# Adaptive Path Following for UAV in Time Varying Unknown Wind Environments

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# Outline

1. Introduction
2. Dynamics of UAV and Control Task
3. Two Control Scenarios:
   - Straight Line Following
   - Orbit Path Following
4. Simulation Results
5. Demo

# Introduction

- Exploration task: military, agriculture, geography
- Rescue in hazard environments: earthquake, forest fire
- Entertainment: photography, video

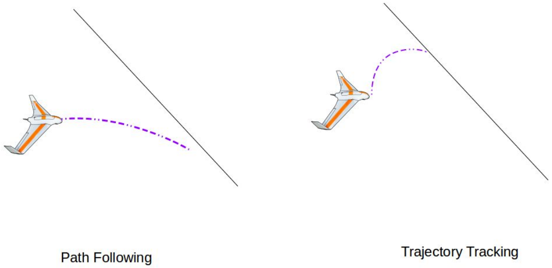
(a) Unmanned aircraft

(b) DJI Phantom 2 [1]

[1] https://www.youtube.com/watch?v=zvvbMxQ9Hj0

# Introduction

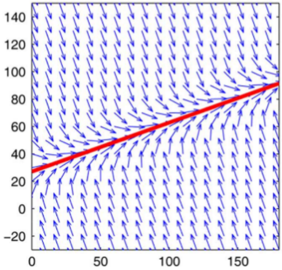
- Path Following (Space)
  1. Following a geometric path in 2D or 3D;
  2. Motivated by the applications in which space error is more crucial than temporal error;
  3. Constant forward velocity.
- Trajectory Tracking (Space × Time)
  1. Tracking a time signal trajectory in 2D or 3D;
  2. More constrained.

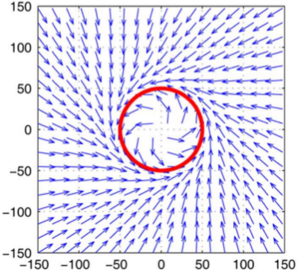


# Introduction

## Vector Field (VF) Path Following

Given a reference path $R^2(R^3)$, build up a group of vectors around the reference path as the control inputs (steering angle, speed) to the UAV so that it can converge to the reference path asymptotically.




**Bad news:**
**Standard VF only works for known, constant wind disturbance**

*Why not using Adaptive Control?*

- Compensate the wind disturbance
- Limit the path following error at least bounded

# Dynamics and Control Task

- UAV kinematics in 2D

$$
\begin{aligned}
\dot{x} &= V_a \cos\psi + W\cos\psi_w + A\cos\psi_A \\
\dot{y} &= V_a \sin\psi + W\sin\psi_w + A\sin\psi_A
\end{aligned} \tag{1}
$$

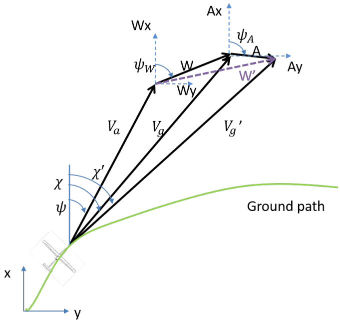



Figure: UAV kinematics

$x, y$: position of UAV
$V_a$: airspeed of UAV
$\psi$: heading angle between airspeed and horizontal axis
$\chi'$: UAV's course angle
$W$: Constant wind amplitude
$A$: Time Varying wind amplitude
$\psi_w$: angle of constant wind in earth frame
$\psi_A$: angle of time varying wind in earth frame

