Optimization of Mechatronic Systems - Exam questions

Contents

1 Introduction

This document gives an overview of all exam questions. These questions are all connected to the ones that we solved during the case study; they are either extensions to, or variations on the ones from the case study. For the exam project, you can form groups of 2 persons and pick questions from the offered set. Each question has a number of points assigned to it. These points indicate the difficulty of the question and/or the amount of work needed to solve it. The goal is to select questions until you reach a score of at least 10 points. Of course it is allowed to solve questions for more than 10 points. While selecting questions, keep in mind the following rules: pick a *maximum* of 2 points from Section 2, pick a *minimum* of 4 points from Section 4. During the exam you will, with two persons, defend your work orally. You don't have to make a complete report, a small presentation or a set of notes and figure suffices. The most important thing is that you can convince us that you understood the concept of formulating and solving an optimization problem, by explaining how you tackled the problem at hand, and by interpreting and critically evaluating your results.

2 Fitting problems

Exercise a.1 *Choose an exercise from the case study in which you used an L2-norm in the objective, and change it into an L-∞ norm. Discuss and compare the results. [**1 pt.**]*

Exercise a.2 *Choose an exercise from the case study in which you used an L2-norm in the objective, and change it into a Huber norm. Find a smooth formulation. Discuss and compare the results. [**2 pts.**]*

3 General Optimal Control Problems

Exercise b.1 *Extend exercise B.7 to collision avoidance with a moving obstacle. Suppose that there is a circular obstacle that is following a known trajectory (propose something yourself). Use a shooting method to compute a collision-free trajectory for the vehicle. [**1 pts.**]*

Exercise b.2 *Suppose that you have a stationary convex polygon as obstacle. Use a shooting method or spline parametrization to compute a collision-free trajectory. Use either a separating hyperplane at each point in time, or come up with your own method [**3 pts.**]*

Exercise b.3 *Change the vehicle model from exercise B.4 to a simple bicycle model:*

$$\begin{aligned}\dot{x} &= V \cdot \cos\theta \\ \dot{y} &= V \cdot \sin\theta \\ \dot{\theta} &= \frac{V}{L} \cdot \tan\delta,\end{aligned} \tag{1}$$

in which $[x, y]$ *is the vehicle position,* θ *is the orientation,* δ *is the steering angle,* L *is the length of the wheel base, and* V *is the total vehicle velocity. Use a shooting method to compute the time-optimal trajectory to move this vehicle from A to B, starting from and ending with an orientation of* $0°$. *[**2 pts.**]*

Exercise b.4 *Propose your own geometric path through 2D space (e.g. Gerono lemniscate). Write an algorithm that, for the simple vehicle model from exercise B.4, follows this path as close as possible. Let it make a trade-off between the velocity and the tracking error. Interpret the results. [**4 pts.**]*

Exercise b.5 *Redo any of the multiple-shooting OCP exercises using fmincon, without modeling in CasADi/Opti. [**2 pts.**]*

Exercise b.6 *Redo any of the multiple-shooting OCP exercises using the solver developed in the exercise sessions of this course, without modeling in CasADi/Opti. [**2 pts.**]*

Exercise b.7 *Redo any of the multiple-shooting OCP using any software specifically designed for OCPs [**2 pts.**]*

Exercise b.8 *Investigate direct collocation:*

a) Redo any multiple-shooting OCP exercise using a direct collocation transcription. Use Legendre collocation points, order 3. Search in literature for more information about this technique. [**3 pts.**] Note: both of the following questions can be chosen as a follow-up of the current question.

b) Redo any of the pendulum exercises using a DAE description as dynamic model. Again use a direct collocation transcription. [**1 pt.**]

b) Discuss the sparsity pattern of the constraint Jacobian and compare it with the multiple shooting code. [**1 pt.**]

Exercise b.9 *Redo any multiple-shooting OCP exercise using single shooting, discuss convergence and compare sparsity patterns of the constraint Jacobians. [**2 pts.**]*

Exercise b.10 *Redo any multiple-shooting OCP using a pseudospectral transcription. Search in literature for more information about this technique. [**3 pts.**]*

Exercise b.11 *Redo any of the pendulum exercises using dynamic programming, compare to the original solution technique. Search in literature for more information about dynamic programming. [**4 pts.**]*

Exercise b.12 *Redo any of the pendulum exercises using an indirect method (Hamilton-Jacobi-Bellman equations), compare to the original solution technique. Search in literature for more information about indirect methods. [**4 pts.**]*

4 Trajectory generation using invariant representations

Exercise c.1 *Check the property of orthonormality of the Frennet-Serret frame at each time point in the solution. Does the Runge-Kutta integrator of order 4 preserve this property? Implement Baumgarte for SO(3) stabilization to make the orthonormality hold more accurately. [**2 pts.**]*

Exercise c.2 *Replace the Runge-Kutta of order 4 integrator by a closed form analytical solution of the integrated Frennet-Serret dynamics. [**2 pts.**]*

Exercise c.3 *Exercise C.3 used finite differences on the invariant trajectory to regularize the rate of change of invariants. Implement an alternative strategy: use the 'rate of change of invariants' directly as control inputs. [**1 pts.**]*

Exercise c.4 *Try out different orders of magnitude for regularization on exercise C.3. What is the effect of convergence of the NLP solver; what is the effect on the quality of the converged solution? [**1 pt.**]*

Exercise c.5 *Redo exercise C.3-C.4 for a screw axis invariant representation. [**3 pts.**]*

Exercise c.6 *For exercise C.4, compare the invariant trajectories. Explain the difference. [**1 pt.**]*

Exercise c.7 *Redo exercise C.3 and C.4 using geometric Frennet-Serret invariants instead of time based: instead of defining the invariants as a function of time, make them a function of the path-length coordinate s. [**3 pts.**]*

Exercise c.8 *Redo exercise C.4 for the case in which there are obstacles in the workspace. [**1 pt.**]*

Exercise c.9 *Redo exercise C.3 and C.4 with the addition of a desired and re-created rotational trajectory: e.g.* demonstrate *the robot how to put a spoon of sugar in your tea, and re-create the motion for a different configuration. [**3 pts.**]*

Exercise c.10 *Suppose that the end position of the trajectory keeps getting moved by a human, while the movement is executed. To this end, extend exercise C.4 with an optimization-based feedback-controller: successively solve OCP problems to keep finding a new trajectory. [**2 pts.**]*

Exercise c.11 *Extend exercise C.3 with robot kinematics. Place the robot base at a position in space, such that part of the trajectory is out of reach. The solver must now find a solution to the OCP that is given by a similar trajectory that is reachable. To simplify things, you may use a 2D trajectory and a planar robot. Therefore, choose a robot with two links, of which each link has only one rotational actuator, realizing a motion in the 2D plane. [**2 pts.**]*

Exercise c.12 *Extend exercise C.3 with robot dynamics and actuation limits. Choose the robot configuration such that part of the trajectory goes too fast for the robot to follow perfectly. To simplify things, you may use a 2D trajectory and a planar robot based on the double pendulum equations, and two torque-controlled rotational actuators. [**4 pts.**]*

Exercise c.13 *In literature, you may find analytical expressions to obtain the Frennet-Serret invariants from a given pose trajectory. Investigate how our numerical method performs relative to these expressions, in the presence of noise on the motion demonstration. [**3 pts.**]*

Exercise c.14 *Redo exercise C.3 and C.4 by using the spline toolbox. Parametrize the invariants as splines. [**2 pts.**]*