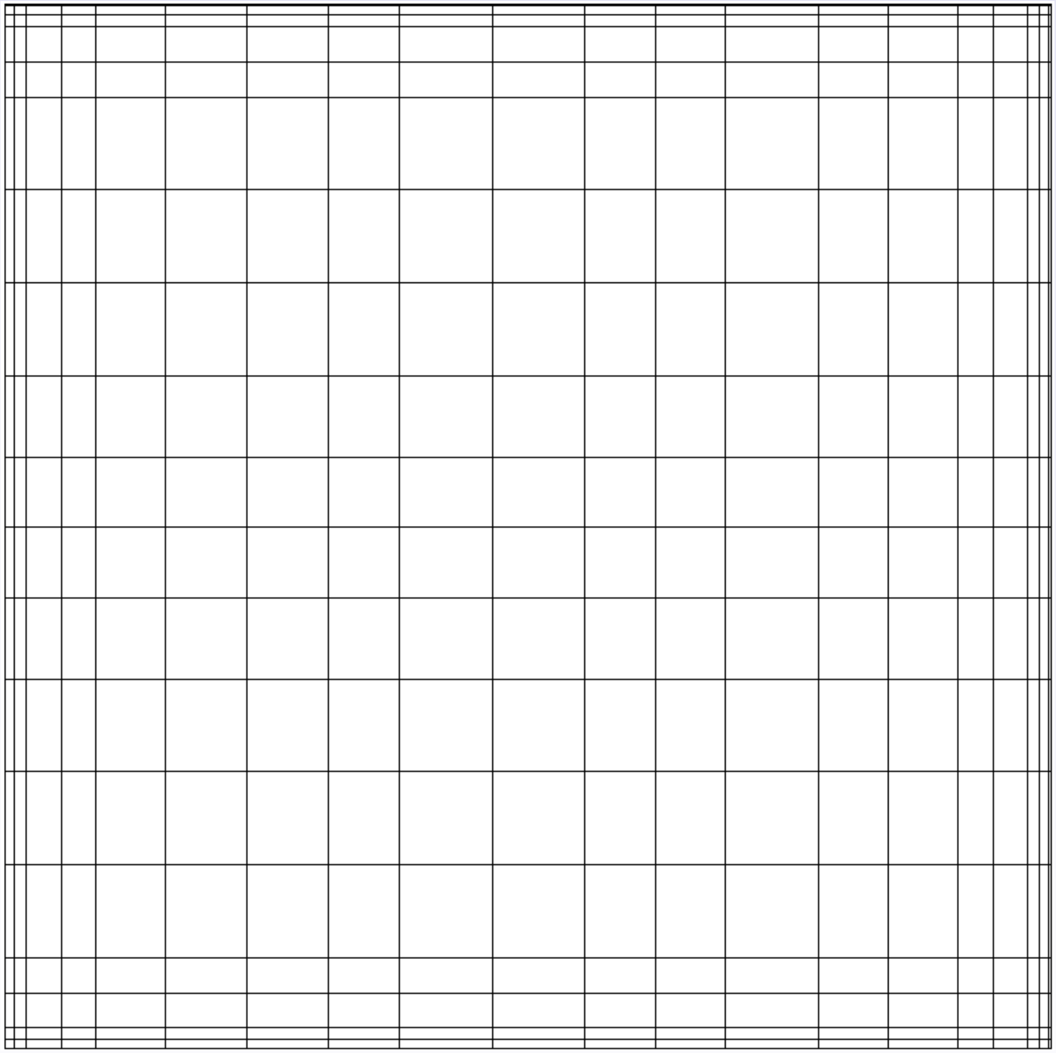
Structured Grid Examples – Uniform

- (i, j, k) indexing
- What are the advantages/disadvantages?