# Dynamics and Control Task

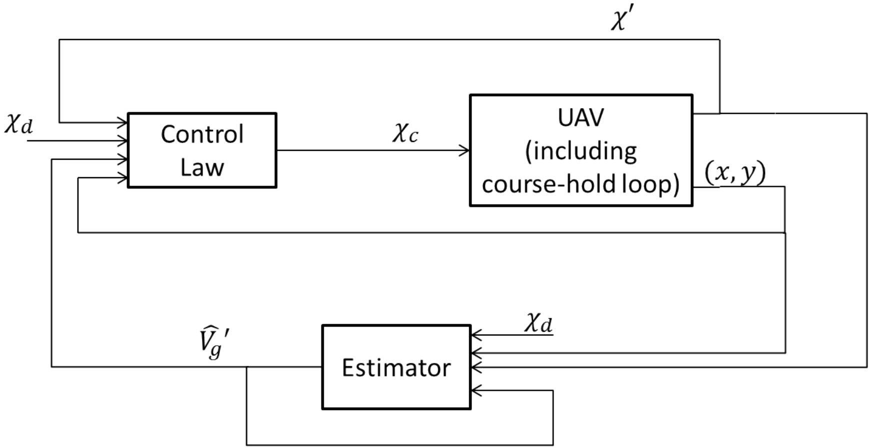
- Assumptions
  1. Altitude and airspeed ($V_a$) are held constant by the longitudinal control of UAV;
  2. The UAV is equipped with the course-hold loop devices whose dynamics can be modeled as the first-order system

$$\dot{\chi}' = \alpha(\chi_c - \chi')$$

  3. The UAV course is measurable;
  4. A slowly time-varying unknown component of wind with amplitude $A(t)$ and angle $\psi_A(t)$.
- Control task

  Build up the control law $\chi_c$ to let the UAV follow the path as accurately as possible under the wind disturbance.

# Dynamics and Control Task



# Path Following Strategies

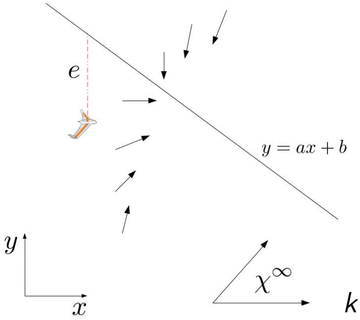
Adaptive Vector Field Path Following Strategy:

- Straight Line Following
- Orbit Path Following

# Straight Line Following

**Task**

Find the control law which can steer the UAV to the reference straight line and keep along with the path.




1. Distance error

$$e = y - (ax + b)$$

2. Course error

$$\tilde{\chi}' = \chi' - \chi_d$$

3. Desired course

$$\chi_d = -\chi^{\infty}\frac{2}{\pi}\tan^{-1}(ke) + \tan^{-1}(a)$$

$k$: a positive constant influences the rate of course transition from $\chi^{\infty}$ to $\tan^{-1}(a)$.

# Straight Line Following

- If $\chi' \to \chi_d$, distance error will converge to zero.

Proof.

Lyapunov function $\mathcal{V}_1 = \frac{1}{2}e^2$

$$\begin{aligned} \dot{\mathcal{V}}_1 &= e(\dot{y} - a\dot{x}) \\ &= eV_g'(\sin\chi_d - a\cos\chi_d) \\ &= eV_g' \frac{\sin(i\chi^\infty \frac{2}{\pi}\tan^{-1}(ke))}{\cos(\tan^{-1} a)} < 0 \end{aligned}$$

$\square$

# Straight Line Following

- Then derive the control law of the course angle
  Define the Lyapunov function $\mathcal{V}_2 = \frac{1}{2}\tilde{\chi}'^2$

$$
\begin{aligned}
\dot{\mathcal{V}}_2 &= \tilde{\chi}'\dot{\tilde{\chi}}' \\
&= \tilde{\chi}'(\alpha(\chi_c - \chi') + \chi^\infty \frac{2}{\pi}\frac{k\dot{e}}{1+(ke)^2}) \\
&= \tilde{\chi}'(\alpha(\chi_c - \chi') + \chi^\infty \frac{2}{\pi}\frac{k}{1+(ke)^2}V_g'(\sin\chi' - a\cos\chi'))
\end{aligned}
$$

Ideally, if we choose the command course as

$$
\chi_c = \chi' - \frac{1}{\alpha}\chi^\infty \frac{2}{\pi}\frac{k}{1+(ke)^2}V_g'(\sin\chi' - a\cos\chi') - \frac{\kappa}{\alpha}sat(\frac{\tilde{\chi}'}{\epsilon}) \quad (2)
$$

$\kappa > 0$: the shape of the trajectories on the sliding surface;
$\epsilon > 0$: the width of the transition region at the sliding surface.

