TABLE VII
IEEE 30-BUS SYSTEM DATA (NOMINAL POWER FLOW)

Bus data					Line data	
Bus #	P inject. (net), p.u.	Q inject. (net), p.u.	Ang., θ rad	Volt., V p.u.	Line #	React. X, p.u.
1	0.1765	0.5084	0.0438	1.0500	1	0.06
2	0.9635	2.0656	0.0441	1.0500	2	0.19
3	-0.1200	-0.0600	0.0102	0.9837	3	0.17
4	-0.3800	-0.0800	0.0075	0.9723	4	0.04
5	0.0000	0.0000	-0.0330	0.9740	5	0.20
6	0.0000	0.0000	-0.0178	0.9530	6	0.18
7	-1.1400	-0.5450	-0.0851	0.9314	7	0.04
8	-1.5000	-0.7500	-0.0685	0.9233	8	0.12
9	0.0000	0.0000	0.0362	0.9759	9	0.08
10	-0.2900	-0.1000	0.0634	0.9890	10	0.04
11	0.0000	0.0000	0.0362	0.9759	11	0.21
12	-0.5600	-0.3750	0.2017	0.9472	12	0.56
13	2.1000	1.2760	0.5018	1.0500	13	0.21
14	-0.3100	-0.0800	0.1444	0.9414	14	0.11
15	-0.4100	-0.1250	0.1697	0.9554	15	0.26
16	-0.1750	-0.0900	0.1032	0.9398	16	0.14
17	-0.4500	-0.2900	0.0476	0.9568	17	0.26
18	-0.1600	-0.0450	0.0468	0.9320	18	0.13
19	-0.4750	-0.1700	-0.0037	0.9300	19	0.20
20	-0.1100	-0.0350	0.0074	0.9423	20	0.20
21	-0.8750	-0.5600	0.1059	1.0280	21	0.19
22	1.5795	2.1958	0.1335	1.0500	22	0.22
23	1.3000	0.8515	0.3294	1.0500	23	0.13
24	-0.4350	-0.3350	0.2010	1.0084	24	0.07
25	0.0000	0.0000	0.3076	1.0115	25	0.21
26	-0.1750	-0.1150	0.2394	0.9638	26	0.08
27	2.0955	1.1650	0.4075	1.0500	27	0.07
28	0.0000	0.0000	0.0193	0.9476	28	0.15
29	-0.1200	-0.0450	0.2871	1.0050	29	0.02
30	-0.5300	-0.0950	0.2047	0.9884	30	0.20
					31	0.18
					32	0.27
					33	0.33
					34	0.38
					35	0.21
					36	0.40
					37	0.42
					38	0.60
					39	0.45
					40	0.20
					41	0.06

to provide feasible initial guesses to the optimization problem (26)–(41), as discussed below.

One can note just by inspection of the three-bus system that the most severe blackout is obtained by removing lines 2–5 from the network as it will isolate the load from generation. Such a situation can be systematically identified by the graph theory based algorithm discussed in [12] for a larger system. For example, a significant blackout is obtained by removing lines 28, 29, 30 and 36 for the IEEE 30-bus system, as identified in [12]. By allowing the line parameter γ associated with only these lines to vary, the initialization process poses the problem: what is the most (locally) severe failure that can be obtained by partially removing only these lines from service? This problem can be described mathematically in an optimization framework as

$$\max_{\theta, V, z, \gamma, \mu_1, \ldots, \mu_6, \lambda} \quad e^T z \tag{47}$$

such that constraints (10)–(13) and (15)–(23) are satisfied along with $0 \leq \gamma \leq \gamma_{\max}$ and $e^T z \geq S_{\min}$. The nominal power flow solution (Tables VI and VII) provides an initial guess for this initialization procedure. Parameter $\gamma_{\max}$ is set to 0.9, a value close to but less than one, and $S_{\min}$ is set to 0.5, a small positive value, to avoid graph partitioning and/or trivial solutions. (One trivial/undesired solution is $\lambda = w_0, \mu_1 = e, \mu_2, \ldots, \mu_6, z = 0$; thus making the objective function zero.) This approach has been used to obtain solutions discussed in Sections III-A and B

We note that there are other ways to obtain feasible initial guesses for the optimization problem (26)–(41). One way features solving the simplified problem

$$\min_{\theta, V, z, \gamma, \mu_1, \ldots, \mu_6, \lambda} \quad \left\| e + \frac{\partial F}{\partial z}^T \lambda - \mu_1 + \mu_2 \right\|_2 \tag{48}$$

such that constraints (10)–(13) and (16)–(23) are satisfied along with $0 \leq \gamma \leq \gamma_{\max}, e^T \gamma \leq L_{\max}$ and $e^T z \geq S_{\min}$. The nominal power flow solution provides an initial guess for this initialization procedure. Random starting guesses have also produced solutions to (48).

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