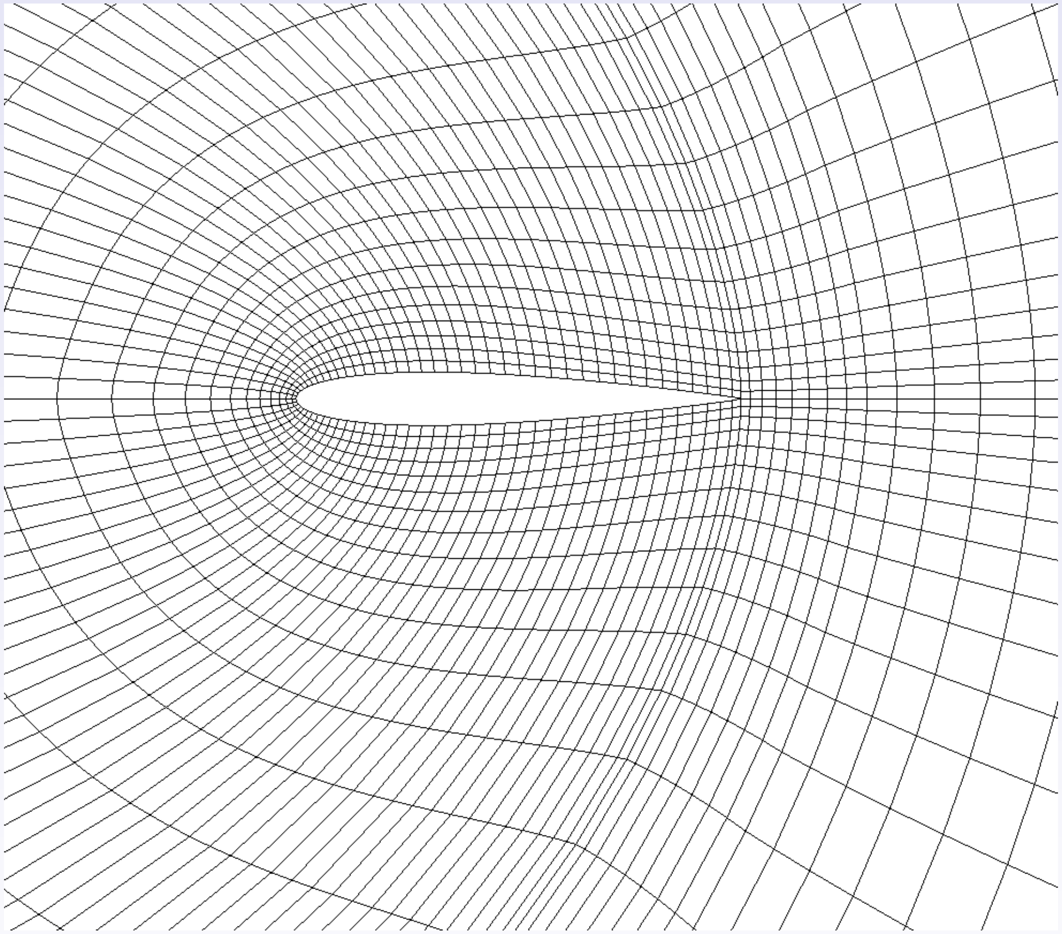
Structured Grid Examples – Rectilinear

- Similar to uniform grid
- What is the main advantage here as compared to a uniform grid?
- What are the disadvantages?