The derivative of Lyapunov function is negative semi-definite.

# Straight Line Following

**Unfortunately, the control law (Eq.(2)) can not be implemented directly!!**

$$\chi_c = \chi' - \frac{1}{\alpha}\chi^{\infty}\frac{2}{\pi}\frac{k}{1+(ke)^2}V_g'(\sin\chi' - a\cos\chi') - \frac{\kappa}{\alpha}sat(\frac{\tilde{\chi}'}{\epsilon})$$

# Straight Line Following

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We need ESTIMATOR for the ground velocity $V_g'$

$$\chi_c = \chi' - \frac{1}{\alpha}\chi^{\infty}\frac{2}{\pi}\frac{k}{1+(ke)^2}\hat{V}_g{}'(\sin\chi' - a\cos\chi') - \frac{\kappa}{\alpha}sat(\frac{\tilde{\chi}'}{\epsilon}) \quad (3)$$

# Straight Line Following

*Time to design the estimator for ground velocity.*

# Straight Line Following

**Theorem**

*In straight line following scenario, the command course (23) and the estimator*

$$\dot{\hat{V}}_g' = \Gamma \rho \tilde{\chi}' \chi^{\infty} \frac{2}{\pi} \frac{k}{1 + (ke)^2} (\sin \chi' - a \cos \chi') - \sigma \Gamma \hat{V}_g{}' \tag{4}$$

*($\Gamma > 0$: the estimation gain, $\sigma > 0$: a switching $\sigma$-modification parameter.)*
*guarantees that the tracking error converges to zero for unknown constant winds and stays bounded for unknown slowly time-varying wind.*

# Straight Line Following

Proof:
Define the estimator error as $\Theta = \hat{V_g}' - V'_g$. The derivative of Lyapunov function $\mathcal{V}_e = \mathcal{V}_1 + \rho\mathcal{V}_2 + \frac{1}{2}\Gamma^{-1}\Theta^2$ is

$$\begin{aligned}
\dot{\mathcal{V}}_e &= \dot{\mathcal{V}}_1 + \rho\dot{\mathcal{V}}_2 + \Gamma^{-1}\Theta\dot{\Theta} \\
&= \dot{\mathcal{V}}_1 + \rho\tilde{\chi}'[-\chi^{\infty}\frac{2}{\pi}\frac{k}{1+(ke)^2}(\hat{V_g}' - V'_g)(\sin\chi' - a\cos\chi') \\
&\quad - \kappa sat(\frac{\tilde{\chi}'}{\epsilon})] + \Gamma^{-1}(\hat{V_g}' - V'_g)(\dot{\hat{V}}'_g - \dot{V_g}')
\end{aligned}$$

$\rho$: positive weight term for course error, which is aimed to make the distance error and course error compatible.

# Straight Line Following

First, we prove the tracking errors ($e$ and $\tilde{\chi}'$) will converge to zero under the assumption that $\dot{V_g}' = 0$.

