Fundamental Concepts in Computational and Applied Mathematics

Juan Meza
School of Natural Sciences
University of California, Merced

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Bayes Formula

Combine (prior) information you have with new data, D, to update your understanding of a model:

$$P(\theta|D) = \frac{P(D|\theta)P(\theta)}{\int P(D|\theta)P(\theta)\mathrm{d}\theta} \tag{1}$$

where

- $P(\theta|D)$ is called the *posterior*
- $P(D|\theta)$ is called the *likelihood*
- $P(\theta)$ is called the *prior*

The term

$$m(x) = \int P(D|\theta)P(\theta)\mathrm{d}\theta$$

is called the *marginal density* of the random variable X.

Basic Idea

- Replaced quantity of interest (posterior) with something that we are more likely to be able to compute
- The prior represents our state of the knowledge before we analyze the data
- The likelihood modifies this state once we've analyzed the new experimental data
- is just a normalization constant in most cases

Bayes

Of interest are various functions of the posterior (e.g. moments, quantiles, etc.), which can be expressed as expectations of functions of θ:

$$E\left[f(\theta)|D\right] = \frac{\int f(\theta)P(D|\theta)P(\theta)\mathrm{d}\theta}{\int P(D|\theta)P(\theta)\mathrm{d}\theta} \tag{2}$$

- Computation of the integrals have been the computational bottlenecks
- In most applications no analytic expressions for the posterior are impossible

Example

Summary

- blah
- blah
- blah