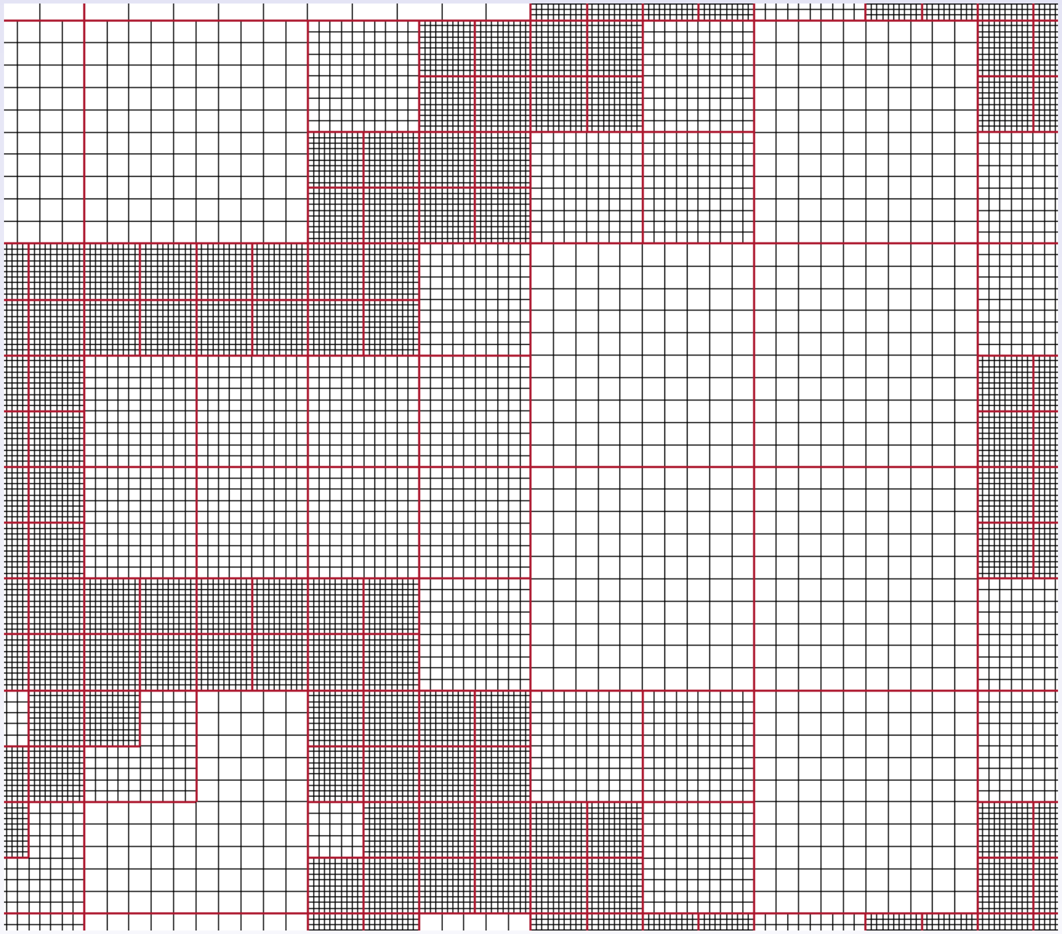
Structured Grid Examples – Curvilinear

- Note that each node still has the same number of neighbors
- What is the main advantage here?
- What are the disadvantages?



Structured Grid Examples - Block Structured

- Used in Adaptive Mesh Refinement methods
- Solves problem of having too much resolution in places that you don't need it
- Software is more complicated
- Error analysis more difficult



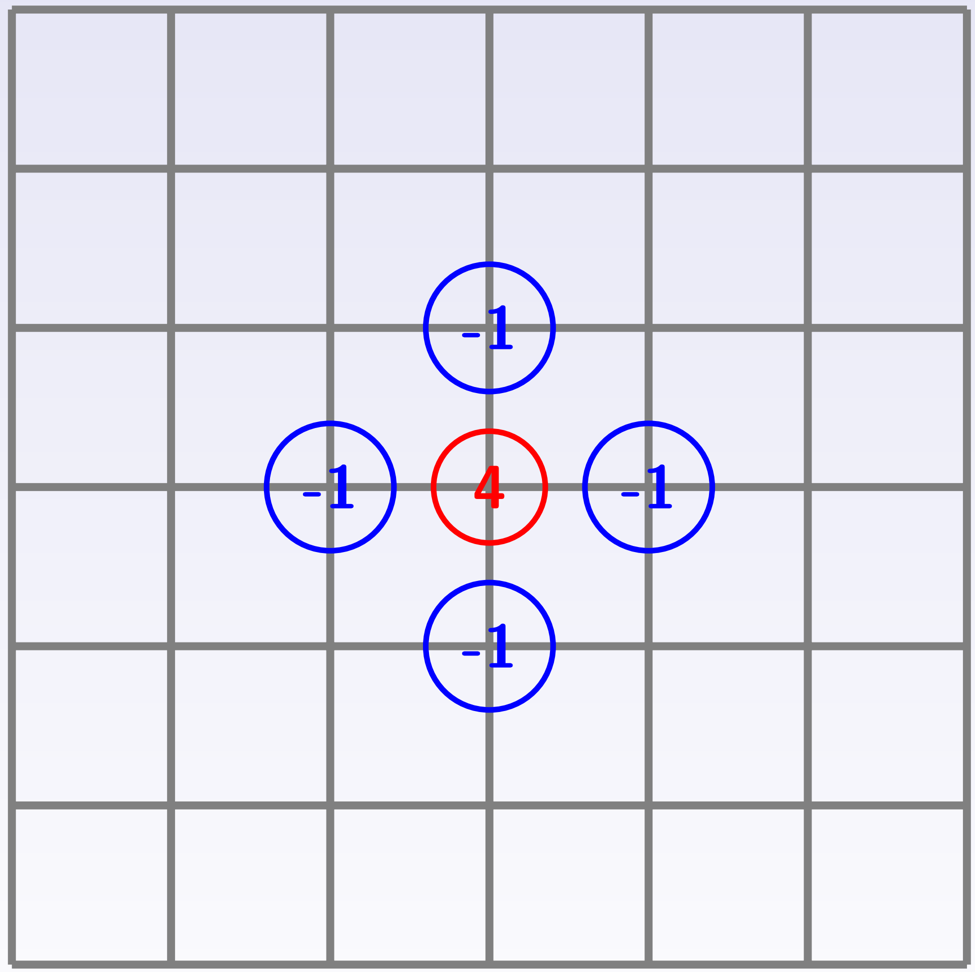
Main properties of structured grids

- Number of adjacent mesh elements is always the same
- Generally more accurate per unknown/dof than unstructured
- Convergence of algorithms (linear solvers) well understood
- Better data layout, which is good for computation

Example: 5-Point Stencil

5-point stencil

- Simplest 2-D case
- Leads directly to a sparse (penta-diagonal) matrix
- Iterative methods easy to apply