$$\dot{\mathcal{V}}_e = \dot{\mathcal{V}}_1 + \rho\tilde{\chi}'[-\chi^\infty \frac{2}{\pi}\frac{k}{1+(ke)^2}(\hat{V_g}' - V_g')(\sin\chi' - a\cos\chi') - \kappa sat(\frac{\tilde{\chi}'}{\epsilon})]\times$$

$$+ \Gamma^{-1}(\hat{V_g}' - V_g')\dot{\hat{V}}_g'$$

$$= \dot{\mathcal{V}}_1 - \rho\kappa\tilde{\chi}' sat(\frac{\tilde{\chi}'}{\epsilon}) + \{\dot{\hat{V}}_g'\Gamma^{-1} - \rho\tilde{\chi}'\chi^\infty\frac{2}{\pi}\frac{k}{1+(ke)^2}(\sin\chi' - a\cos\chi')\}\times$$

$$(\hat{V_g}' - V_g')$$

If the estimator is chosen as

$$\dot{\hat{V}}_g' = \Gamma\rho\tilde{\chi}'\chi^\infty\frac{2}{\pi}\frac{k}{1+(ke)^2}(\sin\chi' - a\cos\chi')$$

Then the derivative of $\mathcal{V}_e$ is negative semi-definite.

$$\dot{\mathcal{V}}_e = \dot{\mathcal{V}}_1 - \rho\kappa\tilde{\chi}' sat(\frac{\tilde{\chi}'}{\epsilon})$$

# Straight Line Following

However, $\dot{\mathcal{V}}_e < 0$ is not enough to prove the asymptotic convergence of tracking errors to zero for time varying systems.
**Barbalat's Lemma** for stability analysis of Time varying systems

$$\begin{aligned}
\ddot{\mathcal{V}}_e &= \ddot{\mathcal{V}}_1 - \rho\kappa sat(\frac{\tilde{\chi}'}{\epsilon})\dot{\tilde{\chi}}' \\
&= \ddot{\mathcal{V}}_1 - \rho\kappa sat(\frac{\tilde{\chi}'}{\epsilon})[-\chi^\infty \frac{2}{\pi}\frac{k}{1+(ke)^2}(\sin\chi' - a\cos\chi')\Theta \\
&\quad - \kappa sat(\frac{\tilde{\chi}'}{\epsilon})]
\end{aligned}$$

$\ddot{\mathcal{V}}_e$ is bounded, $\dot{\mathcal{V}}_e <= 0$, $\mathcal{V}_e >= 0 \Rightarrow \dot{\mathcal{V}}_e \rightarrow 0$ as $t \rightarrow \infty$.
**Conclusion: $e$ and $\tilde{\chi}'$ converge to zero asymptotically.**

# Straight Line Following

Then, we prove the tracking errors will be bounded for unknown *slowly time-varying* wind by using the $\sigma$-modification technique.

$$
\begin{aligned}
\dot{\mathcal{V}} &= -\rho\kappa\tilde{\chi}' sat(\frac{\tilde{\chi}'}{\epsilon}) + \{(\dot{\hat{V}}_g' - \dot{V_g}')\Gamma^{-1} - \rho\tilde{\chi}'\chi^{\infty}\frac{2}{\pi}\frac{k}{1+(ke)^2}\times \\
&\quad (\sin\chi' - a\cos\chi')\}(\hat{V_g}' - V_g') \\
&= -\rho\kappa\tilde{\chi}' sat(\frac{\tilde{\chi}'}{\epsilon}) - \sigma\Theta^2 - \sigma\Theta(-\Gamma^{-1}\dot{V_g}'\sigma^{-1} - V_g')
\end{aligned}
$$

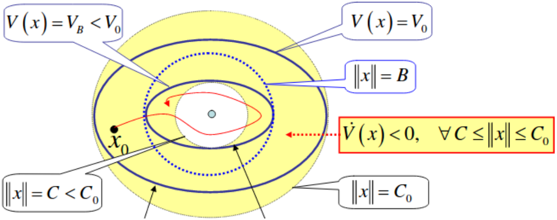
Using the inequality $-a^2 + ab \leq -\frac{a^2}{2} + \frac{b^2}{2}$ for any $a$ and $b$, we write

$$
\begin{aligned}
\dot{\mathcal{V}} &\leq -\rho\kappa\tilde{\chi}' sat(\frac{\tilde{\chi}'}{\epsilon}) - \frac{\sigma}{2}\Theta^2 + \frac{\sigma(V_g' + \dot{V_g}'\Gamma^{-1}\sigma^{-1})^2}{2} \\
&= -\rho\kappa\tilde{\chi}' sat(\frac{\tilde{\chi}'}{\epsilon}) - \frac{\sigma}{2}\Theta^2 + constant
\end{aligned}
$$

**If $\Theta^2 \geq \frac{2C}{\sigma}$, $\dot{\mathcal{V}}$ will be negative definite $\Rightarrow$ $e$, $\tilde{\chi}'$ and $\Theta$ will converge inside a ball around the origin and stay bounded.**

# Straight Line Following

- Uniform Ultimate Boundedness
  The solution of $\dot{x} = f(x,t)$ starting at $x(t_0) = x_0$ are Uniformly Ultimately Bounded (UUB) with ultimate bound B if:
  $\exists C_0 > 0, T = T(C_0, B) > 0 : (\|x(t_0)\| \leq C_0) \Rightarrow (\|x(t)\| \leq B, \forall t \geq t_0 + T$.




All trajectories starting in large ellipse enter small ellipse within finite time $T(C_0, B)$.

# Straight Line Following

Modify the estimator (Eq.(4)) with a feedforward term in practice:

$$
\begin{aligned}
\dot{\hat{V}}_g' &= \dot{V_g}' + \Gamma\rho\tilde{\chi}'\chi^\infty \frac{2}{\pi}\frac{k}{1+(ke)^2}(\sin\chi' - a\cos\chi') - \sigma\Gamma\hat{V_g}' \\
&= \frac{\partial V_g'}{\partial \chi'}[-\chi^\infty \frac{2}{\pi}\frac{k}{1+(ke)^2}(\sin\chi' - a\cos\chi')\hat{V_g}' - \kappa sat(\frac{\tilde{\chi}'}{\epsilon})]\times \\
&\quad \Gamma\rho\tilde{\chi}'\chi^\infty \frac{2}{\pi}\frac{k}{1+(ke)^2}(\sin\chi' - a\cos\chi') - \sigma\Gamma\hat{V_g}'
\end{aligned}
\tag{5}
$$

As $V_g'$ is unknown, the partial derivative is approximated as

$$
\begin{aligned}
\frac{\partial V_g'}{\partial \chi'} &\approx \frac{\partial V_g}{\partial \chi'} \\
&= W\sin(\psi_w - \chi') + [V_a^2 - W^2\sin^2(\psi_w - \chi')]^{-\frac{1}{2}} W^2 \times \\
&\quad \sin(\psi_w - \chi')\cos(\psi_w - \chi')
\end{aligned}
$$

# Straight Line Following

- Reference path

$$y = 0.5x$$

- Initial condition of UAV

$$(x_0, y_0, \chi_0) = (0, 80, \frac{pi}{4})$$

- Control Law

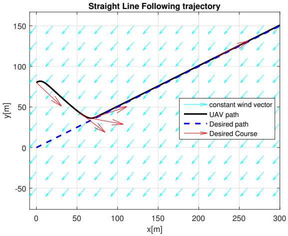
$$\chi_c = \chi' - \frac{1}{\alpha}\chi^{\infty}\frac{2}{\pi}\frac{k}{1+(ke)^2}\hat{V_g}'(\sin\chi' - a\cos\chi') - \frac{\kappa}{\alpha}sat(\frac{\tilde{\chi}'}{\epsilon})$$