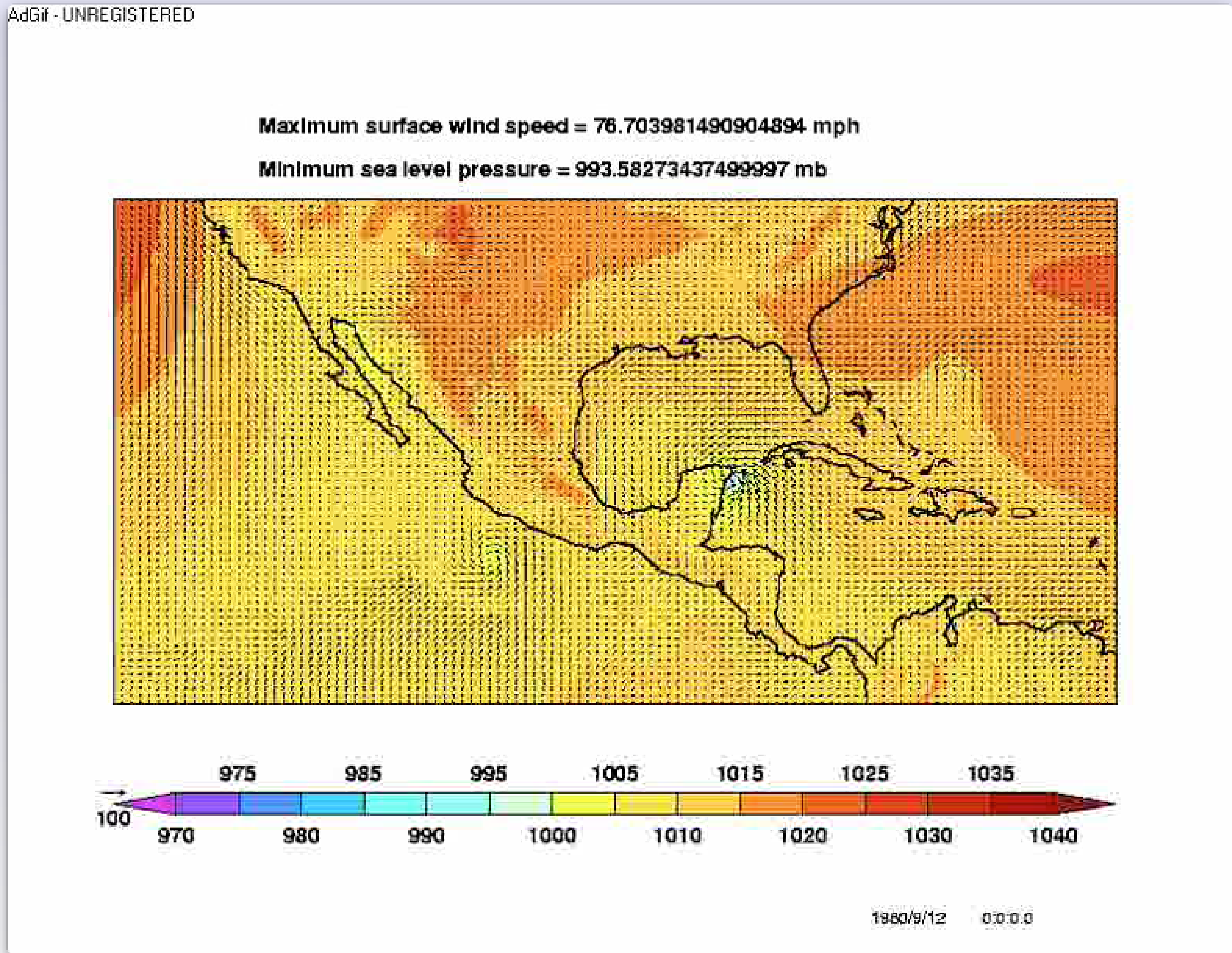
5-Point Stencil Matrix for $N = 3$ grid

$$
A_9 = \left[\begin{array}{ccc|ccc|ccc}
4 & -1 & & -1 & & & & & \\
-1 & 4 & -1 & & -1 & & & & \\
& -1 & 4 & & & -1 & & & \\
\hline
-1 & & & 4 & -1 & & -1 & & \\
& -1 & & -1 & 4 & -1 & & -1 & \\
& & -1 & & -1 & 4 & & & -1 \\
\hline
& & & -1 & & & 4 & -1 & \\
& & & & -1 & & -1 & 4 & -1 \\
& & & & & -1 & & -1 & 4
\end{array}\right]
$$

Question

What matrix properties can you name?

Structured Grid Applications: Climate Modeling



Summary

- Structured grids exist in many shapes and forms
- Well understood methods
- Well developed software available
- Work well on parallel and other high performance computing environments
- Don't work as well with complex geometries
- Also harder to use with multi-material, multi-block problems

More information on meshing

A nice summary of meshing can be found at A. Bakker's web site [1]

References I

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