- Estimator

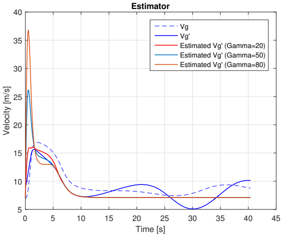
$$\dot{\hat{V}}_g' = \frac{\partial V_g'}{\partial \chi'}[-\chi^{\infty}\frac{2}{\pi}\frac{k}{1+(ke)^2}(\sin\chi' - a\cos\chi')\hat{V_g}' - \kappa sat(\frac{\tilde{\chi}'}{\epsilon})]\times$$
$$\Gamma\rho\tilde{\chi}'\chi^{\infty}\frac{2}{\pi}\frac{k}{1+(ke)^2}(\sin\chi' - a\cos\chi')$$

# Simulation Results

- Performance of controller and estimator

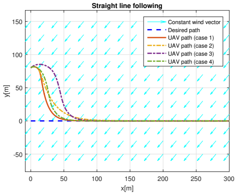



(a) Straight line following performance



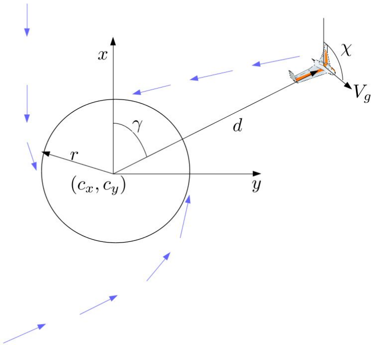

(b) Estimation performance

- Effect of design parameters




Influence of design parameters for straight line following:
Case 1: $k = 0.1, \kappa = \frac{\pi}{2}, \epsilon = 0.5, \Gamma = 50$;
Case 2: $k = 0.05, \kappa = \frac{\pi}{2}, \epsilon = 0.5, \Gamma = 50$;
Case 3: $k = 0.1, \kappa = \frac{\pi}{6}, \epsilon = 0.5, \Gamma = 50$;
Case 4: $k = 0.1, \kappa = \frac{\pi}{2}, \epsilon = 1.5, \Gamma = 50$.

# Orbit Path Following




- UAV kinematics in the polar coordinate

$$\dot{d} = V_g' \cos(\chi' - \gamma)$$
$$\dot{\gamma} = \frac{V_g'}{d} \sin(\chi' - \gamma)$$

- Distance error

$$\tilde{d} = d - r$$

- Course error

$$\tilde{\chi}' = \chi' - \chi_d$$

- Desired course

$$\chi_d = \gamma - [\tfrac{\pi}{2} + \tan^{-1}(k\tilde{d})]$$

# Orbit Path Following

Lyapunov function $\mathcal{V} = \frac{1}{2}\tilde{d}^2 + \frac{1}{2}\rho\tilde{\chi}'^2$

$$
\begin{aligned}
\dot{\mathcal{V}} &= \tilde{d}\dot{\tilde{d}} + \rho\tilde{\chi}'\dot{\tilde{\chi}}' \\
&= -V_g'\tilde{d}\sin(\tan^{-1}(k\tilde{d})) + \\
&\quad \rho\tilde{\chi}'[\alpha(\chi_c - \chi') - \frac{V_g'}{d}\sin(\chi - \gamma) + \beta V_g'\cos(\chi' - \gamma)]
\end{aligned}
$$

If the command course is chosen as

$$\chi_c = \chi' + \frac{V_g'}{\alpha d}\sin(\chi - \gamma) - \frac{\beta}{\alpha}V_g'\cos(\chi' - \gamma) - \frac{\kappa}{\alpha}sat(\frac{\tilde{\chi}'}{\epsilon})$$

The derivative of Lyapunov function is negative semi-definite.
Plug in estimator:

$$\chi_c = \chi' + \frac{\hat{V_g}'}{\alpha d}\sin(\chi - \gamma) - \frac{\beta}{\alpha}\hat{V_g}'\cos(\chi' - \gamma) - \frac{\kappa}{\alpha}sat(\frac{\tilde{\chi}'}{\epsilon}) \qquad (6)$$

where $\beta = \frac{k}{1+(k\tilde{d})^2}$.

# Orbit Path Following

**Theorem**

*In orbit path following scenario, the command course (31) and the estimator*

$$\dot{\hat{V}}_g' = -\Gamma \rho \tilde{\chi}' \left( \frac{\sin(\chi' - \gamma)}{d} - \beta \cos(\chi' - \gamma) \right) - \sigma \Gamma \hat{V}_g{}' \tag{7}$$

*with $\Gamma > 0$ being the estimation gain and $\sigma > 0$ being a switching $\sigma$-modification parameter, guarantees the tracking error converges to zero for unknown constant winds and stays bounded for unknown slowly time-varying wind.*

# Orbit Path Following

Proof:

Lyapunov function $\mathcal{V}_e = \mathcal{V} + \frac{1}{2}\Gamma^{-1}\Theta^2 = \frac{1}{2}\tilde{d}^2 + \frac{1}{2}\rho\tilde{\chi}'^2$

$$\begin{aligned}
\dot{\mathcal{V}}_e &= \dot{\mathcal{V}} + \Gamma^{-1}\Theta\dot{\Theta} \\
&= \rho\tilde{\chi}'(\hat{V}_g{}' - V_g')\left(\frac{\sin(\chi' - \gamma)}{d} - \beta\cos(\chi' - \gamma)\right) \\
&\quad - \rho\tilde{\chi}'\kappa sat\left(\frac{\tilde{\chi}'}{\epsilon}\right) + \Gamma^{-1}(\hat{V}_g{}' - V_g')(\dot{\hat{V}}_g' - \dot{V}_g{}') \\
&\quad - V_g'\tilde{d}\sin(\tan^{-1}(k\tilde{d}))
\end{aligned}$$

# Orbit Path Following

First, we prove the tracking errors will converge to zero under the assumption $\dot{V}_g{}' = 0$.

$$
\begin{aligned}
\dot{\mathcal{V}}_e &\approx \dot{\mathcal{V}} + \Gamma^{-1}\Theta\dot{\Theta} \\
&= -\rho\tilde{\chi}'\kappa sat(\frac{\tilde{\chi}'}{\epsilon}) - V_g'\tilde{d}\sin(\tan^{-1}(k\tilde{d})) \\
&+ (\hat{V}_g{}' - V_g')\{\rho\tilde{\chi}'(\frac{\sin(\chi' - \gamma)}{d} - \beta\cos(\chi' - \gamma)) + \Gamma^{-1}\dot{\hat{V}}_g'\}
\end{aligned}
$$

If the estimator is chosen as

$$\dot{\hat{V}}_g' = -\Gamma\rho\tilde{\chi}'(\frac{\sin(\chi' - \gamma)}{d} - \beta\cos(\chi' - \gamma))$$

Then the derivative of $\mathcal{V}_e$ is negative semi-definite.

$$\dot{\mathcal{V}}_e = -\rho\tilde{\chi}'\kappa sat(\frac{\tilde{\chi}'}{\epsilon}) - V_g'\tilde{d}\sin(\tan^{-1}(k\tilde{d}))$$

$\ddot{\mathcal{V}}_e$ is bounded, $\dot{\mathcal{V}}_e <= 0$, $\mathcal{V}_e >= 0 \Rightarrow \dot{\mathcal{V}}_e \to 0$ as $t \to \infty$.

**Conclusion: $\tilde{d}$ and $\tilde{\chi}'$ converge to zero asymptotically.**

# Orbit Path Following

Then, we prove the tracking errors will be bounded for unknown *slowly time-varying* wind by using the $\sigma$-modification technique.
The same procedure as Straight Line Following.
*Exercise: Work out the proof by yourself!*

# Orbit Path Following

- Reference path

$$(x-50)^2+(y-50)^2=1$$

- Initial condition of UAV

$$(d_0,\gamma_0,\chi_0)=(15,\tfrac{pi}{4},-\tfrac{pi}{4})$$

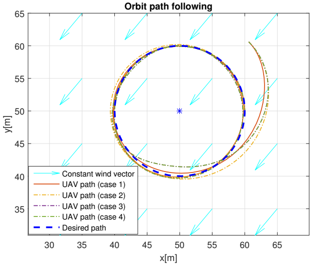
- Control Law

$$\chi_c=\chi'+\frac{\hat{V_g}'}{\alpha d}\sin(\chi-\gamma)-\frac{\beta}{\alpha}\hat{V_g}'\cos(\chi'-\gamma)-\frac{\kappa}{\alpha}sat(\frac{\tilde{\chi}'}{\epsilon})$$

- Estimator

$$\dot{\hat{V}}_g' = \frac{\partial V_g'}{\partial \chi'}(\frac{\hat{V}_g'}{d}\sin(\chi'-\gamma)-\beta\cos(\chi'-\gamma)-\kappa sat(\frac{\tilde{\chi}'}{\epsilon}))$$
$$-\Gamma\rho\tilde{\chi}'(\frac{\sin(\chi'-\gamma)}{d}-\beta\cos(\chi'-\gamma))$$

# Simulation Results




(a) Influence of design parameters for orbit following:
Case 1: $k = 0.1, \kappa = \frac{\pi}{2}, \epsilon = 0.5, \Gamma = 50$;
Case 2: $k = 0.05, \kappa = \frac{\pi}{2}, \epsilon = 0.5, \Gamma = 50$;
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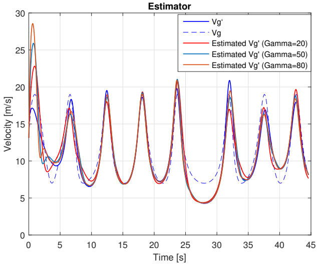



(b) Estimation performance

# Combined Path Following

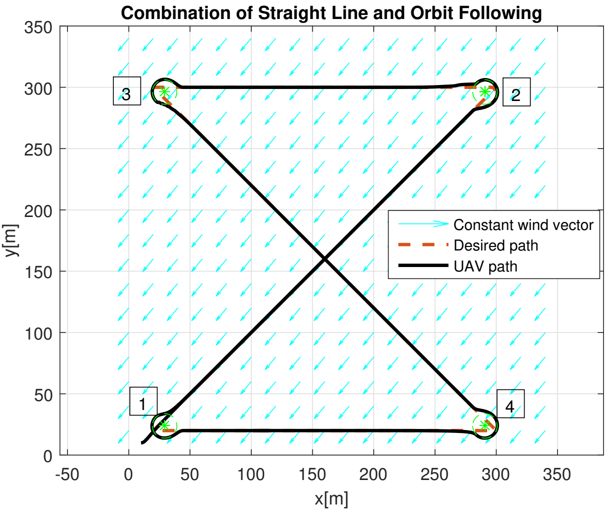
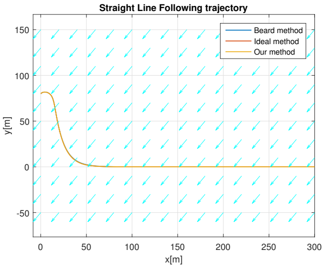
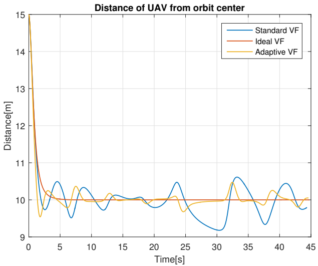



Figure: Combined path following

# Comparison




(a) Straight line following




(b) Orbit path following

Table: Steady state RMS error for straight line following

| | Std. VF | Id. VF | Adap. VF |
|---|---|---|---|
| RMS | 0.2203 | 0.1573 | 0.1434 |

Table: Steady state RMS error for orbit following

| | Std. VF | Id. VF | Adap. VF |
|---|---|---|---|
| RMS | 0.33 | $6.08 \times 10^{-6}$ | 0.1219 |

# Demo

*Demo time*